Project 1: Simulation of projectile trajectory with Forward Euler method

Gabriela Pyda, Computer Physics

October 19, 2025

1 Introduction

Projectile motion is a classical problem in mechanics. In this laboratory project[1], the trajectory of a point-like inert projectile is studied, assuming that it is launched with initial speed v_0 at the angle θ_0 above the horizontal. In addition to gravity, we include aerodynamic drag and, optionally, altitude-dependent air density corrections. The aim is to analyze the flight trajectory and the horizontal range up to impact at ground level $y = 0$ under different conditions in the calculations.

2 Problem formulation

We denote the mass of the projectile by m and its position by $\mathbf{r}(t) = [x(t), y(t)]^{\mathrm{T}}$ with velocity $\mathbf{v}(t) = [v_x(t), v_y(t)]^{\mathrm{T}}$. The gravitational force is

$$\mathbf{F}_{\mathrm{grav}} = -m\,\mathbf{g}, \qquad \mathbf{g} = [0, -g]. \tag{1}$$

The (quadratic) aerodynamic drag is modeled as

$$\mathbf{F}_{\mathrm{drag}} = -D\,v^2\,\hat{\mathbf{v}}, \qquad v = \sqrt{v_x^2 + v_y^2}, \quad \hat{\mathbf{v}} = \frac{\mathbf{v}}{v}, \tag{2}$$

where D is the drag factor (units: Ns^2/m^2). To account for the altitude dependence of air density, an improved model multiplies the drag by a weight function.

$$w(y) = \left(1 - \frac{a\,y}{T_0}\right)^{\alpha}, \tag{3}$$

so that the drag components become

$$F_{\mathrm{drag},x} = -D\,v\,v_x\,w(y), \tag{4}$$
$$F_{\mathrm{drag},y} = -D\,v\,v_y\,w(y). \tag{5}$$

Using Newton's second law yields the equations of motion

$$\ddot{x} = -\frac{1}{m} D\,v\,v_x\,w(y), \tag{6}$$

$$\ddot{y} = -g - \frac{1}{m} D\,v\,v_y\,w(y). \tag{7}$$

Special cases: for $a = 0$, the altitude correction vanishes ($w(y) = 1$). For $D = 0$, we recover free flight without air resistance.

3 Numerical method

The system of two second-order ODEs is transformed into four first-order ODEs by introducing velocity components as independent variables.

$$\dot{x} = v_x, \tag{8}$$

$$\dot{v}_x = -\frac{1}{m} D\, v\, v_x\, w(y), \tag{9}$$

$$\dot{y} = v_y, \tag{10}$$

$$\dot{v}_y = -g - \frac{1}{m} D\, v\, v_y\, w(y). \tag{11}$$

We integrate these equations using the Forward Euler method. For a time step Δt and a discrete time index i, the update rules are:

$$x_{i+1} = x_i + \Delta t\, v_{x,i}, \tag{12}$$

$$v_{x,i+1} = v_{x,i} + \Delta t \left(-\frac{1}{m} D\, v_i\, v_{x,i}\, w(y_i) \right), \tag{13}$$

$$y_{i+1} = y_i + \Delta t\, v_{y,i}, \tag{14}$$

$$\tag{15}$$

Evaluation of this method in n steps provides the evolution of the state of the projectile.

4 Results

4.1 Simulation of the trajectory without drag force

The first step of the laboratory was to recreate the path of the projectile considering that there are no drag forces existing, therefore, assuming $D = 0.0$. The results for different numbers of iterations of the Forward Euler method and their discrepancy from the exact value are presented in Figure 1.

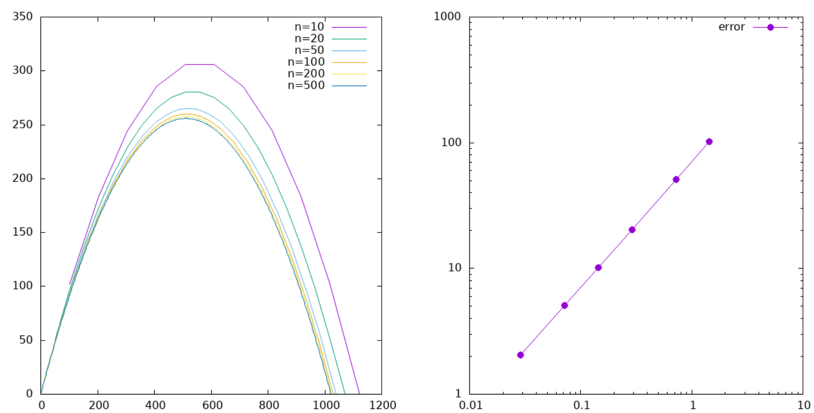

Figure 1: Reconstruction of the trajectories for different numbers of algorithm's iterations n and error of the calculations with respect to the actual value depending on the chosen Δt

It is clearly visible that the increase in the number of iterations significantly improved the quality of the results. The plot of the error agrees with that scenario: for smaller time steps, the discrepancy between the result and the exact value was the smallest.

4.2 Simulation of the trajectory with drag force and without altitude amendment

The next part was to determine the behavior of the projectile's trajectory for different values of constant drag force factor D and the maximal range of the projectile for various initial angle values θ_0. The results are presented in Figure 2 and Figure 3.

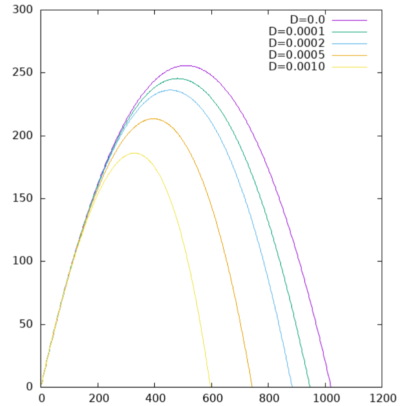

Figure 2: Reconstruction of the trajectories for different constant drag force constants D

Figure 2 clearly shows the predictable result: the range of the projectile depends on the resulting drag force. The higher the drag force factor, the shorter the trajectory.

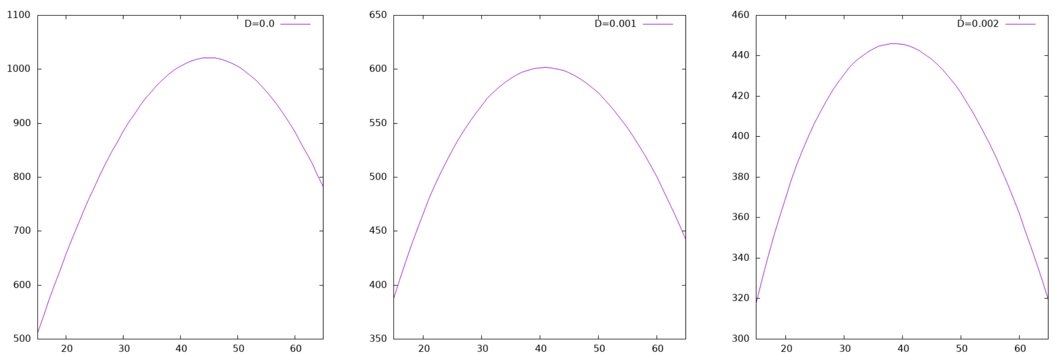

Figure 3: The dependency of the projectile's range on the firing angle θ_0 and drag force factor D

The set of plots in Figure 3 visualizes the impact of the drag force on the choice of firing angle. With an increase in the drag force, it is required to send the projectile with smaller angle θ_0. Moreover, it is again visible that the increase of the drag force results in a smaller maximal range of the projectile.

4.3 Simulation of the trajectory with drag force and altitude amendment

The last part of the laboratory was to compare the trajectory of a real live projectile (comparable to that of the artillery cannon) with or without the correction taken on the altitude change. Its results are shown in Figure 4.

It is clearly visible that the projectile ranges for both test angles are larger when the altitude correction is taken into account. This successfully predicts the change of medium density (and therefore the drag force factor) for larger altitude changes.

5 Conclusions

The laboratory project successfully simulates the trajectory of the projectile considering various factors such as drag force and altitude change.

The Forward Euler method provides sufficient results from 100 of its iterations; nevertheless, the analysis of the obtained errors for multiple iterations' values shows a clear increase of the quality for bigger numbers, which was an expected result.

The drag force has been shown to impact the trajectory curvature and the maximal range of the projectile. Without the altitude correction, the projectile range was decreasing in parallel with the increase of the drag force factor.

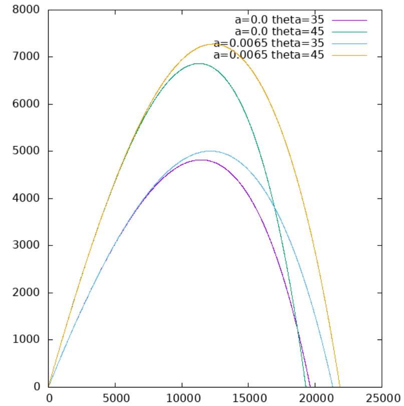

Figure 4: Reconstruction of the trajectories for different firing angles θ_0 and change factor a

What is interesting, yet important, is that for the higher altitudes, the correction on the medium density plays a crucial role in the drag force change. The higher the projectile, the smaller the drag force is; therefore, the actual range of its trajectory is wider than the one calculated without the required altitude amendment.

References

[1] Dr Hab. Eng. Tomasz Chwiej. Simulation of projectile motion with forward euler method. `https://galaxy.agh.edu.pl/~chwiej/comp_phys/labs/1_projectile_launch.pdf`, 2025. Accessed: 2025-10-19.