Determine whether the set $\mathcal{B}$ is a basis for the vector space V.

$$V = M_{22}$$

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \right\}$$

Determine whether the set $\mathcal{B}$ is a basis for the vector space V.

$$V = \mathscr{P}_2$$
$$\mathcal{B} = \left\{x, 1+x, x-x^2\right\}$$

Find the coordinate vector of $p(x) = 1+2x+3x^2$ with respect to the basis $\mathcal{B} = \{l+x, 1-x, x^2\}$ of $\mathscr{P}_2$.

Find the dimension of the vector space V and give a basis for V.

$$V = \{A \text{ in } M_{22} \colon A \text{ is skew-symmetric}\}$$

Determine whether T is a linear transformation.

$$T : M_{nn} \to \mathbb{R} \text{ defined by } T(A) = \operatorname{rank}(A)$$

Let $T : M_{22} \to \mathbb{R}$ be a linear transformation for which

$$T\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = 1, \qquad T\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = 2, \qquad T\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = 3, \qquad T\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 4$$

- Find $T\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$

- Find $T\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Prove Theorem 6.14(b)

Theorem 6.14:

a. $T(\mathbf{0}) = \mathbf{0}$

b. $T(-\mathbf{v}) = -T(\mathbf{v})$ for all $\mathbf{v}$ in V

c. $T(\mathbf{u} - \mathbf{v}) = T(\mathbf{u}) - T(\mathbf{v})$ for all $\mathbf{u}$ and $\mathbf{v}$ in V

For each square matrix below, calculate its eigenvalues and eigenvectors. Then verify that PDP^{-1} is equal to the original matrix, where D is a diagonal matrix with your eigenvalues along its diagonal and P is a matrix with your eigenvectors as its columns.

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -13 & -4 \\ 0 & -3 & 0 \\ 1 & 13 & 0 \end{bmatrix}$