

Find the first- and second-order Taylor polynomials for

$$f(x, y) = \frac{1}{x^2 + y^2 + 1}$$

at  $\mathbf{a} = (1, -1)$ .

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Find the first- and second-order Taylor polynomials for

$$f(x, y) = e^{2x+y}$$

at  $\mathbf{a} = (0, 0)$ .

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Calculate the Hessian matrix  $Hf(\mathbf{a})$  for

$$f(x, y, z) = e^{2x-3y} \sin(5z)$$

at  $\mathbf{a} = (0, 0, 0)$ .

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Determine the total differential of

$$f(x, y) = x^2 y^3$$

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Near the point  $(1, -2, 1)$ , is the function  $g(x, y, z) = x^3 - 2xy + x^2z + 7z$  is most sensitive to changes in  $x$ ,  $y$ , or  $z$ ?

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Identify and determine the nature of the critical points of

$$f(x, y) = y^4 - 2xy^2 + x^3 - x$$

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Identify and determine the nature of the critical points of

$$f(x, y) = e^{-x}(x^2 + 3y^2)$$

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Under what conditions on the constant  $k$  will the function

$$f(x, y) = kx^2 - 2xy + ky^2$$

have a nondegenerate local minimum at  $(0, 0)$ ? What about a local maximum?

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