

Teamwork in Contests*

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Abstract

Why do self-organized teams form in a dynamic contest? We use data from a popular online-contests platform and find that the value of teamwork is uncertain because only some teams result in productive partnerships. We build a structural model to study players' dynamic incentives to form teams throughout a contest. Players weigh the cost of forming a team versus the uncertain value of teamwork, which evolves during the competition. Our estimates show that team formation is costly and that teamwork improves contest outcomes. We find that greater competitive pressure increases team formation. We discuss implications for contest design.

Keywords: Contests, teamwork, collaboration, contest design

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1 Introduction

Over the last decade, firms and government agencies have sponsored thousands of online contests on various platforms. Many skilled players participate in these contests, and depending on the contest’s rules, they can participate as a solo player or as part of a self-organized team. Even if teamwork may enhance the productivity of team members in a contest, some players might choose not to form a team, fearing an unproductive partnership. Our contribution is to empirically investigate the impacts of self-organized teams on contests, shedding light on dynamic incentives to form teams, and drawing implications for contest design. Specifically, why do self-organized teams form in a dynamic contest? Does teamwork enhance productivity? Should a contest designer allow teamwork? We combine policy-evaluation techniques with a novel structural model of team formation in dynamic contests to empirically answer these questions.

We make four contributions. First, we show that forming a team can create performance gains although not all teams result in successful partnerships. The gains of team formation are ex-ante uncertain but have a positive expected value. Second, we propose a structural model to examine players’ dynamic incentives to form teams. Players compare the uncertain payoff of teamwork (which dynamically depends on the status of the competition) with the cost of forming a team. Third, we use our structural estimates to derive contest-design implications such as evaluating the value of facilitating teamwork (e.g., reducing the cost of team formation). Lastly, we investigate factors that motivate or hinder team formation, such as a contest’s competitive pressure, the similarity of players, and the precision of information available to players.

Our empirical setting is Kaggle (www.kaggle.com), the largest platform for hosting online data-science competitions, where players create algorithms to predict the outcome of a random variable conditional on a set of covariates.¹ Our sample includes 149 featured Kaggle competitions, which offer monetary prizes, last several months, and attract thousands of participants who can make multiple submissions over time.² Kaggle is an ideal setting to causally estimate the effect of teamwork on players’ performance. First, we have detailed information about the timing and performance of every submission in a competition, the identity of the player making each submission, the timing of team formation, and the composition of each team. These data allow us to reconstruct the public leaderboard—displaying the performance

¹For instance, the ride-sharing company Lyft is hosting a competition where participants need to predict the movement of traffic agents around an autonomous vehicle.

²Featured competitions are “full-scale machine learning challenges which pose difficult, generally commercially-purposed prediction problems.”

of all players in real time—and the organization of players into teams at every moment of time in each competition. Second, players *must* have made at least one submission prior to forming a team, and teams cannot be disbanded. This allows us to observe the performance of each player before and after they form their team.

Our first contribution is to investigate whether teamwork causes performance gains for team members in a contest. In other settings, evidence suggests that teamwork improves performance (see, e.g., [Hamilton et al., 2003](#); [Jones, 2009](#)). We contribute to this literature by studying the performance of self-organized teams in dynamic contests. We identify the impact of teamwork on team performance by exploiting the timing of team formation. Using a differences-in-differences design, we compare the performance of players who form a team with those who work solo (and never form a team) both before and after the team forms. The identification argument is that the performance of the control solo players and the team members would have followed the same trend had the team not formed. In the estimation, we resort to several methods to deal with endogenous team formation, i.e., players self-select into teams because they expect the benefits of teamwork to exceed the costs of forming a team. First, we use matching on observables (e.g., performance of team members up until the team forms) to accommodate the case in which team formation is a function of observables (e.g., the gains of forming a team are explained by the performance of players), but as good as random among a set of individuals with the same observables. Second, we implement a Heckman-style selection model ([Heckman, 1979](#)) using an instrumental variable to accommodate the case in which team formation responds to unobservables as well as observables.

We find a positive relationship between teamwork and performance using all of these methods. Our baseline estimates imply that teamwork increases a player’s scores by an average of 0.04 to 0.1 standard deviations, which is roughly equivalent to the median score difference between the winner of a competition and the player ranked in the 40th position. When estimating dynamic effects, we find that, team members, prior to forming a team, perform no differently than the comparison group (solo players). However, their performance significantly increases shortly after the team formation, and these performance gains persist over time.

We use a similar research design to study the impact of teamwork on the number of submissions, and we find two results. First, in our data, 8.4 percent of teams “fail,” meaning that they make no submissions after their team forms. Hence, the benefits of teamwork are ex-ante uncertain, since no player would want to form a team that fails. Second, among the teams that remain active (i.e., do not fail), teamwork does not change the team members’ average number of submissions. Thus, for teams that do not fail, the aggregate number of

submissions by all team members is, on average, equal before and after the team forms. This finding indicates that teamwork increases the quality and not the quantity of submissions for teams that remain active.

Armed with the finding that the impact of teamwork on team performance is uncertain, our second contribution is to explore the players' incentives to form a team during the competition. To this end, we build a structural model of team formation in dynamic contests, where players get random opportunities to form teams. Consistent with our findings, we assume that players working in teams are more likely to achieve high scores. This is the driving force that pushes players to form teams. However, there are three factors that discourage team formation: (1) players need to split the prize if they win; (2) a team can fail (i.e., the team can be unproductive); and (3) forming a team is costly. We estimate the primitives of the model, including the distribution of the team-formation cost, and find that the average cost of forming a team across all contests in our data is 52 percent of the prize. While these costs are heterogeneous across players, most players find it too costly to form a team, even knowing that their performance will improve conditional on not failing.

Using the estimates of our structural model, our third contribution is to shed light on whether a contest designer should facilitate teamwork by making team formation less costly. In practice, a contest designer could facilitate team formation by allowing players to communicate, providing easy access other player's profiles (e.g. history of achievements), or incorporating online-collaboration tools. Any of these initiatives would likely reduce the cost of forming teams which, theoretically, has an ambiguous effect on contest outcomes. On the one hand, more teams will form if it is cheaper to do so, generating high-scoring submissions as teamwork improves performance relative to working solo. On the other hand, a fraction of teams will fail, making some competitors unable to make submissions. To empirically compare these countervailing forces, we simulate contests where we reduce the cost of team formation. In the equilibria of these simulated contests, we find that the lower the cost of forming a team, the more teams, the fewer submissions, and the higher the maximum score. In other words, the benefit of facilitating teamwork outweighs the cost. As a corollary, forbidding team formation is detrimental for a contest designer seeking to procure a submission with a score as high as possible.

We also use our estimates of the structural model to investigate the impact of competitive pressure on team formation. We represent an increase of the competitive pressure that players face in a contest by simulating longer contests. We find that longer contests increase the number of teams by increasing the rate of team formation, i.e., the number of teams per

unit of time increases. Thus, anticipating long-lasting competition, players are motivated to form teams. The mechanic effect of more opportunities to form a team in a longer contest contribute to the result of more teams, but this is not fully driving the result as we find that the rate of team formation per unit of time increases in longer contests. These results suggest a greater impact of policies that facilitate teamwork in more competitive environments or in longer contests.

Although teams perform better than solo players, they represent less than 8 percent of all players in the contest (e.g., 92 percent are solo players). The small number of teams can be attributed to a number of factors, such as matching frictions (Boudreau et al., 2017), moral hazard concerns (Bonatti and Hörner, 2011; Georgiadis, 2015), asymmetric information (Lin et al., 2013), or credit allocation (Bikard et al., 2015). In our setting, matching frictions could hinder collaboration because players struggle to find a partner who speaks or writes code in the same language, or has a compatible skill set. Asymmetric information about the type of a potential partner—a player’s ability, commitment to work, or preference over approaches for solving a problem—may prevent partnerships from forming. Any of these problems can trigger the demise of a team (recall that in our data 8.4 percent of teams fail).

We present complementary evidence on factors that might hinder teamwork in contests. First, we find evidence of assortative matching: teams are more likely to form among similarly-ranked players. Forming a team with a “similar” player may alleviate asymmetric-information concerns (ability) and also balance the “power dynamics” inside the team. We observe similar assortative-matching patterns along the dimensions of performance in past competitions and contributions to the community (e.g., code sharing and message posting on public forums). Second, we exploit variation in the precision of the public leaderboard across competitions to assess the role of incomplete information. We find that collaboration occurs earlier in competitions providing more precise performance feedback. We interpret this finding as indicative of rational use of the information content of signals: fewer signals are needed to overcome information asymmetries when signals are more precise.

In summary, our results suggest that a contest designer should facilitate the formation of self-organized teams.³ First, a public leaderboard is vital since it allows players to learn about the performance of prospective partners in the current competition. Second, the leaderboard should be as informative as possible.⁴ Third, information about past performance should be as informative as possible.⁵ Fourth, the contest sponsor should provide opportunities

³Blasco et al. (2013) shows that self-organized teams perform better than randomly-formed teams.

⁴The contest designer needs to consider overfitting concerns with a perfectly informative leaderboard.

⁵Kaggle allocates “medals” based on performance. However, some have questioned the real value of a

to signal skills beyond performance in the current competition. On Kaggle, for example, competitors can develop and share code to analyze a dataset even if they do not participate in a competition. Fifth, the contest sponsor should facilitate the enforcement of prize splits among team members.⁶ While most Kaggle competitions allow the formation of self-organized teams, other platforms that hosts contests do not permit teamwork. Our results suggest that some contest sponsors may have overlooked the potential benefits of facilitating teamwork. Our policy recommendations are low-cost interventions that can greatly enhance the value of contests.

Related Literature. We show that self-assembled teams improve performance in contests, which contributes to the broader literature on teamwork and performance (see, e.g., [Hamilton et al., 2003](#); [Jones, 2009](#); [Ahmadpoor and Jones, 2019](#)).

Teamwork can cause productivity gains from team members’ comparative advantages ([Büyükboyacı and Robbett, 2017](#); [Büyükboyacı and Robbett, 2019](#)), knowledge diversity ([LiCalzi and Surucu, 2012](#)), or by team decisions being less likely influenced by biases, cognitive limitations, and social considerations (see, e.g., [Cooper and Kagel, 2005](#); [Sutter et al., 2013](#); [Müller and Tan, 2013](#); [Feri et al., 2010](#)). We do not observe task allocation and decision making within teams. However, we find that teams are composed by “similar” players, where similarity is measured based on past and current performance.

In our setting, players *must* work independently before forming a team (on average, players send 16 solo submissions before forming a team). [Girotra et al. \(2010\)](#) find that teams formed after players work on their ideas independently perform better than teams where members work together since the team’s inception.

Regarding team size, we find that two- and three-member teams represent 80 percent of all teams, and larger teams do not necessarily perform better. [Wu et al. \(2019\)](#) uses academic papers, patents, and software products to show that smaller teams produce more disruptive research, whereas larger teams expand on the existing knowledge.

We present a structural model in which team formation is driven by reaping the benefits of higher productivity against the cost of forming a team and the possibility that the team could fail. [Bandiera et al. \(2013\)](#) use a field experiment to study the impact of different incentives schemes on workers decision to form a team.

medal, especially if each member of a multiplayer team gets one regardless of their contribution.

⁶In some competitions, it is up to the winning team to reallocate the prize money among its members. In others, the platform allocates the prize in even shares between the team members unless the team requests an alternative prize distribution. See, e.g., some competitions hosted in the platform DrivenData.org.

We provide descriptive evidence suggesting that teamwork benefits players in future contests: players who work in teams are more likely to do better in future contests. [Ahmadpoor and Jones \(2019\)](#) find that teamwork has greater impact than solo work. [Azoulay et al. \(2010\)](#) and [Jaravel et al. \(2018\)](#) show that the premature death of high-skilled team members worsens the future performance of the remaining team members.

Some descriptive articles have also studied teamwork in Kaggle competitions. These articles do not provide causal estimates of the impact of teamwork on contest outcomes nor they model team formation. For example, [Wang et al. \(2019\)](#) discuss repeated participation in Kaggle competitions. [Dissanayake et al. \(2019\)](#) document that members with similar characteristics form most teams, although teams where members have diverse characteristics perform better. [Dissanayake et al. \(2015\)](#) also find that less diverse teams perform worse, unless most of their members are high-skilled. None of these papers structurally estimate a model of team formation.

2 Background and Data

2.1 Kaggle Competitions

Kaggle is a platform that hosts online prediction contests, where participants predict a random variable (e.g., YouTube sponsored a competition where players had to predict video tags for videos). The player with the most accurate predictions wins the competition. We focus on *featured* competitions, which are hosted by a company (e.g., YouTube, Expedia) and pay an average monetary prize of \$48,434 (USD). These competitions usually attract many players, last several months, and participants can submit multiple times before the end of the competition (though there is a limit on the number of submissions that players can make in a given day).

Participants of Kaggle competitions have access to two datasets. The first one, the *training* dataset, includes both an outcome variable and covariates, and is used by the participants to train their algorithms. The second one, the *test* dataset, only includes covariates. When making a submission, the player must submit outcome-variable predictions for each observation in the test dataset. Kaggle partitions the test dataset in two subsets and evaluates the out-of-sample performance of each submission on these two subsets.⁷ The out-of-sample

⁷Players do not know which of these subsets a given observation in the test dataset belongs to.

Table 1: Competition-level summary statistics

	Count	Mean	St. Dev.	Min	Max
	(1)	(2)	(3)	(4)	(5)
Total number of submissions	149	24,787.48	32,416.34	139.00	159,810
Total number of teams	149	1,495.04	1,772.96	29	10,450
Total number of players	149	1,587.93	1,881.41	30	11,111
Average team size	149	1.17	0.13	1.01	1.74
Reward quantity (USD)	149	48,434.21	128,676.46	0	1,200,000

Notes: An observation is a competition.

performance of each submission on the first subset, the *public* score, is instantly posted on a public leaderboard.⁸ The out-of-sample performance of each submission on the second subset, the *private* score, is made public at the end of the competition only and is used to determine the winner. Public and private scores are highly correlated (the correlation in our sample is 0.99), making public scores informative but noisy signals of performance.

Players are free to form teams subject to some restrictions. First, each member of the new team must have made at least one submission prior to the team formation. In our sample, team players submitted an average of 16 submissions prior to the team formation. Second, the cumulative number of submissions by all team members prior to the merger cannot exceed a threshold—the maximum allowed submissions per day times the number of days the competition has been running. Third, they must form their team before the team-formation deadline chosen by Kaggle for each competition. Fourth, players cannot disband teams that have made submissions.

2.2 Data and Descriptive Evidence

We use publicly available information on 149 featured competitions hosted by Kaggle.⁹ An observation in our dataset is a submission in a contest. For each submission, we observe its timestamp, an identifier for the player (and team) who made it, and its public and private scores. We also observe data on team formation: the exact date when a player joins a team, whether the team fails (i.e., stops making submissions). These data allow us to keep track of the performance of a player (or team) during the contest as well as reconstruct both the public and private leaderboard at every instant of time.

⁸The evaluation criterion for the out-of-sample performance of a submission varies across contests. Examples of evaluation criteria include the root mean squared error or R^2 .

⁹<https://www.kaggle.com/kaggle/meta-kaggle>

Table 2: Distribution of team size across competitions

Number of members	Freq. (1)	Percent (2)	Cumulative (3)
<i>Panel A: All teams</i>			
1	205,193	92.11	92.11
2	10,302	4.62	96.74
3	3,873	1.74	98.48
4	1,799	0.81	99.28
5 or more	1,594	0.72	100.00
Total	222,761	100.00	
<i>Panel B: Top 50 teams</i>			
1	5,260	71.35	71.35
2	984	13.35	84.70
3	503	6.82	91.52
4	273	3.70	95.23
5 or more	352	4.77	100.00
Total	7,372	100.00	

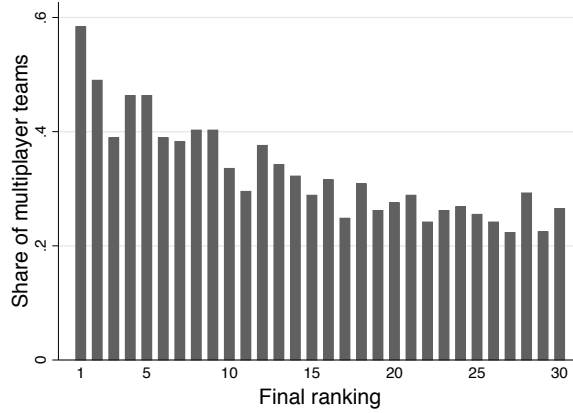
Notes: An observation is a competition–team combination. The top 50 teams are the teams who finished within the first fifty positions of the private leaderboard in each competition.

Table 1 reports competition-level summary statistics. The table shows that these competitions offer a monetary prize of \$48,434 (USD) on average, with some competitions offering as much as \$1,200,000, and attract a large number of participants who make many submissions. On average, 1,495 teams made at least one submission, and the competitions received an average of 24,787 submissions. We standardize the public and private scores of the submissions variables at the competition level (they have mean 0 and standard deviation 1) to facilitate comparison across competitions. Depending on the contest’s metric for evaluating the out-of-sample performance of submissions, players compete to achieve low scores (e.g., RMSE) or high scores (e.g., R^2). We transform scores so that higher scores can always be interpreted as better scores.

Table 2 presents the distribution of team size across competitions.¹⁰ Panel A includes the full sample of teams and shows that 92 percent of them have a single member and 4.6 percent of teams have two members. Panel B restricts attention to the teams that finish the contest within the top 50 and shows that teamwork is more frequent in the top 50: Only 71 percent of these teams have a single member, while 13 percent have two.

¹⁰Figure A.1, in the Online Appendix, shows that distribution of team-formation time is roughly uniform.

Figure 1: Share of teamwork by final ranking



Notes: An observation is a team that finished a competition in the top 30 positions of the final ranking.

One important fact about teams is that 8.4 percent of them “fail,” i.e., they stop making submissions after they form. That is, teamwork appears to be unproductive for some players to the point that they stop participating in the competition.¹¹ Furthermore, most players who form a team do so only once (84.3 percent of players). This suggests that some players may regret forming a team, even though they may have believed it was a good idea *ex ante*, or that players reap all the benefits from teamwork after participating in a team once.

The scant number of teams and non-negligible rate of team failure suggests that forming a team is costly and the prospects of forming a team are uncertain. However, the evidence shows that teams that do not fail do well relative to single players. First, [Table 2](#) shows that teams are relatively more common among the top 50 players. Second, [Figure 1](#) shows the share of teams across contests by player ranking at the end of the competition, and the figure reveals that higher ranked players are likelier to be part of a team. For instance, about 60 percent of the time a team took the first place, while only about 30 percent of the time a team took the 30th place. Thus, top players are far more likely to be part of a team than to work solo.

One of the goals of our paper is to understand if there is a causal relationship between teamwork and performance. Sections 3 and 4 are devoted to studying whether this positive relationship between teamwork and performance is, in fact, causal. In Sections 5 and 6 we explore the cost and incentives of forming teams and discuss implications for contest design.

¹¹To understand team failure, we ask whether teams composed of “similar” players are more likely to succeed. We measure similarity between players in a given contest by the disparity between the number of submissions and the maximum score among team members before the team forms. [Table A.1](#) in the Online Appendix shows that when “dissimilar” players form a team, they are more likely to fail.

3 Empirical Strategy

To measure the impact of teamwork on performance, we exploit variation on the state of a competition at the time of team formation. We compare the performance of team members before and after they join a team with the performance of solo players.

Our main estimating equation is

$$y_{i,j,c,t} = \beta \cdot 1\{\text{post team formation}\}_{i,j,c,t} + h(\mathbf{x}_{i,j,c,t}, \delta) + \mu_{j,c} + \lambda_{c,t} + \varepsilon_{i,j,c,t}, \quad (1)$$

where $y_{i,j,c,t}$ is a measure of an outcome variable i (e.g., score of submission i) by “player” j (a team or a solo player) in competition c at time t , $1\{\text{post team formation}\}_{i,j,c,t}$ is an indicator that takes the value one if player j forms a time at time t , $\mathbf{x}_{i,j,c,t}$ is a vector of time-varying player-level state variables, such as the player’s distance to the maximum score on the public leaderboard, which is a time-dependent variable defined as the difference between the player’s score at time t and the maximum score at time t . The term $h(\cdot, \delta)$ is a quadratic function of the state variables, $\mu_{j,c}$ and $\lambda_{c,t}$ are player–competition and competition–time fixed effects, respectively, and $\varepsilon_{i,j,c,t}$ is an error term clustered at the player level. We also estimate a version of Equation 1 that allows for time-varying effects,

$$y_{i,j,c,t} = \sum_{\tau=-6}^6 1\{\tau \text{ weeks before/after team formation}\}_{i,j,c,t} \beta_{\tau} + h(\mathbf{x}_{i,j,c,t}, \delta) + \mu_{j,c} + \lambda_{c,t} + \varepsilon_{i,j,c,t}, \quad (2)$$

where $\beta_{-\tau}$ and β_{τ} , for $\tau = 1, \dots, 6$ capture, respectively, the performance of a player τ weeks before and τ weeks after the team forms, for players who join a team.¹² In our analysis, *all* the submissions of all members of team j have the same team identifier, even those that are submitted before the team forms. The coefficient of interest, β , therefore, measures the impact of teamwork on the overall performance of all team members. We restrict our analysis to teams that did not fail, i.e., teams that send at least one submission after they form.

Identification. The main identification assumption is that treatment assignment is unconfounded. That is, the probability that a solo player is exposed to the treatment (i.e., forms a team) may depend on player-level state variables ($\mathbf{x}_{i,j,c,t}$) and the player’s ability to produce high scores (captured in the player-level fixed effects), but it does not depend on the potential outcomes (Imbens and Rubin, 2015). In our framework, this can also be interpreted as forming a team being exogenous conditional on player-level state variables and the player’s ability

¹²We normalize the coefficient β_{-1} to zero. β_0 captures the effect of teamwork at the week of the team formation.

to produce high scores, implying that the treatment is uncorrelated with performance-related unobservables in the error term. Under this assumption, β can be identified by comparing the observed scores of treated and non-treated teams that have similar state variables.

The unconfoundedness assumption is compatible with the idea that team formation is endogenous, i.e., a set of players form a team when they expect that the benefits will exceed the costs of forming a team. In particular, unconfoundedness accommodates the cases in which a player’s decision to form a team can be explained based on observable state variables (e.g., their position in the leaderboard) or performance-unrelated unobservables (e.g., the size of their social network). This assumption, however, does not accommodate the case in which performance-related unobservables in the error term affect the decision to form a team. For example, a violation of the unconfoundedness assumption would occur if all participants had perfect foresight about the gains of teamwork and these gains are heterogeneous across players. In this case, team formation would only occur among players expecting sufficiently large gains, and these gains would at least in part appear in the error term.

Plausibility of Unconfoundness. We assess the plausibility of the unconfoundness assumption in two ways. First, we use the estimates of Equation 2 to evaluate whether the performance of treated and non-treated teams, conditional on state variables, exhibit similar trends running up to the time of the team formation. Second, we present descriptive evidence suggesting that collaboration gains are uncertain, from the perspective of a solo player, which implies that post team formation performance-related unobservables are unlikely to be the only driver of team formation.¹³

Estimation Methods. The first approach uses the full sample of solo players and two-member teams. We exclude larger teams to insulate our estimates of the impacts of collaboration from instances of multiple treatments during the competition (i.e., teams that invite multiple players during the competition and thus experience the benefits of collaboration in multiple different occasions). If the treatment assignment is unconfounded, the estimated β coefficient will capture the causal impact of teamwork on outcomes.

We estimate the coefficients of interest in three ways. First, we estimate the equations above using the full sample, which amounts to a differences-in-differences design where we control for observable variables and fixed effects.

¹³Another concern is that we may not observe collaboration instances that are informal. That is, players who share information or code but never formally merge. We note that to the extent that collaboration increases performance, not observing these informal arrangements would lead us to underestimate the impact of collaboration on team performance.

Second, we estimate the equations above using exact matching to alleviate the concern that treated and control players differ in observables. Specifically, we match every team member with a non-treated solo players that have the same state variables at the time of the team formation (e.g., the same number of cumulative submissions and distance to the maximum score on the leaderboard). Although all of our specifications control for these state variables, the matched subsample ensures that we are comparing teams that are observationally equivalent except for being exposed to teamwork. Although players must submit at least one submission prior to forming a team, they are not required to make more submissions after merging. We observe 8,466 teams between two players for which submissions were recorded after the time of the team formation. Our matching procedure matches 7,474 of these teams with solo players with the same characteristics at the time of the team formation (i.e., the same number of cumulative submissions and the same distance to the maximum score on the leaderboard). [Table A.2](#) in the Online Appendix presents a balance analysis for the treated and control teams in the matched subsample.

Third, we use a two-step, Heckman-style selection bias correction ([Heckman, 1979](#)) similar to the one used by [Lee \(1978\)](#). In the first step, we estimate a player-level probit model for the probability of forming a team at time t given a rich set of state variables and an indicator for whether the player is eligible to form a team.¹⁴ Players are eligible to form a team if they join the competition before a preset deadline to form teams. Players must join the competition to download the data and learn about the rules of the competition (one of which is the deadline to form teams). [Lemus and Marshall \(2021\)](#) document that the distribution of entry times of players is roughly uniform throughout a contest, suggesting that players learn about a competition at different times and likely join for reasons that are unrelated to the potential benefits of team formation, making entry time (or the eligibility indicator to be precise) a plausibly exogenous shifter of the probability of forming a team. We use the probit estimates to compute the Mills ratio for every player–time combination, which captures the expected value of unobservables governing the decision to form a team (conditional on treatment). We then incorporate the Mills ratio estimates in a version of [Equation 1](#). This approach has the benefit of relaxing the requirement of the treatment indicator being uncorrelated with performance-related unobservables (i.e., it relaxes the unconfoundedness assumption).

¹⁴Specifically, for every contest, we estimate $\Pr(\text{formed team}_{i,j,c,t} = 1) = \Phi(\alpha + 1\{\text{eligible}\}_{i,j,c,t}\beta + h(\mathbf{x}_{i,j,c,t}, \delta))$, where i is a player; the notation is the same as in [Equation 1](#).

4 The Impact of Teamwork on Performance

4.1 Scores

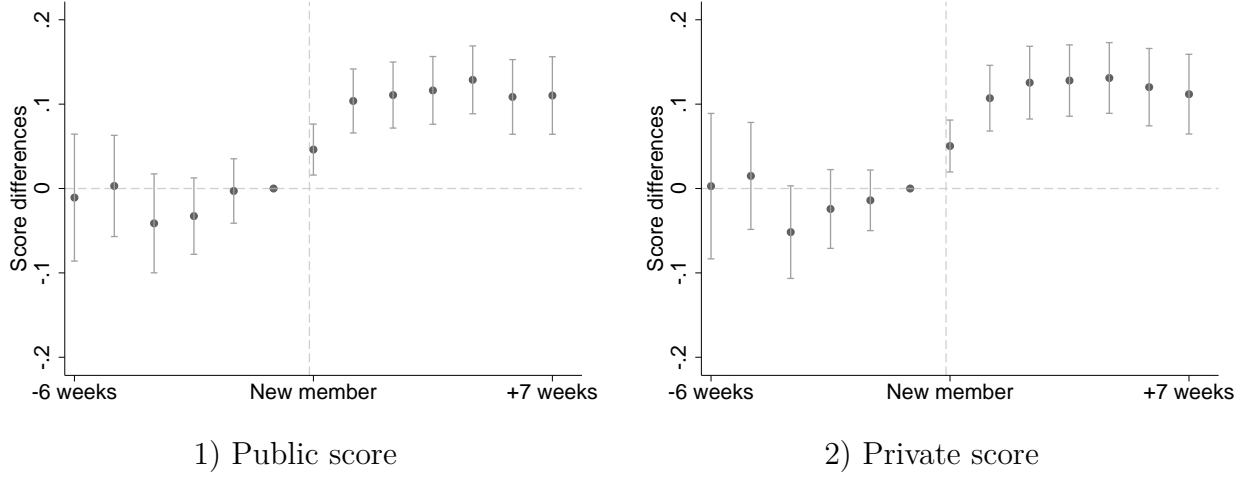
We begin our discussion on the impacts of teamwork by measuring its effect on performance, i.e., the variable y corresponds to scores. As discussed in Section 2, 8.4 percent of teams that are formed “fail” in that their members stop making submissions after team formation. We can say that the impact of teamwork on the performance of the teams that fail is negative, but we are also interested in learning about the impact of teamwork on the performance of the teams that remain active. The analysis that we present next focuses on the teams that remain active.

Figure 2 presents our estimates for Equation 2, which allows us to measure the performance effects of teamwork starting from 6 weeks prior to the actual team formation until 6 weeks after. We conduct the analysis for both the public and private scores on two samples. In Panel A, we make use of the full sample of solo players and two-member teams, which implies that solo players are the control for two-member teams. In Panel B, we further restrict the sample so that every player in a team is matched with a solo player with the same covariates at the time of the team formation (i.e., the same number of cumulative submissions and the same distance to the maximum score on the leaderboard). All specifications include player–competition fixed effects, competition–day fixed effects, and a second-degree polynomial of a number of player-level state variables. These state variables include, at any given time t , the total number of submissions by all players up until t , total number of submissions by the player making the submission up until t , total number of submissions by the team making the submission (possibly a solo player) up until t , the submitting player’s team’s distance to the maximum score on the public leaderboard at t , and the fraction of contest time that had elapsed at t . Although the decision to form a team may respond to state variables, which we are flexibly controlling for, our identification assumption is that team formation does not respond to performance-related unobservables (i.e., treatment assignment is unconfounded).

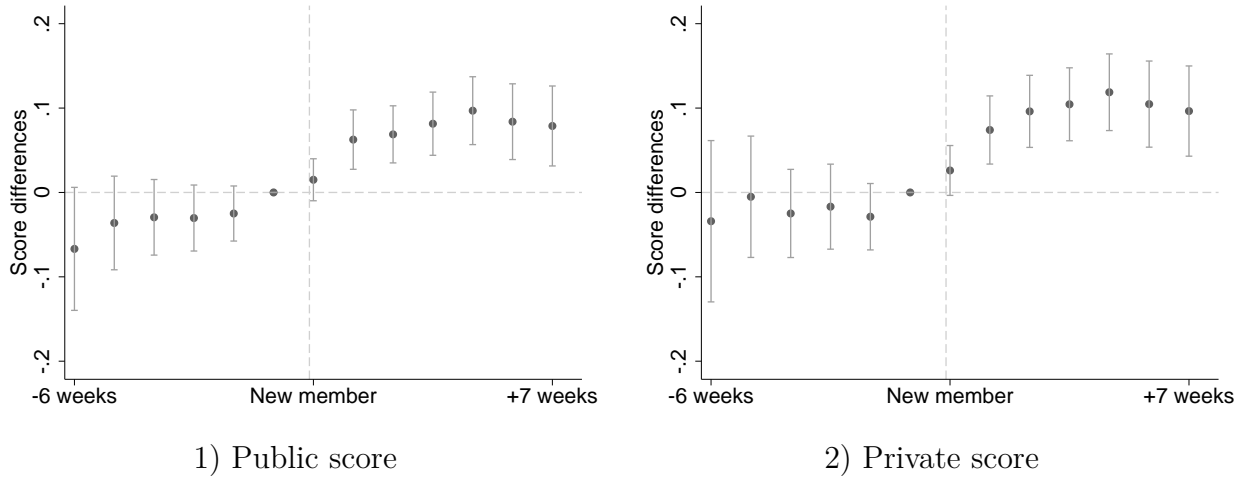
Figure 2 (Panel A) shows that, prior to the actual team formation, public and private scores for treated and non-treated players are statistically indistinguishable, which provides support for our assumption of unconfounded treatment assignment. After the actual team formation, treated players (those who join a team) perform significantly better than non-treated players, with effects that manifest immediately and last for at least 6 weeks after the team formation. In the first week after the team forms, the effect is about 0.05 or 0.06 standard deviations, and

Figure 2: The impact of collaboration on scores: Team-level estimates

Panel A: Baseline estimates



Panel B: Matching estimates



Notes: Standard errors are clustered at the team-level, and 95-percent confidence intervals are depicted in the figures. An observation is a submission made by a team in a competition. All specifications include team fixed effects, competition-day fixed effects, and a second-degree polynomial of variables: total number of submissions by all teams up until the submission time, total number of submissions by the team making the submission up until the submission time, total number of submissions by the member of the team making the submission up until the submission time, the submitting team's distance to the maximum score on the public leaderboard at the submission time, and the fraction of contest time that had elapsed at the submission time. The sample is restricted to include submissions by treated teams that took place six weeks before or after the week in which the team changed its team size, and it also restricts attention to teams with one or two members. Panel B further restricts the sample to ensure balance in observables (measured at the time of treatment).

Table 3: The impact of collaboration on scores: Team-level estimates

	Public score (1)	Private score (2)
<i>Panel A: Baseline estimates</i>		
Teamwork	0.078*** (0.014)	0.085*** (0.016)
Observations	3,248,210	3,179,632
R^2	0.439	0.448
<i>Panel B: Matching estimates</i>		
Teamwork	0.041*** (0.012)	0.050*** (0.015)
Observations	342,716	338,431
R^2	0.335	0.361

Notes: Standard errors clustered at the team-level in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. An observation is a submission made by a team in a competition. All specifications include team fixed effects, competition-day fixed effects, and a second-degree polynomial of variables: total number of submissions by all teams up until the submission time, total number of submissions by the team making the submission up until the submission time, total number of submissions by the member of the team making the submission up until the submission time, the submitting team's distance to the maximum score on the public leaderboard at the submission time, and the fraction of contest time that had elapsed at the submission time. The sample is restricted to include submissions by treated teams that took place six weeks before or after the week in which the team changed its team size, and it also restricts attention to teams with one or two members. Panel B further restricts the sample to ensure balance in observables (measured at the time of treatment).

then it climbs to about 0.11 or 0.12 standard deviations and remains at that level thereafter.¹⁵ Panel B repeats the exercise using the matched subsample. The figures look very similar to those in Panel A, with the exception that the estimated effects are smaller in magnitude than those in Panel A. The smaller magnitudes likely reflect that the control and treatment groups in Panel B are less different in the state variables that predict good performance.

Table 3 presents estimates for Equation 1, which constrains the treatment effect to be constant in the post team formation period. Panel A shows that teamwork causes an increase in public and private scores of 0.078 and 0.085 standard deviations, respectively. When restricting the sample to the matched subsample, these estimates drop to 0.041 and 0.05, respectively. How large are these magnitudes? The median score difference between the winner of the contest and the player who finishes in the 40th position is about 0.05, which suggests that teamwork has an economically significant effect.

Robustness. We also explore whether the impact of teamwork on performance is heteroge-

¹⁵Recall that both public and private scores are standardized (i.e., have mean 0 and standard deviation 1).

neous across different types of players and contests. In [Table A.3](#) in the Online Appendix, we replicate [Table 3](#) using a subsample of “competitive players” and find that our estimates do not significantly change as a result (i.e., the point estimates change by less than 10 percent of the standard error), which suggests that our results are not driven by stronger players.¹⁶ [Table A.4](#), in the Online Appendix, shows that the performance gains of teamwork are no different (in statistical terms) in more difficult contests (e.g., contests where players must analyze image data, contests with larger rewards, or contests with larger datasets).¹⁷

[Table A.5](#) in the Online Appendix replicates [Table 3](#) using indicators for whether a submission has a score that exceeds percentile x of the competition-level score distribution as the dependent variable.¹⁸ The table shows that teamwork has a positive impact on a team’s probability of achieving extreme scores, e.g., the probability of achieving a private score that exceeds the 95th and 99th percentile of the distribution increases by 6.6 and 2.3 percentage points on average as a consequence of teamwork. These findings suggest that the performance gains of teamwork are payoff-relevant by allowing players in a team to score in the upper tail of the score distribution. Moreover, they suggest that teamwork is likely to benefit the contest sponsor in the form of a thicker upper tail of scores. We will explore these questions more in depth in Section 5.

Selection. A remaining concern is whether, after controlling for observables and fixed effects, the impact of teamwork on performance is explained by unobservables driving the incentive to form teams, e.g., players know their skills are complementary so teamwork is beneficial. In other words, the concern is that the results in [Figure 2](#) and [Table 3](#) are driven by selection along the dimension of unobserved performance gains of teamwork that are heterogeneous and players can foresee. As mentioned in Section 2, a number of facts suggest that players face uncertainty about whether teamwork will be productive for them (e.g., 8.4 percent of all teams fail). Nevertheless, we perform a number of robustness checks.

[Table 4](#) presents the results of a correction along the lines of [Lee \(1978\)](#), which relaxes the unconfoundness assumption. Columns 1 and 3 replicate [Table 3](#) (Panel A), with two differences. The first one is that we replace the team-level fixed effects with team-member-level fixed effects, as the Mills ratio estimates are constructed at the individual level, and the second one is that we restrict the sample to those observations for which we can compute the Mills ratio. Columns 1 and 3 show that the gains of teamwork are smaller than those

¹⁶A player is classified as competitive if it achieved a score within the top quartile of the competition-level score distribution by the end of the competition.

¹⁷[Lemus and Marshall \(2021\)](#) present evidence showing that the reward quantity is associated with difficulty.

¹⁸The competition-level score distribution is the final distribution of scores of every competition (i.e., all submissions are used to compute this distribution).

Table 4: The impact of collaboration on scores: Player-level estimates

	Public score		Private score	
	(1)	(2)	(3)	(4)
Teamwork	0.027*** (0.008)	0.020*** (0.008)	0.032*** (0.010)	0.027*** (0.010)
Mills ratio	No	Yes	No	Yes
Observations	2,547,264	2,547,264	2,478,046	2,478,046
R^2	0.385	0.385	0.394	0.394

Notes: Standard errors clustered at the team-level in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. An observation is a submission made by a player in a competition. All specifications include player-competition fixed effects, competition-day fixed effects, and a second-degree polynomial of variables: total number of submissions by all teams up until the submission time, total number of submissions by the team making the submission up until the submission time, total number of submissions by the member of the team making the submission up until the submission time, the submitting team's distance to the maximum score on the public leaderboard at the submission time, and the fraction of contest time that had elapsed at the submission time. The Mills ratio is computed based on estimates of a Probit model on the decision to form a team, which is estimated separately for each competition, and includes a dummy for whether the user entered the competition before the team formation deadline as well as a second-degree polynomial of the variables described above. The sample is restricted to include submissions by treated teams that took place six weeks before or after the week in which the team changed its team size, submissions for which the Mills ratio can be computed, and to teams with one or two members.

in [Table 3](#), possibly because the player-level fixed effects are more flexible and can absorb any changes in the composition of submission authorship caused by teamwork. Columns 2 and 4 show the point estimates after we implement the selection correction, which cuts the impact of teamwork on public scores drop by 25 percent and on private scores by 16 percent. However, the impact of teamwork on scores remains economically relevant after we correct for selection.

In addition, [Table A.6](#), in the Online Appendix, replicates [Table 3](#) restricting the sample of treated players to those who are forming a team for the first time. If players who know the benefit of teamwork select into working in teams, this form of selection is least likely among players who have never worked in teams and have the least amount of information about the benefits of teamwork. That is, selection in performance-related unobservables is less likely among this sample, thus making unconfoundness more plausible. If anything, we find larger estimates of the impact of teamwork on performance, suggesting that this form of selection is not driving our results.

Second, [Table A.7](#) in the Online Appendix shows that the impact of teamwork on performance is unaffected by the timing of the team formation (whether the team was formed early or late in the competition). If players form teams because they know that there are large benefits

from teamwork, one would expect that teams would form as early as possible to maximize the benefits of teamwork.¹⁹ In particular, those who expect the greatest benefits of collaboration should form teams earlier. We do not see this happening. Moreover, on average, players who form teams have sent 16 submissions prior to the team formation.

These pieces of evidence combined points towards a positive causal relationship between teamwork and productivity.

4.2 Number of submissions

We next study the impact of teamwork on the number of submissions by a team. We estimate a version of Equation 1 where the dependent variable, y , corresponds to the number of submissions by each team in every week of the competition. In the analysis, an observation is a team–week–competition combination. We estimate specifications that differ in how we treat the weeks in which a team makes zero submissions. In the first specification, all team–week combinations are included, whereas in the second we only include the team-week combinations that lie between the first and last week with a positive submission count for that team (i.e., the weeks when the team was active).²⁰ The results of the first specification provide an estimate of the impact of teamwork on the number of submissions that includes failed teams (i.e., those that stop making submissions after team formation), whereas those from the second specification provide an estimate for the impact of teamwork on the number of submissions by teams that remain active.

Table 5 presents the estimates of our analysis. Panel A shows that teamwork causes the number of submissions per week by the players that formed the team to decrease by 1.5 submissions when including all player–week combinations (Column 1) or to not decrease at all when considering only the active periods of teams (Column 2). We find similar qualitative effects when looking at the matched subsample (Panel B), but the effects are not statistically different than zero. These estimates suggest that players do not change their behavior significantly after they form teams, unless their team fails.

As mentioned in Section 2, 8.4 percent of teams “fail” and make no submissions after they form. Thus, Table 5 shows that teams that remain active do not seem to be changing the

¹⁹As previously mentioned, Figure A.1 in the Online Appendix shows that team formation occurs throughout the competition and is not concentrated at the beginning.

²⁰For example, if a team makes 0, 1, 0, 1, and 0 submissions in the five weeks of a competition, respectively, we only include weeks 2, 3 and 4 in the estimation sample. This choice is based on the assumption that the team did not enter until week 2 and was already inactive in week 5.

Table 5: The impact of collaboration on the number of submissions: Team-level estimates

	Number of submissions per week (1)	Number of submissions per week (while active) (2)
<i>Panel A: Baseline estimates</i>		
Teamwork	-1.488*** (0.213)	0.162 (0.405)
Observations	1,307,553	424,819
R^2	0.657	0.688
<i>Panel B: Matching estimates</i>		
Teamwork	-0.119 (0.181)	0.279 (0.201)
Observations	75,962	43,690
R^2	0.537	0.573

Notes: Standard errors clustered at the team-level in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. An observation is a competition–team–week combination. All specifications include team fixed effects and competition–week fixed effects. The sample is restricted to include submissions that took place in the first twelve weeks of the competition and teams of up to two members. Panel B further restricts the sample to ensure balance in observables (measured at the time of treatment).

rate at which they make submissions (Column 2). However, a significant share of teams fail, so the number of submissions considering all teams, including those that fail, decreases (Column 1).

4.3 Implications for Contest Design

What do these results imply for contest design? The competition sponsor cares about the best submissions in the competition. Allowing teamwork creates a tradeoff: successful teams increase performance but some of them fail. Thus, a contest that bans teamwork would receive more, lower-quality submissions, while one that permits teamwork would receive fewer, higher-quality submissions.

To determine whether teamwork leads to better outcomes, we would need to compare the equilibrium in a contest that allows teamwork with the equilibrium of a similar contest that bans teamwork. We cannot do this with our data because we only observe contests that allow teamwork. For this reason, in the next section, we develop a structural model to measure the impact of banning teamwork on contest outcomes.

5 Equilibrium Effects of Teamwork

In this section, we present a structural model of team formation. We estimate key structural parameters and use them to investigate the impact of contest design and competition on team formation and contest outcomes.

5.1 Empirical Model

There are N forward-looking players competing in a contest. Time is discrete, the horizon is infinite, and payoffs are undiscounted. Players make submissions over time and can form teams. At every period, only one player (either a solo player or a team) is the leader of the competition and everyone else is a follower. A public leaderboard displays, in real-time, the maximum score and the identity of the leader. The game ends in any given period with probability μ or when the maximum score has reached a value \bar{s} . The leader at the end of the contest earns a prize of π and followers get 0. In the event that a team wins the contest, the team members of the winning team split the prize evenly.

The state space is:

$$\mathcal{S} = \{(s, n) : s = 0, \varepsilon, \dots, \bar{s}; n^a = 0, \dots, N/2; n^f = 0, \dots, n^a; n^{sp} = N - 2(n^a + n^f)\},$$

where s is the current maximum score, $n = (n^{sp}, n^a, n^f)$, n^{sp} is the number of solo players, n^a is the number of active teams, and n^f is the number of failed teams. Team failure is inspired by the descriptive evidence in Section 2: when a team forms and fails its members stop making submissions and become inactive. The constraint $n^{sp} = N - 2(n^a + n^f)$ reflects that all teams consist of two members. Players publicly observe and keep track of these state variables.

At any moment in the contest, there can be four types of players: (1) a follower solo player, (2) a player that belongs to a follower team, (3) a player that belongs to the team leading the competition, and (4) a solo player leading the competition. The terminal values for each type of player are

$$F_{end}^{sp} = 0, \quad F_{end}^{team} = 0, \quad L_{end}^{team} = \frac{\pi}{2}, \quad L_{end}^{sp} = \pi,$$

where we assume that team members split the prize evenly in the event of winning the contest.

For any period in which the contest has not ended, there are two independent and mutually exclusive events. First, with probability λ_1 , one of the *active* players (i.e., a player that does not belong to a failed team) makes a submission. This active player is either a solo player or belongs to a team. When a player of type $\theta \in \{\text{team}, \text{solo player (henceforth, sp)},\}$ makes a submission, the maximum score s increases to $s + \varepsilon$ with probability $q^\theta(s)$, where $q^\theta(\cdot)$ is decreasing (i.e., it becomes harder to advance the maximum score as the maximum score increases). The benefit of teamwork is captured by teams advancing the maximum score with a higher probability: $q^{\text{team}}(s) > q^{\text{sp}}(s)$ for all s .

Second, with probability λ_2 one of the follower solo players can form a team (but cannot make a submission in the current period). As a simplification, we assume that a solo player leading the competition does not consider forming a team. A follower solo player choosing to form a team can always do so provided that $n^{\text{sp}} \geq 2$ (i.e., there are at least two solo players available). The cost of teamwork has two components: i) a direct cost of forming a team (players draw a team-formation cost, c , from the distribution K); and probabilistic success, as a team fails with probability $1 - \gamma$ and becomes unproductive. Our assumption that team formation is costly is motivated by discussions we had with Kaggle users who suggested that screening potential team members is costly, as there is uncertainty regarding who can be a good match.²¹ We assume that the player proposing to form the team bears the team-formation cost. Note that when a solo player benefits from teamwork inclusive of paying the team-formation cost, any other solo player invited to join a team without paying the team formation cost will accept because solo-players' incentives are symmetric.

We next derive the value functions for each type of player and proceed to compute the equilibrium of the game.

Solo Player, Follower. The interim value of a follower solo player (denoted by player i) is

$$F_{s,n}^{\text{sp}} = \mu F_{\text{end}}^{\text{sp}} + (1 - \mu) \left[\psi(n) F_{s,n}^{\text{sp}} + \frac{\lambda_1}{N} F_{s,n}^{\text{sp,own}} + \frac{2n^a}{N} \lambda_1 F_{s,n}^{\text{sp,rival team}} + \frac{(n^{\text{sp}} - 1)}{N} \lambda_1 F_{s,n}^{\text{sp,rival sp}} + \frac{(n^{\text{sp}} - 1)}{N} \lambda_2 F_{s,n}^{\text{sp, team forms}} + \frac{1}{N} \lambda_2 F_{s,n}^{\text{sp, forms team}} \right]. \quad (3)$$

In this expression, with probability μ the contest ends and player i receives $F_{\text{end}}^{\text{sp}}$. If the contest does not end, which occurs with probability $1 - \mu$, there are 6 cases. (1) With probability $\psi(n)$, none of the active players makes a submission and none of the solo players can form a team. Thus, the state does not evolve and player i receives continuation value

²¹See Online Appendix B for details.

$F_{s,n}^{\text{sp}}$. (2) With probability λ_1/N , player i gets to make a submission and receives $F_{s,n}^{\text{sp,own}}$. (3) With probability $\frac{2n^a}{N}\lambda_1$, a team makes a submission and player i receives $F_{s,n}^{\text{sp,rival team}}$. (4) With probability $\frac{(n^{\text{sp}}-1)}{N}\lambda_1$, one of the solo players (other than i) makes a submission, and player i receives $F_{s,n}^{\text{sp,rival sp}}$. (5) With probability $\frac{(n^{\text{sp}}-1)}{N}\lambda_2$, one of the solo players can choose to form a team, and player i receives $F_{s,n}^{\text{sp, team forms}}$. (6) With probability $\frac{1}{N}\lambda_2$, player i can form a team and receives $F_{s,n}^{\text{sp, forms team}}$.

The probability that nobody makes a submission nor forms a team is

$$\psi(n) = (1 - \lambda_1 - \lambda_2 + 2\lambda_1 n^f/N + 2\lambda_2(n^a + n^f)/N),$$

which is the complementary probability of someone making a submission or deciding to form a team. Next, a submission by a player of type $\theta \in \{\text{sp}, \text{team}\}$ transitions the state from (s, n) to (s', n) with probability $q^\theta(s)$. If the submission increases the maximum score and it comes from a follower, then that follower becomes the leader and the former leader becomes a follower. Thus, the continuation values of a follower solo player when that player, a rival team, or a rival solo player makes a submission are given by

$$\begin{aligned} F_{s,n}^{\text{sp,own}} &= q^{\text{sp}}(s)L_{s',n}^{\text{sp}} + (1 - q^{\text{sp}}(s))F_{s,n}^{\text{sp}}, \\ F_{s,n}^{\text{sp,rival team}} &= q^{\text{team}}(s)F_{s',n}^{\text{sp}} + (1 - q^{\text{team}}(s))F_{s,n}^{\text{sp}}, \\ F_{s,n}^{\text{sp,rival sp}} &= q^{\text{sp}}(s)F_{s',n}^{\text{sp}} + (1 - q^{\text{sp}}(s))F_{s,n}^{\text{sp}}, \end{aligned}$$

respectively.

The value of player i when solo player j (with $j \neq i$) can form a team is

$$\begin{aligned} F_{s,n}^{\text{sp, team forms}} &= (1 - p_{s,n})F_{s,n}^{\text{sp}} + p_{s,n} \frac{1}{n^{\text{sp}} - 1} \left(\gamma F_{s,(n^{\text{sp}}-2,n^a+1,n^f)}^{\text{team}} + (1 - \gamma) \cdot 0 \right) \\ &\quad + p_{s,n} \frac{n^{\text{sp}} - 2}{n^{\text{sp}} - 1} \left(\gamma F_{s,(n^{\text{sp}}-2,n^a+1,n^f)}^{\text{sp}} + (1 - \gamma) F_{s,(n^{\text{sp}}-2,n^a,n^f+1)}^{\text{sp}} \right). \end{aligned}$$

Player j chooses to not form a team with probability $1 - p_{s,n}$, in which case player i receives $F_{s,n}^{\text{sp}}$, and where $p_{s,n}$ is an equilibrium object we derive below. With probability $p_{s,n}$, player j chooses to form a team with one of the $n^{\text{sp}} - 1$ solo players.²² The new team includes player i with probability $1/(n^{\text{sp}} - 1)$ (i.e., every available solo player is chosen with equal probability). The team is successful (i.e., the team does not fail) with probability γ , and

²²In the model, players are not keeping track of whether the leader is a solo player or a team. If the leader is a solo player, then one fewer player is available to form a team, but we are not incorporating that into the model, as it only affects the interim payoff of a solo player and the effect is small.

player i receives $F_{s,(n^{\text{sp}}-2,n^{\text{a}}+1,n^{\text{f}})}^{\text{team}}$; with probability $(1-\gamma)$ the team fails, and player i receives 0 (i.e., the value of a failed team). With probability $(n^{\text{sp}}-2)/(n^{\text{sp}}-1)$, player j forms a team with a solo player other than player i . In this case, player i continues being a follower solo player, and there is one more active team with probability γ and one more failed team with probability $1-\gamma$.

Lastly, we have player i 's decision to form a team. There are three factors influencing this decision. First, there is a direct cost of forming a team, $c \sim K$. Second, with probability γ the team will fail and player i will get 0. Third, while a team increases the chances of becoming the leader of the competition (because $q^{\text{team}}(s) > q^{\text{sp}}(s)$, for all s), the prize is evenly split among team members. Thus, a solo player forms a team only if the marginal benefit is larger than the cost, which implies that the probability of team formation is

$$p_{s,n} = \Pr(c < \gamma F_{s,(n^{\text{sp}}-2,n^{\text{a}}+1,n^{\text{f}})}^{\text{team}} - F_{s,n}^{\text{sp}}), \quad (4)$$

and the expected continuation value of forming a team is

$$F_{s,n}^{\text{sp, forms team}} = E_c \left[\max\{\gamma F_{s,(n^{\text{sp}}-2,n^{\text{a}}+1,n^{\text{f}})}^{\text{team}} + (1-\gamma) \cdot 0 - c, F_{s,n}^{\text{sp}}\} \right].$$

Other Players. There are three other types of players: (i) a player that belongs to a follower team, (ii) a member of the team leading the competition, and (iii) a solo player leading the competition. These types of players are not actively choosing whether to form a team because they either already belong to a team, or in the case of a solo player leader, we assume they do not consider teamwork while leading the competition. Nevertheless, the values of these players evolve as the state variables change.

The description of the value functions of these players is similar to the description of the value function of a follower solo player (see [Equation 3](#)). For this reason, and to avoid repetition, we refer the reader to Online Appendix C for a detailed description of these equations.

Equilibrium. We solve the game by backward induction. Although the horizon is infinite, the state variables reach a point at which they stop evolving. Specifically, we assume that the game ends when the maximum score reaches the highest possible value \bar{s} , and there cannot be more teams than $N/2$. We can compute the value functions for all combinations of state variables that are “terminal” and work backwards to fully solve the model.

Discussion of Modeling Assumptions. Our model captures team formation in dynamic contests, where players are presented with opportunities to form teams over time and (dynam-

ically) choose whether to form a team. The benefit of teamwork is to increase the likelihood of becoming the competition leader. This assumption is motivated by our empirical findings in Section 4. On the other hand, forming teams is costly and uncertain (some team fails). Moreover, in the event of winning, team members share the prize equitably. Players take into account the benefit and the cost of forming teams whenever they get a stochastically-arriving opportunity to form a team. In terms of dynamic incentives, a player’s decision to form a team takes into account: (1) the current score; (2) the cost of forming a team; (3) the likelihood that they will have a future opportunity to form a team; (4) the expected composition of players in the rest of the contest (i.e., rivals may form teams in the future).

To keep the model tractable and to focus on the incentives of team formation, we abstract from a number of complexities. We make simplifying assumptions to reduce the state space and facilitate model estimation. These assumptions include a stochastic end of the contest (to avoid keeping track of time), a maximum score at which the contest ends (to solve by backward induction); having only two types of players: leaders and followers (to avoid keeping track of scores of each player), stochastic play (to avoid modelling the decision to play or not whenever an opportunity presents), teams of at most two members (to reduce the number of value functions we need to write). Changing any of these assumptions would likely preserve our results qualitatively but add much computational burden.

5.2 Estimation and Model Fit

We estimate the model using a full-solution method. To compute the equilibrium of the game, we exploit that the state variables are directional (e.g., the maximum score or the number of teams can only increase or stay the same) and that they are capped (e.g., the maximum score and the number of teams cannot exceed \bar{s} and $N/2$, respectively). This allows us to compute the equilibrium by backward induction.

The full set of primitives for a given contest include i) the probability that an active player can play, λ_1 ; ii) the probability that an active solo player can form a team, λ_2 ; iii) the functions $q^{\text{team}}(s)$ and $q^{\text{sp}}(s)$, which indicate the probability of advancing the maximum score given that the current maximum score is s for a team and a solo player, respectively; iv) the probability of team failure, $1 - \gamma$; v) the probability that the contest ends, μ ; and vi) the distribution of team-formation costs, $K(c; \sigma) = c^\sigma$, where $\sigma > 0$ and the support of the distribution is the interval $[0, 1]$.²³ We allow these primitives to vary at the contest level.

²³We normalize the size of the prize to be 1 for every contest.

We use a two-step procedure to estimate the primitives of each contest. In the first step, we estimate primitives i)-v) without using the full structure of the model. In the second step, we use the estimates of these primitives to estimate the cost distribution using a generalized method of moments (GMM) estimator.

We specify the functions $q^\theta(s)$, where $\theta \in \{\text{team}, \text{sp}\}$ as

$$q^\theta(s) = \exp\{\beta_0^\theta + \beta_1 s\} / (1 + \exp\{\beta_0^\theta + \beta_1 s\}),$$

and we estimate β_0^θ and β_1 using a maximum-likelihood estimator, using data on whether each submission increased the maximum score as well as the maximum score at the time of each submission (s). Because in some competitions the maximum score is rather constant, we pool the data from all competitions to gain power in estimating the parameter β_1 , which we constrain to be uniform across contests. We allow β_0^θ to vary across contests.

We also estimate directly from the data the probability that at any given period a player plays, λ_1 , and the probability that a team fails, $1 - \gamma$. We set $\mu = \frac{1}{T}$, so in expectation the contest duration matches the length of the contest in the data. We also set the probability that a player can form a team, λ_2 , to be $1 - \lambda_1$.²⁴

In the second step, we estimate the parameter σ of the cost distribution, $K(c; \sigma) = c^\sigma$, where $\sigma > 0$. We use a GMM estimator, where for each contest k , we estimate σ by minimizing the difference between the number of teams observed in the data and predicted by the model: $m_k(\sigma) = \text{teams}_k^{\text{data}} - \text{teams}_k^{\text{model}}$. The GMM estimator for σ in contest k is then given by

$$\hat{\sigma} = \arg \min_{\sigma} \hat{m}_k(\sigma)^2.$$

We present asymptotic standard errors.

We use the full-solution method to compute the moment $m_k(\sigma)$ for a given value of σ . That is, for a given σ , we compute the equilibrium of the game using backward induction to obtain the matrices of conditional-choice probabilities (CCPs) governing the decision to form teams, \mathbf{p} . \mathbf{p} is of dimensions $S \times N^3$ (S is the size of the set of possible scores and N is the number of players that can be solo players, team players, or failed-team players) where element $(s, n^{\text{sp}}, n^{\text{a}}, n^{\text{f}})$ of \mathbf{p} is $p_{t,n}^j$.²⁵ Using the CCPs, we simulate equilibrium outcomes by

²⁴Both λ_2 and the cost of making a merger impact the equilibrium number of mergers. We set $\lambda_2 = 1 - \lambda_1$ to avoid an identification problem caused by the interplay between λ_2 and the cost of making mergers in explaining the observed number of mergers.

²⁵In the estimation, S varies across contests. In a given contest, the set of scores is set to include all unique maximum scores in the competition as well as the values $\bar{s} + [0.001 : 0.001 : 0.08]$, where \bar{s} is the highest

Table 6: Empirical model estimates*Panel A: Common parameters across contests*

	Estimate	SE
γ	0.74	0.01
$\beta_1(q)$	-1.486	0.035
$\beta_0^{teams} - \beta_0^{sp}(q)$	1.161	0.064

Panel B: Contest-specific parameters (partial list of contests)

	λ_1	SE	σ	SE	$\beta_0^{sp}(q)$	SE	N
TGS Salt Identification Challenge	0.082	0.008	1.4	0.207	-1	0.205	95
Quick, Draw! Doodle Recognition Challenge	0.072	0.007	1.54	0.363	-2.783	0.183	91
RSNA Pneumonia Detection Challenge	0.191	0.016	1.275	0.313	-3.429	0.387	94
Human Protein Atlas Image Classification	0.159	0.012	1.394	0.222	-3.235	0.286	118
Traveling Santa 2018 - Prime Paths	0.071	0.006	1.772	0.762	-1.321	0.212	103
Google Cloud & NCAA ML Competition 2019-Mens	0.12	0.011	1.09	0.086	-1.768	0.25	92
Instant Gratification	0.158	0.014	1.318	0.271	-2.411	0.351	93
Predicting Molecular Properties	0.109	0.009	1.351	0.171	-2.226	0.209	114
SIIM-ACR Pneumothorax Segmentation	0.138	0.014	1.044	0.139	-2.966	0.511	68
Lyft 3D Object Detection for Autonomous Vehicles	0.138	0.011	1.693	0.621	-2.299	0.182	126
Santas Workshop Tour 2019	0.066	0.008	1.149	0.113	-2.925	0.273	67
Predict HIV Progression	0.08	0.008	1.381	0.179	-0.932	0.203	87
Chess ratings - Elo versus the Rest of the World	0.18	0.013	1.271	0.223	-2.072	0.263	136
Tourism Forecasting Part One	0.29	0.023	1.302	0.656	-2.005	0.323	80
Tourism Forecasting Part Two	0.103	0.009	1.309	0.374	-5.346	0.216	108
R Package Recommendation Engine	0.096	0.008	1.264	0.124	-3.4	0.185	112

Notes: SE stands for asymptotic standard errors. See [Table A.8](#) in the Online Appendix for the estimates of the full list of contests.

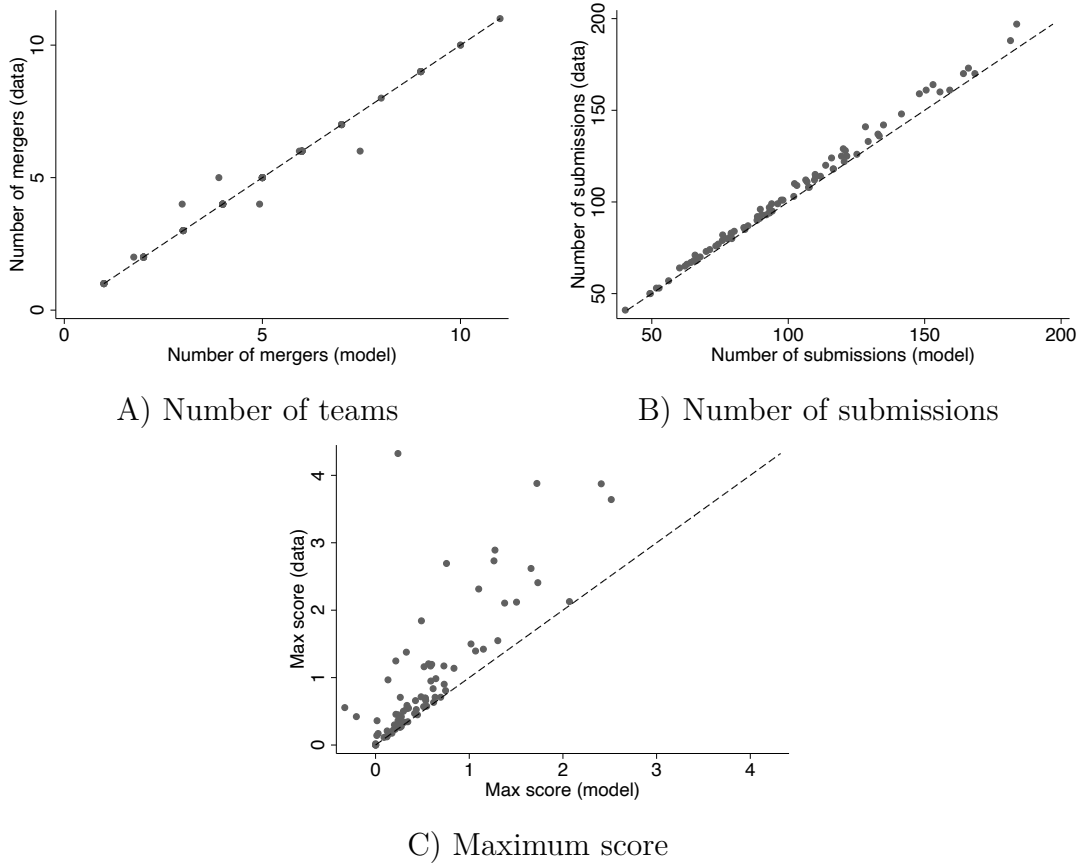
simulating the game $ns = 500$ times and averaging equilibrium outcomes across simulations.

Lastly, we restrict the sample to the top 40 players in each contest (measured by the ranking of players at the end of the competition), i.e., $N = 40$. We make this choice for two reasons: First, these players are more likely to form teams. Second, this group of players is less heterogeneous than the entire pool of players, which allows us to abstract away from modelling player heterogeneity. We also restrict attention to the 80 contests that exhibited team formation among the top 40 players.

Model Estimates and Fit [Table 6](#) shows the model estimates and [Figure 3](#) the fit of the model. Panels A and B of [Figure 3](#) show that the model is able to replicate well both the number of submissions and the number of teams in a contest. Panel C shows that, while the model tends to under-estimate the maximum score, especially for those with large maximum score, the correlation between the data and model predictions is still high (about 78 percent).

[Figure A.2](#) in the Online Appendix shows the distribution of the average cost of forming a observed score in the competition.

Figure 3: Model fit, by equilibrium outcome



Notes: The figures plot equilibrium outcomes in the data against those predicted by the model estimates. Model predictions are computed via simulation. Specifically, we simulate the game $ns = 500$ times and compute the average for each equilibrium outcome across simulations.

team across contests. On average, the mean cost of forming a team is 52 percent of the prize. Given that team members split the prize in two in case of winning, only a few players find forming a team worth it (i.e., those who get a particularly good draw of the cost of forming a team). This explains the rather puzzling finding that only a few players form teams even though there are performance gains.

6 The Equilibrium Impact of Teamwork

In this section, we ask two questions. First, we study the impact of teamwork on contest outcomes. Second, we investigate the impact of competition and team-formation costs on team-formation incentives and contest outcomes. To answer these questions, we use our model estimates to compute the equilibria of each contest under counterfactual scenarios.

Table 7: Equilibrium impact of allowing teamwork

	(1)	(2)
	Number of submissions (in logs)	Maximum score
Teamwork Allowed	-0.036*** (0.002)	0.019*** (0.003)
Observations	160	160
R^2	0.99	0.99

Notes: Standard errors clustered at the competition-level in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. An observation is a competition–treatment combination, where treatment $\in \{\text{no teamwork, teamwork}\}$. All specifications include competition fixed effects.

6.1 Equilibrium Effects of Teamwork

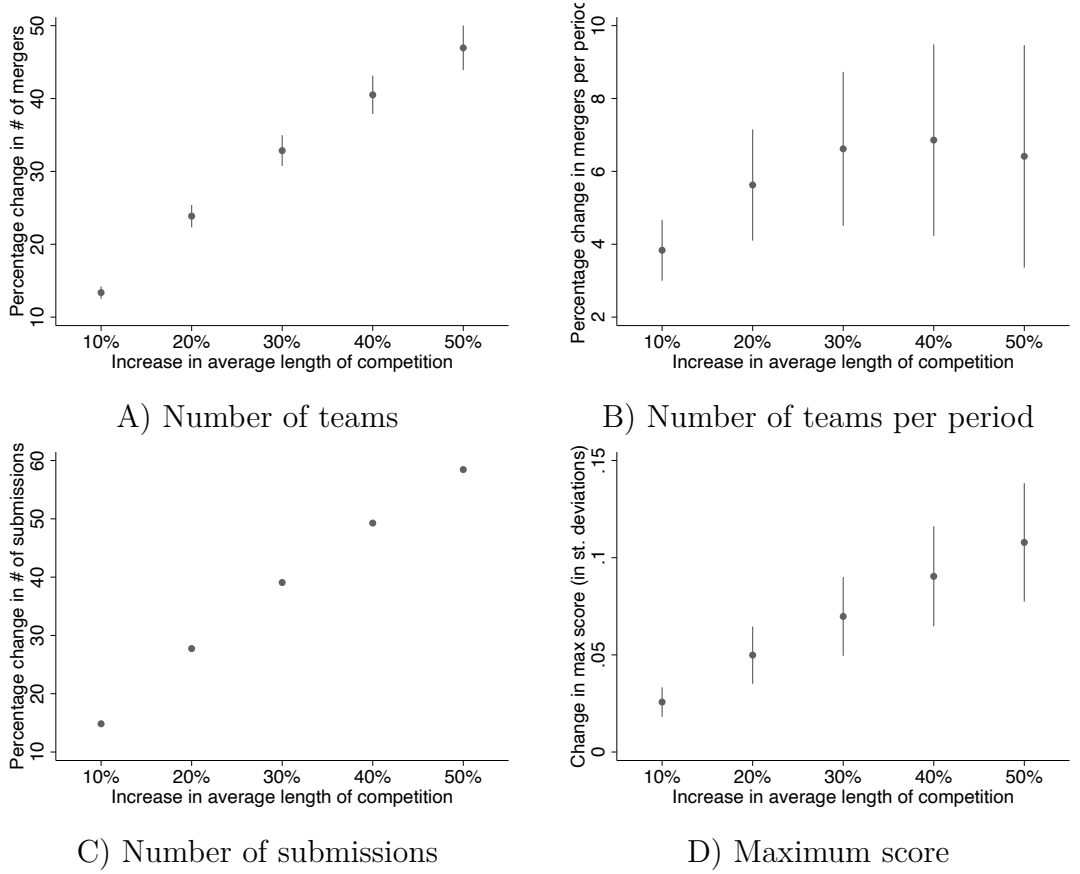
First, most Kaggle competitions allow teamwork but some do not. Why would an online-contest platform, such as Kaggle, allow teamwork? Other online-contest platforms never allow teamwork, nor do some online contests directly sponsored by government agencies. As discussed in Section 4, teamwork creates a tradeoff. On the one hand, we provide evidence suggesting that teamwork causes performance gains among successful teams. On the other hand, some teams fail in that they stop making submissions. Hence, an evaluation of the impact of teamwork must compare the benefit of having higher-performing players in the competition (i.e., teams that succeed) against the cost of fewer submissions (i.e., the missed submissions by the members of failed teams).

To shed light on this question, [Table 7](#) reports a comparison between the equilibrium where teamwork is allowed and the equilibrium where teamwork is forbidden for each contest in our sample. Column 1 shows that allowing teamwork on average *decreases* the number of submissions. As discussed, this is explained by team failures. Despite the fewer submissions overall, Column 2 shows that teamwork on average *increases* the maximum score. The reason for the increase is due to productivity gains by successful teams, which more than compensate the reduction in number of submissions caused by failed teams. That is, teamwork causes an improvement in the best submission in a contest.

6.2 Incentives I: The Role of Competition

Next, we investigate the impact of competition on team formation and contest outcomes. Does greater competitive pressure encourage teamwork? Increasing the length of a contest

Figure 4: Equilibrium outcomes with increased competition levels



Notes: The figures plot equilibrium outcomes predicted by the model estimates (as well as 95 percent confidence bands) when the expected contest length increases by X percent in each contest ($X \in \{10, 20, 30, 40, 50\}$). Model predictions are computed via simulation. Specifically, we simulate the game $ns = 500$ times and compute the average for each equilibrium outcome across simulations.

is one way to capture an increase in competitive pressure. In our model, this is equivalent to reducing the probability that the contest ends at any given period (i.e., reducing μ). A longer contest gives each player more chances to play but it also creates more future competition (i.e., more opportunities for followers to displace the leader).

We take every contest in our sample and compute the equilibrium of the game for counterfactual values of the expected length of the contest. Figure 4 presents the results of the analysis. The figures shows that more competition encourages teamwork. Part of this effect is mechanic because in a longer contest there are more opportunities to form teams. To focus on the change in number of teams caused by incentives and not from the fact that the contest is longer, Figure 4 (Panel B) shows the percentage change in the number of teams per period, i.e., $(\text{number of teams})/(\text{contest length})$. The figure shows that, after controlling for the contest length, the number of teams increases (at least for small increases in competition).

This suggests that teamwork is more valuable when players expect more future competition.

Figure 4 (Panel C) shows that the number of submissions increases when the length of the competition increases. More teams mean that more teams will fail but the teams that do not fail have more time to send submissions. The rate of team failure is lower than the increase in the number of submissions due to a longer contest duration, so the number of submissions increases. Figure 4 (Panel D) shows that the two effects combined (more teams sending more submissions) imply that the maximum score also increases.

These results indicate that competition impacts the incentives to form teams. Players want to form teams to increase their productivity because they are less likely to win by working solo. On the flip side, reducing competition *reduces* the incentive to work in teams because there will be fewer submissions, so a less productive solo player stands a good chance of winning the contest. While forming a team increases a player’s productivity, it also splits the prize in case of winning. Therefore, our results suggest that players will prefer to compete solo in less competitive contests.

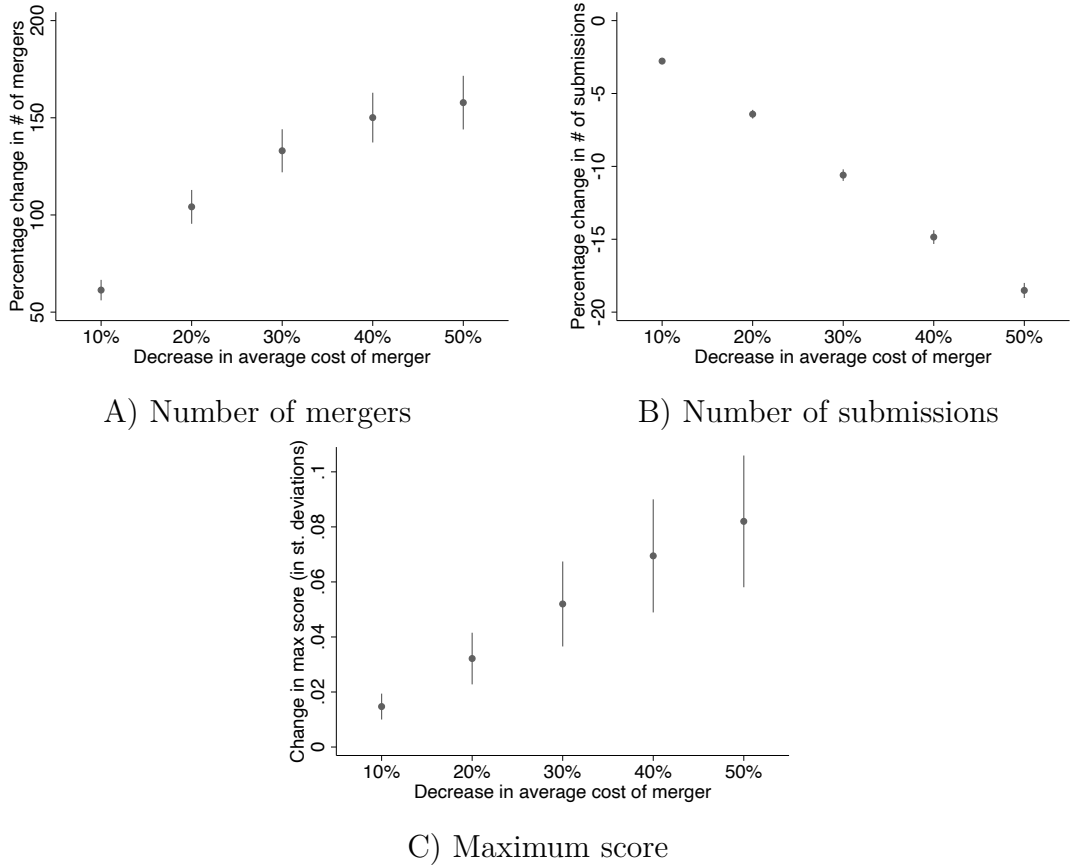
6.3 Incentives II: The Cost of Team Formation

Next, we explore whether a contest platform should make an effort to *facilitate* the formation of teams. For instance, allowing players to communicate, to access other player’s profiles, or incorporating online-collaboration tools may facilitate teamwork by reducing the cost of forming a team.

To investigate the impact of facilitating teamwork, we take every contest in our sample and compute the equilibrium of the game if the average cost of forming a team decreased by a value ranging between 10 percent and 50 percent. The theoretical impact of reducing the cost of team formation on contest outcomes is ambiguous. On the one hand, when team formation is less costly, more teams will form. This implies that high-scoring submissions will be more likely to arise, as teams improve their performance relative to solo players, which is the main finding in Section 4. On the other hand, a fraction of team fails, leaving fewer competitors making submissions, reducing the number of submissions.

Figure 5 presents our results showing that making team formation less costly increases the number of teams (panel A), reduces the number of submissions (panel B) due to failed teams, and has a positive impact on the maximum score. That is, even though the number of submissions decreases the maximum score *increases*. In other words, the performance

Figure 5: Equilibrium outcomes with reduced costs of team formation



Notes: The figures plot equilibrium outcomes predicted by the model estimates (as well as 95 percent confidence bands) when the expected cost of forming a team decreases by X percent in each contest ($X \in \{10, 20, 30, 40, 50\}$). Model predictions are computed via simulation. Specifically, we simulate the game $ns = 500$ times and compute the average for each equilibrium outcome across simulations.

improvement that we identify in Section 4, more than compensates for the reduction in the number of submissions due to failed teams.

How to lower the cost of team formation? Before forming teams, players need to assess the value of collaboration, especially considering that 8.4 percent of all teams fail. Naturally, players will try to *screen* potential partners before forming a team. Because screening potential team members is costly, facilitating this task is one way to decrease the cost of team formation.

One way to screen partners is using the leaderboard to look at other players' performance in the current competition or past ones.²⁶ While the leaderboard is informative about the

²⁶Figure A.3 in the Online Appendix shows that teams usually form among players who are performing similarly at the time of the merger (measured in distance to the leaderboard maximum score) and that the effect is stronger for players in top leaderboard positions. Figure A.4 in the Online Appendix shows a similar

performance of potential teammates, the public score is only a noisy signal of the private score (i.e., the payoff-relevant performance measure). We find evidence suggesting that the noise in the leaderboard makes screening costlier. Specifically, we exploit variation in the precision of the public score as a signal of the private score across contests to measure the impact of asymmetric information on team formation. [Table A.9](#) in the Online Appendix shows that both the number of submissions prior to team formation and the time of the merger decrease with the precision of the information in the leaderboard. We interpret this finding as indicating the informativeness of more precise signals: when information is more precise, fewer signals are needed to form a more precise posterior belief about the type of a potential teammate, which leads to earlier team formation. These findings suggest that players face asymmetric information when forming teams, which suggests that contest platforms should make the leaderboard as informative as possible, and more broadly, make it easier for players to gather information about potential partners to facilitate team formation.

7 Discussion

We investigate teamwork in contests. Why do teams form? What are the benefits of teamwork? Our reduced-form evidence suggest that teamwork causes an economically significant performance improvement (for both high- and low-ranked teams). These performance gains do not come from more quantity but rather from higher quality submissions. We also show that not all teams are successful, which implies that players must compare the uncertain benefits of teamwork with the cost of forming a team.

Motivated by these results, we build and estimate a structural model to shed light on the players’ dynamic incentives to form teams during a contest. Our estimates show that forming a team is quite costly: the average cost of forming a team equals 52 percent of the contest prize. This high cost explains why collaboration is rather scarce in our sample of Kaggle contests, since only 8 percent of participants are part of a team.

Using our estimates, we investigate alternative policies to derive contest-design implications. Should contests allow the formation of self-organized teams? We answer this question by studying the impact of facilitating teamwork (reducing the cost of team formation). We find that the lower the cost of team formation, the higher the maximum score in a contest. This result is not obvious since players will form more teams when team formation is cheaper, meaning that more players will be idle for the rest of the competition, after their team result based on performance in past competitions.

fails (recall that 8.4 percent of teams do not produce submissions). However, successful partnership produce better outcomes, which counter the negative effect of failed teams. We find that the former effect outweighs the latter, leading to an overall enhanced performance. Thus, one implication for contest design from our analysis is that contests should allow the formation of self-organized teams. Similarly, we find that lowering the cost of team formation enhances contest outcomes, as it induces more team formation, leading to higher scores despite a decrease in the number of submissions due to team failure. We propose policies that a contest designer can implement to decrease the cost of forming a team.

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Online Appendix

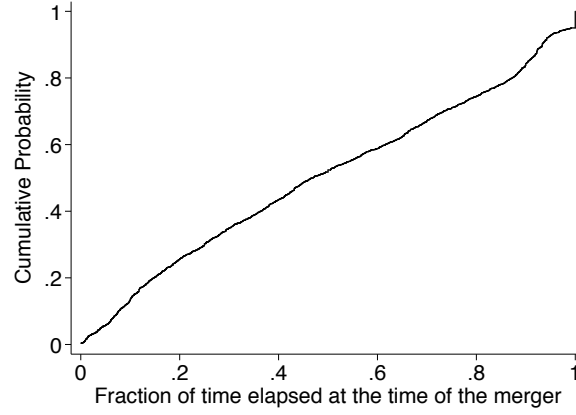
Teamwork in Contests

by Jorge Lemus and Guillermo Marshall

Supplemental Material – Intended for Online Publication

A Additional Tables and Figures

Figure A.1: Timing of team mergers: Cumulative probability function



Notes: An observation is a team that welcomed a new member during the competition.

Table A.1: Probability of a failed team: OLS estimates

	(1) failed
Difference in submissions (in st. dev.)	0.034*** (0.006)
Difference in max score (in st. dev.)	0.112*** (0.006)
Observations	7,578
R^2	0.425

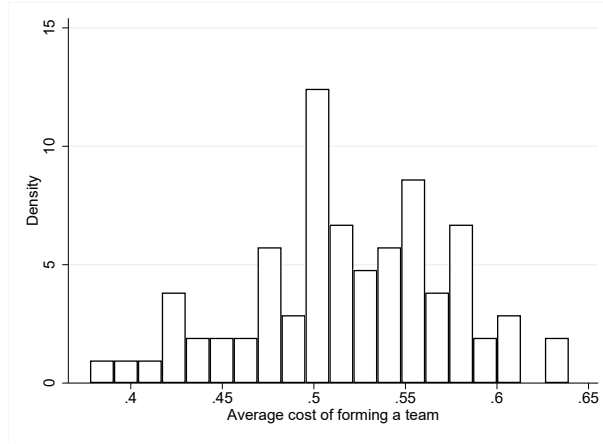
Notes: Robust standard errors in parentheses. An observation is a team. The regression includes competition fixed effects. Difference in submissions (max score) is the difference in the number of submissions (max score) at the time of team formation, which are normalized to have standard deviation 1.

Table A.2: Balance table: pre-merger covariates across treated and non-treated (matched) teams

	Number of submissions up to time of merger (1)	Distance to max score on public leaderboard at time of merger (2)	Team size at time of merger (3)
Non-treated teams	15.955	1.446	1
Treated teams	15.955	1.447	1
p-value	1.000	0.978	1.000

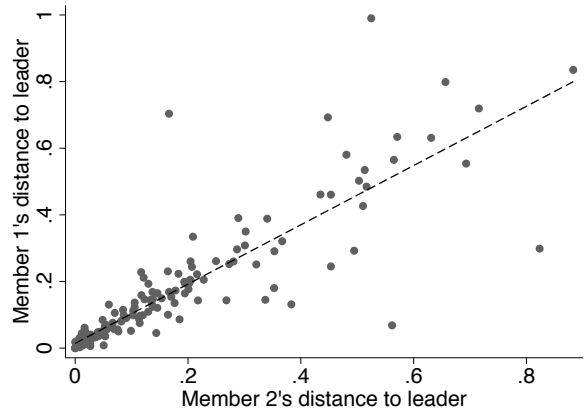
Notes: Treated teams are teams who welcomed a new member during the competition, non-treated teams are teams who did not change their team size during the competition. The last row of the table reports the p-value of a differences-in-mean test.

Figure A.2: Average cost of team formation.



Note: An observation is contest.

Figure A.3: Team member heterogeneity at the time of the merger



Notes: The figure plots the distance of each team member to the leader at the moment of forming a team. These figures restrict attention to teams in which both members had submitted at least 5 submissions prior to the merger.

Table A.3: The impact of collaboration on scores: Team-level estimates, competitive teams sub-sample

	Public score	Private score
	(1)	(2)
Teamwork	0.075*** (0.016)	0.084*** (0.018)
Observations	2,008,287	2,008,287
R^2	0.302	0.308

Notes: Standard errors clustered at the team-level in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. An observation is a submission made by a team in a competition. The sample is restricted to competitive teams, which are defined as teams that obtained scores above the 75th percentile of the score distribution in their competition. All specifications include team fixed effects, competition-day fixed effects, and a second-degree polynomial of variables: total number of submissions by all teams up until the submission time, total number of submissions by the team making the submission up until the submission time, total number of submissions by the member of the team making the submission up until the submission time, the submitting team's distance to the maximum score on the public leaderboard at the submission time, and the fraction of contest time that had elapsed at the submission time. The sample is restricted to include submissions by treated teams that took place six weeks before or after the week in which the team changed its team size, and it also restricts attention to teams with one or two members.

Table A.4: The impact of collaboration on scores: Team-level estimates, heterogeneity analysis with respect to contest characteristics

	(1)	(2)	(3)	(4)	(5)
<i>Panel A. Dependent variable: Public score</i>					
New member	0.078*** (0.014)	0.086*** (0.018)	0.073*** (0.021)	0.038 (0.044)	0.080*** (0.015)
Teamwork * Image data		-0.023 (0.026)			
Teamwork * Large reward			0.012 (0.026)		
Teamwork * Post 2015				0.044 (0.047)	
Teamwork* Large dataset					-0.011 (0.032)
Observations	3,248,210	3,248,210	3,248,210	3,248,210	3,248,210
R^2	0.439	0.439	0.439	0.439	0.439
<i>Panel B. Dependent variable: Private score</i>					
New member	0.085*** (0.016)	0.088*** (0.019)	0.077*** (0.021)	0.041 (0.045)	0.083*** (0.018)
Teamwork * Image data		-0.009 (0.036)			
Teamwork * Large reward			0.019 (0.031)		
Teamwork * Post 2015				0.047 (0.048)	
Teamwork * Large dataset					0.012 (0.038)
Observations	3,179,632	3,179,632	3,179,632	3,179,632	3,179,632
R^2	0.448	0.448	0.448	0.448	0.448

Notes: Standard errors clustered at the team-level in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. An observation is a submission made by a team in a competition. All specifications include team fixed effects, competition-day fixed effects, and a second-degree polynomial of variables: total number of submissions by all teams up until the submission time, total number of submissions by the team making the submission up until the submission time, total number of submissions by the member of the team making the submission up until the submission time, the submitting team's distance to the maximum score on the public leaderboard at the submission time, and the fraction of contest time that had elapsed at the submission time. The sample is restricted to include submissions by treated teams that took place six weeks before or after the week in which the team changed its team size, and it also restricts attention to teams with one or two members. Image data is an indicator for whether the contest requires use of video or image data; large reward is an indicator for contests with above average reward quantity; post 2015 is an indicator for contests taking place after 2015 (when the platform incorporated new features that facilitated communication among players, e.g., notebooks); and large dataset is an indicator for whether the dataset has an above average size (in GBs).

Table A.5: The impact of collaboration on extreme scores: Team-level estimates

	$1\{\text{score} > p75\}$ (1)	$1\{\text{score} > p90\}$ (2)	$1\{\text{score} > p95\}$ (3)	$1\{\text{score} > p99\}$ (4)
<i>Panel A: Public score</i>				
Teamwork	0.074*** (0.008)	0.084*** (0.009)	0.070*** (0.009)	0.023*** (0.005)
Observations	3,248,210	3,248,210	3,248,210	3,248,210
R^2	0.564	0.570	0.633	0.836
<i>Panel B: Private score</i>				
New member	0.067*** (0.008)	0.077*** (0.009)	0.066*** (0.009)	0.023*** (0.005)
Observations	3,179,632	3,179,632	3,179,632	3,179,632
R^2	0.541	0.454	0.433	0.453

Notes: Standard errors clustered at the team-level in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. An observation is a submission made by a team in a competition. $1\{\text{score} > pX\}$ is an indicator that takes the value one if the submission's score exceeded percentile X of the competition-level score distribution. All specifications include team fixed effects, competition-day fixed effects, and a second-degree polynomial of variables: total number of submissions by all teams up until the submission time, total number of submissions by the team making the submission up until the submission time, total number of submissions by the member of the team making the submission up until the submission time, the submitting team's distance to the maximum score on the public leaderboard at the submission time, and the fraction of contest time that had elapsed at the submission time. The sample is restricted to include submissions by treated teams that took place six weeks before or after the week in which the team changed its team size, and it also restricts attention to teams with one or two members.

Table A.6: The impact of collaboration on scores: Team-level estimates, first-time collaborators subsample

	Public score (1)	Private score (2)
Teamwork	0.114*** (0.019)	0.121*** (0.023)
Observations	3,121,785	3,054,446
R^2	0.444	0.453

Notes: Standard errors clustered at the team-level in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. An observation is a submission made by a team in a competition. All specifications include team fixed effects, competition-day fixed effects, and a second-degree polynomial of variables: total number of submissions by all teams up until the submission time, total number of submissions by the team making the submission up until the submission time, total number of submissions by the member of the team making the submission up until the submission time, the submitting team's distance to the maximum score on the public leaderboard at the submission time, and the fraction of contest time that had elapsed at the submission time. The sample is restricted to include untreated teams and treated teams whose members are participating in a multiplayer team for the first time (i.e., in all previous competitions, if any, they participated in a single-member team). Further, the sample is restricted to include submissions by treated teams that took place six weeks before or after the week in which the team changed its team size, and it also restricts attention to teams with one or two members.

Table A.7: The impact of collaboration on scores: Team-level estimates, heterogeneity analysis with respect to time of team formation

	Public score (1)	Private score (2)
Teamwork (early merger)	0.080*** (0.024)	0.081** (0.035)
Teamwork (late merger)	0.078*** (0.017)	0.086*** (0.018)
Observations	3,248,210	3,179,632
R^2	0.439	0.448

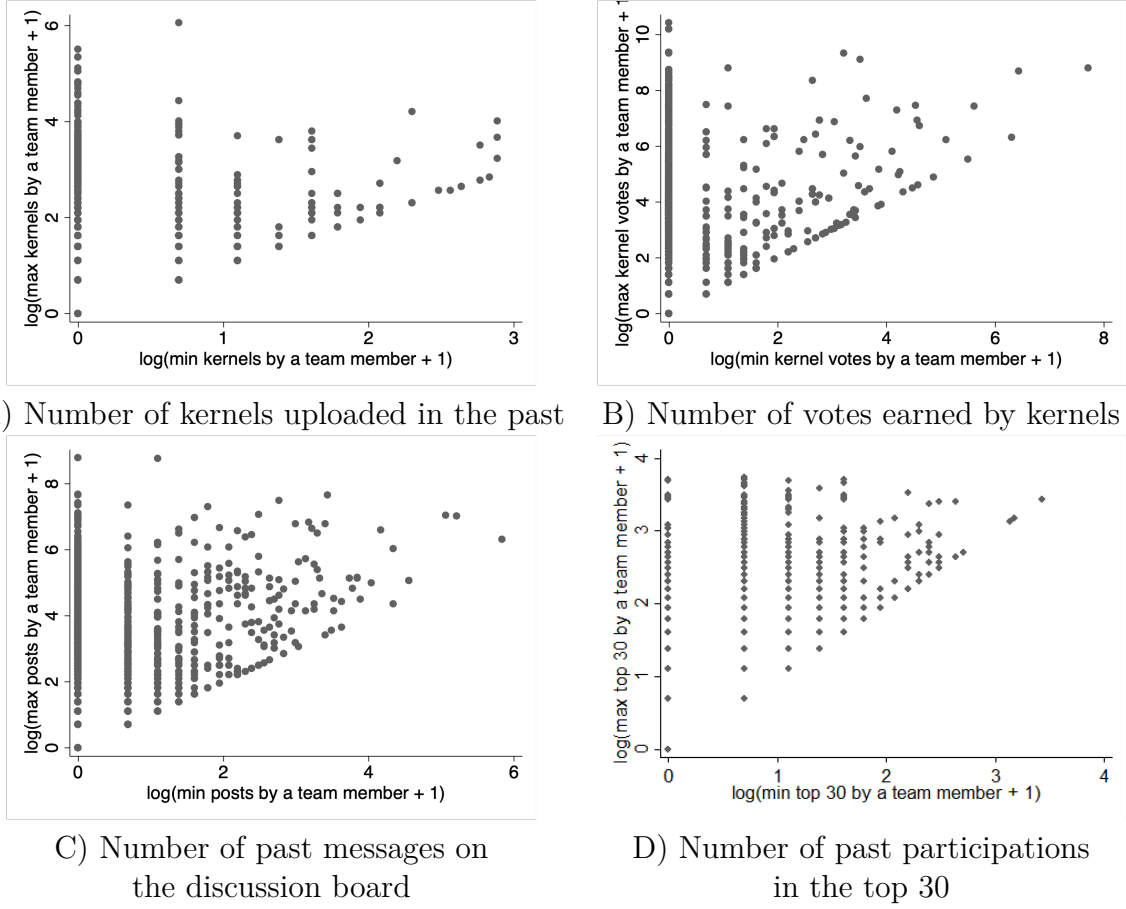
Notes: Standard errors clustered at the team-level in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. An observation is a submission made by a team in a competition. All specifications include team fixed effects, competition-day fixed effects, and a second-degree polynomial of variables: total number of submissions by all teams up until the submission time, total number of submissions by the team making the submission up until the submission time, total number of submissions by the member of the team making the submission up until the submission time, the submitting team's distance to the maximum score on the public leaderboard at the submission time, and the fraction of contest time that had elapsed at the submission time. The sample is restricted to include submissions by treated teams that took place six weeks before or after the week in which the team changed its team size, and it also restricts attention to teams with one or two members. An early merger (late merger) is defined as a merger that took place when less (more) than 50 percent of the contest time had elapsed.

Table A.8: Empirical model estimates: Contest-specific parameters

	λ_1	SE	σ	SE	$\beta_0(q)$	SE	N
TGS Salt Identification Challenge	0.082	0.008	1.4	0.207	-1	0.205	95
Quick, Draw! Doodle Recognition Challenge	0.072	0.007	1.54	0.363	-2.783	0.183	91
RSNA Pneumonia Detection Challenge	0.191	0.016	1.275	0.313	-3.429	0.387	94
Human Protein Atlas Image Classification	0.159	0.012	1.394	0.222	-3.235	0.286	118
Traveling Santa 2018 - Prime Paths	0.071	0.006	1.772	0.762	-1.321	0.212	103
Google Cloud & NCAA ML Competition 2019-Mens	0.12	0.011	1.09	0.086	-1.768	0.25	92
Instant Gratification	0.158	0.014	1.318	0.271	-2.411	0.351	93
Predicting Molecular Properties	0.109	0.009	1.351	0.171	-2.226	0.209	114
SIIM-ACR Pneumothorax Segmentation	0.138	0.014	1.044	0.139	-2.966	0.511	68
Lyft 3D Object Detection for Autonomous Vehicles	0.138	0.011	1.693	0.621	-2.299	0.182	126
Santas Workshop Tour 2019	0.066	0.008	1.149	0.113	-2.925	0.273	67
Predict HIV Progression	0.08	0.008	1.381	0.179	-0.932	0.203	87
Chess ratings - Elo versus the Rest of the World	0.18	0.013	1.271	0.223	-2.072	0.263	136
Tourism Forecasting Part One	0.29	0.023	1.302	0.656	-2.005	0.323	80
Tourism Forecasting Part Two	0.103	0.009	1.309	0.374	-5.346	0.216	108
R Package Recommendation Engine	0.096	0.008	1.264	0.124	-3.4	0.185	112
IJCNN Social Network Challenge	0.045	0.005	1.27	0.108	-0.867	0.282	68
Stay Alert! The Ford Challenge	0.101	0.008	1.27	0.124	-3.546	0.278	125
Mapping Dark Matter	0.61	0.019	1	0.035	-2.613	0.243	161
ICDAR 2011 - Arabic Writer Identification	0.103	0.009	0.981	0.084	-3.179	0.242	99
Dont Overfit!	0.041	0.006	1.472	0.706	-3.09	0.238	50
Wikipedias Participation Challenge	0.092	0.008	1.1	0.09	-4.962	0.201	101
Allstate Claim Prediction Challenge	0.222	0.014	1.119	0.157	-3.84	0.236	160
dunhumby's Shopper Challenge	0.082	0.007	1.557	0.274	-1.718	0.165	118
Semi-Supervised Feature Learning	0.167	0.013	1.402	0.395	-2.524	0.208	122
Give Me Some Credit	0.093	0.009	1.034	0.081	-3.526	0.254	83
Dont Get Kicked!	0.308	0.018	1	0.005	-2.68	0.152	133
CHALEARN Gesture Challenge	0.114	0.01	0.984	0.052	-4.217	0.192	112
What Do You Know?	0.134	0.013	0.868	0.074	-4.739	0.289	79
Photo Quality Prediction	0.201	0.013	0.917	0.053	-3.268	0.237	142
The Hewlett Foundation: Automated Essay Scoring	0.097	0.01	1.118	0.083	-3.774	0.196	80
KDD Cup 2012, Track 2	0.088	0.011	1.211	0.333	-2.731	0.172	57
Predicting a Biological Response	0.24	0.014	1.367	0.19	-3.614	0.166	170
Online Product Sales	0.208	0.014	1.034	0.069	-3.187	0.208	137
Belkin Energy Disaggregation Competition	0.067	0.006	1.03	0.045	-5.004	0.194	101
Merck Molecular Activity Challenge	0.257	0.015	0.943	0.032	-3.978	0.2	173
Predict Closed Questions on Stack Overflow	0.155	0.014	0.72	0.03	-1.989	0.153	82
Traveling Santa Problem	0.069	0.009	1.2	0.259	-3.543	0.371	53
Blue Book for Bulldozers	0.265	0.014	0.712	0.018	-5.867	0.18	197
Job Salary Prediction	0.114	0.009	1.09	0.086	-1.818	0.208	115
The Marinexplore and Cornell University Whale Detection Challenge	0.164	0.012	0.751	0.028	-5.453	0.177	124
KDD Cup 2013 - Author-Paper Identification Challenge (Track 1)	0.25	0.015	0.744	0.032	-5.164	0.165	159
KDD Cup 2013 - Author Disambiguation Challenge (Track 2)	0.163	0.012	0.782	0.031	-4.987	0.171	129
See Click Predict Fix	0.194	0.012	0.849	0.036	-3.729	0.266	161
Packing Santas Sleigh	0.063	0.006	1.243	0.086	-1.672	0.432	97
Higgs Boson Machine Learning Challenge	0.211	0.017	0.716	0.046	-3.164	0.127	96
Liberty Mutual Group - Fire Peril Loss Cost	0.077	0.008	1.054	0.084	-3.763	0.458	84
Helping Santas Helpers	0.106	0.01	1.228	0.602	-6.125	0.179	95
March Machine Learning Mania 2015	0.106	0.008	1.035	0.059	-4.746	0.157	125
Otto Group Product Classification Challenge	0.117	0.011	0.982	0.117	-5.4	0.143	86
ICDM 2015: Drawbridge Cross-Device Connections	0.28	0.015	1.008	0.065	-4.4	0.19	188
Caterpillar Tube Pricing	0.085	0.008	1.193	0.173	-4.071	0.22	93
Liberty Mutual Group: Property Inspection Prediction	0.12	0.011	1.398	0.408	-2.781	0.323	85
Springleaf Marketing Response	0.289	0.016	0.94	0.075	-4.547	0.149	170
Truly Native?	0.19	0.014	0.796	0.039	-4.999	0.152	128
The Allen AI Science Challenge	0.079	0.007	1.286	0.142	-3.238	0.196	99
Santas Stolen Sleigh	0.036	0.004	1	0.006	-4.432	0.36	64
Second Annual Data Science Bowl	0.113	0.009	1.154	0.169	-1.028	0.165	111
BNP Paribas Cardif Claims Management	0.1	0.009	1.046	0.052	-2.872	0.191	109
Home Depot Product Search Relevance	0.102	0.01	0.9	0.064	-6.12	0.191	86
Santander Customer Satisfaction	0.103	0.009	0.923	0.056	-5.988	0.181	114
Expedia Hotel Recommendations	0.117	0.009	1.156	0.075	-2.798	0.185	148
Ultrasound Nerve Segmentation	0.196	0.018	0.793	0.073	-4.249	0.247	73
Draper Satellite Image Chronology	0.083	0.012	1	0.016	-4.151	0.395	41
Predicting Red Hat Business Value	0.11	0.013	1.192	1.145	-4.838	0.718	53
TalkingData Mobile User Demographics	0.111	0.011	1.07	0.067	-1.961	0.218	77
Outbrain Click Prediction	0.176	0.014	1.537	0.571	-0.616	0.341	108
The Nature Conservancy Fisheries Monitoring	0.128	0.009	0.919	0.034	-4.135	0.204	164
Dstl Satellite Imagery Feature Detection	0.07	0.007	1.241	0.19	-2.788	0.241	76
Cdiscounts Image Classification Challenge	0.128	0.009	0.816	0.041	-5.158	0.169	141
Recruit Restaurant Visitor Forecasting	0.145	0.011	0.911	0.069	-4.742	0.199	120
Statoil/C-CORE Iceberg Classifier Challenge	0.09	0.01	1.123	0.118	-1.046	0.403	70
TrackML Particle Tracking Challenge	0.076	0.007	1.424	0.264	-3.937	0.157	90
Santa Gift Matching Challenge	0.121	0.012	0.888	0.046	-4.131	0.14	74
Google Cloud & NCAA ML Competition 2018-Mens	0.077	0.01	1.124	0.206	-4.599	0.329	50
Google Cloud & NCAA ML Competition 2018-Womens	0.27	0.019	0.65	0.037	-5.805	0.169	110
Google AI Open Images - Object Detection Track	0.06	0.007	1	0.043	-5.158	0.166	66
Google AI Open Images - Visual Relationship Track	0.095	0.009	1	0.009	-4.168	0.379	82
Airbus Ship Detection Challenge	0.089	0.01	1.007	0.077	-1.485	0.205	65
Peking University/Baidu - Autonomous Driving	0.121	0.013	0.607	0.027	-5.692	0.265	71

Notes: SE stands for asymptotic standard errors.

Figure A.4: Are players matching up with players who have similar observed outcomes?



Notes: An observation is a multiplayer team. Panel A plots the number of kernels (also known as notebooks) posted by the team members in the past. Kernels are code that players can post so that any user can make use of it. Panel B plots the number of votes earned by the kernels posted by the different team members. Panel C plots the number of discussion board messages posted by the team members in the past. Panel D plots the number of past participations where the team members finished in the top 30 positions.

Table A.9: The impact of performance feedback noise on team formation outcomes: Player-level estimates

	Number of submissions prior to team formation (in logs) (1)	Time of team formation (in logs) (2)
Feedback precision (in St. Dev.)	-0.037** (0.016)	-0.062*** (0.018)
Observations	4,410	4,410
R^2	0.201	0.043

Notes: Robust standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. An observation is a competition–player combination over the sample of team players. “Feedback precision” is a measure in $[0,100]$ readily available in the data. When it is 100, the public and private scores take the same value; when it is 0, the public score is uncorrelated with the private score (see Section 2). We standardize it to simplify the interpretation of our results (the mean and standard deviation before standardizing are 31.6 and 23.4, respectively). All specifications include contest-level controls (i.e., total reward quantity, number of prizes, maximum daily submissions, contest length, dataset size, image data indicator) and player-level controls (i.e., public score of first submission, number of past competitions). Column 1 further controls for the time of the team’s formation, where time is the fraction of the contest time elapsed at the time of team formation.

B Kaggle Users Interviews

To complement our analysis, we informally interview some Kaggle participants with teamwork experience to inquire about team formation, asking them: “How concerned are you that your teammate will not be a good match?” We reproduce verbatim answers below, which align with our findings on screening potential teammates.

“In general, teamwork on kaggle works the following way: At the beginning of the competition everybody participates alone. A few weeks before the end of the competition, you look for somebody close to you on a leaderboard and team up with them. You share your solutions, discuss all the ideas, and decide what to do next. Sometimes everybody brainstorms and works on the new ideas together, sometimes everybody continues to improve their solutions, and then combine them.”

“I just want to team up with someone smart who I’ll enjoy collaborating with. If they’ve done well in other competitions, that’s good enough. If they are doing well in the same competition, it could be do to noise.”

“I would team up with a person only if I am very sure that I will learn something from that person. I would check that person LinkedIn profile and would also have conversations with that person over call before teaming up. LinkedIn and their previous kaggle work can serve as good indicator. Also during the call, I ask them what have they done so far in the competition. I decide based on the answers which they give to this question”

“previous experience at kaggle, posts in the current competition, and the current results. Also it is very important if I already participated in another competition with the person. So I know the capabilities of the person, and how hard he/she can work.”

C Derivation of Value Functions of Structural Model

In this section, we provide further details about the value functions of the different types of players in the structural model.

Team, Leader. The interim value of a player that is a member of the team leading the competition (denoted team i) is

$$\begin{aligned} L_{s,n}^{\text{team}} = & \mu L_{\text{end}}^{\text{team}} + (1 - \mu) \left[\psi(n) L_{s,n}^{\text{team}} + \frac{2\lambda_1}{N} L_{s,n}^{\text{team,own}} + \frac{2(n^a - 1)}{N} \lambda_1 L_{s,n}^{\text{team,rival team}} \right. \\ & \left. + \frac{n^{\text{sp}}}{N} \lambda_1 L_{s,n}^{\text{team,rival sp}} + \frac{n^{\text{sp}}}{N} \lambda_2 L_{s,n}^{\text{team, team forms}} \right] \end{aligned}$$

In this expression, with probability μ , the contest ends and team i receives continuation value $L_{\text{end}}^{\text{team}}$. If the contest does not end, which occurs with probability $1 - \mu$, there are 5 cases. (1) With probability $\psi(n)$, none of the active players is selected to make a submission and none of the solo players can choose to form a team and each member of team i receives $L_{s,n}^{\text{team}}$. (2) With probability $2\lambda_1/N$, one of the members of team i is selected to make a submission, and each member of team i receives $L_{s,n}^{\text{team,own}}$. (3) With probability $\frac{2(n^a - 1)}{N} \lambda_1$, one of the players in a rival team is selected to make a submission, and each member of team i receives $L_{s,n}^{\text{team,rival team}}$. (4) With probability $\frac{n^{\text{sp}}}{N} \lambda_1$, one of the solo players is selected to make a submission, and each member of team i receives $L_{s,n}^{\text{team,rival sp}}$. Lastly, with probability $\frac{n^{\text{sp}}}{N} \lambda_2$, one of the solo players can choose to form a team, and each member of team i receives $L_{s,n}^{\text{team, team forms}}$. The expressions for these values are given by

$$\begin{aligned} \psi(n) &= (1 - \lambda_1 - \lambda_2 + 2\lambda_1 n^f / N + 2\lambda_2 (n^a + n^f) / N), \\ L_{s,n}^{\text{team,own}} &= q^{\text{team}}(s) L_{s',n}^{\text{team}} + (1 - q^{\text{team}}(s)) L_{s,n}^{\text{team}}, \\ L_{s,n}^{\text{team,rival team}} &= q^{\text{team}}(s) F_{s',n}^{\text{team}} + (1 - q^{\text{team}}(s)) L_{s,n}^{\text{team}}, \\ L_{s,n}^{\text{team,rival sp}} &= q^{\text{sp}}(s) F_{s',n}^{\text{team}} + (1 - q^{\text{sp}}(s)) L_{s,n}^{\text{team}}, \\ L_{s,n}^{\text{team, team forms}} &= p_{s,n} [\gamma L_{s,(n^{\text{sp}} - 2, n^a + 1, n^f)}^{\text{team}} + (1 - \gamma) L_{s,(n^{\text{sp}} - 2, n^a, n^f + 1)}^{\text{team}}] + (1 - p_{s,n}) L_{s,n}^{\text{team}}, \end{aligned}$$

where $p_{s,n}$ is the conditional probability that a solo player decides to form a team, which is the equilibrium object given by [Equation 4](#). In $L_{s,n}^{\text{team, team forms}}$, with probability γ the composition of teams and solo players changes: there will be one more team and two fewer solo players. With probability $1 - \gamma$, two solo players become “inactive” and the number of failed teams increase by one. The last term, $(1 - p_{s,n}) L_{s,n}^{\text{team}}$, corresponds to the case where a solo player can form a team but chooses not to do so. In these expressions, whenever a

player makes a submission, the player becomes the leader of the competition with probability $q^{\text{team}}(s)$ if the player is in a team, and with probability $q^{\text{sp}}(s)$ if the player is a solo player.

Team, Follower. The interim value of a follower team is

$$F_{s,n}^{\text{team}} = (1 - \mu) \left[\psi(n) F_{s,n}^{\text{team}} + \frac{2\lambda_1}{N} F_{s,n}^{\text{team,own}} + \frac{2(n^a - 1)}{N} \lambda_1 F_{s,n}^{\text{team,rival team}} + \frac{n^{\text{sp}}}{N} \lambda_1 F_{s,n}^{\text{team,rival sp}} + \frac{n^{\text{sp}}}{N} \lambda_2 F_{s,n}^{\text{team, team forms}} \right]$$

When the contest does not end, there are 5 cases analogous to the cases for a team leading the competition. The expressions for these values are given by

$$\begin{aligned} \psi(n) &= (1 - \lambda_1 - \lambda_2 + 2\lambda_1 n^f / N + 2\lambda_2 (n^a + n^f) / N), \\ F_{s,n}^{\text{team,own}} &= q^{\text{team}}(s) L_{s',n}^{\text{team}} + (1 - q^{\text{team}}(s)) F_{s,n}^{\text{team}}, \\ F_{s,n}^{\text{team,rival team}} &= q^{\text{team}}(s) F_{s',n}^{\text{team}} + (1 - q^{\text{team}}(s)) F_{s,n}^{\text{team}}, \\ F_{s,n}^{\text{team,rival sp}} &= q^{\text{sp}}(s) F_{s',n}^{\text{team}} + (1 - q^{\text{sp}}(s)) F_{s,n}^{\text{team}}, \\ F_{s,n}^{\text{team, team forms}} &= p_{s,n} [\gamma F_{s,(n^{\text{sp}}-2,n^a+1,n^f)}^{\text{team}} + (1 - \gamma) F_{s,(n^{\text{sp}}-2,n^a,n^f+1)}^{\text{team}}] + (1 - p_{s,n}) F_{s,n}^{\text{team}}, \end{aligned}$$

Solo Player, Leader. The interim value of a solo player who leads the competition is

$$L_{s,n}^{\text{sp}} = \mu L_{\text{end}}^{\text{sp}} + (1 - \mu) \left[(\psi(n) + \lambda_2 / N) L_{s,n}^{\text{sp}} + \frac{\lambda_1}{N} L_{s,n}^{\text{sp,own}} + \frac{2n^a}{N} \lambda_1 L_{s,n}^{\text{sp,rival team}} + \frac{(n^{\text{sp}} - 1)}{N} \lambda_1 L_{s,n}^{\text{sp,rival sp}} + \frac{(n^{\text{sp}} - 1)}{N} \lambda_2 L_{s,n}^{\text{sp, team forms}} \right]$$

Again, when the contest does not end, there are 5 cases analogous to the cases for a team leading the competition. The expressions for these values are given by

$$\begin{aligned} \psi(n) &= (1 - \lambda_1 - \lambda_2 + 2\lambda_1 n^f / N + 2\lambda_2 (n^a + n^f) / N), \\ L_{s,n}^{\text{sp,own}} &= q^{\text{team}}(s) L_{s',n}^{\text{sp}} + (1 - q^{\text{team}}(s)) L_{s,n}^{\text{sp}}, \\ L_{s,n}^{\text{sp,rival team}} &= q^{\text{team}}(s) F_{s',n}^{\text{sp}} + (1 - q^{\text{team}}(s)) L_{s,n}^{\text{sp}}, \\ L_{s,n}^{\text{sp,rival sp}} &= q^{\text{sp}}(s) F_{s',n}^{\text{sp}} + (1 - q^{\text{sp}}(s)) L_{s,n}^{\text{sp}}, \\ L_{s,n}^{\text{sp, team forms}} &= p_{s,n} [\gamma L_{s,(n^{\text{sp}}-2,n^a+1,n^f)}^{\text{sp}} + (1 - \gamma) L_{s,(n^{\text{sp}}-2,n^a,n^f+1)}^{\text{sp}}] + (1 - p_{s,n}) L_{s,n}^{\text{sp}}, \end{aligned}$$