

Find the gradient $\nabla f(\mathbf{a})$ for

$$f(x, y, z) = \cos z \ln(x + y^2)$$

$$\mathbf{a} = \left(e, 0, \frac{\pi}{4} \right)$$

Find the matrix $D\mathbf{f}(\mathbf{a})$ of partial derivatives, for

$$\mathbf{f}(s, t) = (s^2, st, t^2)$$

$$\mathbf{a} = (-1, 1)$$

Find equations for the planes tangent to $z = x^2 - 6x + y^3$ that are parallel to the plane $4x - 12y + z = 7$.

Suppose that you have the following information concerning a differentiable function f :

$$f(2, 3) = 12, \quad f(1.98, 3) = 12.1, \quad f(2, 3.01) = 12.2.$$

- (a) Give an approximate equation for the plane tangent to the graph of f at $(2, 3, 12)$.
-

- (b) Use the result of part (a) to estimate $f(1.98, 2.98)$.
-

Verify the product and quotient rules (Proposition 4.2) for the pair of functions given below.

$$f(x, y) = x^2y + y^3$$

$$g(x, y) = \frac{x}{y}$$

For the function given below determine all second-order partial derivatives (including mixed partials).

$$f(x, y) = x^2 e^y + e^{2z}$$

Let $f(x, y) = ye^{3x}$. Give general formulas for $\frac{\partial^n f}{\partial x^n}$ and $\frac{\partial^n f}{\partial y^n}$ where $n \geq 2$.

The three-dimensional heat equation is the partial differential equation

$$k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) = \frac{\partial T}{\partial t},$$

where k is a positive constant. It models the temperature $T(x, y, z, t)$ at the point (x, y, z) and time t of a body in space.

- (a) We examine a simplified version of the heat equation. Consider a straight wire “coordinatized” by x . Then the temperature $T(x, t)$ at time t and position x along the wire is modeled by the one-dimensional heat equation

$$k\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

Show that the function $T(x, t) = e^{-kt} \cos x$ satisfies this equation. Note that if t is held constant at value t_0 , then $T(x, t_0)$ shows how the temperature varies along the wire at time t_0 . Graph the curves $z = T(x, t_0)$ for $t_0 = \{0, 1, 10\}$, and use them to understand the graph of the surface $z = T(x, t)$ for $t \geq 0$. Explain what happens to the temperature of the wire after a long period of time.
