Research Question:
How does the surface area of a paper airplane affect the lift force that the plane experiences as it is horizontally projected from a certain height above the ground?

Introduction:
I vividly recall the memory of when I was in the fourth grade, holding onto my origami paper airplane that we had just made in class. All the students were lined up as we all threw the paper airplanes we had made as hard as possible. I watched as my airplane flew the farthest and for the longest time, as I was then crowned the champion of this childish contest. As a child, I did not understand what I had done to craft the winning plane. This mystery was not uncovered until I entered physics class, learning about how different forces act on different objects and the way that these laws can answer how airplanes can come to fly in the air. I learned about the effect of a lift force on the projection of an object, and how this would vary between objects. Now, with my understanding of the physical relationships between surface area and lift force, I will endeavour to investigate the relationship between the surface area of the wings of a paper airplane and the lift force that it experiences, measured through its initial horizontal velocity and its range of projection.

Hypothesis:
The hypothesis is that increasing the surface area of the airplane wings will increase the magnitude of lift force experienced. This would therefore counteract the effects of gravity and allow the plane to take longer to fall due to gravity's effects. This is made with the assumption that the paper airplane will be "perfectly" folded; the wings of the plane would be perfectly symmetrical to one another.

The null hypothesis is that the surface area of the paper airplanes will not influence the lift force experienced by the paper airplanes.

Background Information
The paper airplane, during its flight, will be assumed to have four forces acting on it: the thrust force, the drag force, the force of gravity, and the lift force (Connor). Figure 1 below shows a diagram of the forces acting on the paper airplane.

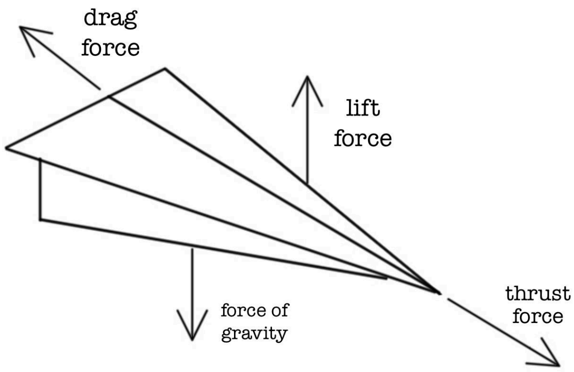

Figure 1: Forces acting on a paper airplane

The thrust force is the force exerted by the source producing its initial velocity. It acts in the direction of motion. In the experiment, the thrust force is produced by an airplane launcher (see Fig. 4) crafted from cardstock and a rubber band ("OrigamiFly").

The paper airplane first gains a thrust force, and then, the drag force acts on the plane in the opposite direction. It is the friction that the plane experiences in the air that is given by:

$$F_{drag} = \frac{1}{2}\rho v^2 C_D A_C$$

This is where ρ is the density of air, v is the speed of the object relative to the air, C_D is the drag coefficient, and A_C is the object's cross-sectional area (Tsokos 52).

In terms of the vertical forces acting on the plane, the force of gravity is relevant for any object in this context. The force of gravity will be acting straight down. The force of gravity is dependent on the mass of the paper airplane and the acceleration due to gravity, $9.81ms^{-2}$. Its equation is given by:

$$F_{gravity} = mg$$

While the force of gravity will be accelerating the paper airplane downwards, a paper airplane is able to remain in flight for an observably longer period of time before landing on the ground a certain range away from its launch. If there is a force downwards produced by the acceleration due to gravity, there must be an opposing force acting on the paper airplane if it were not to fall almost immediately. The force is known as the lift force. The lift force does not act upwards relative to gravity, it will act normally to the surface of the wings. The lift force can be explained by Newton's third law of motion and by Bernoulli's principle. As the airplane flies through the air, the wings of the paper airplane exert a certain force downwards on the passing airflow. Therefore, by Newton's third law, an equal and opposite force will be exerted upwards on the wings of the plane, also known as the lift force. Bernoulli's principle is a more complicated explanation of this phenomenon. Figure 2 below shows how one wing of the paper airplane would flow through the air:

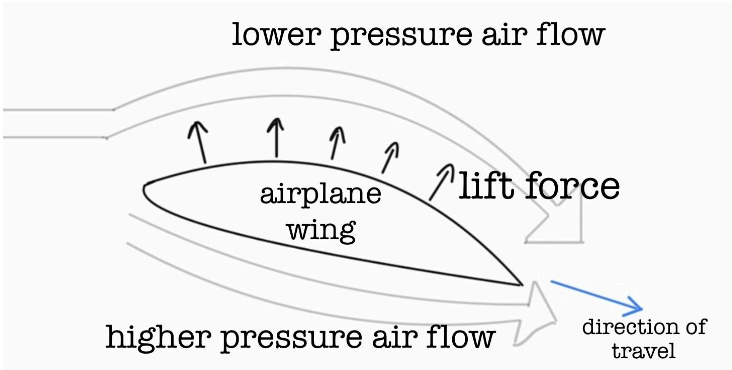

Figure 2: How the particles in the air flow in order to produce lift force

As the figure shows, the shape of the wing will cause the air streamlines to be deflected on the top and the bottom of the wing. The shape of the wing causes air to flow faster on the top, relative to the speed of air on the bottom of the wing. According to Bernoulli's principle of fluid dynamics, the fluid with the greater flow velocity, in this case, the top of the wing will exert less pressure (Tsokos 41). As a result of this pressure difference, the pressure exerted upwards by the slower-moving air will be greater. Hence, this establishes a net upward force on the wings of the paper airplane. This lift force can balance the force of gravity to some extent and allow it to remain in flight for a prolonged period. This lift force is given by:

$$F_{Lift} = C_{Lift}\frac{1}{2}\rho V^2 A$$

Where C_{Lift} is a constant lift coefficient dependent on the shape of the wing, ρ is the air density, V is the initial velocity of the paper airplane, and A is the surface area (Connor). In this experiment, the variable of surface area will be manipulated, ultimately varying the pressure exerted on the wings and the lift force of the paper airplanes. Velocity in the horizontal direction is assumed to be a constant variable because the paper airplane is being launched horizontally with no vertical component of the initial velocity (Tsokos 45). While all other variables are kept constant, the equation can then be rewritten as a constant multiplied by the surface area of the wings:

$$F_{Lift} = CA$$

As the lift force is acting upwards, normal to the wings of the paper airplane and the force of gravity is acting vertically downwards, the net vertical force on the paper airplane will be:

$$F_{Net} = ma_y = mg\text{-}CA$$

In order to determine how the range of projection relates to the lift force, and hence the surface area of the plane's wings, I will model the paper airplane as a particle in projectile motion, ignoring the effects of the drag force, so that the initial horizontal velocity remains constant.

$t = \frac{d_x}{u_x}$

$d_y = u_y t + \frac{1}{2}a_y t^2$

$a_y = \frac{2d_y}{t^2}$

$ma_y = \frac{m2d_y}{t^2}$

$ma_y = \frac{m2d_y}{(\frac{d_x}{u_x})^2}$

$ma_y = \frac{m2d_y u_x^2}{d_x^2}$

$mg - CA = \frac{m2d_y u_x^2}{d_x^2}$

$CA = mg - \frac{m2d_y u_x^2}{d_x^2}$

$CA \propto -\frac{u_x^2}{d_x^2}$

The effect of drag has been assumed to be negligible for the convenience of observing the relationship between the lift force and the surface area of the plane. This is a reasonable assumption that has been drawn according to research cross-referenced from several sources. Below, table 1 cites data from Ng et al. in their experiment to investigate the lift force to drag force ratio.

Table 1: Data comparing the lift force to the drag force of paper airplanes (Ng et al.)

	Average C_L, Lift Coefficient	Average C_D, Drag Coefficient	Ratio of Lift force : Drag Force
Plane 1	0.19	0.04	4.92
Plane 2	0.23	0.05	4.88
Plane 3	0.12	0.02	4.94
Plane 4	0.22	0.04	5.44
Plane 5	0.11	0.02	5.09
Total	0.17	0.03	5.05

Data from Ng et al. suggest that the effects of lift force to drag force are about 5:1. While this difference is not enough to conclude that the drag force has no effect on the paper airplane, it is reasonable to say that the effects of the lift force are much more significant in the context of this investigation for the sake of simplicity. Drag effects are minimal relative to the effects of the lift force because of the area variable involved in their respective equations:

$$F_{drag} = \frac{1}{2}\rho v^2 C_D A_C \text{ and } F_{Lift} = C_{Lift}\frac{1}{2}\rho V^2 A$$

The variable A_c in the drag equation represents the cross-sectional area of the paper airplane whereas the variable A in the lift equation represents the surface area of the paper airplane. When comparing their relative areas, it holds true that drag force is minimal compared to lift force.

Methodology:

Variables:

Independent Variable: The surface area of the wings of the paper airplane. The different surface areas of the square-shaped paper used to fold the paper airplanes were: 16 cm^2, 19 cm^2, 22 cm^2, 25 cm^2, and 28 cm^2 Using paper of these sizes to fold the paper airplanes gives the surface area of the wings to be: 51.4 $cm^2 \pm 0.5$ cm^2, 82.2 $cm^2 \pm 0.5$ cm^2, 97.5 $cm^2 \pm 0.5$ cm^2, 127 $cm^2 \pm 0.5$ cm^2, 160.5 $cm^2 \pm 0.5$ cm^2

Figure 3 shows the dimensions used to calculate the surface area of the paper airplane:

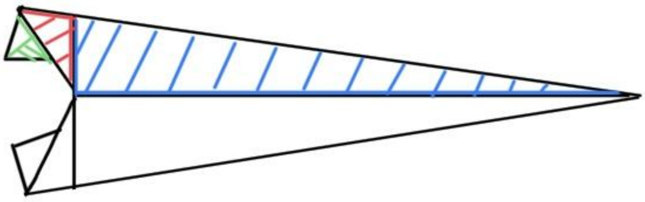

Figure 3: Dimensions measured and used to obtain the surface area of the paper airplane
The uncertainty of the paper airplanes has been assessed given that one measurement has an uncertainty of ± 0.1cm^2. From the diagram, five separate measurements were recorded in order to calculate the surface area. The sum of the uncertainties is therefore ± 0.5 cm^2.

Dependent Variable: The initial horizontal velocity of the paper airplane (ms^{-1}) and the range (m) from the distance of launching to the point that the plane has fallen.

Table 2: Controlled variables

Variable	How it could affect data collection	How it was controlled for
The type of paper used	Different types of paper vary between their thickness and weight. A paper with different dimensions could influence the surface area between the different types of paper.	For the experiment, North American A4 paper was used.
The height that the plane is thrown from	Throwing it from different heights would not account for the time that the plane will be flying. If the plane was thrown from a greater height, it would be in the air for a longer period of time than if it were thrown from a lower height.	The plane will be thrown horizontally from the same height above the ground as this will allow me to isolate the variable of its range without the variation in flight time that the height above the ground could cause.
The thrust force	Variation in the thrust force would affect the initial velocity of the paper airplane. This would affect the horizontal range that the plane would travel.	A simple paper airplane launcher, resembling a slingshot, was created (OrigamiFly). This keeps the thrust force constant as the spring force is equal to kx, given by Hooke's law.

Table 3: Uncontrolled variables

Variable	Why it could not have been controlled
The angle of projection	The angle of projection is difficult to control without a suitable apparatus. With the apparatus used, there may be slight human errors in the angle at which the plane is projected at.
Variations in the mass	The paper airplanes of varying surface areas required paper of varying sizes. This means that the mass of the planes varied by about 2 grams between conditions.
The speed of airflow	Airflow speed depends on the pressure of the air and the temperature of the room. This could not have been completely controlled, as pressure and temperature do not remain constant and were likely to fluctuate between the days of trials. This was controlled to the greatest extent by conducting the experiment in the same room at around the same time each day.
Variation in the surface area	As the paper airplanes are being folded by hand, it is very likely that there is some variation between the folding of the planes. An example of this would be that some folds may not have been perfectly aligned. This would cause variation in the precision of the airplane's surface areas.

Variable	Why it could not have been controlled
The rubber band is not perfectly elastic	While the same rubber band is used throughout the experiment to control for the spring force = kx, this rubber band is not known to obey Hooke's law perfectly. That is, after stretching out the rubber band several times, it is likely to not be able to return to its original position of equilibrium. The rubber will become more and more "stretched out".

Apparatus/Materials:

- Cardstock paper
- Rubber band
- A4 sheets of paper
- Photogates
- Scissors
- Ruler
- Measuring tape

Procedure:

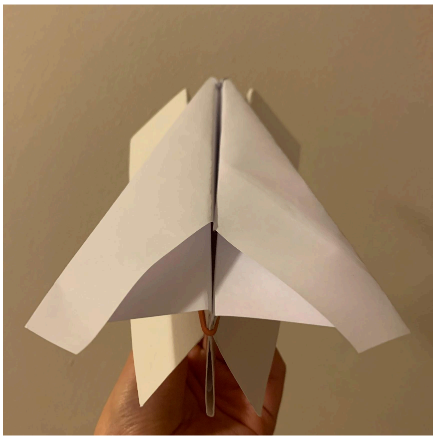

Figure 4: Paper airplane and its launcher

Making the paper airplane and its launcher (OrigamiFly)

1. Fold the square sheet of cardstock paper four times as shown in figure 4
2. Staple a rubber band to the end of the paper
3. The launcher will be made as the rubber band is extended over the opposite end of the folded paper

1. Measure out squares of: 16 cm^2, 19 cm^2, 22 cm^2, 25 cm^2, and 28 cm^2 on A4 paper
2. Cut out the squares of paper
3. An origami paper airplane can be folded by watching an online tutorial

Determine the initial velocity of the paper airplanes

1. Set up two photogates a fixed distance apart as in figure 5
2. Set up the paper airplane in the launcher
3. Launch the paper airplane through the photo gates
4. Determine the initial velocity with the distance between the two photogates divided by the time recorded by the photogate

Determine the range of flight of the paper airplanes

1. Locate a flat, elevated surface, a certain height above the ground. Ensure that there is clear space within the area of experimentation
2. Place the paper airplanes in the launcher one at a time and launch them by releasing the rubber band
3. Measure the horizontal distance travelled from the base of the surface of projection to where the paper airplane landed

Figure 5: Setup for calculating the initial velocity

Ethical, Safety, and Environmental Concerns:

There are very minimal risks to this experiment. The experiment was conducted when there were no people blocking the path of the paper airplane. Not only will people obstruct the path of the planes, but this also eliminates any possible safety concerns that may arise from a person being hit by one of the paper planes. If a person were to be hit by one of the planes, it would not cause them any injuries as the paper planes weigh very little and are quite fragile themselves. An environmental concern to take into consideration is the large use of paper in this experiment. This concern was reduced to the greatest extent possible by recycling or reusing all of the used paper. There are not any serious ethical considerations in this experiment.

Analysis:

Raw Data:

Table 4: Measured range of flight of the paper airplanes

Trial: A, (m^2 ± 0.5 cm^2)	Range of flight d_x, (m ± 0.002 m)				
	1	2	3	4	5
51.4 cm^2	7.624	7.440	7.124	7.475	7.096
82.2 cm^2	5.642	6.119	6.280	6.583	6.554
97.5 cm^2	6.030	5.980	5.775	5.846	6.019
127 cm^2	5.670	5.106	4.980	5.082	5.381
160.5 cm^2	4.953	5.131	5.235	5.282	5.492

Table 3: Measured initial horizontal velocity of the paper airplanes

Trial: A, (m^2 ± 0.5 cm^2)	Initial horizontal velocity u_x, (ms^{-1} ± 0.03)				
	1	2	3	4	5
51.4 cm^2	6.10	5.82	5.84	5.93	6.71
82.2 cm^2	5.43	4.43	5.36	5.17	4.36
97.5 cm^2	4.89	3.69	3.58	3.26	4.06
127 cm^2	2.63	3.59	2.78	3.50	3.35
160.5 cm^2	3.36	2.69	2.26	2.99	2.97

Analysis:

Table 5: Processed data

Average d_x, (m ± 0.02 m)	Average u_x, (ms^{-1} ± 0.03)	$\frac{Average\ u_x}{Average\ d_x}$, $(\frac{1}{s})$	$\sqrt{A}$, (m ± 0.007 m)
7.352	6.08	0.827 ± 0.004	0.0717
6.236	4.95	0.854 ± 0.005	0.0907
5.930	3.90	0.500 ± 0.004	0.0987
5.244	3.17	0.605 ± 0.006	0.113
5.219	2.85	0.547 ± 0.006	0127

Table 6: Sample calculations from row one

Calculation for average d_x: $\frac{\Sigma\ trials}{\#\ of\ trials}$ $= \frac{7.624+7.440+7.124+7.475+7.096}{5}$ $= 7.352$	Assessment for the uncertainty of d_x: The measuring tape used in the experiment had an uncertainty of 0.01 m at both the start of the ruler and the number measured to. The sum of the two uncertainties gives ± 0.02 m. The trials for which the planes slid across the floor were discarded.
Calculation for average u_x: $\frac{\Sigma\ trials}{\#\ of\ trials}$ $= \frac{6.10+5.82+5.84+5.93+6.71}{5}$	Assessment for the uncertainty of u_x: An uncertainty of ± 0.025 s, rounded to ± 0.03 ("What").
Calculation for $\frac{Average\ u_x}{Average\ d_x}$: $= \frac{6.08}{7.352}$	Uncertainty propagation of $\frac{Average\ u_x}{Average\ d_x}$: $\frac{0.03}{6.08} = 0.00493$

$= 0.827$	$\frac{0.002}{7.352} = 0.000272$ $0.00493 + 0.000272 = 0.00521$ $(0.00521)(0.827)$ $= 0.004$
Calculation for the square root of A: $51.4\ cm^2 x \frac{10^{-4} m}{1 cm^2}$ $= 0.00514$ $\sqrt{A}$ $= \sqrt{0.00514}$ $= 0.00717$	Assessment for the uncertainty of $\sqrt{A}$: $\pm$ 0.5 cm^2 (previously calculated) $0.5\ cm^2 x \frac{10^{-4} m}{1 cm^2}$ $= 0.00005$ $\sqrt{0.00005}$ $= 0.007$

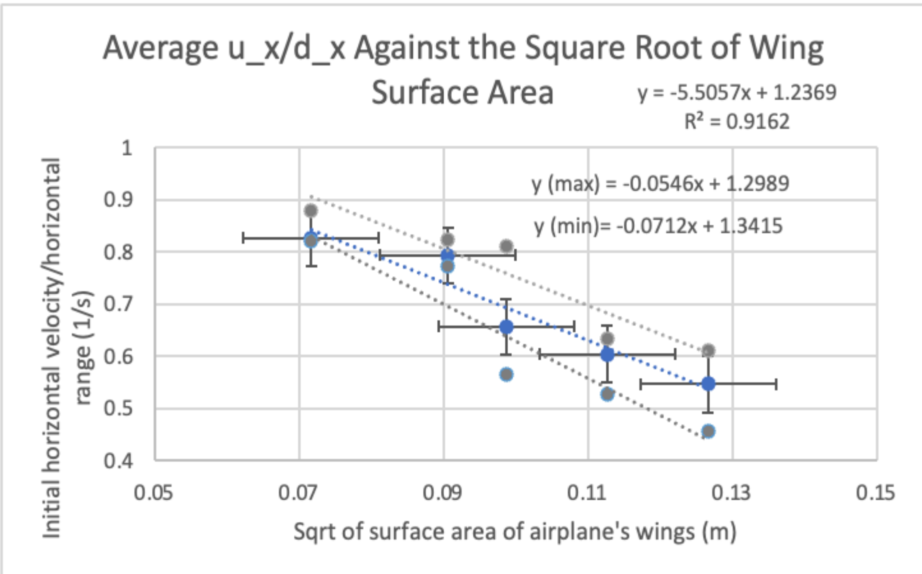

Figure 6: Graph of $\frac{Average\ u_x}{Average\ d_x}$ against $\sqrt{wing\ surface\ area}$

Discussion

The initial horizontal velocity and the range are proportional to the surface area of the paper airplanes given by:

$$\sqrt{A} \propto -\frac{u_x}{d_x} \text{ or } A \propto -\frac{u_x^2}{d_x^2}$$

As denoted earlier, lift force and area are related by:

$$F_{Lift} = C_{Lift}\frac{1}{2}\rho V^2 A$$

Where $C_{Lift}\frac{1}{2}\rho V^2$ are constants. Therefore:

$$F_{Lift} \propto A \propto -\frac{u_x^2}{d_x^2}$$

Therefore, by plotting $-\frac{u_x}{d_x}$ against $\sqrt{A}$, the line of best fit shows a linear relationship. This suggests that the greater the surface area of the paper airplane wings, the greater the lift force experienced. This hypothesis is supported by the data collection plotted in the graph above. As the surface areas of the paper airplanes were increased, the value of $\frac{u_x}{d_x}$ decreased. The smaller the value of $\frac{u_x}{d_x}$, the greater the lift force will be. The line of best fit gives a linear relationship with an R^2 value of 0.9162, suggesting a strongly correlated relationship where the points on the graph fall closely on the line of best fit.

Evaluation

Conclusion

The aim of this experiment was to determine how the surface area of paper airplane wings will affect the lift force experienced, calculated by measuring the plane's initial horizontal velocity and its range of flight. The conclusion can be drawn that the lift force experienced by the paper airplanes was generally proportional to the surface area of the wings. The conclusion supports the hypothesis that the lift force experienced by the plane increases as the surface area of the wings increase. However, it was not necessarily accurate to say that the greater lift force will result in a greater time of flight. This is because lift force is influenced by the initial velocity of the plane as well as its range. The plane could have flown far and fallen fast or vice versa. While the lift force may be greater for the planes with larger surface areas, that is not to say that they flew for a longer period of time.

Table 7: Strengths of the methodology and their significance

Strengths of the methodology	*Significance*
The force exerted (thrust force) on the paper airplane was controlled.	While the initial velocities varied with the planes of various sizes, the initial thrust force being kept constant was significant. This allowed the differences in mass to be overlooked because it was recognized that the horizontal velocities between conditions varied, but were constant for the planes of the same size.
Several variables were controlled including the angle of projection and the height above the ground that the plane was projected from when measuring how far the planes flew.	This allowed for simplicity and accuracy in the calculations. The angle of projection (0°) did not differ between trials in measuring the planes' flight distance. The height above the ground was only constant, simplifying the calculations for the horizontal range of the plane.
When considering the lift force experienced, both the initial velocity and the range of the projection were taken into consideration.	While it is generally hypothesised that the greater the wing span, the further the plane will fly, this is generally untrue considering the variation in the mass of the planes. It was more accurate to measure the effect of increasing wing span on the lift force experienced. This makes the theory and methodology behind this experiment much more relevant because this experiment was able to answer why surface area affects the flight of planes.
The values of the initial horizontal velocity were averaged over 5 trials per condition.	While the initial velocities were not able to be calculated at the same time as its ranges of projection, this problem was controlled to the greatest extent by taking the values across several trials and determining an average velocity per condition.
Discarding the trials in which the plane would glide across the ground after landing.	When the plane would glide across the ground after having hit the floor, it would affect the data of the plane's range, making the values larger than how far the planes flew in reality.

Table 8: Limitations to the methodology and their improvements

Limitations to the methodology	*Improvements to the investigation*
The initial velocity of each trial was not calculated at the same time as the range of projection, and therefore the initial velocity data does not correspond with the data of how far the planes flew.	Several different methodologies were undertaken in an attempt to find a way to calculate both the initial velocity of the airplane and its range at the same time. Motion sensors were used, however, they did not work since both the hand launching the plane and the paper airplane launcher blocked the motion sensor. The option used in the experiment was the use of photogates. The initial velocity was calculated by sending the paper airplane through two photogates a certain distance apart and averaged over 5 trials. To improve this, a suitable apparatus could be built that is able to control precisely the initial velocity. Photogates with much larger widths could also be used as well since the larger airplanes had difficulty passing through the small photogates used, resulting in numerous failed attempts when gathering data.
When calculating the initial velocity of the plane, the angle of projection was not kept entirely constant due to human error. This was due to the way that the photogates were set up, preventing the launcher from being placed on a flat surface.	Using the photogates at hand was not the most suitable option for calculating the initial velocity as aforementioned. Since the photogates were set up so high, it would have been difficult to set up a flat surface between where I stood and the photogates. It would have been equally difficult for me to be able to reach over the flat surface and launch the plane. This can be improved by putting a table in front of the photogates and having two people pull either side of the launcher while standing on either side of the photogates.
The rubber band used in the launcher was not perfectly inextensible. Over several trials, the rubber band likely "loosened up" and would not fully return to its original equilibrium position. This means that the thrust force delivered by the launcher was likely to decrease over trials.	This can be improved by creating a new paper airplane launcher for every few trials that are conducted. This was not done in this experiment because there likely would have been variation between the spring constant of the different rubber bands, which ultimately affects the thrust force as well. Using a stiffer rubber band could have been helpful as it would be less prone to easily being stretched out. Lastly, there are more complex designs for which a launcher could be built that do not require the use of rubber bands.
The smaller paper airplanes were more prone to not being able to fly in a straight trajectory, therefore having greater differences in the ranges measured	Asymmetries in the paper airplanes explain the reasoning behind it not being able to fly in a straight line. This lack of precision in folding is more likely to occur in the smaller planes because it becomes increasingly more difficult to fold over a small area of space ("Fold"). To improve this, the planes could have been folded with the use of rulers and protractors to increase the precision of the folding.
The effects of drag force were ignored in measuring how the surface area of the plane's wings related to the lift force experienced by the planes due to the insignificance of the cross-sectional area of the plane compared to its surface area.	Drag force is friction that the plane will experience during its flight, due to air resistance. This force acting on the plane will reduce how far the plane flies. To improve on this disparity, it must be considered that the area of the planes influences both the lift force and the drag force. The greater the cross-sectional area of the plane, the greater the magnitude of the drag force will be. A formula could be derived to calculate how much the drag force decreases the range of projection for each plane. This measured distance that has been reduced due to the effects of drag force can then be added to the actual measured ranges of the planes. This will give an estimation of the distance that the planes would have travelled without the effects of drag force. The new range can then be used in the derived formula of lift force.

Extensions

The investigation of the phenomenon of lift force and Bernoulli's principle had its strengths in the methodology by keeping the variables involved in the experiment controlled to the greatest extent given the experimental conditions. One key limitation was the simplicity of the apparatus used. The paper airplane launcher was created with cardstock and a rubber band. This resulted in the limitation of the thrust force being inconsistent. Furthermore, because of the way in which the launcher was created and its placement in the setup, it would have been very difficult to calculate both the initial horizontal velocity of the plane and its range of flight at the same time- even more difficult carrying this procedure out alone. Because of this, this experiment can be extended on a greater scale to increase its accuracy. Extensions include the use of a suitable apparatus that is able to control precisely the thrust force. This would also include a setup where the paper airplane is able to successfully pass through the photogates while flying in a straight trajectory. The quality of this experiment can also be extended by changing the material that the planes are made out of. While no significant error was detected during the experiment, using paper to make the planes results in less structural integrity. This could have caused slight problems with the trajectory in which the planes flew. To improve on this, the plane could be constructed out of durable plastics or potentially wood. Lastly, another key limitation of this experiment was the simplicity of the relationship established between the area of the planes and the distance that they would fly. This experiment negated the drag force in order to simplify the calculations for the lift force. A much more complex experiment could be done in order to investigate the effects of lift force on the range that the planes flew.

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