Find the first- and second-order Taylor polynomials for

$$f(x,y) = \frac{1}{x^2+y^2+1}$$

at $\mathbf{a} = (1,-1)$.

Find the first- and second-order Taylor polynomials for

$$f(x,y) = e^{2x+y}$$

at $\mathbf{a} = (0,0)$.

Calculate the Hessian matrix $Hf(\mathbf{a})$ for

$$f(x,y,z) = e^{2x-3y}\sin(5z)$$

at $\mathbf{a} = (0,0,0)$.

Determine the total differential of

$$f(x, y) = x^2 y^3$$

Near the point $(1,-2,1)$, is the function $g(x,y,z) = x^3 - 2xy + x^2z + 7z$ is most sensitive to changes in x, y, or z?

Identify and determine the nature of the critical points of

$$f(x,y) = y^4 - 2xy^2 + x^3 - x$$

Identify and determine the nature of the critical points of

$$f(x,y) = e^{-x}(x^2 + 3y^2)$$

Under what conditions on the constant k will the function

$$f(x,y) = kx^2 - 2xy + ky^2$$

have a nondegenerate local minimum at $(0,0)$? What about a local maximum?