

Evaluate $\oint_C (x^4 y^5 - 2y) dx + (3x + x^5 y^4) dy$, where C is the oriented curve pictured in Figure 6.29

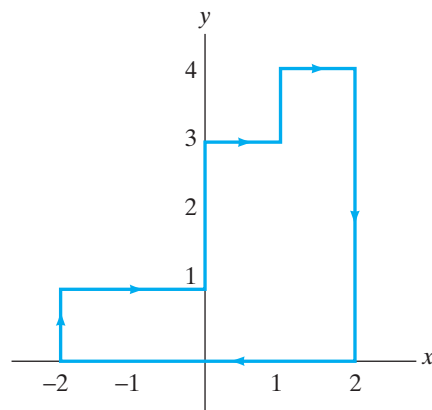


FIGURE 6.29. The oriented curve C of Exercise 13.

- (a) Sketch the curve given parametrically by $x(t) = (1 - t^2, t^3 - t)$.
- (b) Find the area inside the closed loop of the curve.
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Consider the line integral $\int_C z^2 \, dx + 2y \, dy + xz \, dz$.

- (a) Evaluate this integral, where C is the line segment from $(0, 0, 0)$ to $(1, 1, 1)$.
 - (b) Evaluate this integral, where C is the path from $(0, 0, 0)$ to $(1, 1, 1)$ parametrized by $\mathbf{x}(t) = (t, t^2, t^3)$, $0 \leq t \leq 1$.
 - (c) Is the vector field $\mathbf{F} = [z^2, \ 2y, \ xz]$ conservative? Why or why not?
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Determine whether the vector field $\mathbf{F} = [e^{x+y}, e^{xy}]$ conservative. If it is, find a scalar potential function for \mathbf{F} .

Determine whether the vector field $\mathbf{F} = [2x \sin y, x^2 \cos y]$ conservative. If it is, find a scalar potential function for \mathbf{F} .

Let $\mathbf{F} = [x^2, \cos y \sin z, \sin y \cos z]$.

(a) Show that \mathbf{F} is conservative and find a scalar potential function f for \mathbf{F} .

(b) Evaluate $\int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s}$ along the path $\mathbf{x}[0, 1] \rightarrow \mathbb{R}^3$, $\mathbf{x}(t) = (t^2 + 1, e^t, e^{2t})$.

- (a) Determine where the vector field

$$\mathbf{F} = \left[\frac{x + xy^2}{y^2} \quad \frac{x^2 + 1}{y^3} \right]$$

is conservative.

- (b) Determine a scalar potential for \mathbf{F} .

- (c) Find the work done by \mathbf{F} in moving a particle along the parabolic curve $y = 1 + x - x^2$ from $(0, 1)$ to $(1, 1)$.
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