Find $\iint_S (x^2 + y^2)\, \mathrm{d}s$, where S is the lateral surface of the cylinder of radius a and height h whose axis is the z-axis.

In Exercises 10-18, let S denote the closed cylinder with bottom given by $z = 0$, top given by $z = 4$, and lateral surface given by the equation $x^2 + y^2 = 9$. Orient S with outward normals. Determine

$$\iint_S \begin{bmatrix} x, & y \end{bmatrix} \cdot d\mathbf{s}$$

In Exercises 19-22, find the flux of the given vector field $\mathbf{F}$ across the upper hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$. Orient the hemisphere with an upward-pointing normal.

$$\mathbf{F} = \left[x^2, \quad xy, \quad xz\right]$$

The glass dome of a futuristic greenhouse is shaped like the surface $z = 8 - 2x^2 - 2y^2$. The greenhouse has a flat dirt floor at $z = 0$. Suppose that the temperature T, at points in and around the greenhouse, varies as

$$T(x, y, z) = x^2 + y^2 + 3(z - 2)^2.$$

Then the temperature gives rise to a **heat flux density field H** given by $\mathbf{H} = -k\nabla T$. (Here k is a positive constant that depends on the insulating properties of the particular medium.) Find the total heat flux outward across the dome and the surface of the ground if $k = 1$ on the glass and $k = 3$ on the ground.

Verify Stokes's theorem for the surface and vector field

S is defined by $x^2 + y^2 + z^2 = 4$, $z \le 0$, oriented by downwards normal;

$$\mathbf{F} = \left[2y - z, \quad x + y^2 - z, \quad 4y - 3x \right]$$

In Exercises 6-9, verify Gauss's theorem for the given three-dimensional region D and vector field $\mathbf{F}$.

$$\mathbf{F} = \begin{bmatrix} x, & y, & z \end{bmatrix},$$
$$D = \left\{(x, y, z) \,\middle|\, 0 \le z \le (9 - x^2 - y^2)\right\}$$

6. If S is the unit sphere centered at the origin, then $\iint_S x^3 \, dS = 0$.

10. $\iint_S \begin{bmatrix} -y, & x \end{bmatrix} \cdot d\mathbf{s} = 0$, where S is the cylinder $x^2 + y^2 = 9$, $0 \leq z \leq 5$.

18. $\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{s}$ has the same value for all piecewise smooth, oriented surfaces S that have the same boundary curve C. *Hint:* think about what the field is doing.