Contingent Prizes in Dynamic Contests*

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Abstract

Firms and government agencies are increasingly using online contests to procure solutions to problems. In these contests, players can make multiple attempts, and their performance is publicly displayed on a real-time leaderboard. Although these contests are *dynamic*, they only reward players based on their final rankings, which we argue is a missed opportunity to shape incentives. We propose the use of prizes that depend on the history of the game—*contingent prizes*—which can shape players' incentives, encouraging them to make more attempts throughout the contest. We empirically evaluate the impact of contingent prizes on participants' performance by estimating a structural model using observational data from large online competitions. We complement these results with experimental evidence, where we randomize prize structures across competitions. Evidence from both methodologies shows that contingent prizes can significantly improve contest outcomes.

Keywords: Contests, Tournament Design, Contingent Prizes, Dynamic Games

JEL codes: C51, C57, C72, O31.

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1 Introduction

Firms and government agencies are increasingly using online contests to procure solutions to problems. In particular, online data science competitions have gained popularity over the last decade. These competitions are *dynamic*: players try many attempts before a pre-determined deadline, and a public leaderboard discloses their performance in real-time. Notably, these competitions only reward players based on their final rankings, which we argue is a missed opportunity to shape incentives during the contest and improve participants' performance. We study *contingent* prizes—pre-announced prize allocation rules based on the contest's *history*—which give the contest designer a more flexible set of tools for motivating players throughout the contest.[1] Specifically, we ask, should *interim leaders* in a contest receive prizes? Should the contest sponsor set *milestones*? How much money is left on the table by only awarding prizes based on final rankings?

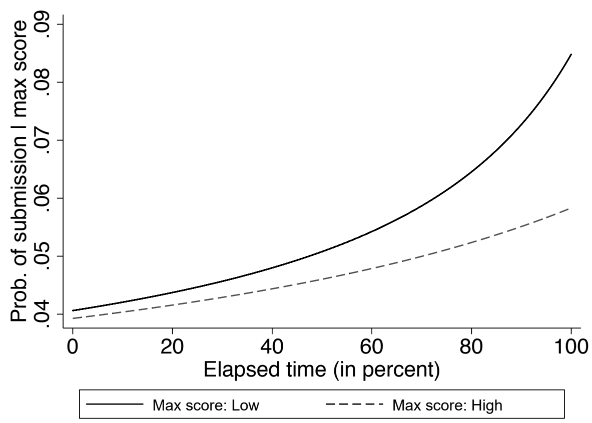
Our contribution is to empirically investigate the impact of contingent prizes on performance, measured as the best submission in the contest. An example of contingent prizes is a prize for interim competition leaders at predetermined times. Another example is a prize for the first player to reach a milestone. Our results suggest that contest sponsors are wasting resources by awarding prizes based only on final rankings, overlooking the role of contingent prizes in shaping players' incentives. We quantify that simple contingent prizes (like splitting the budget to reward interim leaders) can achieve the same performance as a prize based only on final rankings but using merely half of the budget.

Contingent prizes matter because they can dynamically motivate players to make a costly submission (henceforth, "playing"). Two salient economic forces shape incentives during the competition. First, at any point during the competition, a player's chance of rising to the top of the leaderboard could be small ("current-competition" effect), discouraging her from playing. Second, even if a player believes that she can become the competition leader at the current time, she anticipates that future submissions by her rivals could threaten her lead ("future-competition" effect). The current-competition effect is strongest near the end of the competition, when most of the alternatives to improve scores have been exhausted. In contrast, the future-competition effect is strongest at the beginning of the competition, when many rival submissions are yet to come.

[1] Some dynamic contests award prizes contingent on milestones or to interim leaders. For instance, the XPRIZE Carbon Removal competition, the largest incentive prize in history, splits a $100 million budget over time: After one year of competition, up to 15 competitors will receive prizes of $1 million each. At the end of the competition, the winner will get $50 million, and three runner-ups will share $30 million

Figure 1 illustrates these forces for a prize structure that only rewards the leader of the competition based on the final ranking (henceforth, a "final-ranking" prize). The figure shows the probability of playing conditional on two fixed scores (one lower than the other) at different times for a given contest. When the maximum score is low, the probability of playing at any time (solid line) is larger than that probability when the maximum score is high (dashed line). This downward shift captures the discouragement effect of the maximum score, i.e., the current-competition effect. Fixing the maximum score, which corresponds to moving along one of the curves, the probability of making a submission increases over time (i.e., both curves slope upwards). Part of this effect is explained by future competition, which discourages players from making submissions earlier in the competition.

Figure 1: Probability of making a submission



Notes: The figure plots the probability of making a submission for given parameter values. Each curve plots this probability over time given a maximum score (fixed over time).

Altering the importance of current- and future-competition effects throughout the contest affects the equilibrium number of submissions. A strong future-competition effect discourages early submissions. When there are few submissions at the beginning of the contest, the current-competition effect is mild (scores are low), which encourages players to play. Incentives to play hinge on the relative importance of the current- and future-competition effects. By awarding contingent prizes, the contest designer can change the balance of these effects, so a carefully chosen contingent-prize structure can potentially increase the number of submissions relative to a final-ranking prize.

For instance, consider a final-ranking prize and an alternative time-contingent prize structure, one that splits the budget and allocates 50% to the interim leader at the first half of the competition and 50% to the leader at the end of the competition. A final-ranking prize induces a high future-competition effect earlier in the contest. In contrast, the future-competition effect is weaker in the second prize structure since the interim leader at half of the competition receives 50% of the budget independently of the number of submissions in the second half of the contest. However, because players have more incentives to play early on, the current competition-effect will be larger in the second prize structure, since scores will tend to rise faster during the first half of the contest. On top of this tradeoff, a fixed budget introduces additional considerations for a designer who chooses among contingent-prize structures that do not exceed the budget in *any* realized history. Apart from shifting the relative importance of current- and future-competition effects, allocating a larger prize early on reduces the available budget for later in the contest.

We illustrate these tradeoffs in a two-period, two-player setting, showing that a final-ranking prize might be optimal, but often is not, which motivates our empirical work. We also illustrate that a "simple" prize structure can capture most of the performance gains from a fully-flexible contingent-prize structure, a result that we later show to hold using estimates of our empirical model in a setting with many periods and players.

We combine two empirical methodologies to investigate the impact of contingent prizes on performance: structural estimation and experimental evidence. In the first part of our analysis, we estimate a structural model with observational data from Kaggle.com, the largest platform for hosting data-science competitions.[2] In these competitions, players can make multiple submissions that are scored based on an objective criterion (e.g., prediction accuracy). A public leaderboard displays these scores in real time, and the leader based on final ranking receives a prize. Thus, these data allow us to estimate structural parameters of a dynamic contest where a prize is allocated based on final rankings only. Using these estimates, we simulate the equilibrium of each contest under counterfactual prize structures.

Our analysis focuses on *simple* prize structures, which include rewards for interim leaders at a predetermined set of times, or the first player to achieve a milestone, or the first player to achieve a milestone before a predetermined time. In particular, a final-ranking prize *is* a simple prize structure, since it only rewards the leader based on rankings at one specific time (at the end of the contest). Simple prize structures are both computationally manageable and relatively easy to implement in practice. Fully-flexible, budget-constrained, contingent-prize structures require solving an optimization problem with millions of variables and exponentially-large number of constraints, which is computationally unfeasible.[3] Aside from being tractable, our choice to focus on simple prize structures is motivated by evidence

[2] Expedia, Google, Two Sigma, the NFL, among others, have sponsored competitions in Kaggle.

[3] Specifically, there are $T(T+1)$ variables to optimize over and 2^T constraints (histories), where T is the number of periods in the empirical model, which in our baseline estimate is 10,000.

showing that they can approximate the benefits of fully-flexible contingent prize structures. Specifically, we use the estimates of our structural model to simulate "short contests," and find that simple prize structures capture a large fraction of the performance of the optimal contingent prize structure in short contests. Armed with these findings, we evaluate simple prize structures using the sample of contests in our data (i.e., the full scale contests). In this exercise, we compute the equilibrium of each contest under counterfactual prize structures and compare it to the observed equilibrium of each contest.

More specifically, using our model estimates, for each contest we consider six classes of counterfactual designs, each one corresponding to a "simple" prize structure. The first three designs award the interim leader of the contest at time t_k a prize of size π_k, for $k = 1, ..., K$, with $K \in \{2, 3, 6\}$. The fourth design ("2 timed prizes") allocates two prizes, one to the leader at the end of the competition and another the leader at an optimally-chosen time before the end of the contest, with the magnitude of both prizes chosen optimally. The fifth design ("milestone") awards the full prize pool to the first player who surpasses an optimally-chosen milestone score. The last design ("hybrid") awards a prize to the first player who surpasses a milestone score and another prize to the leader at the end of the competition, where the milestone score and the magnitude of both prizes are optimally chosen. To assess the performance of each design, we compute the equilibrium of the game under these alternative prize structures and compare it to the baseline equilibrium (i.e., when all the prize money is awarded at the end of the contest).

For each contest, we find the parameters for each one of these six prize structure that maximize the expected maximum score given the contest's primitives. For instance, within the class of time-contingent prizes with 2 equally-spaced prizes (i.e., one at the middle of the competition and one at the end), we find the optimal split of the budget for each contest. These results rely on the designer knowing the contest's primitives, such as the contest's difficulty, player's submission cost, and the distribution of scores for a given submission.

A contest designer, however, may not know these primitives. In that case, the contest designer would like to have a notion of the "robustness" of different prize structures. To shed light on the robustness of using one design over another, we find parameters that a designer can use "blindly" for each one of the six different prize structures. That is, we find parameters that maximize the average maximum score across all the contests in our data. For instance, with two time-contingent prizes equally spaced over time, the best uniform split is to allocate 30% of the budget at the middle of the contest and 70% at the end of the contest. However, when using a prize structure where two prizes are optimally allocated over time, the best uniform prize structure is to allocate 25% of the budget when 68% of the competition time has elapsed, and the remaining 75% of the budget at the end date.

Our estimates show that the contest designer can achieve a large fraction of the gains of a simple prize structure tailored to each contest by simply using a uniform prize structure. That is, even when contests are heterogeneous in their primitives, a simple uniform policy can substantially improve contest outcomes. We also find that using a fraction (52 percent) of the total budget, an optimal, simple prize structure achieves the same performance, on average, than a final-ranking prize. This result suggest that contest sponsors, which have paid million of dollars in prizes, may be leaving money on the table by using a suboptimal prize structure.

In the second part of our analysis, we complement our model-based analysis with a randomized control experiment, which provides an answer that is independent of our modeling choices. In our experiment, we organized competitions for students from the University of British Columbia and the University of Illinois. The students competed in groups of up to five in a prediction competition on Kaggle.com. We randomly assigned each group to one of three conditions: (1) the leader at the end of the competition received the full prize pool (baseline); (2) the leader two days before the end of the competition received 30% of the prize pool, and the leader at the end of the competition received the remaining 70% of the prize pool, (time-contingent prizes); (3) the first player to surpass a milestone score received 30% of the prize pool, and the leader at the end of the competition received the remaining 70% of the prize pool (hybrid prizes). We based these experimental conditions on optimal designs according to our structural estimation.

Our model-based and experimental evidence align in showing that contingent prizes can significantly improve contest outcomes relative to a prize based on final standings. Our model estimates show that a hybrid design—one that provides one prize to the first player to reach a milestone score and another prize to the leader at the end of the competition—produces the best outcomes among six counterfactual simple prize structures. Our experimental results show that the hybrid prize structure causes similar gains in contest outcomes, but with larger magnitudes than our empirical model predicts.

A contingent prize can encourage players early on by reducing the future competition effect but can discourage them by increasing the current competition effect. Our results show that simple contingent prizes, which are easy to implement in practice, resolve this tradeoff favorably for the designer and achieve better performance than a prize based on final rankings.

Related Literature. A recent empirical literature uses structural estimation to investigate the design of dynamic contests. For instance, Bhattacharya (2021) estimate large gains from alternative designs of multi-stage contests hosted by the U.S. Department of Defense. Gross (2017) and Lemus and Marshall (2021) study the impact of feedback on outcomes. Our analysis focuses on *contingent* prizes, fixing all other design dimensions including the number of players, contest length, evaluation criterion, and information provision. In contrast to most of the literature, we make use of two complementary methodologies: structural estimation using observational data from large contests, and a randomized control trial that provides model-free estimates.

The literature has (mostly theoretically) investigated the number of participants that should enter a contest (Taylor, 1995; Fu and Lu, 2010; Aycinena and Rentschler, 2019), or earn a prize (Moldovanu and Sela, 2001; Olszewski and Siegel, 2020; Kireyev, 2020). In dynamic settings, researchers have studied whether contests should be divided in multiple stages (Moldovanu and Sela, 2006; Sheremeta, 2011) and whether participants should receive performance feedback during the contest (Mihm and Schlapp, 2019). Moldovanu and Sela (2006), Fu and Lu (2009), Fu and Lu (2012), Chowdhury and Kim (2017), and Clark and Nilssen (2020), among others, study the design of both the number of stages and prizes.

As a dynamic competition unfolds, players can suffer discouragement (see, e.g., Harris and Vickers, 1987; Konrad and Kovenock, 2009). Feng and Lu (2018) show that optimal time-contingent prizes depend on how effort impacts the probability of winning a stage battle. In a two-stage contest, Klein and Schmutzler (2021) show that a single prize at the end of the contest dominates intermediate prizes, and in a laboratory setting, find that a single prize increases effort by 41%. Alshech and Sela (2021) characterize optimal split of the budget in a two-stage Tullock contest where the designer chooses a "bonus" prize for the player who wins both stages, in addition to prizes for each battle (i.e., a rank-and-time-contingent prize).

Stracke et al. (2014) show that under risk aversion, two prizes may dominate one, and present laboratory evidence supporting this prediction. Cason et al. (2020) study the optimal allocation of prizes and, experimentally, find that a noisy performance measure increases effort. Güth et al. (2016) study milestone prizes, and show, both theoretically and experimentally, that they outperform fixed-prize tournaments and piece rates. Liu et al. (2018) show that negative prizes for players with low effort can increase the expected total effort.

Other settings have theoretically examined score-contingent prizes. For instance, in Ely et al. (2021) the designer observe the players' progress and chooses prizes, feedback, and when to end the contest. The optimal design features a sequence of contests of fixed length. The competition ends when (at least) one player surpasses an exogenous milestone score (i.e., not set by the designer). In Benkert and Letina (2020) players privately observe their progress and choose when to report it to the principal. The optimal prize structure makes interim transfers to all players while the competition has not ended and rewards the first player who reveals success upon which the contest ends. Unlike these papers, we empirically evaluate contingent prizes in contests with a fixed duration and a public leaderboard.

2 Dynamic Prize Allocation: A Simple Model

The model in this section is a simplified version of the model we use in our empirical model. It serves the purpose of illustrating tradeoffs for different contingent prize allocations. We show that contingent prizes can improve outcomes relative to a prize only based on final standings. Hence, the performance of different contingent prize structures is ultimately an empirical question.

Two players, A and B, compete in two stages, $t \in \{t_1, t_2\}$. The identity and score of the leader of the competition at each stage is public information. The competition starts with player A as the leader with a score of $\bar{s} \geq 0$ and player B as the follower with a score of 0. At stage 1, nature selects at random only one of the two players. The player selected by nature is the only one who can make a submission (i.e., "play"), in which case she may become the competition's interim leader. At $t = t_2$, nature again selects only one of the players: the same one selected at $t = t_1$ is chosen with probability $1 - \alpha$ and the other player is selected with probability α. The parameter α is a reduced-form of the *future competition* effect: A higher value of α increases the probability that natures selects at t_2 the rival of the player selected at t_1.[4]

When the score of the current leader is s, a submission increases the score to $s' = s + \varepsilon$ with probability $q(s)$ and does not change it with probability $1 - q(s)$. The function $q(\cdot)$ is *decreasing*: it is harder to replace the leader when her score is higher. We call this the *current competition* effect. Playing is costly: players draw a submission cost, c, where $c \sim K(\cdot)$ is i.i.d among players and stages. After observing this cost, the player selected by nature chooses whether to play. Different contingent prizes influence a player's incentive to play through their impact on the current and future competition effects.

[4] In the empirical model of Section 4, future competition depends on a parameter λ that governs play opportunities as well as conditional-choice probabilities that depend on all the parameters of the model. That is, future competition is endogenous in the full model.

The player who leads the competition at the end of stage t with a score of s_t after history $h_t = (s_0, s_1, ..., s_{t-1})$ receives the prize $\pi(s_t|h_t)$. The contest designer has a fixed budget, normalized to 1, and decides the size of each prize such that for each possible history the sum of prizes along that history does not exceed the budget. In this simple model, there are four possible histories:

$$\mathcal{H} = \{(\bar{s}, \bar{s}), (\bar{s}, \bar{s}+\varepsilon), (\bar{s}+\varepsilon, \bar{s}+\varepsilon), (\bar{s}+\varepsilon, \bar{s}+2\varepsilon)\}.$$

Score $\bar{s}+\varepsilon$ at t_2 is the only state that can be reached by multiple histories (in other cases, there is a unique history leading to the final score). For notational convenience, we make history explicit only in this case. The designer chooses six prizes: Two prizes at t_1, $\pi(\bar{s}, t_1)$ and $\pi(\bar{s}+\varepsilon, t_1)$; and four prizes at t_2, $\pi(\bar{s}, t_2)$, $\pi(\bar{s}+\varepsilon, t_2|\bar{s})$, $\pi(\bar{s}+\varepsilon, t_2|\bar{s}+\varepsilon)$, $\pi(\bar{s}+2\varepsilon, t_2)$. For each history, the sum of prizes cannot exceed the total budget, i.e.,

$$\begin{aligned}
\pi(\bar{s}, t_1) + \pi(\bar{s}, t_2) &\leq 1, \\
\pi(\bar{s}, t_1) + \pi(\bar{s}+\varepsilon, t_2|\bar{s}) &\leq 1, \\
\pi(\bar{s}+\varepsilon, t_1) + \pi(\bar{s}+\varepsilon, t_2|\bar{s}+\varepsilon) &\leq 1, \\
\pi(\bar{s}+\varepsilon, t_1) + \pi(\bar{s}+2\varepsilon, t_2) &\leq 1.
\end{aligned}$$

Since the number of periods is small, $T = 2$, this setting is tractable: there are $T(T+1)$ variables and 2^T constraints. As T grows, the problem becomes computationally unfeasible. Focusing on simple contingent prize structure require imposing additional constraints, which reduce the number of variables in the problem.

Time-contingent prizes do not depend on scores, which imposes the constraints

$$\pi(\bar{s}, t_1) = \pi(\bar{s}+\varepsilon, t_1) \text{ and } \pi(\bar{s}, t_2) = \pi(\bar{s}+\varepsilon, t_2|\bar{s}) = \pi(\bar{s}+\varepsilon, t_2|\bar{s}+\varepsilon) = \pi(\bar{s}+2\varepsilon, t_2).$$

These constraints, of course, reduce the number of variables to T, since we only need to determine the size of the prize for each time period, subject to a single constraint. In contrast, a score-contingent (milestone) prize rewards the first player to increase the current score, imposing the constraints

$$\pi(\bar{s}, t_1) = \pi(\bar{s}, t_2) = \pi(\bar{s}+\varepsilon, t_2|\bar{s}+\varepsilon) = 0.$$

Figure 2 shows the designer's expected payoff (expected maximum score) for the optimal-contingent, time-contingent, and score-contingent prizes as a function of the level of future competition. Of course, the optimal-contingent prizes allows more flexibility and dominates time- and score-contingent. Figure 2 also shows that score-contingent prizes dominate time-contingent prizes, and they capture a large fraction of the gains achieved by the optimal-contingent prize.

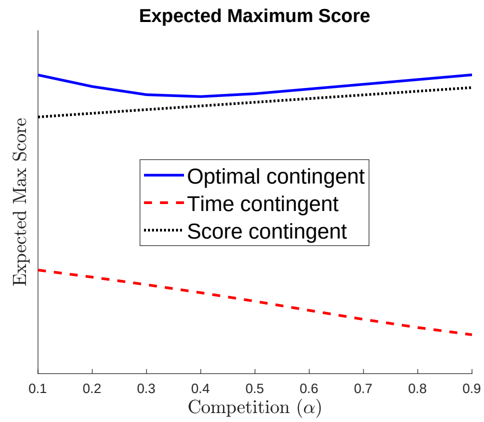


Figure 2: Simulation of expected maximum score as a function of α. Parameters: $q(\bar{s}) = 0.8$, $q(\bar{s}+\varepsilon) = 0.7$, and cost distribution $K(c) = c^{0.85}$.

	Future competition	
Prize Type	$\alpha = 0.2$ (Low)	$\alpha = 0.8$ (High)
Contingent	$\pi(s, t_2 \vert h_{t_2}) = 1,\ s > \bar{s}$ otherwise, $\pi(s,t) = 0$	$\pi(\bar{s}+\varepsilon, t_1) = 0.78$ $\pi(\bar{s}+2\varepsilon, t_2) = 0.22$ $\pi(\bar{s}+\varepsilon, t_2 \vert \bar{s}) = 1$
Time-contingent	$\pi(t_1) = 0$ $\pi(t_2) = 1$	$\pi(t_1) = 0.25$ $\pi(t_2) = 0.75$
Score-contingent	$\pi(\bar{s}+\varepsilon) = 0.98$ $\pi(\bar{s}+2\varepsilon) = 0.02$	$\pi(\bar{s}+\varepsilon) = 0.95$ $\pi(\bar{s}+2\varepsilon) = 0.05$

Table 1: Optimal prize structures for different levels of future competition. Parameters: $q(\bar{s}) = 0.8$, $q(\bar{s}+\varepsilon) = 0.7$, and cost distribution $K(c) = c^{0.85}$.

Table 1 shows the optimal prize structures for two different levels of future competition. When $\alpha = 0.2$, the player selected to play at t_1 is able to play again with probability 0.8, so the impact of future competition is relatively small. The optimal contingent prize structure allocates the full budget to the player that leads the competition at time t_2, only if the final score is above $\bar{s}$. The reason is that with low future competition, the player that plays first expects to play again. So the designer can motivate both players by allocating the prize at the end of the contest. The optimal time-contingent prize allocates all the budget at t_2. The main difference with the optimal structure is that the time-contingent prize is awarded *even* when no player increases the score. Optimal score-contingent prizes allocate most of the prize (98%) to the first player that beats the score $s+\varepsilon$. This is because after increasing the score once it becomes more difficult to increase it again because $q(\cdot)$ decreases from 0.8 to 0.7.

When $\alpha = 0.8$, the player selected to play at t_1 is able to play again with probability 0.2, so the impact of future competition is relatively large. In this case, the optimal contingent prize structure allocate 78% of the budget to the player that increases the score at t_1 and, conditional on this outcome, 22% to the player that further increases the score at t_2. Otherwise, if there is no progress at t_1, it allocates the full budget to the player that increases the score at t_2. Optimal time-contingent prizes allocate 25% of the prize at t_1 and 75% at t_2. Since future competition is high, the intermediate prize motivates the player at t_1. Optimal score-contingent prizes allocate most of the prize (95%) to the first player that beats the score $s+\varepsilon$. This prize is lower than in the previous case. The reason is that the first player is not likely to play again, so it plays to get 95% of the budget, which leaves a higher prize for the player who can play at t_2.

3 Data and Background Information

We use publicly available data on 57 featured competitions hosted by Kaggle.[5] These competitions received thousands of submissions, coming from an average of 894 players per contest, and offered an average prize of $30,489. A partial list of competition characteristics are summarized in Table 2 (see Table A.1 in the Online Appendix for the full list).[6]

In the competitions, participants have access to a training and a test dataset. The training dataset includes both an outcome variable and covariates, while the test dataset only includes covariates. The goal of the contest is to generate the most accurate predictions of the outcome variables for the covariates in the test dataset. A submission in a contest must include an outcome variable prediction for each observation in the test dataset. Kaggle objectively scores each submission based on its out of sample performance, and posts its score in a public leaderboard. Prizes are awarded to top players based on the leaderboard ranking at the end of the contest.[7]

[5] https://www.kaggle.com/kaggle/meta-kaggle

[6] Lemus and Marshall (2021) provide a detailed overview of the dataset as well as descriptive evidence.

[7] Kaggle partitions the test dataset into two subsets and does not inform participants which observations correspond to each subset. The first subset is used to generate the *public score*; the second subset is used to generate the *private score*. The public score is posted in real-time on a public leaderboard, whereas the private score is never made public during the contest. The private score is used to determine the winner of the competition, so the public score is a noisy signal of performance. The correlation between public and

Table 2: Summary of competitions (partial list of competitions)

Competition	Total reward	Submissions	Start date	Deadline
Heritage Health Prize	500,000	2,687	04/04/2011	04/04/2013
Allstate Purchase Prediction Challenge	50,000	1,204	02/18/2014	05/19/2014
Higgs Boson Machine Learning Challenge	13,000	1,776	05/12/2014	09/15/2014
Acquire Valued Shoppers Challenge	30,000	2,347	04/10/2014	07/14/2014
Liberty Mutual Group - Fire Peril Loss Cost	25,000	1,057	07/08/2014	09/02/2014
Driver Telematics Analysis	30,000	1,619	12/15/2014	03/16/2015
Crowdflower Search Results Relevance	20,000	1,645	05/11/2015	07/06/2015
Caterpillar Tube Pricing	30,000	1,938	06/29/2015	08/31/2015
Liberty Mutual Group: Property Inspection Prediction	25,000	1,271	07/06/2015	08/28/2015
Coupon Purchase Prediction	50,000	631	07/16/2015	09/30/2015
Springleaf Marketing Response	100,000	1,567	08/14/2015	10/19/2015
Homesite Quote Conversion	20,000	2,557	11/09/2015	02/08/2016
Prudential Life Insurance Assessment	30,000	818	11/23/2015	02/15/2016
Santander Customer Satisfaction	60,000	1,138	03/02/2016	05/02/2016
Expedia Hotel Recommendations	25,000	436	04/15/2016	06/10/2016

Notes: The table only considers submissions by the top 10 teams of each competition. The total reward is measured in US dollars at the moment of the competition. See Table B.1 in the Online Appendix for the complete list of competitions.

We have contest-level information on all submissions, including the time of the submission, who made them (team identity), and their score (public and private scores). We are able to reconstruct both the public and private leaderboard at every time for every contest. Using the same approach as in Lemus and Marshall (2021), we standardize the score distribution to have a mean of zero a standard deviation of one.

4 Empirical Model

N players compete in a contest of length T. We divide the length of the contest into time intervals of length δ, so time is discrete $t = 0, \delta, 2\delta, ..., T$. Payoffs are undiscounted. Players enter at an exogenously given time, so that at time t, N_t players have already entered. They have perfect foresight regarding entry times of rivals and stay until the end of the contest. Making a submission when the current maximum score is s increases the maximum score to $s' = s + \varepsilon$ with probability q_s and leaves the maximum score at s with probability $1 - q_s$. As in the simple model in Section 2, q_s decreases in s, i.e., it becomes increasingly difficult to replace the leader as the maximum score increases.[8]

Players publicly observe and keep track of four state variables: the current time period (t),

private scores is 0.99, which motivates us to abstract away from the noise in the public signals for tractability.

[8]The assumption of small increments of size ε is motivated by the data.

the maximum score in the previous period (s_P), the current maximum score (s), and the identity of the current leader (ℓ). The state space is

$$\mathcal{S} = \{(t, s, s_P, \ell) : t = 0, \delta, ..., T,\ s_P = 0, \varepsilon, ..., T\varepsilon,\ s \in \{s_P, s_P + \varepsilon\} \text{ and } \ell = 1, ..., N\}$$

At period $t = 0, ..., T-1$, nature selects with probability $\lambda \in (0, 1)$ one (randomly selected) player, and with probability $1-\lambda$ no one is selected. The selected player is the only one who can choose to make a submission ("play") in period t. Submissions are instantaneous at cost c, where c is a random variable distributed according to $K(\cdot)$ and i.i.d. across players and time. Players observe their cost realization *before* choosing whether to play.

The contest awards **contingent prizes**. For tractability, we restrict to "Markovian" contingent prizes rather than prizes that depend on the whole history of the contest. That is, the leader of the competition in period t, receives a prize $\pi(s, t|s_P)$, where s_P is the previous period's maximum score and s is the current period's maximum score. Contingent prize structures permit to allocate different prizes for reaching the same maximum score in the current period depending on maximum score in the previous period. That is, we can have $\pi(s, t|s_P) \neq \pi(s, t|s'_P)$, when $s_P \neq s'_P$.

At time t, a player is either the leader or one of the $N_t - 1$ followers. Let L_{t,s,s_P} be the value of being the leader at time t when the current maximum score is s and the previous maximum score is s_P. Let $F_{t,s}$ be the value of being one of the followers. At time T, when the competition ends and the maximum score is s_T, the leader gets $\pi(s_T, T|s_{T-1})$ and the followers get 0. We denote $t' = t + \delta$ and $s' = s + \varepsilon$.

The expected payoff of a leader with score s at $t = 0, 1, ..., T-1$ is

$$L_{t,s,s_P} = \pi(s, t|s_P) + \left(1 - \lambda \frac{N_t}{N}\right) L_{t',s,s} + \frac{\lambda}{N} L_{t,s}^{\text{own play}} + \frac{\lambda(N_t - 1)}{N} L_{t,s}^{\text{rival play}} \tag{1}$$

That is, when the maximum score in the previous and current periods are s_p and s, the leader at time t receives the prize $\pi(s, t|s_P)$. There are three continuation payoffs. First, with probability $1 - \lambda \frac{N_t}{N}$ none of the players who have entered the contest can play, so the current leader remains the leader, the score continues to be s, and the leader receives the continuation payoff $L_{t',s,s}$. Second, with probability $\frac{\lambda}{N}$ the current leader can choose to play, in which case she receives the continuation payoff $L_{t,s}^{\text{own play}}$, defined in equation (2). Third, with probability $\frac{\lambda(N_t-1)}{N}$ one of the $N_t - 1$ followers can choose to play, in which case the current leader receives the continuation payoff $L_{t,s}^{\text{rival play}}$, defined in equation (5).[9]

[9] Note that the probability that any given player gets to play does not depend on the number of players

The value $L_{t,s}^{\text{own play}}$ is given by

$$L_{t,s}^{\text{own play}} = E_c \left[\max\{ q_s L_{t',s',s} + (1-q_s) L_{t',s,s} - c, L_{t',s,s} \} \right], \tag{2}$$

If selected by nature, the leader chooses whether to play after observing the cost realization, c, playing if only if the expected marginal value of increasing the score is larger than the cost of making the submission, i.e.,

$$q_s(L_{t',s',s} - L_{t',s,s}) \geq c. \tag{3}$$

From this condition, the probability that the leader plays is

$$p_{t,s}^L = K(q_s(L_{t',s',s} - L_{t',s,s})), \tag{4}$$

where $K(\cdot)$ is the distribution function of the submission cost. The last term in equation (1), $L_{t,s}^{\text{rival play}}$, is the leader's continuation payoff when one of the current $N_t - 1$ followers can play, which happens with probability $\frac{\lambda(N_t-1)}{N}$. We have

$$L_{t,s}^{\text{rival play}} = p_{t,s}^F \left(q_s F_{t',s'} + (1-q_s) L_{t',s,s} \right) + (1 - p_{t,s}^F) L_{t',s,s}. \tag{5}$$

If a follower is selected by nature, she plays and replaces the current leader with probability $p_{t,s}^F q_s$, in which case the maximum score increases to s', and the current leader becomes a follower, obtaining $F_{t',s'}$. With probability $p_{t,s}^F(1-q_s)$, the follower plays but fails to replace the leader, so the current leader remains the leader, obtaining $L_{t',s,s}$. With probability $1-p_{t,s}^F$ the follower chooses not to play, so the current leader remains the leader, obtaining $L_{t',s,s}$. The probability $p_{t,s}^F$ is an equilibrium object that we derive later on.

We next specify the value of being a follower, $F_{t,s}$, which depends on the current score and time but not on the previous maximum score. All the followers are symmetric, so the value for any one particular follower is identical. Let j denote one of the $N_t - 1$ followers. The value of being a follower is

$$F_{t,s} = \left(1 - \lambda \frac{N_t}{N}\right) F_{t',s} + \frac{\lambda}{N} F_{t,s}^{\text{leader play}} + \frac{\lambda}{N} F_{t,s}^{j \text{ play}} + \frac{\lambda(N_t - 2)}{N} F_{t,s}^{\text{follower play}} \tag{6}$$

Followers do not receive prizes. In our model, prizes for followers is suboptimal because there is no entry margin and all followers have the same probability of becoming the leader. At

that have actually entered (N_t), as any one player gets the opportunity to play with probability $1/N$, where N is the time-independent number of players that will participate in the competition. Empirically, N_t converges quickly to N.

time t, there are four cases. First, with probability $1 - \lambda \frac{N_t}{N}$, none of the players who have entered the contest can play, so followers remain followers and receive the continuation payoff $F_{t',s}$. Second, nature selects the leader with probability $\frac{\lambda}{N}$, in which case followers receive the continuation payoff $F_{t,s}^{\text{leader play}}$. Third, nature selects follower j with probability $\frac{\lambda}{N}$, in which case she receives the continuation payoff $F_{t,s}^{j\text{ play}}$. Fourth, nature selects one of the other $N_t - 2$ followers (not j) with probability $\frac{\lambda(N_t-2)}{N}$, in which case follower j receives the continuation payoff $F_{t,s}^{\text{follower play}}$.

The value $F_{t,s}^{\text{leader play}}$ is given by

$$F_{t,s}^{\text{leader play}} = p_{t,s}^L \left(q_s F_{t',s'} + (1 - q_s) F_{t',s}\right) + (1 - p_{t,s}^L) F_{t',s} \tag{7}$$

When nature selects the leader, she plays and increases the maximum score with probability $p_{t,s}^L q_s$, in which case the followers receive $F_{t',s'}$. With probability $p_{t,s}^L(1 - q_s)$, the leader plays but fails to increase the maximum score, so followers receive $F_{t',s}$. With probability $1 - p_{t,s}^L$ the leader chooses not to play, so the followers receive $F_{t',s}$.

The value $F_{t,s}^{j\text{ play}}$ is given by

$$F_{t,s}^{j\text{ play}} = E_c \left[\max\{q_s L_{t',s',s} + (1 - q_s) F_{t',s} - c, F_{t',s}\}\right]. \tag{8}$$

If selected by nature, follower j chooses between playing or not after observing cost of making a submission, c, playing if and only if

$$q_s(L_{t',s',s} - F_{t',s}) \geq c. \tag{9}$$

The condition above means that the expected marginal gain from becoming the leader must be sufficiently larger than the cost. From here, the probability that a follower makes a submission is

$$p_{t,s}^F = K\left(q_s(L_{t',s',s} - F_{t',s})\right). \tag{10}$$

The value $F_{t,s}^{\text{follower play}}$ is given by

$$F_{t,s}^{\text{follower play}} = p_{t,s}^F \left(q_s F_{t',s'} + (1 - q_s) F_{t',s}\right) + (1 - p_{t,s}^F) F_{t',s} \tag{11}$$

When nature selects a follower other than player j, follower j always remains a follower but the maximum score can change. The follower other than j plays with probability $p_{t,s}^F$ and increases the maximum score with probability q_s. In that case, follower j gets $F_{t',s'}$. In any other case, only time progresses and follower j receives $F_{t',s}$.

Key Driving Forces. Conditional on the current score and time, leaders and followers have different incentives to play. For the leader, the expected marginal benefit of a successful play is $L_{t',s',s} - L_{t',s,s}$, whereas for a follower the benefit is $L_{t',s',s} - F_{t',s}$. Consider first the incentives of the leader. First, by increasing the score, the leader collects a prize $\pi(t', s'|s)$ instead of $\pi(t', s, |s)$. This effect could be positive, negative, or zero, depending on the contingent prize structure; if it is positive, the future prize motivates the leader to play. Second, a higher score makes it harder for every player in future periods to replace the leader, since $q_{s'} < q_s$. This is a negative externality on all players, including the leader. So while the leader benefits from deterring rivals from future plays, it also deteriorates its own incentives to play in the future.

Consider next the incentives for the follower. First, by increasing the score, the follower collects a prize $\pi(t', s'|s)$ instead of 0 (a weakly positive effect) and becomes the leader. By playing, the follower also exerts a negative externality on all players by increasing the score.

Objective of the Contest Designer. The contest designer chooses the prize structure to maximize the expected maximum score at the end of the contest.[10] We assume that any prize structure must not violate the budget constraint, i.e., there is no history for which the sum of prizes exceeds the budget (normalized to 1). Our dynamic game has 2^T possible histories, which we can write as:

$$\mathcal{H} = \{(a_1, ..., a_T) \,:\, a_t \in \{0, \varepsilon\}\}$$

In history $h = (a_1^h, ..., a_T^h) \in H$, a_t^h indicates the change in score at time t. In this history, the maximum score at time t is $s_t^h = \sum_{j=1}^{t} a_j^h$.

Prizes are conditional on the maximum score at $t-1$, the maximum score at t, and the current time, t. Thus, there are $T(T+1)$ variables that define a contingent prize structure $\{\pi(s', t|s)\}$ for $t = 1, ..., T$, $0 \leq s \leq T\varepsilon$ and $s' \in \{s, s+\varepsilon\}$. A *feasible* contingent prize structure must satisfy that the sum of prizes in history $h \in H$ is less than or equal to one. These are 2^T constraints, one for each history $h \in H$, with

$$\pi(s_1^h, 1) + \pi(s_2^h, 2|s_1^h) + ... + \pi\left(s_t^h, t|s_{t-1}^h\right) + ... + \pi\left(s_T^h, T|s_{T-1}^h\right) \leq 1 \qquad \text{(BC-}h\text{)}$$

When the goal of the designer is to maximize the expected maximum score at the end of the

[10] Implicitly, we are assuming the risk-neutral contest designer with monotone preferences on the maximum score

contest, the designer solves

$$\max_{\{\pi(\cdot)\}} \int s_T^h \, dF(h|\pi(\cdot)) \text{ subject to (BC-}h\text{) for all } h \in H. \tag{12}$$

In equilibrium, contingent prize structures change the balance of different histories by dynamically manipulating competitors incentives to play throughout the competition.

4.1 Computational Burden

Finding the optimal contingent prize structure that solves problem (12) is, in general, computationally demanding. It requires to choose $T(T+1)$ variables (a prize structure) subject to 2^T budget constraints.

For small instances (e.g., $T \leq 15$), the problem can be solved. In fact, in Section 6, we use the estimates of our structural model to simulate "short contests" ($T \in \{5, 10, 15\}$ instead of the longer durations in our empirical model) for which we can solve for the optimal contingent-prize structure. We compare the optimal structure with *simple* prize structures, which are constrained problems that require to optimize over a small subset of variables, and can be solved for large values of T. We use short contests to measure the percentage of the gains from an optimal prize structure achieved by simple prize structures.

Upper Bound on Gains. Even though we cannot explicitly compute the optimal prize structure when T is large, we can find an upper bound for the value of using it. To this end, we use duality theory to provide an upper bound for the expected maximum score. Specifically, for any vector $\mu = (\mu_h)_{h \in H}$, with $\mu_h \geq 0$, an upper bound to (12) is

$$\sup_{\{\pi(\cdot)\}} \left\{ \int s_T^h \, dF(h|\pi(\cdot)) - \sum_{h \in H} \mu_h \left(\sum_{t=1}^{T} \pi(s_t^h, t | s_{t-1}^h) - 1 \right) \right\}. \tag{13}$$

The optimal bound is found by minimizing (13) over $(\mu_h)_{h \in H}$. This entails solving an unconstrained problem with 2^T variables, which is computationally unfeasible. However, fixing the variables $(\mu_h)_{h \in H}$ and solving (13) over $T(T+1)$ variables, which is computationally feasible, delivers an upper bound.

Finding the Optimal Budget. The main computational burden of finding the optimal solution is the large number of constraints. Instead of solving the constrained problem for a fixed budget, we could solve an alternative unconstrained problem. Let the designer's value of obtaining a maximum score s be $V(s)$. The designer chooses the prizes to maximize her expected payoff by solving the following unconstrained problem

$$\max_{\{\pi(\cdot)\}} \int V(s_T^h)\, dF(h|\pi(\cdot)) - \int \sum_{t=1}^{T} \pi(s_t^h, t|s_{t-1}^h)\, dF(h|\pi(\cdot)) \tag{14}$$

While this problem is computationally tractable (unconstrained with $T(T+1)$ variables), it requires us to take a stand on the shape of $V(\cdot)$, which endogenously determines the size of the budget.

To show that the use of flexible prize structures—rather than a final-ranking prize—is optimal for different designer's preferences, we solve problem (14) for different contests using two functional forms of $V(\cdot)$: $V(x) = x^2$ and $V(x) = \sqrt{x}$.[11] We choose these two functional forms to see whether a final-ranking prize is optimal for a convex function (representing that the designer is "risk loving" on the maximum score) or for a concave function (representing that the designer is "risk averse" on the maximum score). We find that flexible prize structures dominate a final-ranking prize in both of these two cases.

4.2 Simple Prize Structures

One way to deal with the high dimensionality of the problem is to restrict the set of prize structures. We investigate different classes of *simple* prize structures, where prizes are zero for most states and positive for a small subset of states. Simple prize structures are appealing not only for being computationally tractable, but also for being easy to implement in practice.[12] We focus on three classes of simple prize structures: time-contingent, score-contingent, and hybrid.

Time-contingent prizes. The prize structure is time-contingent if $\pi(s_t, t|s_{t-1}) = \pi_t$ for all $s_t \in \{s_{t-1}, s_{t-1} + \varepsilon\}$ and $s_{t-1} \in \{0, \varepsilon, ..., T\varepsilon\}$. That is, the interim leader at time t receives a prize that is independent of the current or the previous maximum score.

Score-contingent prizes (milestones). The prize structure is score-contingent if $\pi(s_t, t|s_{t-1}) = 0$ for $s_t = s_{t-1}$ and $\pi(s_t, t|s_{t-1}) \geq 0$ for $s_t = s_{t-1}+\varepsilon$. Furthermore, for any $t' \neq t$ and $s' = s+\varepsilon$, $\pi(s', t|s) = \pi(s', t'|s)$. That is, the first player to reach a milestone receives a prize, and the size of the prize is independent of the time at which the milestone was reached.

[11] The simulated contests make use of the same parameters estimated for the main model (Table 4).

[12] In regulation, simple menus can capture large gains of optimal menus (Rogerson, 2003) .

Hybrid prizes. The hybrid structure is a combination of milestones and time-contingent prizes. Here we have $\pi(s', t|s) = 0$ unless $s \in \{s_1, ..., s_m\}$ and $s' = s + \varepsilon$ or $t = T$, in which case $\pi(s', t|s) \geq 0$. In contrast to score-contingent prizes, the time at which the milestone is reached matters; also, there can be a final-ranking prize even if the milestone is not reached.

4.3 Discussion of Modeling Assumptions

Our model hinges on several assumptions to facilitate estimation. Some of these assumptions simplify the behavior of players while others reduce the choice set of the contest designer.

1. *Learning and experimentation.* One motivation to participate in a Kaggle competition is the opportunity for players to learn and experiment. Even experienced players can benefit from learning from their performance on earlier submissions. Our model accommodates one form of learning: each player has the same probability of becoming a leader, even when that probability decreases as the maximum score increases. That is, the function $q(s)$ captures that all players learn in the same manner when the score increases since playing gives them the same chance of becoming the leader.

   A different way of modeling learning would be to allow for players' performance (or cost) to depend on the number of past submissions. Under this form of learning, prize structures that motivate players to play early allow the designer to reap the benefit of learning (enhanced performance or lower cost). This approach increases the state space by m^N, where m is the number of types associated to different levels of learning and N is the number of players. If, for instance, learning gives rise to two types, our state-space is $2^{10} = 1,024$ times larger. We refrain from such approach for tractability.

2. *Incentives to withhold submissions.* A player could be concerned about increasing the maximum score because it may inform rivals that are "stuck" that "something else" is possible, encouraging them to exert effort. In such case, players could withhold their submissions and send them near to the end of the competition to prevent encouraging their rivals. We argue that at least three facts alleviate this concern. First, players benefit from submitting their solutions as soon as possible to receive feedback, which allows them to improve their current solutions. Second, there is a limit on the number of submissions players can send each day. Third, Lemus and Marshall (2021) use the same sample of contests as in this article and do not find empirical evidence suggesting strategic withholding.

3. *Leader, followers, and prizes.* At each instant during the contest, Kaggle's leaderboard shows the best score for each player. Furthermore, multiple players receive prizes at the end of the contest based on the final ranking (usually, three prizes). In our model, there is no notion of "being close" to the leader, as each follower is symmetric. We also assume that only the leader receives a prize. These assumptions are for convenience, to reduce the dimensionality of the state space. Keeping track of each player's scores at each point in time increases our state space by $|S|^{N-1}$, where $|S|$ is the number of possible scores and N is the number of players. Also, the number of variables for contingent prizes would increase by m, where m is the number of "places" that receive an award. Qualitatively, the optimal contingent prize structures in this augmented model should be similar to what we find. In our setting, the incentive to play is driven by the prize $\pi(s', t|s)$ for becoming the leader. With multiple prizes, the players' decisions are more involved as they assess the probability of ending the contest in a different ranking, which determines the their expected reward. With m prizes, players' incentives are driven by the expected prize $\sum_{j=1}^{m} \pi(s, t, j|s') \times \Pr(\text{rank } j \text{ at time } t)$.

4. *Player heterogeneity.* Our model assumes players are symmetric. In our estimation, we focus on top performers, who are arguably more symmetric than two randomly-selected players in the contest, which alleviates heterogeneity concerns. A simple test reveals that the top 10 players are fairly homogeneous. Specifically, less than 20 percent of the variance of scores can be explained by competition and team fixed effects. Moreover, most of the coefficients on players' fixed effects are statistically insignificant. These facts combined suggest that most of the differences in performance among this group of players are random.

5. *Open or restricted entry.* Kaggle contests are typically open contests (anyone can participate). We do not model entry or exit. Lemus and Marshall (2021) discuss at length the exogenous entry assumption. Here, we analyze the strategic behavior of the top performers in each contest both to alleviate concerns about player heterogeneity and focus on the players who are likely to influence the contest outcomes.

6. *Length and number of stages.* Most Kaggle contests are single-stage contests with a fixed length. We normalize the length of the contests to make meaningful comparisons across contests. Dividing the contest into multiple stages can reduce discouragement. In our setting, there is no notion of being "close" to the leader because each player that is not a leader is a follower. Furthermore, if a contest was divided in stages, players could always resubmit their last solution to restore the leaderboard at the beginning of

each stage. Thus, in our setting, it is unfeasible to reduce discouragement by splitting the contest in multiple stages.

5 Estimation

Given that only the state is payoff relevant, we use the Markov-perfect equilibrium concept. Computationally, we find it using backwards induction.

The full set of primitives for a given contest include i) the probability that a player can play at time t, λ; ii) the entry times of each player; iii) the function q_s, which indicates the probability of advancing the maximum score given that the current maximum score is s; and iv) the distribution of submission costs, $K(c;\sigma) = c^\sigma$, where $\sigma > 0$ and the support of the distribution is the interval $[0, 1]$.[13] We allow these primitives to vary at the contest level.

We use a two-step procedure to estimate the primitives of each contest. In the first step, we recover or estimate primitives i)-iii) without using the full structure of the model. In the second step, we use the estimates of these primitives to estimate the cost distribution using a generalized method of moments (GMM) estimator.

We make use of a feature of the platform Kaggle to estimate the probability that a player can play at a given time period, λ. Specifically, players face a cap on the number of daily submissions, which in conjunction with the length of the contest, gives us a player's maximum number of submissions during the competition (i.e., daily cap $\cdot$ competition length (days)). We set λ to be $N \cdot$daily cap$\times$competition length (days)$/T$, where N is the number of players and T is the number of time periods in the model. This expression gives a measure of the fraction of periods in which a player can play, which is what we intend to capture with λ.

The entry times of each player are assumed to be exogenous in the model and we recover them directly from the data. Next, we specify the function q_s as

$$q_s = \exp\{\beta_0 + \beta_1 s\}/(1 + \exp\{\beta_0 + \beta_1 s\}),$$

and we estimate β_0 and β_1 using a maximum-likelihood estimator, using data on whether each submission increased the maximum score as well as the maximum score at the time of each submission (s). Because in some competitions the maximum score is rather constant, we pool the data from all competitions to gain power in estimating the parameter β_1, which

[13] Recall that we normalize the size of the prize to be 1 for every contest.

we constrain to be uniform across contests. We allow β_0 to vary across contests.

In the second step, we estimate the parameter σ of the cost distribution, $K(c;\sigma) = c^\sigma$, where $\sigma > 0$, using a GMM estimator, where the moments are based on comparisons of the number of observed submissions and the number of submissions predicted by the model. Specifically, we divide the length of each contest in five periods of equal length (henceforth, time quintiles) and we compute the number of submissions observed in the data and predicted by the model in each time quintile k: $m_k(\sigma) = \text{submissions}_k^{data} - \text{submissions}_k^{model}$. We also include a sixth moment that compares the overall number of submissions in the data and predicted by the model. The GMM estimator is then given by

$$\hat{\sigma} = \arg\min_{\sigma} \hat{\mathbf{m}}(\sigma)' \mathbf{W} \hat{\mathbf{m}}(\sigma),$$

where $\mathbf{W}$ is a weighting matrix. We present bootstrapped standard errors.

We use the full-solution method to compute the moments for a given value of σ. That is, for a given σ, we compute the equilibrium of the game using backward induction to obtain the matrices of conditional-choice probabilities (CCPs) $\mathbf{p}^L$ (leader) and $\mathbf{p}^F$ (followers) of dimensions $S^2 \times T$ (S is the size of the set of possible scores and T is the number of periods) where element (s, t, s^P) of $\mathbf{p}^j$ is p^j_{s,t,s^P}.[14] Using the CCPs, we can also compute the equilibrium distribution of maximum scores at every period of time, $\mathbf{G}$ (of dimensions $S^2 \times T$), where column t gives the distribution of maximum scores at time t. Element-wise multiplication of $\lambda(\mathbf{p}^L + (N-1)\mathbf{p}^F)/N$ (i.e., the probability of play when a player is chosen at random) and $\mathbf{G}$, followed by summation of the product over the first dimension, gives us a $1 \times T$ vector with the expected probabilities of a submission at every moment of time, which we use to compute the moments.

Lastly, we restrict the sample to the top 10 players in each contest (measured by the ranking of players at the end of the competition), i.e., $N = 10$. We make this choice for two reasons: i) this is the set of players achieving scores that trigger changes in the top positions of the leaderboard, and ii) this group of players is less heterogeneous than the entire pool of players, which allows us to abstract away from modelling player heterogeneity.

[14] In the estimation, we set $T = 10,000$ and S varies across contests. In a given contest, the set of scores is set to include all unique maximum scores in the competition as well as the values $-2, -1.5, -1, -0.5$, and $\bar{s} + [0.001 : 0.001 : 0.045]$, where $\bar{s}$ is the highest observed score in the competition.

Table 3: GMM estimates of model parameters (partial list of competitions)

Competition	σ	SE	λ	β_0 (q)	SE	β_1 (q)	SE	Obj. Fun	N
Heritage Health Prize	0.0279	0.0001	0.7306	-3.5412	0.0461	-1.3618	0.1938	0.0138	2687
Allstate Purchase Prediction Challenge	0.0555	0.0001	0.4519	-3.1441	0.0473	-1.3618	0.1938	0.0006	1204
Higgs Boson Machine Learning Challenge	0.0648	0.0001	0.6307	-3.3141	0.0994	-1.3618	0.1938	0.0042	1776
Acquire Valued Shoppers Challenge	0.0148	0.0003	0.4759	-2.3999	0.1707	-1.3618	0.1938	0.0045	2347
Liberty Mutual Group - Fire Peril Loss Cost	0.0367	0.0001	0.2825	-2.075	0.1316	-1.3618	0.1938	0.0011	1057
Driver Telematics Analysis	0.0735	0.0001	0.4571	-4.2049	0.1214	-1.3618	0.1938	0.0018	1619
Crowdflower Search Results Relevance	0.0129	0.0001	0.2806	-2.8256	0.105	-1.3618	0.1938	0.0012	1645
Caterpillar Tube Pricing	0.0172	0.0001	0.3166	-3.0875	0.0277	-1.3618	0.1938	0.0004	1938
Liberty Mutual Group: Property Inspection Prediction	0.0233	0.0001	0.2667	-3.1092	0.06	-1.3618	0.1938	0.0009	1271
Coupon Purchase Prediction	0.1222	0.0003	0.3848	-2.0006	0.2093	-1.3618	0.1938	0.0027	631
Springleaf Marketing Response	0.0326	0.0001	0.332	-3.0862	0.0708	-1.3618	0.1938	0.0009	1567
Homesite Quote Conversion	0.0223	0.0001	0.4559	-3.1196	0.0355	-1.3618	0.1938	0.0009	2557
Prudential Life Insurance Assessment	0.0749	0.0006	0.4219	-3.0026	0.0449	-1.3618	0.1938	0.002	818
Santander Customer Satisfaction	0.0218	0.0001	0.3059	-2.2694	0.0479	-1.3618	0.1938	0.0021	1138
Expedia Hotel Recommendations	0.1011	0.0001	0.2814	-1.7786	0.0485	-1.3618	0.1938	0.0005	436

Notes: The table reports the GMM estimates with bootstrapped standard errors. See Table B.2 in the Online Appendix for the complete list of competitions.

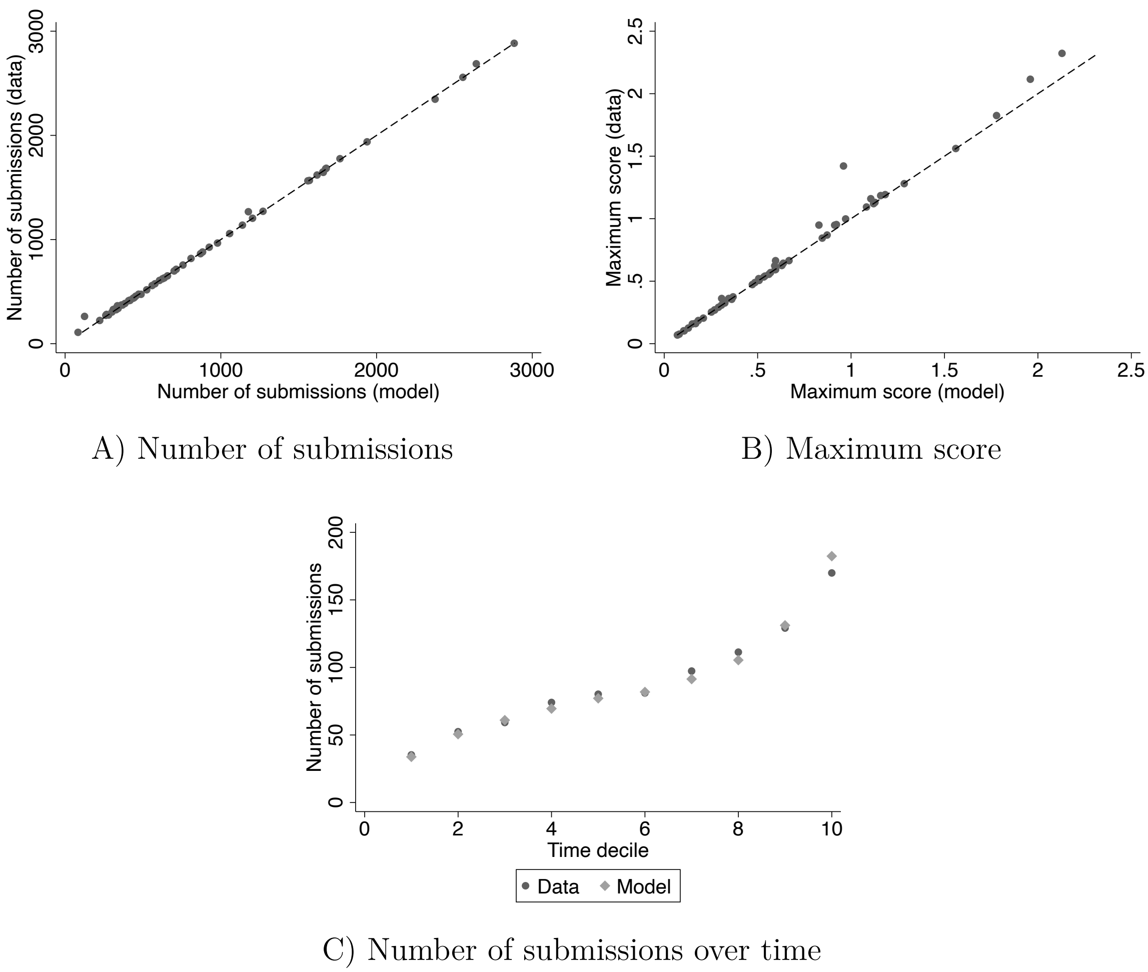
5.1 Estimation Results and Model Fit

Table 3 reports the GMM estimates for a partial list of contests (see Table B.2 in the Online Appendix for the full list).

Regarding the goodness of fit of the model, Figure 3.A and Figure 3.B plot the actual versus the predicted maximum score and number of submissions for every contest. The figures show that the model estimates are able to replicate the data in both cases. Figure 3.C plots the actual and predicted number of submissions over time, averaged across contests, where time is divided in 10 periods of equal length. The figure shows that the model estimates are able to replicate the submission dynamics in the data without systematically under- or over-predicting the observed values.

Figure B.1 in the Online Appendix plots the cross-contest distribution of the expected cost of making a submission. Given the distribution of costs that we use in our model, i.e., $K(x) = x^{\sigma}$, the expected cost is given by $\sigma/(1+\sigma)$. Since the model normalizes the value of the prize pool to 1, we have to scale this up by the size of the prize in order to translate costs to dollars. The figure shows that the median expected cost of making a submission is \$642 dollars and 75 percent of contests have an expected cost that is less than \$1,280. If a player needs 10 hours to prepare to make a submission, the median hourly cost is \$64.2 dollars. Note however that these are averages for the unconditional distribution of costs—when a player chooses to make a submission, there is selection in that the player will only choose to do so when getting a low enough cost draw.

Figure 3: Model fit



Notes: Panels A and B are scatter plots of the number of submissions and the maximum score in every contest, where the data is on the y-axis and the model predictions on the x-axis. Panel C plots the average number of submissions across contests for every time decile in the data and predicted by the model.

6 Prize Structure and Contest Outcomes

How do different the prize structures impact contest outcomes? Using our model estimates, we use counterfactual simulations to provide an answer using two exercises. In the first exercise, we use the model estimates in Table 3 to simulate contests that are shorter than the ones observed in our sample. The shorter length of these simulated contests allows us to solve for the optimal prize structure and compare contest outcomes under the optimal prize structure with outcomes under alternative prize structures. Because the dimensionality of the optimal prize structure explodes as we increase the length of the contest, we use this exercise to explore whether simple prize structures can approximate the gains of the optimal prize structure. In the second exercise, we use our model estimates and compare the observed

Table 4: Percentage change in the expected number of score increments under alternative prize structures in simulated contests (relative to a prize structure where all the prize money is awarded at the end of the competition)

	$\bar{s} = 0$			$\bar{s} = 0.4$			$\bar{s} = 0.8$		
	Optimal	SC	2-parameter SC	Optimal	SC	2-parameter SC	Optimal	SC	2-parameter SC
$T = 5$	2.52	2.3	2.25	2.76	2.61	2.58	3.01	2.91	2.89
	(0.25)	(0.23)	(0.23)	(0.27)	(0.26)	(0.26)	(0.29)	(0.28)	(0.28)
$T = 10$	1.87	1.5	1.41	2.1	1.79	1.73	2.38	2.14	2.1
	(0.19)	(0.16)	(0.16)	(0.21)	(0.19)	(0.19)	(0.23)	(0.22)	(0.22)
$T = 15$	1.68	1.29	1.14	1.83	1.47	1.38	2.09	1.78	1.73
	(0.17)	(0.15)	(0.13)	(0.19)	(0.16)	(0.16)	(0.21)	(0.19)	(0.19)

Notes: The table reports the average percentage change in the expected number of score increments (i.e., the number of times the maximum score increases during the contest) under alternative prize structures, relative to a final-ranking prize. The optimal column captures the gains from using the optimal prize structure, the 'SC' shows the gains of a hybrid a prize structure, and the '2-parameter SC column' shows the gains of an optimally-calibrated milestone in combination with a prize to the leader at the end. These outcomes were computed based on simulated contests that make use of the model estimates in Table 3. In these simulated contests, we vary three parameters relative to the "full" model: T (number of periods), $\bar{s}$ (the score at the beginning of the contest), and ε (the size of the score increments). ε is fixed at 0.1, all other parameters are set as described in the table. Standard errors are in parentheses.

equilibrium of the contest in our sample with the equilibria of these contests under alternative prize structure, where our focus is on simple prize structures.

6.1 Optimal Prize Structure

As described, we begin our analysis by simulating short contests for which we can solve for the optimal prize structure. We simulate a contest for each set of primitives in Table 3 but vary three parameters relative to what is observed in reality: T (number of periods), $\bar{s}$ (the score at the beginning of the contest), and ε (the size of the score increments). We fix ε at 0.1 and vary T and $\bar{s}$ to consider the sensitivity of our results to these parameters.

For every combination of parameters, we compare the performance of a final-ranking prize with the performance of three alternative prize structures: (1) the optimal prize structure, which solves problem (12); (2) the optimal hybrid prize structure ("SC") that rewards players who advance the maximum score as well as the player who leads the competition at the end; (3) a "2-parameter SC" that awards one prize from reaching a milestone and one prize for the leader at the end of the contest; the size of the milestone and the size of the two prizes are chosen optimally. We measure the performance of each prize structure by the expected number of score increments during the contest (i.e., the number of times the maximum score increases during the contest). We use this variable because it contains all the information needed to compute the final maximum score (together with ε and $\bar{s}$) and is unaffected by the scale of scores.

Table 4 shows that the optimal prize structure always increases performance relative to a final-ranking prize in short contests. Specifically, it increases the number of score increments between 1.68 percent (when $T = 15$ and $\bar{s} = 0$) and 3.01 percent (when $T = 5$ and $\bar{s} = 0.8$) on average. We capture the intensity of the current-competition effect by the initial score: A higher value of $\bar{s}$ implies that the probability of increasing the maximum score is lower. We capture the intensity of the future-competition effect by the length of the contest: a higher value of T means that the future-competition effect early in the contest is stronger. The table shows two salient patterns related to the current- and the future-competition effects.

First, the gains from using the optimal prize structure are larger when the current-competition effect is stronger. For instance, when $T = 5$, the gain increases from 2.52 percent to 3.01 percent. Intuitively, a stronger current-competition effect discourages a players, so with a final-ranking prize the leader can "rest on her laurels" and wait for the competition to end to receive a prize. The optimal structure must reward players that increase the score but do not necessarily lead the competition at the end. We can see that rewarding score increments (column 'SC') capture a large fraction of the gains of the optimal prize structure (between 77 and 97 percent). Perhaps more surprisingly, a milestone prize in combination with a prize to the leader based on final standings (column '2 parameter SC') achieves between 68 and 96 percent of the gains of the optimal prize structure. This is surprising because the '2 parameter SC' is very sparse. For example, in a contest with 15 periods, the optimal prize structure can award up to 210 prizes whereas a 2-parameter SC awards only 2 prizes.

Second, the gains from using the optimal prize structure are smaller when the future-competition effect is stronger. For instance, when $\bar{s} = 0$ the gain decreases from 2.52 percent to 1.68 percent. Here, the optimal prize structure can use contingent prizes to motivate players early on. However, it cannot reward early plays too much otherwise the remaining budget will be too small, discouraging players later on. One concern regarding the gains from the optimal structure when T becomes very large, is that they could converge to zero, since there is a decreasing pattern. In the next section, we show this is not the case. While we cannot compute the optimal prize structure for large values of T, we show that the gains from simple structures is bounded away for zero and, in fact, can be substantial.

In summary, studying short contests suggests that the gains of using contingent prizes can be economically relevant and that simple prize structures can approximate the gains of the optimal prize structure despite being sparse.

6.2 The Gains of Simple Prize Structures

We next turn to comparing the contests we observe in our sample (i.e., the observed equilibria) with the equilibrium of each of these contests under alternative prize structures. We consider six counterfactual designs, each of which have a simple structure. The first three designs consider k prizes to the interim leaders at k equally spaced times, with the size of each of the k prizes chosen optimally and $k \in \{2, 4, 6\}$. The fourth design ("2 timed prizes") allocates two prizes, one to the leader based on final standings and another to the leader based on standings at an optimally-chosen time, with the magnitude of both prizes chosen optimally. The fifth design ("milestone") awards the full prize pool to the first player surpassing an optimally-chosen milestone score B. The last design ("hybrid") awards one prize to the first player surpassing a milestone score and one prize to the leader of the competition based on final standings, where the milestone and the magnitude of both prizes are optimally chosen.

Table 5 presents a comparison of equilibrium outcomes. For each contest, we compute the optimal prize structure within each counterfactual design. The columns labeled "Optimal" compare the equilibrium number of submissions and maximum score (in expectation) under the within-class optimal prize structure relative to the equilibrium outcomes under the baseline design (i.e., when all the prize money is awarded based on the final ranking). The table shows that awarding intermediate prizes can increase submissions by up to an average of 36.5 percent and the maximum score by 0.047 standard deviations (the case with the hybrid prize structure). The prize structure with 6 intermediate prizes achieves the second highest gains, with an average increase in the number of submissions and maximum score of 30.5 percent and 0.039 standard deviations, respectively. The hybrid prize structure achieves the greatest gains in 82 percent of the competitions, while the 6-prize structure is optimal in the remaining 18 percent.[15] These results suggest that flexible prize structures can increase incentives to make submissions by economically significant magnitudes.

To further illustrate the impacts of contingent prizes, Figure 4 shows how the average number of submissions changes over time when implementing a 4-prize prize structure (measured relative to the baseline design). As the figure shows, the 4-prize structure boosts incentives early in the competition, especially around the times the interim prizes are given, and decreases incentives near the end. Although the decrease near the end is small, incentives to participate are greatest near the end (see Figure 3.C). Nevertheless, the total number of

[15] Table B.3 in the Online Appendix presents estimates of a probit model for the probability of the hybrid prize structure being optimal for a contest as a function of contest primitives. The table shows that the hybrid design is more likely to be optimal when the frequency of submission opportunities is higher (i.e., stronger future competition).

Table 5: Equilibrium outcomes under alternative prize structures

	Change in # of submissions (in %)		Change in max score (in st. dev.)	
	Optimal	Uniform	Optimal	Uniform
2 prizes	13.378	12.985	0.017	0.016
	(1.676)	(1.668)	(0.004)	(0.004)
4 prizes	25.061	24.742	0.032	0.032
	(2.353)	(2.338)	(0.008)	(0.007)
6 prizes	30.535	30.389	0.039	0.039
	(2.67)	(2.67)	(0.008)	(0.008)
2 timed prizes	21.096	15.368	0.03	0.02
	(2.185)	(1.805)	(0.008)	(0.005)
Milestone	30.802	-84.503	0.04	-0.168
	(3.282)	(13.715)	(0.009)	(0.052)
Hybrid	36.528	8.45	0.047	0.01
	(3.149)	(2.542)	(0.009)	(0.003)

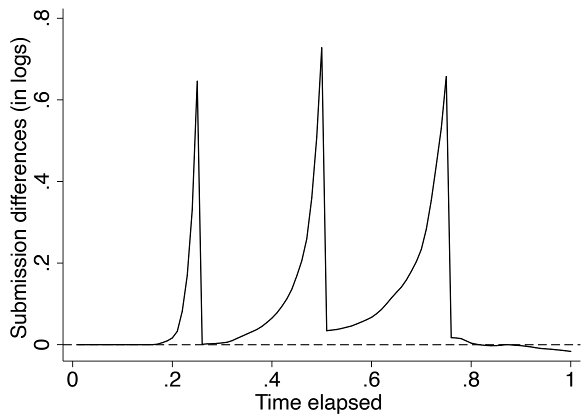
Notes: The table reports the average change in the number of submissions and maximum score when using alternative prize structures. The outcome differences compare the optimal design for each prize structure class. The column "Optimal" reports results for when the optimal design is computed for each contest, whereas the column "Uniform" for when the optimal design is constrained to be uniform across all contests.

submissions increases on average by 8.9 percent.

Figure 5 presents details about the parameters governing the optimal prize structures within each prize class. Panels B and C show that the 4- and 6-prize optimal designs are similar across contests in that the prizes increase over time and the ranges of each of the prizes are somewhat narrow. Panel D shows that in the design with 2-timed prizes, about 80 percent of the probability mass of the distribution of the optimal time of the first prize is in between times 0.6 and 0.8. The optimal milestone score and hybrid designs (Panels E and F) feature more heterogeneity across contests, which reflects underlying differences in the score distributions and the probability of increasing the maximum score across contests.

We note that to compute the optimal prize structure within each counterfactual design for a contest, the contest designer needs to know all the primitives of the model. The designer may not have that information before the contest, which motivates us to ask: are the gains in contest outcomes similar if the prize structure is constrained to be uniform across all contests? We answer this question in the columns of Table 5 that are labeled "Uniform", where we compute the gains for each contest using a prize structure that was chosen by optimizing the average maximum score across contests subject to the prize structure being identical

Figure 4: Submission dynamics in a 4-prize prize structure relative to the baseline design



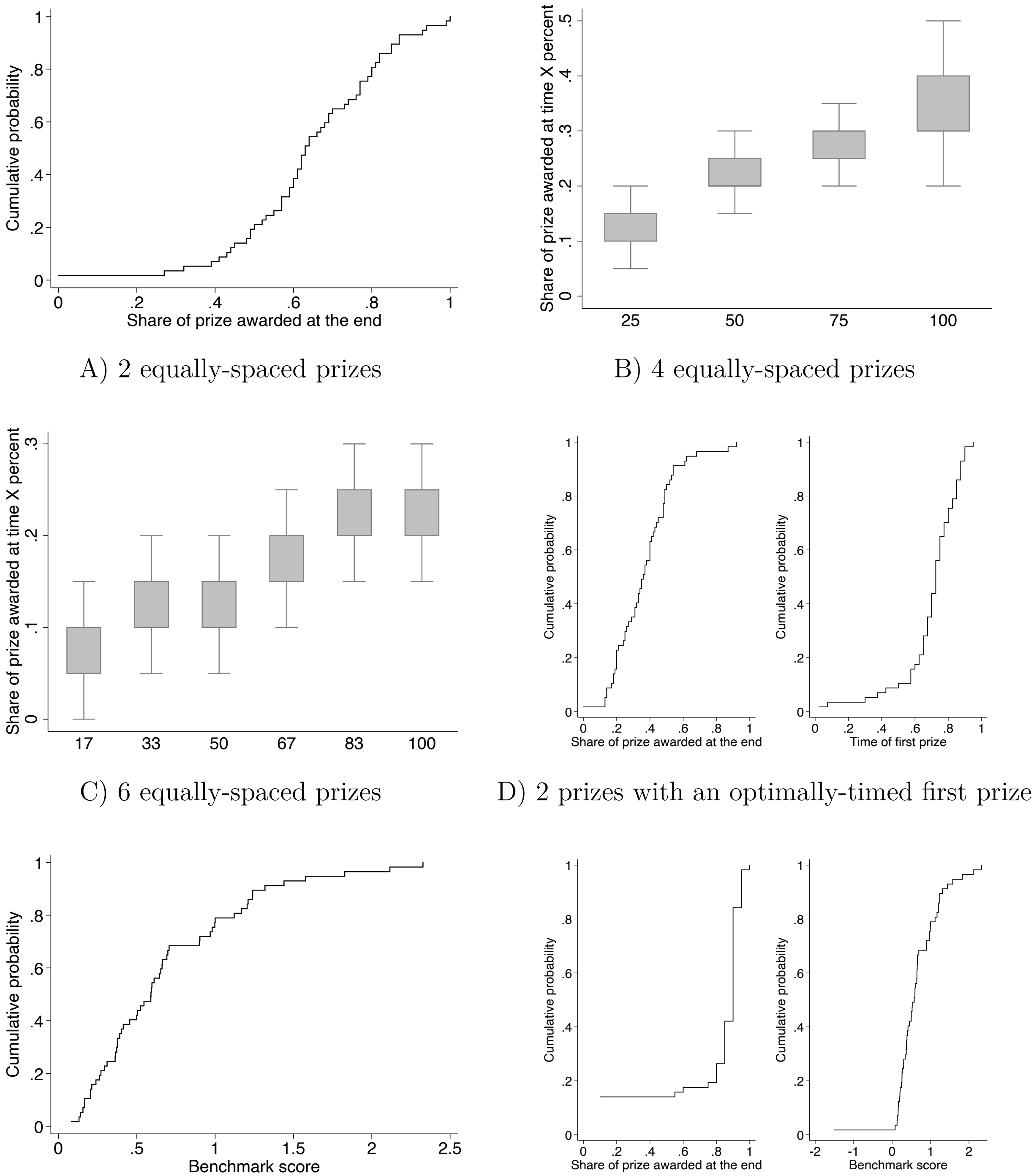
Notes: The figures present the change in the equilibrium number of submissions predicted by the model (in logs) when implementing the optimal 4-prize prize structure (relative to the baseline design).

for all contests.[16] The table reveals that when a uniform prize structure is imposed on all contests, the gains of intermediate prizes are generally not too different from when prize structures are optimized contest by contest, with the exceptions being the milestone and hybrid prize structures. For example, in the design with 6 prizes, the gains with a uniform prize structure are less than 1 percent smaller than those with the contest-by-contest optimal prize structure. The uniform prize structures perform worse in the milestone and hybrid prize structures because of the heterogeneity in the optimal milestone scores across contests we show in Figure 5.

Lastly, to measure the gains of using the within-class optimal design in US dollars, we perform the following exercise. For every competition, we compute the equilibrium of the game using the within-class optimal prize structure but with a prize pool that is scaled down so that the equilibrium outcomes match the outcomes observed in the data. We do this using the optimal hybrid prize structure for each contest and using the optimal 6-prize structure under the constraint of uniformity across contests, as these are the designs that perform best in each column of Table 5. The results suggest that the contest designer could achieve the same outcomes in the data and save an average of \$14,894 if they used the optimal hybrid structure or \$14,355 if they used the uniform 6-prize structure. That is, the contest designer could achieve the same outcomes while saving nearly half of the prize money by better managing the discouragement effects with more flexible prize structures.

[16] Table B.4 in the Online Appendix presents the parameters of the optimal uniform prize structure for each design.

Figure 5: Optimal prize structures, by prize class



A) 2 equally-spaced prizes

B) 4 equally-spaced prizes

C) 6 equally-spaced prizes

D) 2 prizes with an optimally-timed first prize

E) Milestone prize

F) Hybrid prize

Notes: The different panels present the distribution of the parameters of the optimal prize structures within each prize class. In every plot, a data point is a contest.

Combined, these results suggest that both discouragement effects (i.e., future competition and current competition) impact participation incentives. Contingent prizes can boost incentives by counteracting the discouragement effects (in particular, the future competition effect) and cause economically significant gains in contest outcomes.

7 Experimental Evidence

7.1 Description of the Experiment

To complement our model-based evidence, we recruited University of British Columbia (UBC) and University of Illinois at Urbana-Champaign (UIUC) students for a randomized control trial, which we ran on Kaggle.[17] We exploit experimental variation in prize structure to measure the impact of the prize structure on contest outcomes and competition dynamics. This exercise is meant to provide model-free evidence of the value of contingent prizes.

We recruited 405 students (both undergraduates and graduates) via emails, department newsletters, and flyers. Registration required participants to create a Kaggle account and complete a short survey, which we use as our baseline survey. The survey asked participants whether they had participated in an online data science competition prior to the study, had statistics knowledge, or had machine learning skills.

In the experiment, we created 81 groups (or contests) of 5 participants. Of these groups, 27 out of 81 featured UBC students, with the remaining 54 being composed of UIUC students. Each group was randomly assigned to one of three prize structures and all other aspects of the contests were identical (e.g., difficulty, reward budget, duration, and number of participants). We ran the competitions simultaneously. In the competitions, players had 11 days to solve a simple prediction problem: interpolate a function (see Online Appendix C for details). Players were allowed to submit up to 10 sets of predictions per day and all competitions displayed a real-time leaderboard providing information about the performance of all participants. The objective of the competition was to achieve prediction accuracy, as measured by RMSE. Unlike the rest of the paper, a better submission here is one with a *smaller* score (i.e., lower RMSE). The prize pool in every competition, regardless of the prize structure, was $100 (in Amazon gift cards).

[17] Approval from the University of Illinois Human Subjects Committee, IRB22154, and the University of British Columbia's Behavioral Research Ethics Board, H21-01835.

Table 6: Baseline summary statistics and test of balance

	Control ($N = 27$)	2 prizes ($N = 27$)		Hybrid ($N = 27$)		F-test
Variable	Mean	Coeff.	p-value	Coeff.	p-value	p-value
	(1)	(2)	(3)	(4)	(5)	(6)
participated_past	0.096	0.027	0.504	0	1	0.723
machine_learning	0.467	-0.036	0.569	0.022	0.727	0.562
stat_tools	0.733	-0.018	0.775	0.037	0.533	0.636

Notes: An observation is a contest. All variables are defined at the contest level as follows: 'participated_past' is the share of players in the contest who have participated in a prediction contest in the past, 'mach_learning' is the share of players in the contest who have machine learning skills, and 'stat_tools' is the share of players in the contest who have learned statistics. Columns 2-6 report the coefficients and p-values from OLS regressions of each covariate on two indicators: '2 prizes' and 'hybrid'. Column 7 reports the p-value from a joint test of statistical significance of both indicators.

As mentioned, each group was randomly assigned to one of three prize structures. In the first, the leader at the end of the competition received \$100 (control). In the second, the leader at 80 percent of the competition time—at the end of day 9 (out of 11 days)—received \$30, and the leader at the end of the competition received \$70 (treatment "2 prizes"). Lastly, the third one awarded \$30 to the first player to surpass a predetermined milestone score and \$70 to the leader at the end of the competition (treatment "hybrid"). We set the milestone score at 0.15, which was the median score of the winning submission in an experiment that we ran in the past (Lemus and Marshall, 2021), where different participants had to solve the same problem.

Table 6 shows the outcome of the randomization. For every covariate in the baseline survey, we ran an OLS regression with indicators for every treatment assignment, where the control group is the omitted category. Column 1 reports the average value of the covariates in the control group and columns 2 to 5 report the coefficients on the treatment indicators as well as the p-values from statistical significance tests. Column 6 reports the p-value from a joint test of statistical significance of both indicators. The table shows that about 10 percent of participants had prior experience in online data science competitions, 75 percent had knowledge of statistical tools, whereas only about half reported knowing machine learning techniques. There are no statistically significant differences in these covariates across treatment groups.

Table 7: Prize structure impacts on contest outcomes

	Min. score (1)	# of submissions (2)	Min. score (3)	# of submissions (4)
2 prizes	0.026	4.268	0.026	5.375
	(0.063)	(10.229)	(0.064)	(10.529)
Hybrid	-0.054**	6.000	-0.055**	6.492
	(0.026)	(10.811)	(0.027)	(11.072)
Controls	No	No	Yes	Yes
N	80	80	80	80
R^2	0.030	0.005	0.082	0.042
Mean dep. variable	0.174	39.375	0.174	39.375
Std. dev. dep. variable	0.191	37.549	0.191	37.549

Notes: An observation is a contest. Robust standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Controls include all covariates reported in Table 6.

7.2 Results

Table 7 reports the main results on the impacts of the prize structure on contest outcomes. We consider two contest-level outcome variables: the minimum score (i.e., the best score) and the number of submissions. For every outcome variable we run an OLS regression with indicators for each treatment assignment. The first two columns exclude controls, whereas the second two include the covariates in Table 6 as controls.[18]

Columns 1 and 3 suggest that the hybrid prize structure caused the minimum score to decrease by 0.05, which is about a third of the mean of the dependent variable.[19] These columns also suggest no statistical difference in the average minimum score between the control group and the contests assigned to a 2-prize design.[20] To shed light on heterogeneity, Figure 6 plots the minimum scores across all contests, by treatment assignment. Figure 6.A shows that the cumulative distribution functions of the control and the 2-prize groups cross each other, whereas Figure 6.B shows that the cumulative distribution function of minimum scores in the control groups first-order stochastically dominates that of the hybrid contests.

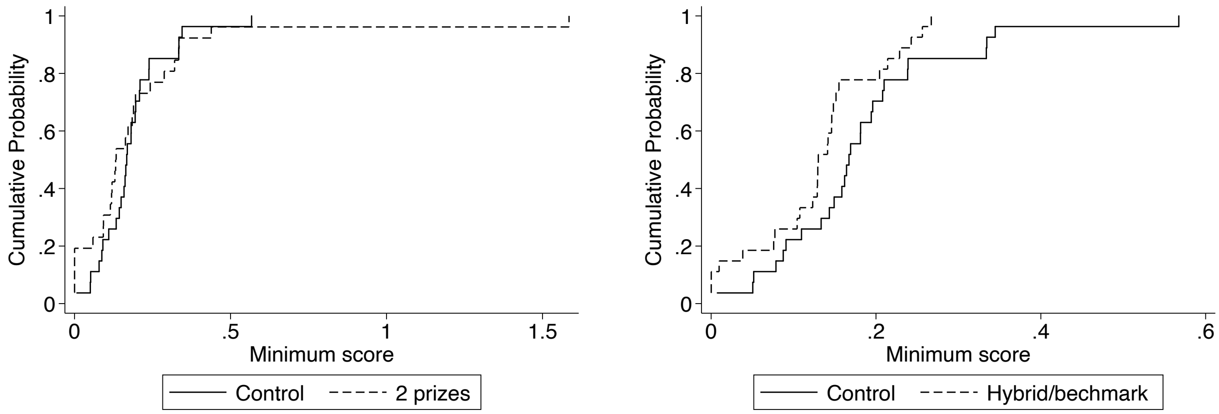
Columns 2 and 4 of Table 7 show that at the end of the competition there are no statistical differences across treatment assignments in the average number of submissions. This compar-

[18] The table has 80 observations because 1 contest received no submissions.

[19] We replicate these results using a quantile regression for the median in Table B.5 in the Online Appendix.

[20] We had one outlier in our experiment. After removing this outlier, the 2-prizes design features lower minimum scores on average.

Figure 6: Distribution of minimum scores, by prize structure



A) 2 prizes

B) Hybrid

Notes: The figures present the cumulative distribution functions of the maximum score for the different prize structure. A data point in every figure is a contest.

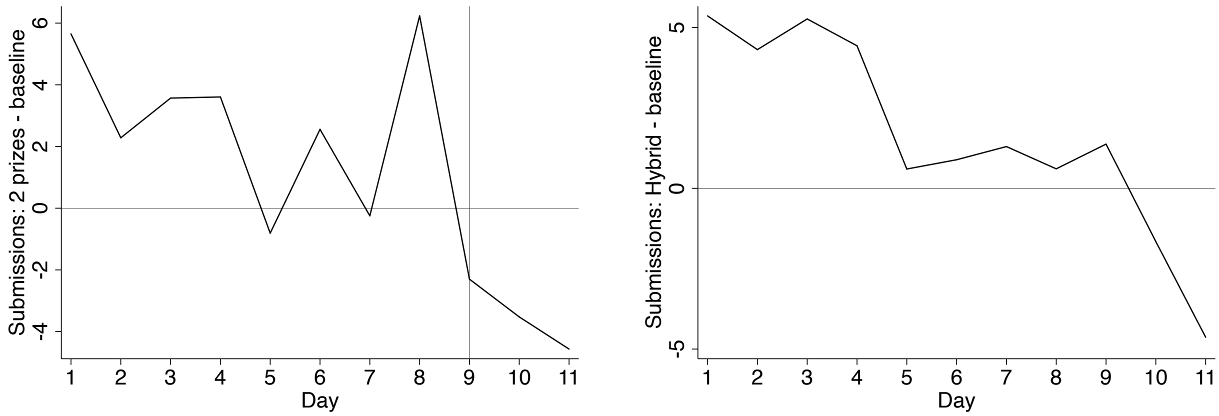
ison, however, obscures how the prize structure shapes incentives to exert effort throughout the competition. Figure 7 plots the difference in the average number of submissions between treatment X and the control group, by day. Figure 7.A shows that early in the competition, the 2-prize contests had an average number of submissions that was greater than that of the control contests, with the difference peaking at day 8 (a day before the first of the two prizes was awarded). The difference becomes negative in the last three days of the competition, which is as expected: after the first of the prizes is awarded, the control contests have a greater continuation prize (all else equal), which should lead to greater effort provision in those contests.[21] A similar pattern is observed in Figure 7.B, where the difference in submissions was greatest in the first four days, which reflects the effort of participants to surpass the milestone score in the hybrid contests.

8 Summary

We study dynamic contests with public, real-time performance feedback. These type of contests are widely used in practice, so it is valuable to understand simple ways in which a contest designer can improve outcomes, on a fixed the budget. We identify two central forces governing incentives to play at any point during the competition: the future-competition

[21] Along these lines, Table B.6 in the Online Appendix shows that conditional on the minimum score at day 9, the number of submissions was on average lower in the 2-prizes and hybrid contests.

Figure 7: Submissions over time, by prize structure



A) 2 prizes

B) Hybrid

Notes: The figures present the average difference between the daily number of submissions in treatment X and the control contests. An observation is a contest. The vertical line in Panel A marks the day in which the first prize was awarded in the 2-prize contests.

effect (i.e., plays that are yet to unfold) and current-competition effect (i.e., current maximum score).

Contingent-prizes, rather than prizes based on the final ranking only, can affect the balance of these effects and, therefore, can improve outcomes. To empirically shed light on this issue, we measure the performance of various contest designs featuring time- or score-contingent prizes using both an empirical structural model and an experiment. Our model estimates and experimental results show that a combination of score- and time-contingent prizes generate significant gains relative to the baseline design where all the prize money is awarded to the leader at the end of the competition. We characterize the optimal prize structure among a set of simple prize structures for each contest in the data. We also find parameters for each prize structure that the designer can use when she does not know the primitive parameters of a contest, and we show that this "robust" design captures a large portion of the gains from using a tailored prize structure in each contest.

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Supplemental Material – Intended for Online Publication

Contingent Prizes in Dynamic Contests

Jorge Lemus and Guillermo Marshall

A Simple Model: Analysis

Lemma 1. *At $t = t_2$, if the score is $s \in \{\bar{s}, \bar{s}+\varepsilon\}$ and the leader can play, she plays whenever*

$$q(s)(\pi(s', t_2|s) - \pi(s, t_2|s)) \geq c.$$

If the follower can play, she plays if and only if

$$q(s)\pi(s', t_2|s) \geq c.$$

The follower has (weakly) more incentives to play than the leader, because the cannibalization of the certain prize $\pi(s, t)$ discourages the leader. Note that if the prize is independent of the score, i.e., time-contingent prizes, the leader has no incentive to play in the last period.

Hence, the payoff of being the leader and the payoff of being the follower at $t = t_2$ when the score is s are, respectively,

$$V_L(s, t_2) = E_c[\max\{q(s)\pi(s', t_2|s) + (1 - q(s))\pi(s, t_2|s) - c, \pi(s, t_2|s)\}] \quad (15)$$

$$V_F(s, t_2) = E_c[\max\{q(s)\pi(s', t_2|s) - c, 0\}] \quad (16)$$

Let $Q_F^2(s) \equiv \Pr(c \leq q(s)\pi(s', t_2|s))$ be the probability that a follower who can play at $t = t_2$ actually plays, and similarly, we define $Q_L^2(s) \equiv \Pr(c \leq q(s)(\pi(s', t_2|s) - \pi(s, t_2|s)))$ for the leader.

At $t = t_1$, if the current follower (player B) can and does play, her expected payoff from successfully increasing the score is

$$R(\bar{s}') \equiv \pi(\bar{s}', t_1) + (1 - \alpha)V_L(\bar{s}', t_2) + \alpha[1 - Q_F^2(\bar{s}') + Q_F^2(\bar{s}')(1 - q(\bar{s}'))]\pi(\bar{s}', t_2|\bar{s})$$

In the expression above, by increasing the score from $\bar{s}$ to $\bar{s}'$, the follower becomes the leader at the end of period t_1, and receives $\pi(\bar{s}', t_1)$. In the next period, with probability $1 - \alpha$ this player will again have the option to play, from which the player derives a payoff of $V_L(\bar{s}', t_2)$; and with probability α, her rival can play, in which case the player who was leading the competition at the end of period t_1 will remain a leader when her rival chooses not to play (which happens with probability $1 - Q_F^2(\bar{s}')$) or chooses to play but fails to replace the leader (which happens with probability $Q_F^2(\bar{s}')(1 - q(\bar{s}'))$). In either of these cases, the player leading the competition at the end of t_1 will also be the leader at the end of t_2, in which case she receives $\pi(\bar{s}', t_2)$.

Instead, if at $t = t_1$, if the current follower (player B) can but chooses not to play, or chooses to play but fails to replace the leader, her expected payoff is

$$N(\bar{s}) \equiv (1-\alpha)V_F(\bar{s}, t_2).$$

Here, the player's only chance to receive a positive expected payoff in the next period is to be selected to play again, which occurs with probability $(1-\alpha)$, in which case she is a follower facing a current score of $\bar{s}$.

Now, consider the case that at $t = t_1$, if the current leader (player A) can and does play. Her expected payoff from successfully increasing the score is exactly $R(\bar{s}')$. However, if she chooses not to play, or chooses to play but fails to increase the score, her expected payoff is $R(\bar{s})$. Thus, we have the following result:

Lemma 2. *At $t = 1$,*

1. *If the current follower (player B) can play, she chooses to play if and only if*

$$q(\bar{s})(R(\bar{s}') - N(\bar{s})) \geq c, \tag{17}$$

2. *If the current leader (player A) can play, she chooses to play if and only if*

$$q(\bar{s})(R(\bar{s}') - R(\bar{s})) \geq c, \tag{18}$$

Thus, given an opportunity to play, player A has more incentives to play at $t = 1$ than player B if and only if $R(\bar{s}) < N(\bar{s})$. Let $Q^1_F(\bar{s}) \equiv \Pr(c \leq q(\bar{s})(R(\bar{s}') - N(\bar{s})))$ be the probability that player B plays at $t = t_1$ actually plays, and similarly, we define $Q^1_L(\bar{s}) \equiv \Pr(c \leq q(\bar{s})(R(\bar{s}') - R(\bar{s})))$ the probability that player A plays.

A.1 Optimal Contingent Prize Structures

The maximum score at the end of the competition is the random variable:

$$s^* = \begin{cases} \bar{s} & \text{with prob. } p_{\bar{s}} \\ \bar{s} + \varepsilon & \text{with prob. } p_{\bar{s}+\varepsilon} \\ \bar{s} + 2\varepsilon & \text{with prob. } p_{\bar{s}+2\varepsilon} \end{cases}$$

The expected maximum score is therefore

$$\begin{aligned} E[s^*] &= \bar{s}p_{\bar{s}} + (\bar{s}+\varepsilon)p_{\bar{s}+\varepsilon} + (\bar{s}+2\varepsilon)p_{\bar{s}+2\varepsilon} \\ &= \bar{s} + \varepsilon + \varepsilon(p_{\bar{s}+2\varepsilon} - p_{\bar{s}}) \end{aligned}$$

Thus, to maximize the expected maximum score, the contest designer selects a prize structure $\{\pi(s,t)\}_{s,t}$ to maximize the difference $p_{\bar{s}+2\varepsilon} - p_{\bar{s}}$, i.e., the designer solves

$$\max_{\{\pi(s,t)\}_{s,t}} p_{\bar{s}+2\varepsilon} - p_{\bar{s}}.$$

Let us define $f_i^t(s)$, the probability that player i conditional on having an opportunity to play when the score is s at time t, does not increase the score, i.e., $f_i^t(s) = 1 - Q_i^t(s) + Q_i^t(s)(1-q(s))$. We also define $r_i^t(s)$, the probability that player i conditional on having an opportunity to play when the score is s at time t, increases the score, i.e., $r_i^t(s) = Q_i^t(s)q(s)$.

Using these definitions, $p_{\bar{s}}$ is the probability that the score never increases, which is

$$p_{\bar{s}} = \frac{1}{2}\left\{f_F^{t_1}(\bar{s})[(1-\alpha)f_F^{t_2}(\bar{s}) + \alpha f_L^{t_2}(\bar{s})] + f_L^{t_1}(\bar{s})[(1-\alpha)f_L^{t_2}(\bar{s}) + \alpha f_F^{t_2}(\bar{s})]\right\},$$

and the probability that the score reaches $\bar{s}+2\varepsilon$ is

$$p_{\bar{s}+2\varepsilon} = \frac{1}{2}\left\{r_F^{t_1}(\bar{s})[(1-\alpha)r_L^{t_2}(\bar{s}') + \alpha r_F^{t_2}(\bar{s}')] + r_L^{t_1}(\bar{s})[(1-\alpha)r_L^{t_2}(\bar{s}') + \alpha r_F^{t_2}(\bar{s}')]\right\}.$$

Each prize-contingent scheme $\{\pi(s,t)\}_{s,t}$ determines the contest designer's expected payoff. We consider three possible contingent prize rules: (a) a *time-contingent* scheme, where the player who leads the competition at the end of stage t receives the prize π_t (i.e., $\pi(s,t) = \pi(s',t)$ for all t, $s \neq s'$; (b) a *simple milestone* score, where the first player to surpass a preset milestone score wins the whole prize and ends the contest; (c) a *hybrid* scheme, where the first player to surpass a milestone score wins a fraction π_B of the prize, and the player with the highest score at the end of the competition wins $1-\pi_B$.

B Additional Tables and Figures

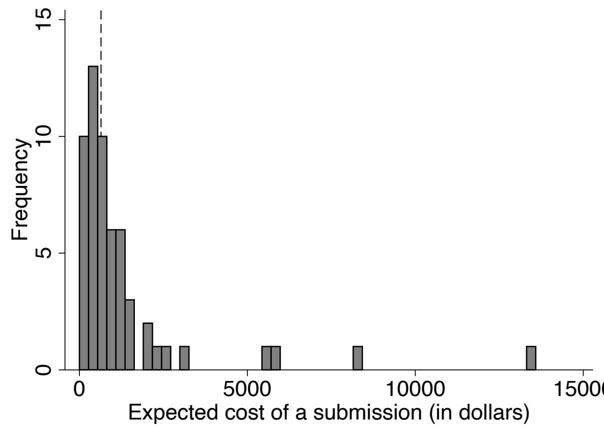


Figure B.1: Estimates for the cost of making a submission

Note: An observation is a contest. “Cost of a submission” is the expected cost of making a submission (i.e., given the distribution of costs we use in our model, the average cost is given by $\sigma/(1+\sigma)$). Since the model normalizes the value of the prize pool to 1, we have to scale this up by the size of the prize in order to translate costs to dollars. The median across contests (dashed vertical line) is \$642 (USD).

Table B.1: Summary of competitions

Competition	Total reward	Submissions	Start date	Deadline
Predict Grant Applications	5,000	371	12/13/2010	02/20/2011
RTA Freeway Travel Time Prediction	10,000	386	11/23/2010	02/13/2011
Deloitte/FIDE Chess Rating Challenge	10,000	342	02/07/2011	05/04/2011
Heritage Health Prize	500,000	2,687	04/04/2011	04/04/2013
Wikipedia's Participation Challenge	10,000	338	06/28/2011	09/20/2011
Allstate Claim Prediction Challenge	10,000	338	07/13/2011	10/12/2011
dunnhumby's Shopper Challenge	10,000	304	07/29/2011	09/30/2011
Give Me Some Credit	5,000	413	09/19/2011	12/15/2011
Don't Get Kicked!	10,000	880	09/30/2011	01/05/2012
Algorithmic Trading Challenge	10,000	442	11/11/2011	01/08/2012
What Do You Know?	5,000	371	11/18/2011	02/29/2012
Photo Quality Prediction	5,000	223	10/29/2011	11/20/2011
Benchmark Bond Trade Price Challenge	17,500	456	01/27/2012	04/30/2012
KDD Cup 2012, Track 1	8,000	1,267	02/20/2012	06/01/2012
KDD Cup 2012, Track 2	8,000	864	02/20/2012	06/01/2012
Predicting a Biological Response	20,000	651	03/16/2012	06/15/2012
Online Product Sales	22,500	418	05/04/2012	07/03/2012
EMI Music Data Science Hackathon - July 21st - 24 hours	10,000	109	07/21/2012	07/22/2012
Belkin Energy Disaggregation Competition	25,000	607	07/02/2013	10/30/2013
Merck Molecular Activity Challenge	40,000	415	08/16/2012	10/16/2012
U.S. Census Return Rate Challenge	25,000	272	08/31/2012	11/11/2012
Amazon.com - Employee Access Challenge	5,000	755	05/29/2013	07/31/2013
The Marinexplore and Cornell University Whale Detection Challenge	10,000	326	02/08/2013	04/08/2013
See Click Predict Fix - Hackathon	1,000	262	09/28/2013	09/29/2013
KDD Cup 2013 - Author Disambiguation Challenge (Track 2)	7,500	623	04/19/2013	06/12/2013
Influencers in Social Networks	2,350	281	04/13/2013	04/14/2013
Personalize Expedia Hotel Searches - ICDM 2013	25,000	517	09/03/2013	11/04/2013
StumbleUpon Evergreen Classification Challenge	5,000	328	08/16/2013	10/31/2013
Personalized Web Search Challenge	9,000	275	10/11/2013	01/10/2014
See Click Predict Fix	4,000	575	09/29/2013	11/27/2013
Allstate Purchase Prediction Challenge	50,000	1,204	02/18/2014	05/19/2014
Higgs Boson Machine Learning Challenge	13,000	1,776	05/12/2014	09/15/2014
Acquire Valued Shoppers Challenge	30,000	2,347	04/10/2014	07/14/2014
The Hunt for Prohibited Content	25,000	966	06/24/2014	08/31/2014
Liberty Mutual Group - Fire Peril Loss Cost	25,000	1,057	07/08/2014	09/02/2014
Tradeshift Text Classification	5,000	714	10/02/2014	11/10/2014
Driver Telematics Analysis	30,000	1,619	12/15/2014	03/16/2015
Diabetic Retinopathy Detection	100,000	698	02/17/2015	07/27/2015
Click-Through Rate Prediction	15,000	1,679	11/18/2014	02/09/2015
Otto Group Product Classification Challenge	10,000	926	03/17/2015	05/18/2015
Crowdflower Search Results Relevance	20,000	1,645	05/11/2015	07/06/2015
Avito Context Ad Clicks	20,000	558	06/02/2015	07/28/2015
ICDM 2015: Drawbridge Cross-Device Connections	10,000	364	06/01/2015	08/24/2015
Caterpillar Tube Pricing	30,000	1,938	06/29/2015	08/31/2015
Liberty Mutual Group: Property Inspection Prediction	25,000	1,271	07/06/2015	08/28/2015
Coupon Purchase Prediction	50,000	631	07/16/2015	09/30/2015
Springleaf Marketing Response	100,000	1,567	08/14/2015	10/19/2015
Truly Native?	10,000	474	08/06/2015	10/14/2015
Rossmann Store Sales	35,000	1,684	09/30/2015	12/14/2015
Homesite Quote Conversion	20,000	2,557	11/09/2015	02/08/2016
Prudential Life Insurance Assessment	30,000	818	11/23/2015	02/15/2016
BNP Paribas Cardif Claims Management	30,000	1,648	02/03/2016	04/18/2016
Home Depot Product Search Relevance	40,000	2,884	01/18/2016	04/25/2016
Santander Customer Satisfaction	60,000	1,138	03/02/2016	05/02/2016
Expedia Hotel Recommendations	25,000	436	04/15/2016	06/10/2016
Avito Duplicate Ads Detection	20,000	1,564	05/06/2016	07/11/2016
Draper Satellite Image Chronology	75,000	475	04/29/2016	06/27/2016

Note: The table only considers submissions by the top 10 teams of each competition. The total reward is measured in US dollars at the moment of the competition.

Table B.2: GMM estimates of model parameters

Competition	σ	SE	λ	β_0 (q)	SE	β_1 (q)	SE	GMM Obj. Fun	N
Predict Grant Applications	0.0799	0.0005	0.1391	-2.6669	0.0909	-1.3618	0.1938	0.0085	371
RTA Freeway Travel Time Prediction	0.1001	0.0004	0.1658	-0.9835	0.1434	-1.3618	0.1938	0.0013	386
Deloitte/FIDE Chess Rating Challenge	0.054	0.0001	0.1733	-1.5192	0.0541	-1.3618	0.1938	0.0036	342
Heritage Health Prize	0.0279	0.0001	0.7306	-3.5412	0.0461	-1.3618	0.1938	0.0138	2687
Wikipedia Participation Challenge	0.1496	0.0009	0.1686	-3.7402	0.0637	-1.3618	0.1938	0.0149	338
Allstate Claim Prediction Challenge	0.0956	0.0004	0.1837	-0.5781	0.2179	-1.3618	0.1938	0.0012	338
dunnhumbys Shopper Challenge	0.0919	0.0004	0.1263	-1.6724	0.1829	-1.3618	0.1938	0.009	304
Give Me Some Credit	0.0621	0.0003	0.1749	-2.6001	0.0454	-1.3618	0.1938	0.0048	413
Dont Get Kicked!	0.052	0.0001	0.2917	-2.2272	0.107	-1.3618	0.1938	0.0022	880
Algorithmic Trading Challenge	0.0452	0.0001	0.1165	-3.0026	0.0133	-1.3618	0.1938	0.0009	442
What Do You Know?	0.0707	0.0001	0.2062	-2.256	0.0923	-1.3618	0.1938	0.0123	371
Photo Quality Prediction	0.0168	0.0003	0.0444	-1.7916	0.0974	-1.3618	0.1938	0.0008	223
Benchmark Bond Trade Price Challenge	0.0653	0.0002	0.1899	-2.8435	0.0187	-1.3618	0.1938	0.003	456
KDD Cup 2012, Track 1	0.0873	0.0001	1	-1.6961	0.1663	-1.3618	0.1938	0.0177	1267
KDD Cup 2012, Track 2	0.1245	0.0003	1	-2.134	0.1743	-1.3618	0.1938	0.0005	864
Predicting a Biological Response	0.0382	0.0002	0.1824	-3.432	0.0259	-1.3618	0.1938	0.005	651
Online Product Sales	0.0552	0.0001	0.1202	-2.9822	0.0333	-1.3618	0.1938	0.0062	418
EMI Music Data Science Hackathon - July 21st - 24 hours	0	0.0001	0.023	-1.3017	0.1215	-1.3618	0.1938	0.0913	109
Belkin Energy Disaggregation Competition	0.0605	0.0002	0.2418	-1.7264	0.0767	-1.3618	0.1938	0.0021	607
Merck Molecular Activity Challenge	0.0337	0.0384	0.1222	-1.8376	0.1146	-1.3618	0.1938	0.0011	415
U.S. Census Return Rate Challenge	0.0599	0.0001	0.1435	-2.1342	0.0548	-1.3618	0.1938	0.0497	272
Amazon.com - Employee Access Challenge	0.0158	0.0001	0.1263	-2.5885	0.0863	-1.3618	0.1938	0.0008	755
The Marinexplore and Cornell University Whale Detection Challenge	0.1225	0.0004	0.236	-2.0254	0.0968	-1.3618	0.1938	0.0022	326
See Click Predict Fix - Hackathon	0	0.0001	0.0183	-2.3371	0.1248	-1.3618	0.1938	0.1398	262
KDD Cup 2013 - Author Disambiguation Challenge (Track 2)	0.0095	0.0001	0.1081	-1.7064	0.0846	-1.3618	0.1938	0.0005	623
Influencers in Social Networks	0	0.0001	0.04	-2.1596	0.0896	-1.3618	0.1938	0.0046	281
Personalize Expedia Hotel Searches - ICDM 2013	0.0213	0.0001	0.1246	-1.2842	0.1194	-1.3618	0.1938	0.0043	517
StumbleUpon Evergreen Classification Challenge	0.0857	0.0001	0.1523	-2.5777	0.096	-1.3618	0.1938	0.0496	328
Personalized Web Search Challenge	0.2166	0.0007	0.9134	-2.1926	0.0646	-1.3618	0.1938	0.002	275
See Click Predict Fix	0.0046	0.0001	0.1199	-1.5334	0.0946	-1.3618	0.1938	0.0002	575
Allstate Purchase Prediction Challenge	0.0555	0.0001	0.4519	-3.1441	0.0473	-1.3618	0.1938	0.0006	1204
Higgs Boson Machine Learning Challenge	0.0648	0.0001	0.6307	-3.3141	0.0994	-1.3618	0.1938	0.0042	1776
Acquire Valued Shoppers Challenge	0.0148	0.0003	0.4759	-2.3999	0.1707	-1.3618	0.1938	0.0045	2347
The Hunt for Prohibited Content	0.032	0.0001	0.2731	-2.7914	0.0649	-1.3618	0.1938	0.0043	966
Liberty Mutual Group - Fire Peril Loss Cost	0.0367	0.0001	0.2825	-2.075	0.1316	-1.3618	0.1938	0.0011	1057
Tradeshift Text Classification	0.0372	0.0005	0.197	-1.9588	0.0297	-1.3618	0.1938	0.0018	714
Driver Telematics Analysis	0.0735	0.0001	0.4571	-4.2049	0.1214	-1.3618	0.1938	0.0018	1619
Diabetic Retinopathy Detection	0.0889	0.0001	0.8012	-1.6264	0.1974	-1.3618	0.1938	0.0114	698
Click-Through Rate Prediction	0.0395	0.0001	0.4162	-3.2616	0.0314	-1.3618	0.1938	0.0012	1679
Otto Group Product Classification Challenge	0.0232	0.0001	0.187	-2.5233	0.0229	-1.3618	0.1938	0.0006	926
Crowdflower Search Results Relevance	0.0129	0.0001	0.2806	-2.8256	0.105	-1.3618	0.1938	0.0012	1645
Avito Context Ad Clicks	0.0612	0.0001	0.2814	-2.2102	0.0146	-1.3618	0.1938	0.0004	558
ICDM 2015: Drawbridge Cross-Device Connections	0.0684	0.0001	0.1687	-0.9735	0.192	-1.3618	0.1938	0.0051	364
Caterpillar Tube Pricing	0.0172	0.0001	0.3166	-3.0875	0.0277	-1.3618	0.1938	0.0004	1938
Liberty Mutual Group: Property Inspection Prediction	0.0233	0.0001	0.2667	-3.1092	0.06	-1.3618	0.1938	0.0009	1271
Coupon Purchase Prediction	0.1222	0.0003	0.3848	-2.0006	0.2093	-1.3618	0.1938	0.0027	631
Springleaf Marketing Response	0.0326	0.0001	0.332	-3.0862	0.0708	-1.3618	0.1938	0.0009	1567
Truly Native?	0.0526	0.0002	0.2073	-2.5242	0.0917	-1.3618	0.1938	0.0067	474
Rossmann Store Sales	0.0185	0.0001	0.3775	-3.3451	0.0452	-1.3618	0.1938	0.0015	1684
Homesite Quote Conversion	0.0223	0.0001	0.4559	-3.1196	0.0355	-1.3618	0.1938	0.0009	2557
Prudential Life Insurance Assessment	0.0749	0.0006	0.4219	-3.0026	0.0449	-1.3618	0.1938	0.002	818
BNP Paribas Cardif Claims Management	0.026	0.0001	0.3757	-3.2909	0.0187	-1.3618	0.1938	0.0013	1648
Home Depot Product Search Relevance	0.0147	0.0001	0.4918	-2.9788	0.0701	-1.3618	0.1938	0.002	2884
Santander Customer Satisfaction	0.0218	0.0001	0.3059	-2.2694	0.0479	-1.3618	0.1938	0.0021	1138
Expedia Hotel Recommendations	0.1011	0.0001	0.2814	-1.7786	0.0485	-1.3618	0.1938	0.0005	436
Avito Duplicate Ads Detection	0.0242	0.0001	0.3346	-2.4246	0.0583	-1.3618	0.1938	0.0006	1564
Draper Satellite Image Chronology	0.0857	0.0002	0.1188	-3.6358	0.0019	-1.3618	0.1938	0.0025	475

Notes: The table reports the GMM estimates with bootstrapped standard errors.

Table B.3: Probability of the hybrid prize structure being the optimal structure: Probit estimates

	hybrid
	(1)
σ (submission cost parameter)	-17.906**
	(7.713)
λ (frequency of submission opportunities)	6.135**
	(2.528)
β_0 (q_s function)	0.806**
	(0.407)
Observations	57
R^2	

Notes: Robust standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. This probit model is for the probability that the hybrid prize structure is the optimal prize structure for a contest (with the alternative being that the optimal prize structure is the one with 6 prizes). The covariates are parameter estimates. An observation is a contest.

Table B.4: Uniform prize structure

2 prizes	30% of prize at $t = 0.5$, 70% of prize at $t = 1$
4 prizes	10%, 20%, 30%, and 40% of prize at times
	$t = .25$, $t = .5$, $t = .75$, and $t = 1$, respectively
6 prizes	5%, 10%, 15%, 20%, 25%, and 25% of prize at times
	$t = 1/6$, $t = 2/6$, $t = 3/6$, $t = 4/6$, $t = 5/6$, and $t = 1$, respectively
2 timed prizes	25% of prize at $t = 0.68$ and 75% of prize at $t = 1$
Benckmark	100% of the prize to the first player
	who surpasses the milestone score 1.175
Hybrid	70% of prize at $t = 1$, 30% to the first player
	who surpasses the milestone score 0.375

Notes: The table summarizes the optimal prize (within each class) subject to the constraint that all contests have the same prize structure.

Table B.5: Prize structure impacts on contest outcomes

	Min. score			
	OLS	Quant. Reg.	OLS	Quant. Reg.
	(1)	(2)	(3)	(4)
2 prizes	0.026	-0.034	0.026	-0.043
	(0.063)	(0.027)	(0.064)	(0.034)
Hybrid	-0.054**	-0.037*	-0.055**	-0.040*
	(0.026)	(0.019)	(0.027)	(0.022)
Controls	No	No	Yes	Yes
N	80	80	80	80
R^2	0.030	0.005	0.082	0.042
Mean dep. variable	0.174	0.174	0.174	0.174
Std. dev. dep. variable	0.191	37.549	0.191	37.549

Notes: An observation is a contest. Robust standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. "Quant. Reg." is a quantile regression for the median. Controls include all covariates reported in Table 6.

Table B.6: Number of submissions in the last two days of the contests (i.e., days 10 and 11)

	# of submissions	
	(1)	(2)
Min. score at day 9	-9.835*	-11.819*
	(5.601)	(6.274)
2 prizes	-8.381**	-7.578*
	(4.063)	(4.095)
Hybrid/bechmark	-9.002**	-9.586**
	(4.124)	(4.217)
Controls	No	Yes
N	78	78
R^2	0.095	0.134

Notes: An observation is a contest. Robust standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Controls include all covariates reported in Table 6.

C Description of the Experiment

In this section, we reproduce the instructions given to all contest participants, regardless of the group they were randomly assigned to.

Description of the Competition

A large restaurant chain owns restaurants located along major highways. The average revenue of a restaurant located at distance x from the highway is $R(x)$. For simplicity, the variable distance to the highway is normalized to be in the interval [1,2]. The function $R(x)$ is unknown. The goal of this competition is to predict the value of $R(x)$ for several values of distances to the highway. Currently, the restaurant chain is located at 40 different locations. You will have access to

$$\{(x_i, R(x_i))\}_{i=1}^{30},$$

i.e., the distance to the highway and average revenue for 30 of these restaurants. Using these data, you must submit a prediction of average revenue for the remaining 10 restaurants, using their distances to the highway.

You will find the necessary datasets in the Data tab. You can send up to 10 different submission each day until the end of the competition. The deadline of the competition is Sunday September 26th at 23:59:59.

Evaluation

We will compare the actual revenue and the revenue predictions for each value

$$(x_j)_{j=31}^{40}.$$

The score will be calculated according to the Root Mean Square Deviation:

$$\text{RMSD} = \sqrt{\frac{\sum_{j=31}^{40}(\hat{R}(x_j) - R(x_j))^2}{10}},$$

which is a measure of the distance between your predictions and the actual values $R(x)$. The deadline of the competition is Sunday September 27th at 23:59:59.

Note. Following the convention used throughout the paper, we multiplied the $RSMD$ scores by minus one to make the winning score maximize private score in the competition.

Description of the Data

The goal of this competition is to predict the value of $R(x)$ for a number of values of distance to the highway. The csv file "train" contains data on the distance to the highway and average revenue for 30 restaurants

$$\{(x_i, R(x_i))\}_{i=1}^{30},$$

You can use these data to create predictions of average revenue for the remaining 10 restaurants. For these 10 restaurants you only observe their distances to the highway in the csv file "test." You can find an example of how your submission must look like in the csv file "sample_submission."

File descriptions:

- **train.csv** - the training set
- **test.csv** - the test set
- **sample_submission.csv**- an example of a submission file in the correct format

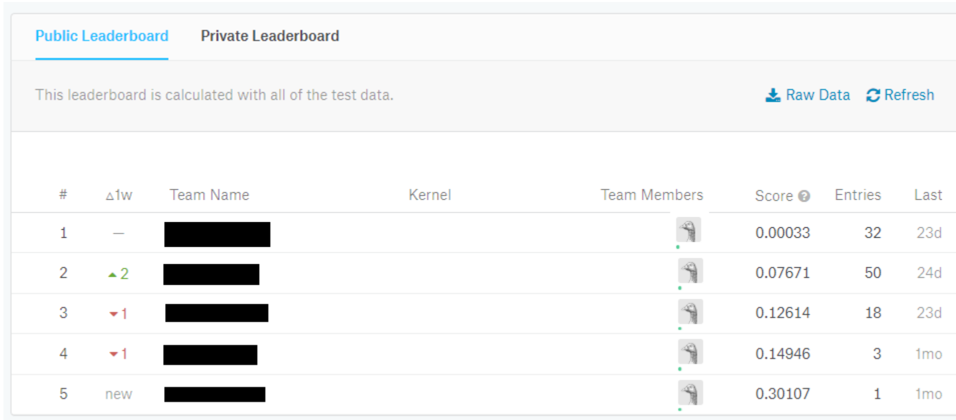
Submission File:

The submission file must be in csv format. For every distance to the highway of the 10 restaurants, your submission files should contain two columns: distance to the highway (x) and average revenue prediction (R). The file should contain a header and have the following format:

x	R
1.047579	34.43375
1.926801	36.83077
etc.	

A correct submission must be a csv file with one row of headers and 10 rows of numerical data, as displayed above. To ensure that you are uploading your predictions in the correct format, we recommend that you upload your predictions by editing the sample submission file. There is a limit of 10 submissions per day.

Figure C.1 shows a screenshot of the leaderboard in one of our student competitions hosted on Kaggle.



Public Leaderboard   Private Leaderboard

This leaderboard is calculated with all of the test data.   Raw Data   Refresh

#	△1w	Team Name	Kernel	Team Members	Score	Entries	Last
1	—	[redacted]			0.00033	32	23d
2	▲2	[redacted]			0.07671	50	24d
3	▼1	[redacted]			0.12614	18	23d
4	▼1	[redacted]			0.14946	3	1mo
5	new	[redacted]			0.30107	1	1mo

Figure C.1: Snapshot of the leaderboard in one of our competitions with a leaderboard. Names are hidden for privacy reasons.