Determine whether the set $\mathcal{B}$ is a basis for the vector space V.

$$V = M_{22}$$

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \right\}$$

Determine whether the set $\mathcal{B}$ is a basis for the vector space V.

$$V = \mathscr{P}_2$$
$$\mathcal{B} = \left\{1, 2 - x, 3 - x^2, x + 2x^2\right\}$$

Find the coordinate vector of $p(x) = 1 + 2x + 3x^2$ with respect to the basis $\mathcal{B} = \{1 + x, 1 - x, x^2\}$ of $\mathscr{P}_2$.

Find the dimension of the vector space V and give a basis for V.

$$V = \{A \text{ in } M_{22} \colon A \text{ is skew-symmetric}\}$$

With

$$\begin{aligned} p(x) &= 1 + x^2 \\ \mathcal{B} &= \{1 + x + x^2, x + x^2, x^2\} \\ \mathcal{C} &= \{1, x, x^2\} \end{aligned}$$

in $\mathscr{P}_2$,

(a) Find the coordinate vectors $[p(x)]_{\mathcal{B}}$ and $[p(x)]_{\mathcal{C}}$ of $p(x)$ with respect to the bases $\mathcal{B}$ and $\mathcal{C}$ respectively.

(b) Find the change-of-basis matrix $P_{C \leftarrow B}$ from $\mathcal{B}$ to $\mathcal{C}$.

(c) Use your answer from part (b) to compute $[p(x)]_{\mathcal{C}}$, and compare you answer with the one found in part (a).

(d) Find the change-of-basis matrix $P_{\mathcal{B}\leftarrow\mathcal{C}}$ from $\mathcal{C}$ to $\mathcal{B}$.

(e) Use your answers to parts (c) and (d) to compute $[p(x)]_{\mathcal{C}}$, and compare you answer with the one found in part (a).

Express $p(x) = 1 + 2x - 5x^2$ as a Taylor polynomial about $a = -2$

Let $\mathcal{B}, \mathcal{C}, \mathcal{D}$ be bases for a finite-dimensional vector space V. Prove that

$$P_{\mathcal{D}\leftarrow\mathcal{C}} P_{\mathcal{C}\leftarrow\mathcal{B}} = P_{\mathcal{D}\leftarrow\mathcal{B}}$$

For each square matrix below, calculate its eigenvalues and eigenvectors. Then verify that PDP^{-1} is equal to the original matrix, where D is a diagonal matrix with your eigenvalues along its diagonal and P is a matrix with your eigenvectors as its columns.

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -13 & -4 \\ 0 & -3 & 0 \\ 1 & 13 & 0 \end{bmatrix}$