
CS 20 PROJECT 2 DOCUMENTATION

H-E-L-L-O Digits

Second Semester, A.Y. 2022-2023

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Lab 2
Group 4

25 June 2023

Link for documentation video:

<https://drive.google.com/drive/folders/1sEoHxDQJxpABbEZbp8wBDdK-M6UXjeik?usp=sharing>

1. Project Overview

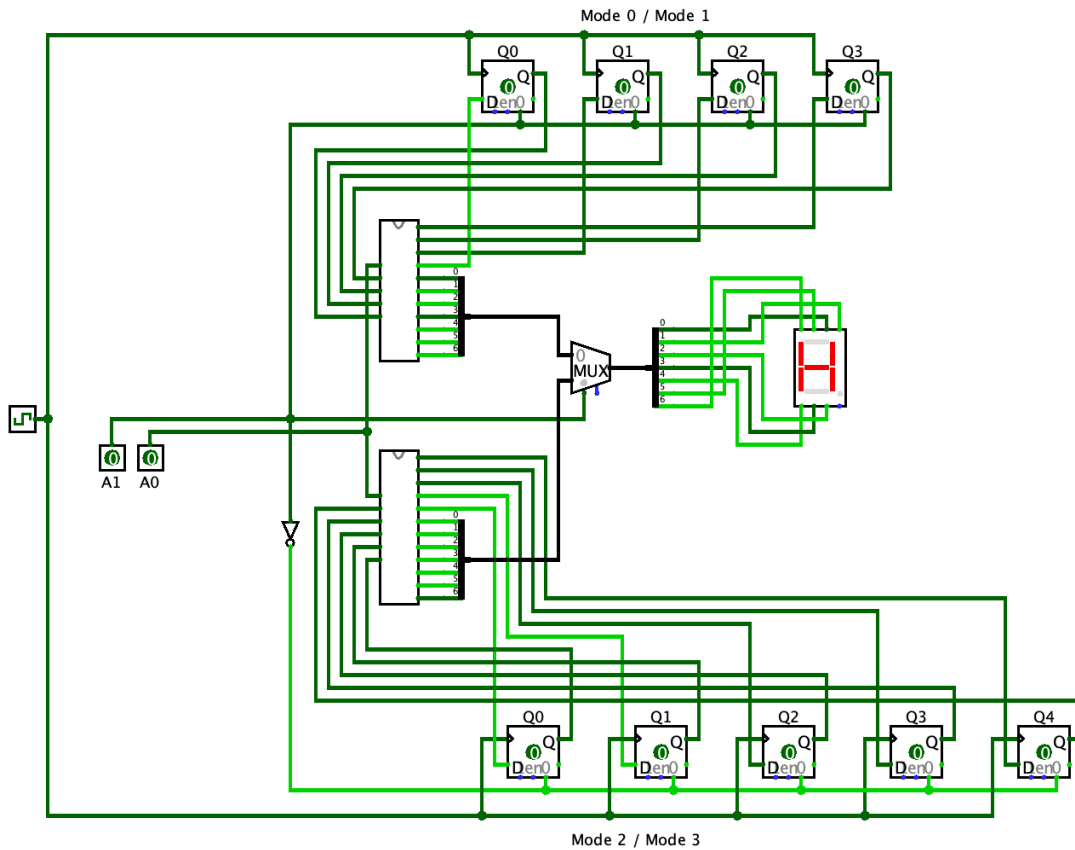


Figure 1. H-E-L-L-O Digits circuit diagram.

This project involves a 4-mode circuit with its functionalities shown in Table 1. The modes are determined using a 2-bit input, i.e. A1 (MSB) and A0. The circuit has two main sub-circuits to implement its modes. In particular, Modes 0/1 and Mode 2/3 are built using two Moore circuits shown in Figure 2.

To combine 4 modes into one main circuit, the 7-segment outputs, A7 to G7, from modes 0/1 and modes 2/3 are grouped together by a splitter and fed into a multiplexer. The output of the multiplexer is connected to a splitter that will then be connected to a 7-segment display. Furthermore, the behavior of the display when switching between modes is as follows:

- Modes 0 and 1 can be selected consecutively without resetting the current display.
- Modes 2 and 3 can be selected consecutively without resetting the current display.
- Changing from mode 0/1 to 2/3, and vice versa, asynchronously resets the display to the default value of the new mode

| | A1 | A0 | Description |
|---------------|----|----|---|
| Mode 0 | 0 | 0 | Display the letter sequence 'H-E-L-L-O' |
| Mode 1 | 0 | 1 | Display the letter sequence 'O-L-L-E-H' |
| Mode 2 | 1 | 0 | Count by 2's |
| Mode 3 | 1 | 1 | Count by 3's |

Table 1. The 4-Mode Circuit Functionalities.

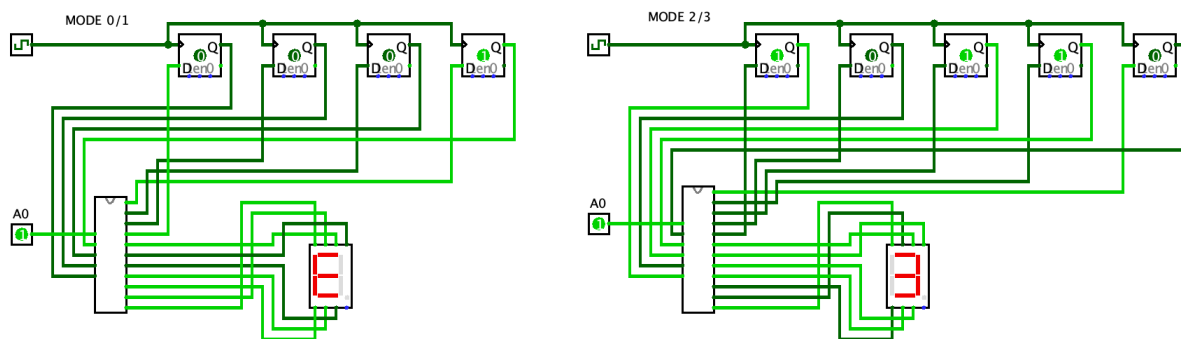


Figure 2. Modes 0/1 and 2/3 implemented using two Moore Circuits.

2. Mode 0/1

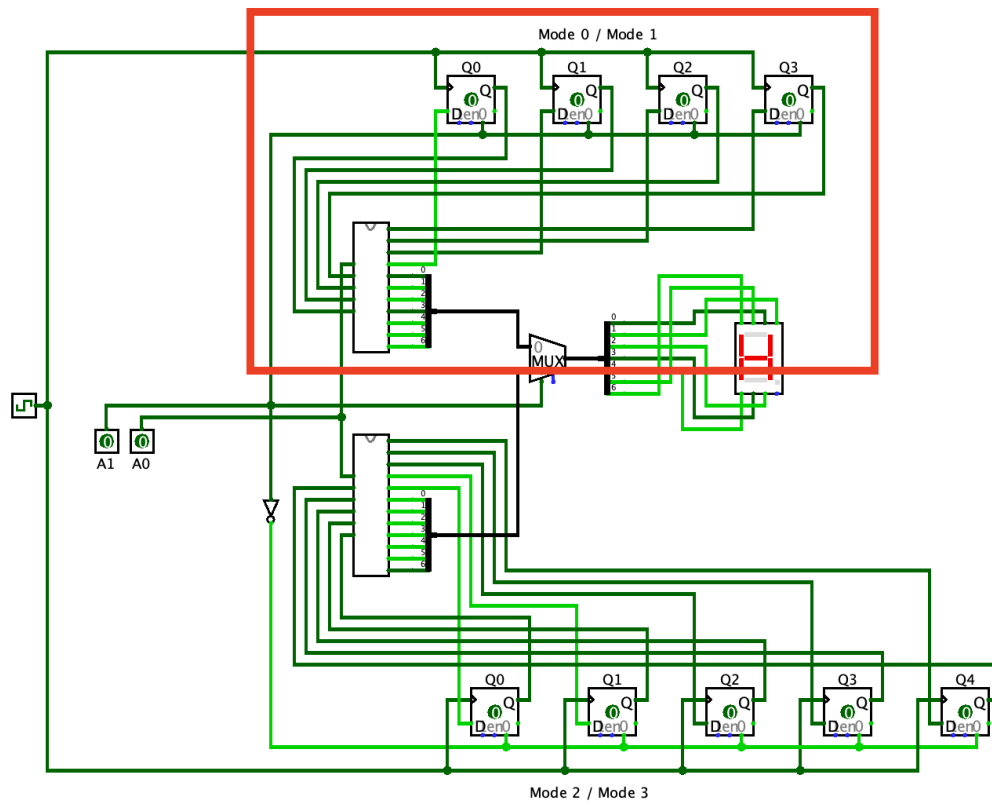


Figure 3. Highlighted mode 0/1 in the main circuit.

The Mode 0/1 sub-circuit displays the letter sequence “H-E-L-L-O” and “O-L-L-E-H”, respectively, depending on the input of A0; hence, we need 10 states to achieve this. For clear visualization of states and its transitions, a Moore state diagram of mode 0/1 can be seen in Figure 4. The truth table of mode 0/1 seen in Table 2 takes in a 5-bit input– A0 (MSB) represents whether it is mode 0 or 1, while Q3, Q2, Q1, and Q0 represent the states. The circuit outputs D3, D2, D1, and D0 represent the next states for the connected D flipflops, and the outputs A7 to G7 represent the inputs for the 7-segment display. Implementation of the mentioned mode 0/1 can be seen in Figure 5.

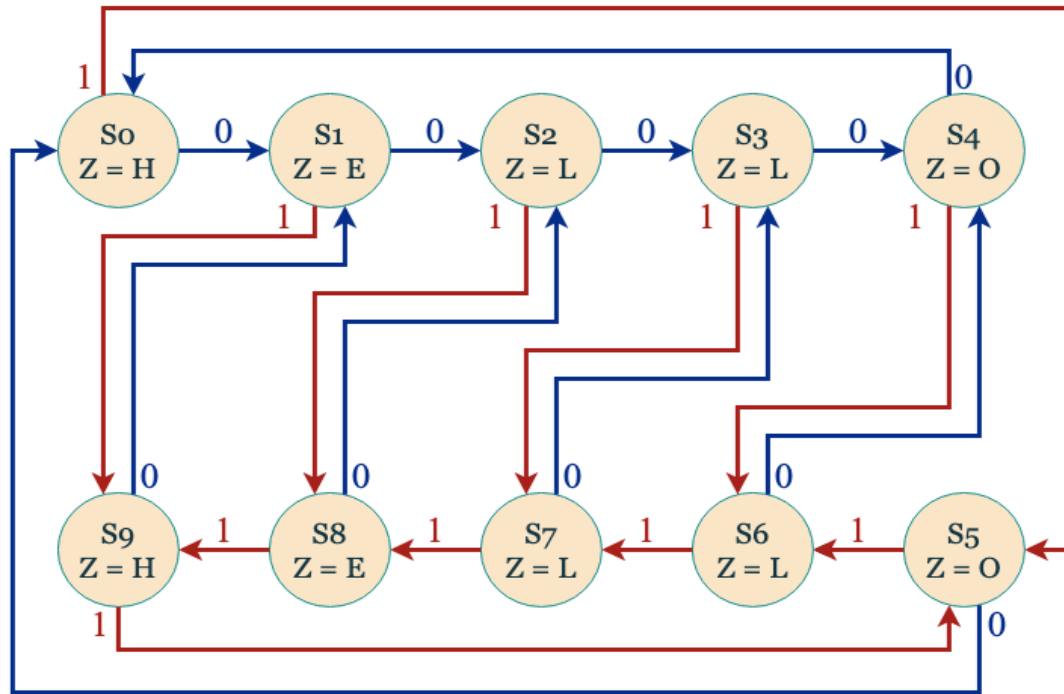


Figure 4. Mode 0/1 Moore state diagram.

| INPUTS | | | | | OUTPUTS | | | | | | | | | | |
|--------|----|----|----|----|---------|----|----|----|----|----|----|----|----|----|----|
| A0 | Q3 | Q2 | Q1 | Q0 | D3 | D2 | D1 | D0 | A7 | B7 | C7 | D7 | E7 | F7 | G7 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 2. Truth table of the sub-circuit mode 0/1.

Furthermore, the following tables are the K-Maps for every output in the truth table. For simplicity, let $A_0 = A$, $Q_3 = B$, $Q_2 = C$, $Q_1 = D$, and $Q_0 = E$. Therefore,

| ABC \ DE | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 000 | 0 | 0 | 0 | 0 |
| 001 | 0 | 0 | 0 | 0 |
| 011 | 0 | 0 | 0 | 0 |
| 010 | 0 | 0 | 0 | 0 |
| 100 | 0 | 1 | 0 | 1 |
| 101 | 0 | 0 | 1 | 0 |
| 111 | 0 | 0 | 0 | 0 |
| 110 | 1 | 0 | 0 | 0 |

Table 3. K-Map for the output D3 with SOP:
 $AB'C'D'E + AB'C'DE' + AB'CDE + ABC'D'E'$

| ABC \ DE | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 000 | 0 | 0 | 1 | 0 |
| 001 | 0 | 0 | 0 | 1 |
| 011 | 0 | 0 | 0 | 0 |
| 010 | 0 | 0 | 0 | 0 |
| 100 | 1 | 0 | 1 | 0 |
| 101 | 1 | 1 | 0 | 1 |
| 111 | 0 | 0 | 0 | 0 |
| 110 | 0 | 1 | 0 | 0 |

Table 4. K-Map for the output D2 with SOP:
 $B'C'DE + B'CDE' + AB'D'E' + AB'CD' + ABC'D'E'$

| ABC \ DE | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 000 | 0 | 1 | 0 | 1 |
| 001 | 0 | 0 | 1 | 0 |
| 011 | 0 | 0 | 0 | 0 |
| 010 | 1 | 0 | 0 | 0 |
| 100 | 1 | 0 | 1 | 0 |
| 101 | 1 | 1 | 0 | 1 |
| 111 | 0 | 0 | 0 | 0 |
| 110 | 0 | 0 | 0 | 0 |

Table 5. K-Map for the output D1 with SOP: $A'B'C'D'E + A'B'C'DE' + A'B'CDE + A'BC'D'E' + AB'D'E' + AB'C'DE + AB'CD' + AB'CE'$.

| ABC \ DE | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 000 | 1 | 0 | 0 | 1 |
| 001 | 0 | 0 | 1 | 0 |
| 011 | 0 | 0 | 0 | 0 |
| 010 | 0 | 1 | 0 | 0 |
| 100 | 0 | 1 | 1 | 0 |
| 101 | 0 | 0 | 0 | 1 |
| 111 | 0 | 0 | 0 | 0 |
| 110 | 1 | 1 | 0 | 0 |

Table 6. K-Map for the output D0 with SOP: $A'B'C'E' + A'B'CDE + BC'D'E + AB'C'E + AB'CDE' + ABC'D'$.

| ABC \ DE | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 000 | 0 | 1 | 0 | 0 |
| 001 | 1 | 1 | 0 | 0 |
| 011 | 0 | 0 | 0 | 0 |
| 010 | 1 | 0 | 0 | 0 |
| 100 | 1 | 1 | 0 | 0 |
| 101 | 1 | 1 | 0 | 0 |
| 111 | 0 | 0 | 0 | 0 |
| 110 | 1 | 0 | 0 | 0 |

Table 7. K-Map for the output A7 with SOP: $B'D'E + B'CD' + AB'D' + BC'D'E'$.

| ABC \ DE | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 000 | 1 | 0 | 0 | 0 |
| 001 | 1 | 1 | 0 | 0 |
| 011 | 0 | 0 | 0 | 0 |
| 010 | 0 | 1 | 0 | 0 |
| 100 | 1 | 0 | 0 | 0 |
| 101 | 1 | 1 | 0 | 0 |
| 111 | 0 | 0 | 0 | 0 |
| 110 | 0 | 1 | 0 | 0 |

Table 8. K-Map for the output B7 with SOP: $B'D'E' + B'CD' + BC'D'E$.

| ABC \ DE | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 000 | 1 | 0 | 0 | 0 |
| 001 | 1 | 1 | 0 | 0 |
| 011 | 0 | 0 | 0 | 0 |
| 010 | 0 | 1 | 0 | 0 |
| 100 | 1 | 0 | 0 | 0 |
| 101 | 1 | 1 | 0 | 0 |
| 111 | 0 | 0 | 0 | 0 |
| 110 | 0 | 1 | 0 | 0 |

Table 9. K-Map for the output C7 with SOP: $B'D'E' + B'CD' + BC'D'E$.

| ABC \ DE | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 000 | 0 | 1 | 1 | 1 |
| 001 | 1 | 1 | 1 | 1 |
| 011 | 0 | 0 | 0 | 0 |
| 010 | 1 | 0 | 0 | 0 |
| 100 | 1 | 1 | 1 | 1 |
| 101 | 1 | 1 | 1 | 1 |
| 111 | 0 | 0 | 0 | 0 |
| 110 | 1 | 0 | 0 | 0 |

Table 10. K-Map for the output D7 with SOP: $B'E + B'D + B'C + AB' + BC'D'E'$.

| ABC \ DE | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 000 | 1 | 1 | 1 | 1 |
| 001 | 1 | 1 | 1 | 1 |
| 011 | 0 | 0 | 0 | 0 |
| 010 | 1 | 1 | 0 | 0 |
| 100 | 1 | 1 | 1 | 1 |
| 101 | 1 | 1 | 1 | 1 |
| 111 | 0 | 0 | 0 | 0 |
| 110 | 1 | 1 | 0 | 0 |

Table 11. K-Map for the output E7 with SOP: $B' + C'D'$.

| ABC \ DE | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 000 | 1 | 1 | 1 | 1 |
| 001 | 1 | 1 | 1 | 1 |
| 011 | 0 | 0 | 0 | 0 |
| 010 | 1 | 1 | 0 | 0 |
| 100 | 1 | 1 | 1 | 1 |
| 101 | 1 | 1 | 1 | 1 |
| 111 | 0 | 0 | 0 | 0 |
| 110 | 1 | 1 | 0 | 0 |

Table 12. K-Map for the output F7 with SOP: $B' + C'D'$.

| ABC \ DE | 00 | 01 | 11 | 10 |
|----------|----|----|----|----|
| 000 | 1 | 1 | 0 | 0 |
| 001 | 0 | 0 | 0 | 0 |
| 011 | 0 | 0 | 0 | 0 |
| 010 | 1 | 1 | 0 | 0 |
| 100 | 0 | 1 | 0 | 0 |
| 101 | 0 | 0 | 0 | 0 |
| 111 | 0 | 0 | 0 | 0 |
| 110 | 1 | 1 | 0 | 0 |

Table 13. K-Map for the output G7 with SOP: $A'C'D' + C'D'E + BC'D'$.

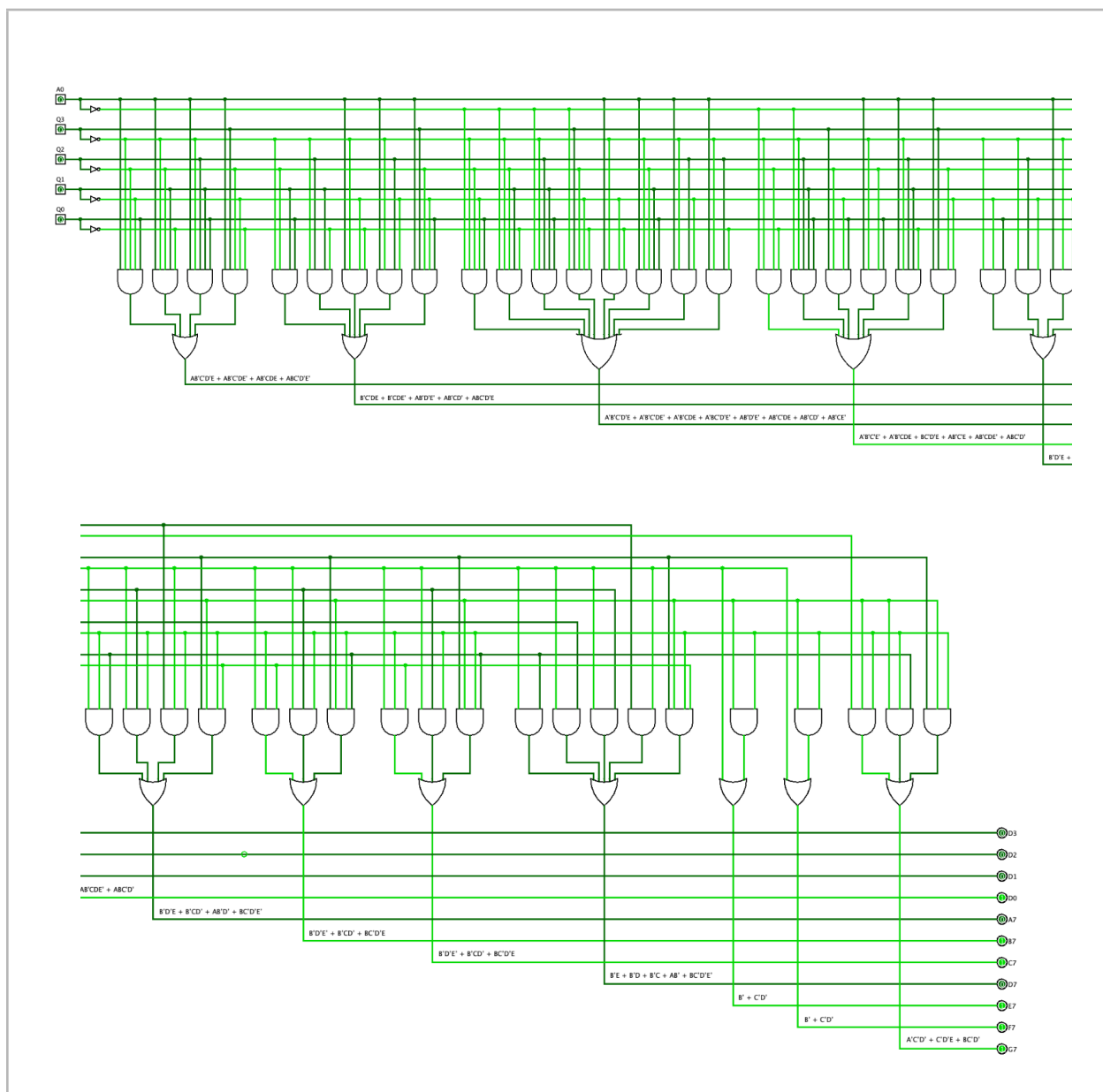


Figure 5. Circuit implementation of mode 0/1 sub-circuit.

3. Mode 2/3

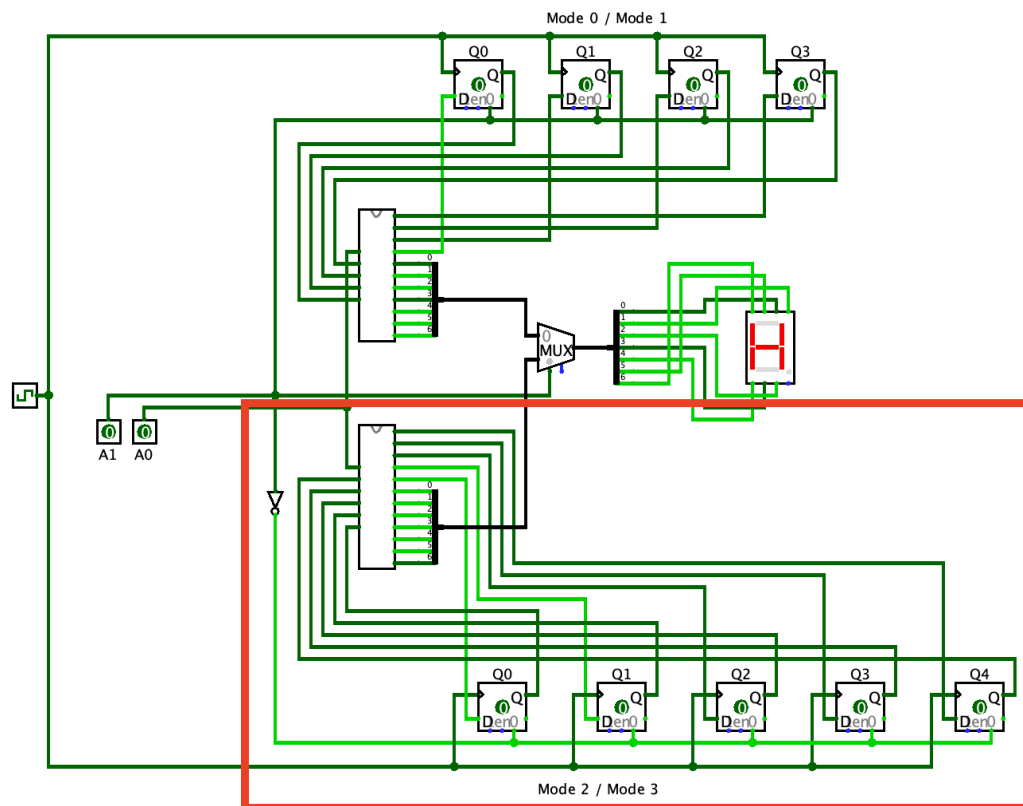


Figure 6. Highlighted Mode 2/3 in Main Circuit.

The Mode 2/3 sub-circuit displays count by 2's and by 3's, respectively, depending on the input of A0.; hence, we need 20 states to achieve this. For clear visualization of states and its transitions, a Moore state diagram of mode 2/3 can be seen in Figure 6. The truth table of mode 2/3, seen in Table 14, takes in a 6-bit input – A0 (MSB) represents whether it is mode 2 or 3, while Q4, Q3, Q2, Q1, and Q0 represent the states. The circuit outputs D4, D3, D2, D1, and D0 represent the next states for the connected D flipflops, and the outputs A7 to G7 represent the inputs for the 7-segment display.

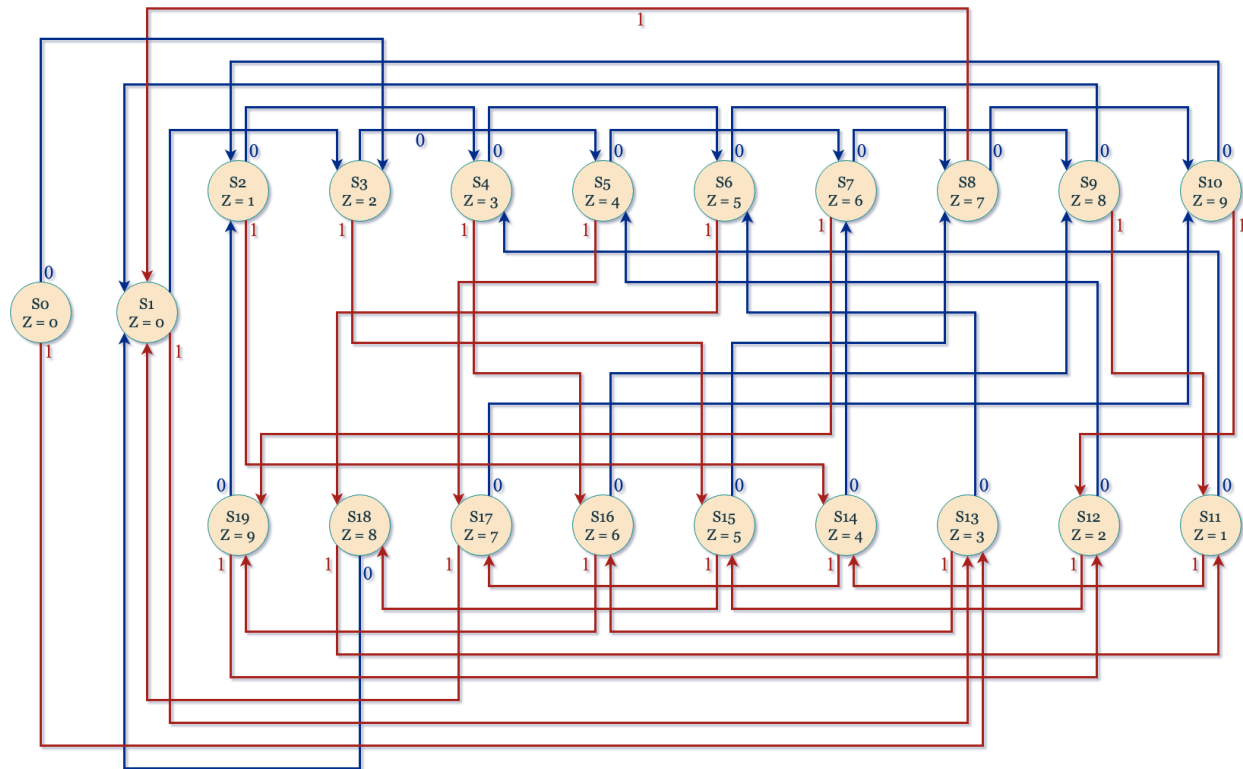


Figure 7. Mode 2/3 Moore State Diagram.

| INPUTS | | | | | | OUTPUTS | | | | | | | | | | | |
|--------|----|----|----|----|----|---------|----|----|----|----|----|----|----|----|----|----|----|
| A0 | Q4 | Q3 | Q2 | Q1 | Q0 | D4 | D3 | D2 | D1 | D0 | A7 | B7 | C7 | D7 | E7 | F7 | G7 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 14. Truth table of sub-circuit mode 2/3.

The following tables are the K-Maps for every output in the truth table. For simplicity, let $A0 = A$, $Q4 = B$, $Q3 = C$, $Q2 = D$, $Q1 = E$, and $Q0 = F$. Therefore,

| ABC \ DEF | 000 | 001 | 011 | 010 | 100 | 101 | 111 | 110 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|
| 000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 001 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 011 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 010 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 100 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 101 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 111 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 110 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 15. K-Map for the output D4 with SOP: $AB'C'D + AB'DF + AB'DE + ABC'D'E'F'$.

| ABC \ DEF | 000 | 001 | 011 | 010 | 100 | 101 | 111 | 110 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|
| 000 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 001 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 011 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 010 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 100 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 101 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 111 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 110 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |

Table 16. K-Map for the output D3 with SOP: $AB'C'D' + AB'D'F + AB'D'E + AC'D'E + A'B'C'DE + A'B'DEF + A'BC'D'E' + A'B'CD'E'F' + AB'CDE'F'$.

| ABC \ DEF | 000 | 001 | 011 | 010 | 100 | 101 | 111 | 110 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|
| 000 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 001 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| 011 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 010 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 100 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 101 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 111 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 110 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |

Table 17. K-Map for the output D2 with SOP: $B'C'D'E + B'D'EF + A'B'DE' + AB'C'D' + AB'D'E + A'B'CDF' + B'CDE'F' + AC'D'EF$.

| ABC \ DEF | 000 | 001 | 011 | 010 | 100 | 101 | 111 | 110 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|
| 000 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 001 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| 011 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 010 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 100 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 101 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 111 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 110 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |

Table 18. K-Map for the output D1 with SOP: $A'B'C'E' + AB'C'E + AB'EF + A'B'D'E'F' + A'B'DE'F + A'B'CEF' + A'BC'D'F + AB'CD'F + ABC'D'F' + AB'CDE'F'$.

| ABC \ DEF | 000 | 001 | 011 | 010 | 100 | 101 | 111 | 110 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|
| 000 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 001 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 011 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 010 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 100 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 101 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 111 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 110 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |

Table 19. K-Map for the output D0 with SOP:
 $C'D'E'F' + B'C'F + B'D'E'F + B'CDF' + BC'D'F' + AB'D'E' + AC'D'E'$.

| ABC \ DEF | 000 | 001 | 011 | 010 | 100 | 101 | 111 | 110 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|
| 000 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| 001 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 011 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 010 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 100 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| 101 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 111 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 110 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |

Table 20. K-Map for the output A7 with SOP:
 $B'D'E' + B'C'EF + B'C'DF' + B'CD'F' + B'CE' + B'CDF + BC'D'$.

| ABC \ DEF | 000 | 001 | 011 | 010 | 100 | 101 | 111 | 110 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|
| 000 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 001 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |
| 011 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 010 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 100 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 101 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |
| 111 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 110 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |

Table 21. K-Map for the output B7 with SOP: $B'D' + B'E' + C'D'F + C'D'E + B'CF'$.

| ABC \ DEF | 000 | 001 | 011 | 010 | 100 | 101 | 111 | 110 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|
| 000 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 001 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 011 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 010 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 100 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 101 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 111 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 110 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |

Table 22. K-Map for the output C7 with SOP: $B'C'E' + B'C'F' + B'E'F + B'DE + B'CD' + BC'D'$.

| ABC \ DEF | 000 | 001 | 011 | 010 | 100 | 101 | 111 | 110 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|
| 000 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| 001 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 011 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 010 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 100 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| 101 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 111 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 110 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |

Table 23. K-Map for the output D7 with SOP:

$$B'C'D'E' + B'C'EF + B'C'DF' + B'DE'F' + B'CE'F + B'CDF + BC'D'F'.$$

| ABC \ DEF | 000 | 001 | 011 | 010 | 100 | 101 | 111 | 110 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|
| 000 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 001 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 011 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 010 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 100 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 101 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 111 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 110 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |

Table 24. K-Map for the output E7 with SOP:

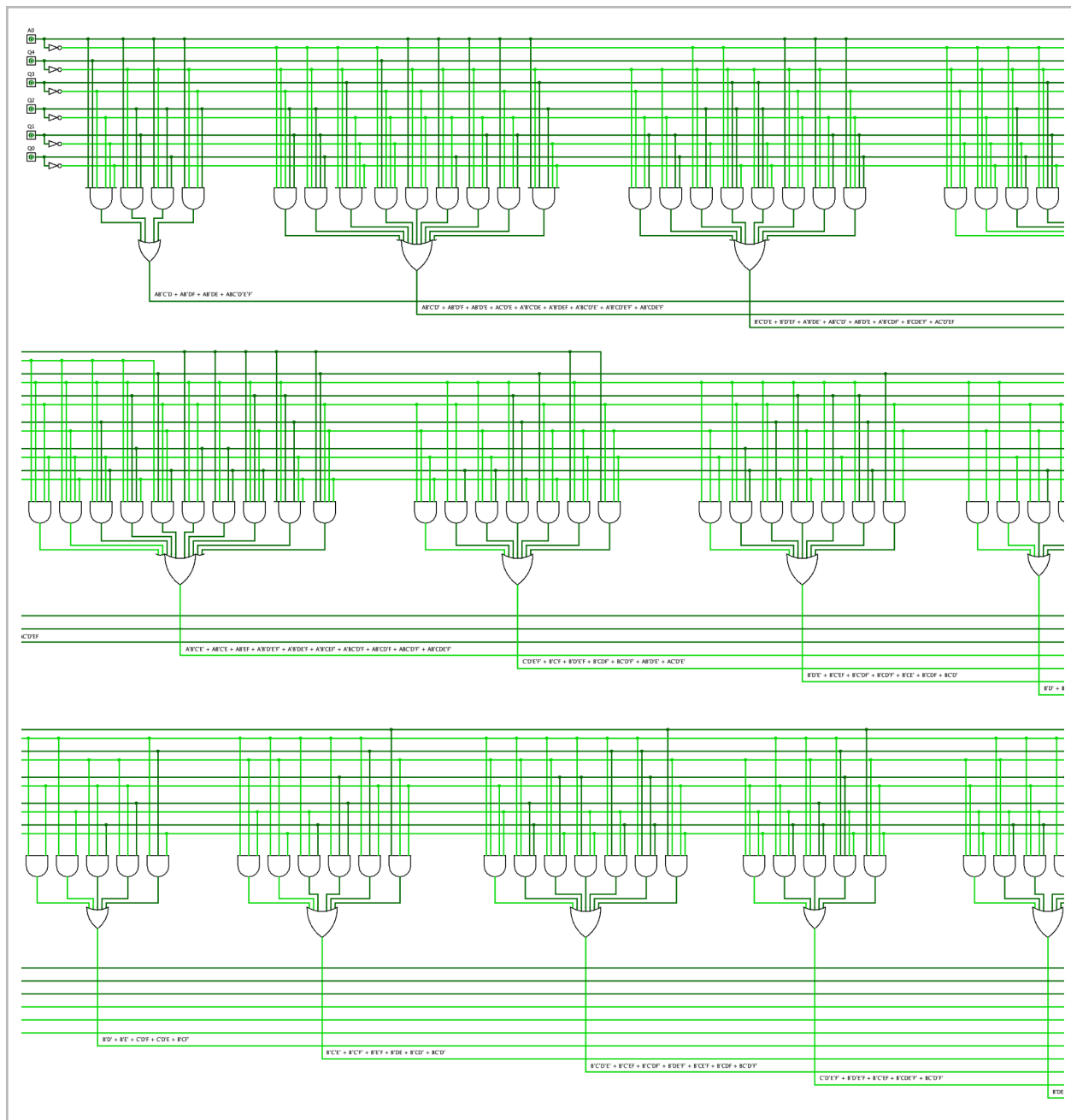
$$C'D'E'F' + B'D'E'F + B'C'EF + B'CDE'F' + BC'D'F'.$$

| ABC \ DEF | 000 | 001 | 011 | 010 | 100 | 101 | 111 | 110 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|
| 000 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 001 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 011 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 010 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 100 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 101 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 111 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 110 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |

Table 25. K-Map for the output F7 with SOP: $B'DE + C'D'E'F' + B'C'E'F + B'D'E'F + B'CEF' + BC'D'E$.

| ABC \ DEF | 000 | 001 | 011 | 010 | 100 | 101 | 111 | 110 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|
| 000 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 001 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 011 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 010 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 100 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 101 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 111 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 110 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |

Table 26. K-Map for the output G7 with SOP: $B'D + C'D'EF + B'CE'F + B'CEF' + BC'D'F'$.



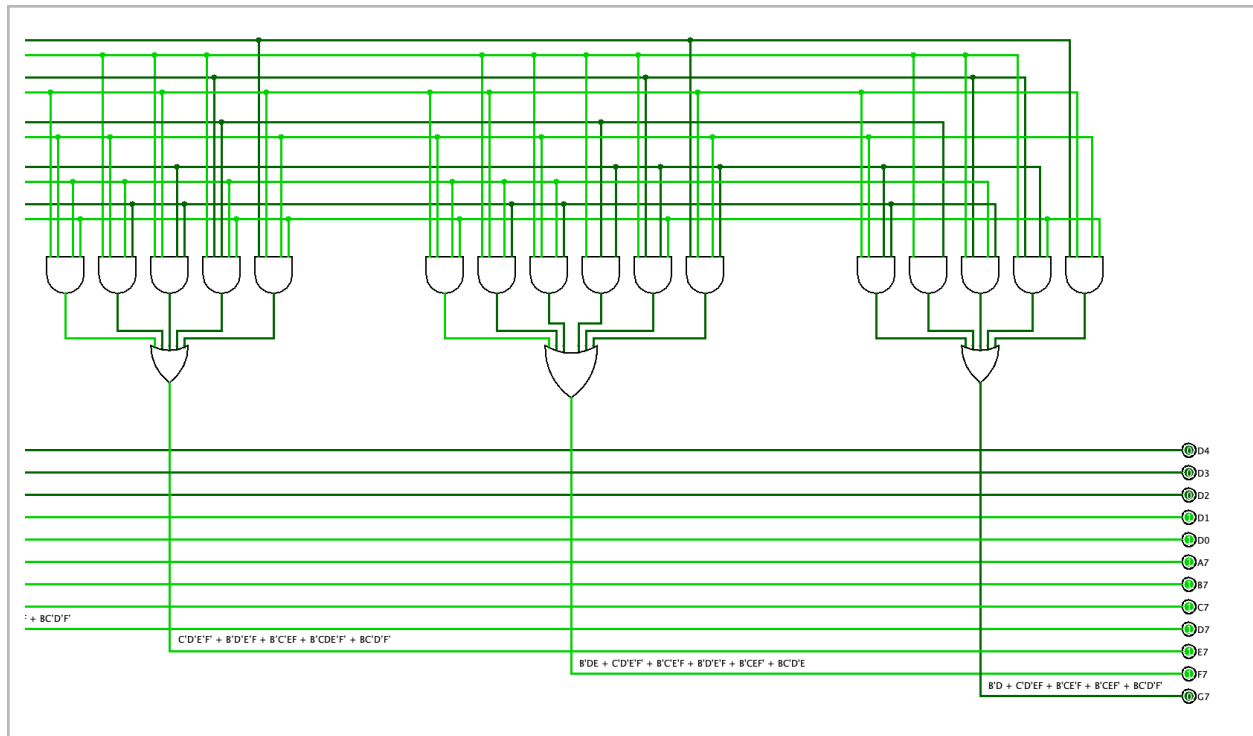


Figure 8. Circuit implementation of mode 2/3 sub-circuit.