Evaluate $\oint_C (x^4y^5 - 2y)\,dx + (3x + x^5y^4)\,dy$, where C is the oriented curve pictured in Figure 6.29.

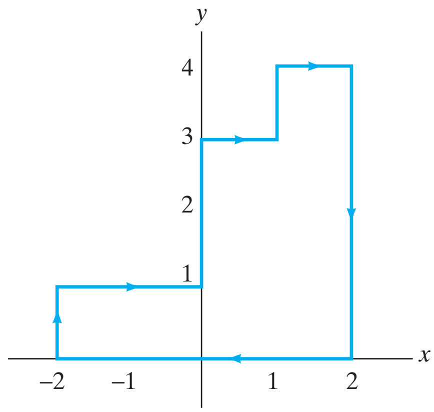

FIGURE 6.29. The oriented curve C of Exercise 13.

(a) Sketch the curve given parametrically by $\mathbf{x}(t) = \left[1 - t^2, \quad t^3 - t\right]$.

(b) Find the area inside the closed loop of the curve.

Consider the line integral $\int_C z^2\,\mathrm{d}x + 2y\,\mathrm{d}y + xz\,\mathrm{d}z$.

(a) Evaluate this integral, where C is the line segment from $(0,0,0)$ to $(1,1,1)$.

(b) Evaluate this integral, where C is the path from $(0,0,0)$ to $(1,1,1)$ parametrized by $\mathbf{x}(t) = \begin{bmatrix} t, & t^2, & t^3 \end{bmatrix}$, $0 \le t \le 1$.

(c) Is the vector field $\mathbf{F} = \begin{bmatrix} z^2, & 2y, & xz \end{bmatrix}$ conservative? Why or why not?

Determine whether the vector field $\mathbf{F} = \begin{bmatrix} e^{x+y}, & e^{xy} \end{bmatrix}$ is conservative. If it is, find a scalar potential function for $\mathbf{F}$.

Determine whether the vector field $\mathbf{F} = \begin{bmatrix} 2x\sin y, & x^2\cos y \end{bmatrix}$ is conservative. If it is, find a scalar potential function for $\mathbf{F}$.

Let $\mathbf{F} = \left[x^2, \quad \cos y \sin z, \quad \sin y \cos z\right]$.

(a) Show that $\mathbf{F}$ is conservative and find a scalar potential function f for $\mathbf{F}$.

(b) Evaluate $\int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s}$ along the path $\mathbf{x} : [0,1] \to \mathbb{R}^3$, $\mathbf{x}(t) = \left[t^2 + 1, \quad e^t, \quad e^{2t}\right]$.

(a) Determine where the vector field

$$\mathbf{F} = \begin{bmatrix} \dfrac{x + xy^2}{y^2} & -\dfrac{x^2+1}{y^3} \end{bmatrix}$$

is conservative.

(b) Determine a scalar potential for **F**.

(c) Find the work done by **F** in moving a particle along the parabolic curve $y = 1 + x - x^2$ from $(0, 1)$ to $(1, 1)$.