

Determine whether the set \mathcal{B} is a basis for the vector space V .

$$V = M_{22}$$

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \right\}$$

Determine whether the set \mathcal{B} is a basis for the vector space V .

$$V = \mathcal{P}_2$$

$$\mathcal{B} = \{x, 1 + x, x - x^2\}$$

Find the coordinate vector of $p(x) = 1 + 2x + 3x^2$ with respect to the basis $\mathcal{B} = \{1 + x, 1 - x, x^2\}$ of \mathcal{P}_2 .

Find the dimension of the vector space V and give a basis for V .

$$V = \{A \text{ in } M_{22} : A \text{ is skew-symmetric}\}$$

With

$$p(x) = 1 + x^2$$

$$\mathcal{B} = \{1 + x, 1 - x\}$$

$$\mathcal{C} = \{1, x, x^2\}$$

in \mathcal{P}_2 ,

- (a) Find the coordinate vectors $[\mathbf{x}]_{\mathcal{B}}$ and $[\mathbf{x}]_{\mathcal{C}}$ of \mathbf{x} with respect to the bases \mathcal{B} and \mathcal{C} respectively.
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- (b) Find the change-of-basis matrix $P_{\mathcal{C} \leftarrow \mathcal{B}}$ from \mathcal{B} to \mathcal{C} .
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- (c) Use your answer from part (b) to compute $[\mathbf{x}]_{\mathcal{C}}$, and compare your answer with the one found in part (a).
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(d) Find the change-of-basis matrix $P_{B \leftarrow C}$ from \mathcal{C} to \mathcal{B} .

(e) Use your answers to parts (c) and (d) to compute $[\mathbf{x}]_{\mathcal{C}}$, and compare your answer with the one found in part (a).

Express $p(x) = 1 + 2x - 5x^2$ as a Taylor polynomial about $a = -2$

Let $\mathcal{B}, \mathcal{C}, \mathcal{D}$ be bases for a finite-dimensional vector space V . Prove that

$$P_{\mathcal{D} \leftarrow \mathcal{C}} P_{\mathcal{C} \leftarrow \mathcal{B}} = P_{\mathcal{D} \leftarrow \mathcal{B}}$$

For each square matrix below, calculate its eigenvalues and eigenvectors. Then verify that PDP^{-1} is equal to the original matrix, where D is a diagonal matrix with your eigenvalues along its diagonal and P is a matrix with your eigenvectors as its columns.

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -13 & -4 \\ 0 & -3 & 0 \\ 1 & 13 & 0 \end{bmatrix}$
