Evaluate the given iterated integral. In addition, sketch the region $D$ that is determined by the regions of integration.

$$\int_{-1}^{3}\int_{x}^{2x+1} xy \, \mathrm{d}y \, \mathrm{d}x$$

Figure 5.43 shows the level curves indicating the varying depth (in feet) of a 25 ft by 50 ft swimming pool. Use a Riemann sum to estimate, to the nearest 100 $\text{ft}^3$, the volume of water that the pool contains.

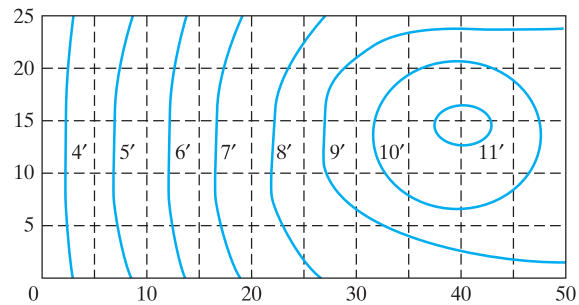


Figure 5.43

Consider the integral

$$\int_0^2 \int_{x^2}^{2x} (2x+1)\, dy\, dx$$

(a) Evaluate this integral

(b) Sketch the region of integration

(c) Write an equivalent iterated integral with the order of integration reversed. Evaluate this new integral and check that your answer agrees with part (a).

Rewrite the sum of iterated integrals as a single iterated integral by reversing the order of integration, and evaluate.

$$\int_0^8 \int_0^{\sqrt{\frac{y}{3}}} y \, \mathrm{d}x \, \mathrm{d}y + \int_8^{12} \int_{\sqrt{y-8}}^{\sqrt{\frac{y}{3}}} y \, \mathrm{d}x \, \mathrm{d}y$$

Evaluate the iterated integral

$$\int_0^2 \int_{\frac{y}{2}}^1 e^{-x^2}\,\mathrm{d}x\,\mathrm{d}y$$

Find the value of $\iiint_W z \, \mathrm{d}V$, where $W = [-1,2]\times[2,5]\times[-3,3]$, without resorting to explicit calculation.

Evaluate the iterated integral

$$\int_{-1}^{2}\int_{1}^{z^2}\int_{0}^{y+z} 3yz^2 \, dx \, dy \, dz$$

Integrate the function $f(x,y,z) = z$ over the region $W$ bounded by $z = 0$, $x^2 + 4y^2 = 4$, and $z = x + 2$.

Consider the integral

$$\int_{-2}^{2}\int_{0}^{\frac{1}{2}\sqrt{4-x^2}}\int_{x^2+3y^2}^{4-y^2}(x^3+y^3)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$$

(a) This integral is equal to a triple integral over a solid region $W$ in $\mathbb{R}^3$. Describe $W$.

(b) Set up an equivalent iterated integral by integrating first with respect to z, then with respect to x, then with respect to y. Do not evaluate your answer.