Determine whether the set $\mathcal{B}$ is a basis for the vector space V.

$$V = M_{22}$$

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \right\}$$

Determine whether the set $\mathcal{B}$ is a basis for the vector space V.

$$V = \mathscr{P}_2$$
$$\mathcal{B} = \left\{ x, 1 + x, x - x^2 \right\}$$

Find the coordinate vector of $p(x) = 1 + 2x + 3x^2$ with respect to the basis $\mathcal{B} = \{1 + x, 1 - x, x^2\}$ of $\mathscr{P}_2$.

Find the dimension of the vector space V and give a basis for V.

$$V = \{A \text{ in } M_{22} : A \text{ is skew-symmetric}\}$$

With

$$p(x) = 1 + x^2$$
$$\mathcal{B} = \{1 + x + x^2, x + x^2, x^2\}$$
$$\mathcal{C} = \{1, x, x^2\}$$

in $\mathscr{P}_2$,

(a) Find the coordinate vectors $[p(x)]_\mathcal{B}$ and $[p(x)]_\mathcal{C}$ of $p(x)$ with respect to the bases $\mathcal{B}$ and $\mathcal{C}$ respectively.

(b) Find the change-of-basis matrix $P_{C\leftarrow B}$ from $\mathcal{B}$ to $\mathcal{C}$.

(c) Use your answer from part (b) to compute $[p(x)]_\mathcal{C}$, and compare you answer with the one found in part (a).

(d) Find the change-of-basis matrix $P_{B \leftarrow C}$ from C to B.

(e) Use your answers to parts (c) and (d) to compute $[p(x)]_C$, and compare you answer with the one found in part (a).

Express $p(x) = 1 + 2x - 5x^2$ as a Taylor polynomial about $a = -2$

Let $\mathcal{B}, \mathcal{C}, \mathcal{D}$ be bases for a finite-dimensional vector space V. Prove that

$$P_{\mathcal{D} \leftarrow \mathcal{C}} P_{\mathcal{C} \leftarrow \mathcal{B}} = P_{\mathcal{D} \leftarrow \mathcal{B}}$$

For each square matrix below, calculate its eigenvalues and eigenvectors. Then verify that PDP^{-1} is equal to the original matrix, where D is a diagonal matrix with your eigenvalues along its diagonal and P is a matrix with your eigenvectors as its columns.

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -13 & -4 \\ 0 & -3 & 0 \\ 1 & 13 & 0 \end{bmatrix}$