A rectangular stick of butter is placed in the microwave oven to melt. When the butter's length is 6 in and its square cross section measures 1.5 in on a side, its length is decreasing at a rate of 0.25 in/ min and its cross-sectional edge is decreasing at a rate of 0.125 in/ min. How fast is the butter melting (i.e., at what rate is the solid volume of butter turning to liquid) at that instant?

Suppose that $z = f(x+y, x-y)$ has continuous partial derivatives with respect to $u = x+y$ and $v = x-y$. Show that

$$\frac{\partial z}{\partial x}\frac{\partial z}{\partial y} = \left(\frac{\partial z}{\partial u}\right)^2 - \left(\frac{\partial z}{\partial v}\right)^2$$

With

$$\mathbf{f}(x,y,z) = (x^2y + y^2z, xyz, e^z),$$
$$\mathbf{g}(t) = (t-2, 3t+7, t^3),$$

calculate $D(\mathbf{f} \circ \mathbf{g})$ in two ways:

(a) By first evaluating $\mathbf{f} \circ \mathbf{g}$

(b) By using the chain rule and the derivative matrices $D\mathbf{f}$ and $D\mathbf{g}$.

Suppose that you are given an equation of the form

$$F(x,y,z) = 0,$$

for example, something like $x^3 z + y\cos z + \frac{\sin y}{z} = 0$. Then we may consider z to be defined implicitly as a function $z(x,y)$.

(a) Use the chain rule to show that if F and $z(x,y)$ are both assumed to be differentiable, then

$$\frac{\partial z}{\partial x} = -\frac{F_x(x,y,z)}{F_z(x,y,z)},$$
$$\frac{\partial z}{\partial y} = -\frac{F_y(x,y,z)}{F_z(x,y,z)}$$

(b) Use part (a) to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ where z is given by the equation $xyz = 2$. Check your result by explicitly solving for z and then calculating the partial derivatives.

Calculate the directional derivative of the given function f at the point $\mathbf{a}$ in the direction parallel to the vector $\mathbf{u}$.

$$f(x,y,z) = xyz, \qquad \mathbf{a} = (-1,0,2), \qquad \mathbf{u} = \frac{2\mathbf{k}-\mathbf{i}}{\sqrt{5}}$$

A ladybug (who is very sensitive to temperature) is crawling on graph paper. She is at the point $(3, 7)$ and notices that if she moves in the **i**-direction, the temperature *increases* at a rate of 3 deg/cm. If she moves in the **j**-direction, she finds that her temperature *decreases* at a rate of 2 deg/cm.

(a) In what direction should the ladybug move if she wants to warm up most rapidly?

(b) In what direction should the ladybug move if she wants to cool off most rapidly?

(c) In what direction should the ladybug move if she desires her temperature *not* to change?

Find an equation for the tangent plane to the surface given by the equation

$$2xz + yz - x^2y + 10 = 0$$

at the point $(x_0, y_0, z_0) = (1, -5, 5)$.