Calculate $\int_{\mathbf{x}} f \, \mathrm{d}s$, where

$$f(x,y,z) = xyz$$
$$\mathbf{x}(t) = (t, 2t, 3t), \quad 0 \le t \le 2$$

Calculate $\int_{\mathbf{x}} \mathbf{F} \cdot \, \mathrm{d}s$, where

$$\mathbf{F}(x, y) = (y + 2, x)$$
$$\mathbf{x}(t) = (\sin t, -\cos t), \quad 0 \le t \le \frac{\pi}{2}$$

Let $\mathbf{F}(x, y) = (x^2 + y, y - x)$ and consider the two paths

$$\mathbf{x}(t) = (t, t^2), \quad 0 \le t \le 1$$

$$\mathbf{y}(t) = (1 - 2t, 4t^2 - 4t + 1), \quad 0 \le t \le \frac{1}{2}$$

(a) Calculate $\int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s}$ and $\int_{\mathbf{y}} \mathbf{F} \cdot d\mathbf{s}$.

(b) By considering the image curves of the paths $\mathbf{x}$ and $\mathbf{y}$, discuss your answers in part (a).

Evaluate $\int_C yz\,\mathrm{d}x - xz\,\mathrm{d}y + xy\,\mathrm{d}z$, where C is the line segment from $(1,1,2)$ to $(5,3,1)$.

Tom Sawyer is whitewashing a picket fence. The bases of the fenceposts are arranged in the xy-plane as the quarter circle $x^2 + y^2 = 25$, $x, y \geq 0$, and the height of the fencepost at point (x, y) is given by $h(x, y) = 10 - x - y$ (units are feet). Use a scalar line integral to find the area of one side of the fence. (See Figure 6.16.)

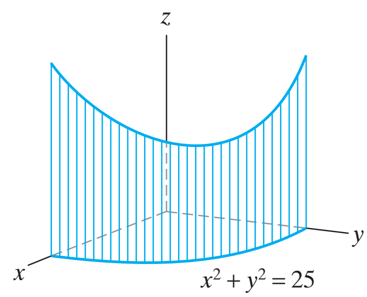

FIGURE 6.16. The picket fence of Exercise 34. The base of the fence is the quarter circle $x^2 + y^2 = 25$, $x, y \geq 0$.

You are traveling through Cleveland, famous for its lake-effect snow in winter that makes driving quite treacherous. Suppose that you are currently located 20 miles due east of Cleveland and are attempting to drive to a point 20 miles due west of Cleveland. Further suppose that if you are s miles from the center of Cleveland, where the weather is the worst, you can drive at a rate of at most $v(s) = 2s + 20$ miles per hour.

(a) How long will the trip take if you drive on a straight-line path directly through Cleveland? (Assume that you always drive at the maximum speed possible.)

(b) How long will the trip take if you avoid the middle of the city by driving along a semicircular path with Cleveland at the center? (Again, assume that you drive at the maximum speed possible.)

(c) Repeat parts (a) and (b), this time using $v(s) = \frac{s^2}{16} + 25$ miles per hour as the maximum speed that you can drive.