A Reachability Index for Recursive Label-Concatenated Graph Queries

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ABSTRACT

Reachability queries checking the existence of a path from a source node to a target node are fundamental operators for querying and processing graph data. Current approaches for index-based evaluation of reachability queries either focus on plain reachability or constraint-based reachability with alternation only. In this paper, for the first time we study the problem of index-based processing for recursive label-concatenated reachability queries, referred to as RLC queries. These queries check the existence of a path that can satisfy the constraint defined by a concatenation of at most k edge labels under the Kleene plus. Many practical graph database and network analysis applications exhibit RLC queries. However, their evaluation remains prohibitive in current graph database engines.

We introduce the RLC index, the first reachability index to efficiently process RLC queries. The RLC index checks whether the source vertex can reach an intermediate vertex that can also reach the target vertex under a recursive label-concatenated constraint. We propose an indexing algorithm to build the RLC index, which guarantees the soundness and the completeness of query execution and avoids recording redundant index entries. Comprehensive experiments on real-world graphs show that the RLC index can significantly reduce both the offline processing cost and the memory overhead of transitive closure while improving query processing up to six orders of magnitude over online traversals. Finally, our open-source implementation of the RLC index significantly outperforms current mainstream graph engines for evaluating RLC queries.

1 INTRODUCTION

Graphs have been the natural choice of data representation in various domains [39], *e.g.,* social, biochemical, fraud detection and transportation networks. Plain reachability queries are fundamental operators to process graphs, *i.e.,* checking whether there exists a path from a source vertex to a target vertex, for which various indexing techniques have been proposed [7, 15, 16, 18, 19, 24, 26, 28–30, 44, 45, 47, 50, 52, 54]. To facilitate the representation of different types of relationships in real-world applications, *edge-labeled graphs* and *property graphs*, where labels can be assigned to edges, are more widely adopted nowadays than unlabeled graphs. Such advanced graph models allow users to add path constraints when defining reachability queries, which play a key role in graph analytics. However, current index-based approaches focus on constraint-based reachability with alternation only [17, 27, 42, 48, 57]. In this paper, we consider for the first time reachability queries with a path constraint corresponding to a *concatenation* of edge labels under the Kleene plus, referred to as *recursive label-concatenated queries* (RLC queries). RLC queries call for a novel indexing technique due to inherently different path constraints compared to either plain reachability queries or alternation-based reachability, respectively. To further motivate RLC queries, we present a running example in the following.

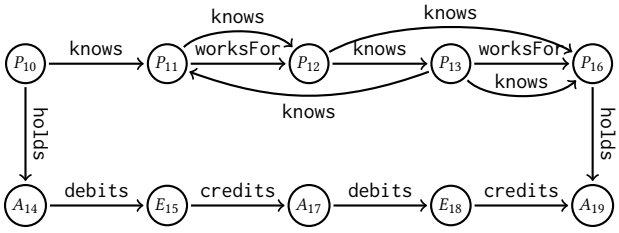

Figure 1: An edge-labeled graph with nodes of persons (P), accounts (A), and external entities (E), and five edge labels: knows, worksFor, holds, debits, and credits.

Running example. Figure 1 shows a property graph inspired by a real business use case at a company, encoding an interleaved social and professional network along with information of bank accounts of social peers. Navigating such a graph by means of RLC queries leads to interesting results, *e.g.,* identifying fraud and money laundering patterns among financial transactions. For instance, the query $Q1(A_{14}, A_{19}, (\texttt{debits, credits})^+)$ asks whether there is a path from A_{14} to A_{19} such that the label sequence of the path is a concatenation of an arbitrary number (one or more) of occurrences of (debits,credits). Queries like $Q1$ can be used to identify and investigate suspicious patterns of money transfers between account A_{14} and A_{19}. The RLC query $Q1((A_{14}, A_{19}, (\texttt{debits, credits})^+)$ evaluates to *true* because of the existence of the path (A_{14}, debits, E_{15}, credits, A_{17}, debits, E_{18}, credits, A_{19}). Another example is $Q2(P_{10}, P_{13}, (\texttt{knows,knows,worksFor})^+)$ that evaluates to false because there is no path from P_{10} to P_{13} satisfying the constraint.

RLC queries are also frequently occurring in real-world query logs, *e.g.,* Wikidata Query Logs [14], which is the largest repository of open-source graph queries (of the order of 500M queries). In particular, RLC queries often time out in these logs [14] thus showing the limitations of graph query engines to efficiently evaluate them. Moreover, Neo4j(v4.3) [4] and TigerGraph(v3.3) [20], two of the mainstream graph data processing engines, do not yet support RLC queries in their current version. On the other hand, these systems have already identified the need to support these queries in the near future by following the developments of the Standard Graph Query Language (GQL) [3]. RLC queries

can be expressed in Gremlin supported by TinkerPop-Enabled Graph Systems [2], *e.g.*, Amazon Neptune [12], in PGQL [49] supported by Oracle PGX [25, 46], and in SPARQL 1.1 (ASK query) supported by Virtuoso [6] and Apache Jena [1], among the others. However, many of these systems cannot efficiently evaluate RLC queries yet as shown in our experimental study. In addition, indexing is one of the open problems for future graph processing systems, as it permits to improve and predict the performance of graph queries [43]. Thus, in our work we address the design of an index-based solution to evaluate RLC queries efficiently.

To the best of our knowledge, little research has been carried out on this topic. Although there exist two different kinds of reachability indexes, neither of them can be used to process RLC queries. More precisely, plain reachability indexes have been well studied [7, 15, 16, 18, 19, 24, 26, 28–30, 44, 45, 47, 50, 52, 54], but are not usable to evaluate RLC queries because of the lack of edge label information. Recent indexing methods have been focused on alternation of edge labels [17, 27, 42, 48, 57], instead of concatenation in RLC queries. While such indexes include edge label information, they are still unable to process RLC queries due to completely different path constraints (alternation versus concatenation). In addition, we are also aware that modern graph engines have internal indexes by default to allow fast access to vertices or edges, but such non-reachability indexes are not sufficient to speed up the processing of RLC queries as shown in our experiments.

In this work, we propose the RLC index, the first reachability index tailored for processing RLC queries. In the RLC index, we assign to each vertex v two sets $\mathcal{L}_{in}(v)$ and $\mathcal{L}_{out}(v)$ of pairs of vertices and sequences, where $\mathcal{L}_{in}(v)$ contains vertices u that can reach v and the (sub-)sequences of edge labels from u to v, and $\mathcal{L}_{out}(v)$ contains vertices w that are reachable from v and the (sub-)sequences of edge labels from v to w. Then, the RLC index processes an RLC query with source s and target t by checking *(i)* whether there exists vertex u, such that s can reach u and t is reachable from u, and *(ii)* whether the concatenation of the two recorded sequences, *i.e.*, ones from s to u and u to t, can satisfy the label constraint of the query.

One of the challenges for building an index to process RLC queries is that there can be infinite sequences of edge labels from vertex s to vertex t due to the presence of cycles on paths from s to t. Thus, building an index for arbitrary RLC queries can be hard since the number of label sequences for cyclic graphs is exponentially growing and potentially infinite, which leads to unrealistic indexing time and index size. We overcame this issue by leveraging a practical observation: the number of edge labels concatenated under the Kleene plus (or star) is typically bounded in real-world query logs, *e.g.*, [14]. We refer to the maximum number of concatenated labels in a workload of RLC queries as k. Given an arbitrary input parameter k, we show that the RLC index can be correctly built to evaluate any RLC query with a label concatenation of *at most* k labels.

Another key challenge for designing an index-based solution lies in how to strike the balance between offline indexing cost (indexing time and index size) and online query processing time. We design an indexing algorithm that builds the RLC index efficiently over large and highly cyclic graphs with millions of vertices and edges as shown in our experiments. In general, the indexing algorithm conducts both backward and forward BFS from each vertex. During each BFS traversal, the algorithm identifies situations where redundant traversals and index entries can be avoided and leverages pruning rules to speed up index construction as well as reduce index size.

Contribution. Our contributions are summarized as follows:

- We introduce RLC queries, reachability queries with the path constraint of the Kleene plus over a concatenation of edge labels. We analyze the problem of building a reachability index for RLC queries, and propose a novel instantiation of the problem based on an input parameter k to overcome the underlying issue of cycle traversals.
- We propose the RLC index, the first reachability index for processing RLC queries. Our key contribution is an indexing algorithm for this novel RLC index. We prove that the indexing algorithm builds a sound and complete RLC index for an arbitrary parameter k, where redundant index entries can be greedily removed.
- Our comprehensive experiments using highly cyclic real-world graphs show that the RLC index can be efficiently built and can significantly reduce the memory overhead of the (extended) transitive closure. Meanwhile, the RLC index is capable of answering RLC queries efficiently, up to six orders of magnitude over online traversals. We also demonstrate the speed-up and the generality of using the RLC index to accelerate query processing on mainstream graph engines. Our complete codebase[1] is available as open source for future research.

We discuss related works in Section 2 and formally define RLC queries in Section 3. We present the theoretical foundation of the RLC index in Section 4, and the RLC index and the indexing algorithm in Section 5. Experimental results are presented in Section 6. We conclude in Section 7. Our online technical report [5] provides proofs that are not included in this paper due to space limit.

2 RELATED WORK

Plain Reachability Index. Given an unlabeled graph $G = (V, E)$ and a pair of vertices (s, t), a plain reachability query asks whether there exists a path from s to t. The existing approaches lie between two extremes, *i.e.*, online traversals and the transitive closure, and try to find a good balance between query time, indexing time, and index size. We briefly review the literature, whereas comprehensive surveys can be found in [13, 45, 55]. They mainly fall into two categories [45, 52]: *(1)* Index-Only approaches; *(2)* Index+Graph approaches. The former can answer queries using only the index, while the latter requires an online graph traversal, which can be guided by auxiliary data, *i.e.*, partial index, and can deal with large graphs. The Index-Only approaches can be further classified into two sub-categories: *(1.1)* cover-based approaches, which use simple graph structures to cover a graph, such as *Chain Cover* [16, 26], *Tree Cover* [7], and *Path-Tree Cover* [30]; *(1.2)* hop-based approaches, which decompose the path between a reachable pair of vertices into sub-paths passing through intermediate vertices, such as 2-hop labeling [19] and 3-hop labeling [29]. As the minimal 2-hop cover problem is NP-hard, various approaches have been proposed for improving the index construction of 2-hop labeling, *e.g.*, *TF-Label* [18], *HL* [28], *DL* [28], and *TOL* [56]. The Index+Graph approaches can fall into two sub-categories: *(2.1)* position-based approaches, *e.g.*, *Tree+SSPI* [15], *GRIPP* [47], *GRAIL* [54], and *Ferrari* [44]; *(2.2)* set-containment-based approaches, *e.g.*, *IP* [52] and *BFL* [45].

[1] Open Source Link: https://github.com/g-rpqs/rlc-index

RLC queries are different from plain reachability queries because they are evaluated on labeled graphs to find the existence of a path satisfying additional recursive label-concatenated constraints. Thus, the approaches used to evaluate plain reachability queries, *e.g.*, 2-hop labeling [19], are not suitable for RLC queries. More precisely, indexing techniques for plain reachability queries only record information about graph structure but ignore information of edge labels.

Alternation-Based Reachability Index. Reachability queries with a path constraint that is based on alternation of edge labels (instead of concatenation as in our work), are known as LCR queries in the literature. Index-based solutions for LCR queries have been extensively studied in the last decade. We provide an overview of this line of works below.

Jin *et. al* [27] presented the first result on LCR queries. To compress the generalized transitive closure that records reachable pairs and sets of path-labels, the authors proposed sufficient label sets and a spanning tree with partial transitive closure. The main idea is to record in a partial transitive closure only the minimal label sets for paths with non-tree edges as the starting and ending edge, then the generalized transitive closure can be recovered by traversing the spanning tree and looking up the partial transitive closure. The Zou *et. al* [57] method finds all strongly connected components (SCCs) in an input graph and replaces each SCC with a bipartite graph to obtain an edge labeled DAG. Zou *et. al* also proposed an efficient algorithm to compute generalized transitive closures for the DAG using its topological order. To handle a large graph, the algorithm is applied to graph partitions instead of SCCs. Valstar *et. al* [48] proposed a landmark-based index. In a nutshell, the method computes the generalized transitive closure for a subset of vertices called landmarks that have the highest total degree and applies an online BFS to answer LCR queries that can be accelerated by hitting landmarks. The method is also optimized by adding a fixed number of index entries for non-landmark vertices, such that the search space for negative queries can be pruned. The state-of-the-art indexing techniques for LCR queries are the Peng *et. al* [42] method and the Chen *et. al* [17] method. Peng *et. al* [42] proposed the LC 2-hop labeling, which extends the 2-hop labeling framework through adding minimal sets of path-labels for each entry in the 2-hop labeling. Chen *et. al* [17] proposed a recursive method to handle LCR queries, where an input graph is recursively decomposed into spanning trees and graph summaries, and LCR queries are decomposed into sub-queries evaluated using spanning trees and graph summaries.

LCR queries are similar to RLC queries in the sense that path-label information is mandatory to check reachability. Their major difference is in the type of the regular expression given in queries. The expression in LCR queries is an alternation of edge labels, while the one in RLC queries is a concatenation of edge labels. The completely different path constraint makes LCR indexes infeasible for processing RLC queries. More precisely, LCR indexes store label sets for a pair of vertices (s, t), but such a set is not sufficient to answer an RLC query with (s, t) because the order and the number of occurrences of edge labels are missing, *i.e.*, the stored index entries are label sets, instead of label sequences required by processing RLC queries. The difference in path constraint also makes indexing algorithms for LCR queries and RLC queries fundamentally different. Specifically, for an LCR indexing algorithm, it is sufficient to traverse any cycle in a graph only once. Then based on a snapshot of an LCR index, any further traversal along the cycle can be skipped because the reachability information related to the cycle under an alternation-based path constraint has already been recorded. However, it is not true for the case of RLC queries, where a cycle might need to be traversed multiple times depending on label sequences along paths. Therefore, indexing approaches for LCR queries are not applicable to indexing RLC queries.

Regular Path Queries. Regular path queries correspond to queries generating node pairs according to path constraints specified using regular expressions. Under the *simple* path semantics (non-repeated vertices or edges in a path), it is NP-complete to check the existence of a path satisfying a regular expression [37]. Thus, a recent work [51] focused on an approximate solution for evaluating regular simple path queries. By restricting regular expressions or graph instances, there exist tractable cases [10, 37]. When it comes to the *arbitrary* path semantics (allowing repeated vertices or edges in a path), regular path queries for generating node pairs can be processed by using automata-based techniques [11, 53] or bi-directional BFSs from rare labels [32]. Optimal solutions can be used for sub-classes of regular expressions, *e.g.*, a matrix-based method [21] for the case without recursive concatenation, and a B+tree index for the case with a bounded path length (non-recursive path constraints) [23] and a fixed path pattern [33]. There also exist in the literature partial evaluation for distributed graphs [22], and incremental approaches for streaming graphs [40, 41]. Compared to all these works, we focus on designing an index-based solution to handle reachability queries with path constraints of recursive concatenation over a static and centralized graph under the arbitrary path semantics, which is an open challenge in the design of reachability indexes. To the best of our knowledge, our work is the first of its kind focusing on the design of a reachability index for such queries.

3 PROBLEM STATEMENT

An edge-labeled graph is $G = (V, E, \mathbb{L})$, where $\mathbb{L}$ is a finite set of labels, V a finite set of vertices, and $E \subseteq V \times \mathbb{L} \times V$ a finite set of labeled edges. For the graph in Fig. 1, we have $\mathbb{L}$ = {knows, worksFor, debits, credits, holds}, and that e_1=(P_{10}, knows, P_{11}) is a labeled edge. We use $\lambda : E \rightarrow \mathbb{L}$ to denote the mapping from an edge to its label, *e.g.*, $\lambda(e_1)$ = knows. The frequently used symbols are summarized in Table 1.

In this paper, we consider the *arbitrary paths* semantics [8], *i.e.*, allowing for duplicate vertices along the path. An arbitrary path p in G is a vertex-edge alternating sequence $p(v_0, v_n) = (v_0, e_1, ..., e_n, v_n)$, where $n \geq 1$, and $v_0, ..., v_n \in V$, $e_1, ..., e_n \in E$, and $|p(v_0, v_n)| = n$ that is the length of the path. For $p(v_0, v_n)$, v_0 is the source vertex and v_n is the target vertex. If there exists a path from v_0 to v_n, then v_0 reaches v_n, denoted as $v_0 \rightsquigarrow v_n$. The label sequence of the path $p(v_0, v_n)$ is $\Lambda(p(v_0, v_n)) = (\lambda(e_1), ..., \lambda(e_n))$. When the context is clear, we also use $\Lambda(u, v)$ to denote the sequence of edge labels of a path from u to v.

3.1 Minimum Repeats

We use l_i to denote an edge label, $L = (l_1, ..., l_n)$ a label sequence, $|L| = n$ the length of L, and ϵ the empty label sequence, *i.e.*, $|\epsilon| = 0$. We use $\circ$ to denote the concatenation of label sequences (or labels), *i.e.*, $(l_1, ..., l_i) \circ (l_{i+1}, ..., l_n) = (l_1, ..., l_n)$, or $L \circ L = L^2$. For the empty label sequence ϵ, we define $L \circ \epsilon = \epsilon \circ L = L$.

The label sequence $L' = (l'_1, ..., l'_{n'}), |L'| = n'$ is a *repeat* of $L = (l_1, ..., l_n)$, if there exists an integer z, such that $\frac{n}{n'} = z \geq 1$, and $l'_j = l_{j+i\times n'}$ for every $j \in (1, ..., n')$ and $i \in (0, ..., z-1)$. A repeat L' of L is minimum if L' has the shortest length of all the repeats of L. The *minimum repeat* (MR) of L is denoted as $MR(L)$

Table 1: Frequently used symbols.

Notation	Description
p, or $p(u,v)$	a path, or the path from u to v
$\circ$	concatenation of labels or label sequences
$\Lambda(u,v)$, or $\Lambda(p(u,v))$	the label sequence of a path from u to v
L	a label sequence
L^+	a label constraint
$MR(L)$	the minimum repeat of a label sequence L
k	the upper bound of the number of labels in L^+
$S^k(u,v)$	the concise set of minimum repeats from u to v
$u \overset{L^+}{\rightsquigarrow} v$, or $u \overset{L^+}{\not\rightsquigarrow} v$	u reaches v through an L^+-path, or otherwise
$u \rightsquigarrow v$, or $u \not\rightsquigarrow v$	u reaches v, or otherwise
$in(v)$, or $out(v)$	the set of vertices that can reach v, or v can reach
$aid(v)$	the access id of vertex v by the indexing algorithm

that is also a sequence of edge labels. For example, given the path $p = (P_{10}, \texttt{knows}, P_{11}, \texttt{worksFor}, P_{12}, \texttt{knows}, P_{13}, \texttt{worksFor}, P_{16})$ in Fig. 1, we have $MR(p(P_{10}, P_{16})) = (\texttt{knows},\texttt{worksFor})$. If $L = MR(L)$, we also say L itself is a minimum repeat. Given a positive integer k and a label sequence L, if $|MR(L)| \leq k$, then we say L has a non-empty k-MR that is $MR(L)$.

Lemma 3.1. *For a label sequence L, $MR(L)$ is unique.*

Suppose L has two minimum repeats, then only the shorter is $MR(L)$. Therefore, we have Lemma 3.1.

3.2 RLC Query

In this work, we consider the path constraint $L^+ = (l_1, ..., l_k)^+$, where '+' is the Kleene plus, *i.e.*, one-or-more concatenations of the sequence $L = (l_1, ..., l_k)$. We focus on a path-constraint L, s.t. $L = MR(L)$, *e.g.*, $L^+ = (\texttt{knows},\texttt{worksFor})^+$. A label sequence $\Lambda(u,v)$ of a path $p(u,v)$ satisfies a label-constraint L^+, if and only if $MR(\Lambda(u,v)) = L$. If such a path $p(u,v)$ exists, then u can reach v with the constraint L^+, denoted as $u \overset{L^+}{\rightsquigarrow} v$, otherwise $u \overset{L^+}{\not\rightsquigarrow} v$.

Definition 3.2 (RLC Query). Given an edge-labeled directed graph $G = (V, E, \mathbb{L})$ and a constant k, an RLC query is a triple (s, t, L^+), where $s, t \in V$, $L = MR(L)$, and $|L| \leq k$. If $s \overset{L^+}{\rightsquigarrow} t$, then the answer to the query is *true*. Otherwise, the answer is *false*.

Notice that for the sake of simplicity in this paper we focus on the RLC queries with the Kleene plus. Queries (s, t, L^*) with the Kleene star can be trivially reduced to queries with the Kleene plus (s, t, L^+) through checking whether s is equal to t. Our method is thus applicable to RLC queries with the Kleene star. It is also worth noting that path queries that do not belong to the above fragment, *i.e.*, with L^+ s.t. $L \neq MR(L)$ impose an additional constraint on the length of paths boiling down to the complicated even-path problem (NP-complete [10, 35, 37] for simple paths). For these reasons, these queries with additional constraints are not considered further in our work.

Given an RLC query $Q(s, t, L^+)$, under the arbitrary path semantics, two naive approaches can be used to evaluate Q. The first approach is using an online traversal, *e.g.*, BFS, where each visited edge should satisfy the label (or state) transition of L^+, aka a finite automata-based approach. The second approach is precomputing the transitive closure, where for each pair of vertices (s, t) we record whether $s \rightsquigarrow t$ and all the label sequences from s to t. Note that the naive approach to build the transitive closure is not usable in our case because cycles may exist on the path from s to t such that the BFS from s can generate infinite label sequences for paths to t. We adopt an *extended transitive closure*, which is presented in Section 6. However, as demonstrated in our experiments, these two solutions require either too much query time or storage space, which are impractical for a large graph.

3.3 Indexing Problem

Our goal is to build an index to efficiently process RLC queries. The corresponding indexing problem is summarized as follows.

Problem 3.1. *Given an edge-labeled graph G, the indexing problem is to build a reachability index for processing* RLC *queries on G, such that the storage of label sequences in the index is minimal and the correctness of query processing is preserved.*

We first observe that recording MRs for a pair of vertices, instead of raw label sequences of paths in G, can reduce the storage space, and such a strategy does not violate the correctness of query processing. The main benefits are twofold: (1) MRs are not longer than raw label sequences; (2) different raw label sequences may have the same MR. For example, in Fig. 1, there exist two paths from P_{10} to P_{16} having the label sequence (knows, knows, knows, knows) and (knows, knows, knows), which have the same MR knows.

Definition 3.3 (Concise Label Sequences). Let $\mathbb{P}(s, t)$ be the set of all paths from s to t. The concise set of label sequences from vertex s to t, denoted as $S^k(s, t)$, is the set of k-MRs of all label sequences from s to t, *i.e.*, $S^k(s, t) = \{L | p \in \mathbb{P}(s, t), MR(\Lambda(p)) = L, |L| \leq k\}$.

To deal with RLC queries, we need to compute and record the concise label sequences. We have Proposition 3.4 by definition.

Proposition 3.4. $s \overset{L^+}{\rightsquigarrow} t, |L| \leq k$ *in G if and only if* $L \in S^k(s, t)$.

For example, in Fig. 1, we have $S^2(P_{12}, P_{16}) = \{(\texttt{knows}), (\texttt{knows}, \texttt{worksFor})\}$. With $S^2(P_{12}, P_{16})$, RLC queries with P_{12} as source and P_{16} as target can be processed correctly.

4 KERNEL-BASED SEARCH

In this section, we deal with the following question: *how to compute concise label sequences?* The problem for computing a concise label sequence is that if a cycle exists on a path from s to t, there exist infinite paths from s to t, which makes the computation of $S^k(s, t)$ infeasible, *e.g.*, $|\mathbb{P}(P_{11}, P_{13})|$ in Fig. 1 is infinite. We overcome this issue by leveraging the upper bound of concatenated labels in a constraint, *i.e.*, k. We observed that we don't have to compute all possible label sequences for paths going from P_{11} to P_{13} as the set of label sequences L such that $|MR(L)| \leq k$ is actually finite. In general, let v be an intermediate vertex that a forward breadth-first search from s is visiting. The main idea is that when the path from s to v reaches a specific length, we can decide whether we need to further explore the outgoing neighbours of v. Moreover, if the outgoing neighbours of v are worth exploring, the following search can be guided by a specific label constraint. In the following, we first provide an illustrating example, and then formally define the specific constraint that is used to guide the subsequent search.

Example 4.1 (Illustrating Example). Consider the graph in Fig. 1. Assume we need to compute $S^2(P_{11}, P_{13})$, *i.e.*, $k = 2$, and we perform a breadth-first search from P_{11}. When P_{13} is visited for the first time, we add (knows) and (worksFor, knows) into $S^2(P_{11}, P_{13})$. After that, when the search depth reaches $2k = 4$, *i.e.*, P_{12} is visited for the second time, we can have 4 different label sequences, which are $L_1 = (\texttt{knows}, \texttt{knows}, \texttt{knows}, \texttt{knows})$, $L_2 = (\texttt{knows}, \texttt{knows}, \texttt{knows}, \texttt{worksFor})$, $L_3 = (\texttt{worksFor},$

knows, knows, knows), and L_4 = (worksFor, knows, knows, worksFor). Given this, all the 4 label sequences except L_1 do not need to be expanded anymore, because their expansions cannot produce a minimum repeat whose length is not larger than 2. After this, the following search continued with L_1 is guided by $(\texttt{knows})^+$ that is computed from L_1. However, because there already exists (knows) in $S^2(P_{11}, P_{13})$, the search does not need to continue.

Definition 4.2 (Kernel and Tail). If a label sequence L can be represented as $L = (L')^h \circ L''$, where $h \geq 2$, and L' and L'' are two label sequences, such that $L' \neq \epsilon$ and $MR(L') = L'$, and L'' is ϵ or a proper prefix of L', then L has the *kernel* L' and the *tail* L''.

For example, the label sequence (knows, knows, knows, knows) from P_{11} has a kernel knows and a tail ϵ.

Kernel-based search. When a kernel has been determined at a vertex that is being visited, the subsequent search to compute $S^k(s,t)$ can be guided by the Kleene plus of the kernel, *e.g.*, $(\texttt{knows})^+$ is used to guide the search in Example 4.1. We call this approach KBS (*kernel-based search*) in the remainder of this paper. In a nutshell, KBS consists of two phases: (1) *kernel-search* and (2) *kernel-BFS*, where the first phase is to compute kernels, and the second to perform kernel-guided BFS. We show in Theorem 4.3 that KBS can compute a sound and complete $S^k(s,t)$. The proof of Theorem 4.3 is included in our technical report [5] due to the space limit.

Theorem 4.3. *Given a path p from u to v and a positive integer k, p has a non-empty k-MR if and only if one of the following conditions is satisfied,*

- *Case 1: $|p| \leq k$. $MR(\Lambda(p))$ is the k-MR of p;*
- *Case 2: $k < |p| \leq 2k$. If $|MR(\Lambda(p))| \leq k$, $MR(\Lambda(p))$ is the k-MR of p;*
- *Case 3: $|p| > 2k$. Let x be the intermediate vertex on p, s.t. $|p(u,x)| = 2k$. If $\Lambda(p(u,x))$ has a kernel L' and a tail L'', and $MR(L'' \circ \Lambda(p(x,v))) = L'$, then L' is the k-MR of p.*

In Case 3 of Theorem 4.3, if $\Lambda(u,x)$ of p does not have a kernel, then p does not have a non-empty k-MR. In other words, Theorem 4.3 says that although $|p(u,v)|$ can be very large or infinite, we can determine the possible k-MRs of p by applying a search up to a length of $2k$ from u. In addition, with kernels determined in Theorem 4.3, we will not miss any k-MRs for paths from u to v.

We discuss below two strategies to compute kernels based on Theorem 4.3, namely *lazy* KBS and *eager* KBS, and explain why eager KBS is better than lazy KBS, which is used in our indexing algorithm presented in Section 5.2.

Lazy KBS. Theorem 4.3 can be transformed into an algorithm to find kernels, *i.e.*, for a source vertex we generate all paths of length $2k$, and then compute all the kernels of these paths. This strategy is referred to as lazy KBS, which means kernels are correctly determined when the length of paths reaches $2k$, *e.g.*, lazy KBS is used in Example 4.1.

Eager KBS. In contrast to the lazy strategy, we can determine kernel candidates earlier, instead of valid kernels that require the length of paths to be $2k$. The main idea is to treat any k-MR that is computed using any path p, $|p| \leq k$ as a kernel candidate, and then kernel candidates will be used to guide the subsequent search. As KBS is a breadth-first search, the set of kernel candidates does not miss any valid kernels. Although an invalid kernel may be included, the search guided by the invalid kernel will not reach a target vertex through a path of which the k-MR is the invalid kernel. Therefore, the set of concise label sequences computed by the eager strategy is still sound and complete.

Example 4.4. Consider the example of computing $S^2(P_{10}, P_{13})$ in Fig. 1. Using the eager strategy, when P_{12} is visited for the first time, two kernel candidates can be determined , *i.e.*, (knows) and (knows, worksFor). Although an invalid kernel (knows, worksFor) is included, the search guided by $(\texttt{knows, worksFor})^+$ cannot reach P_{13} through a path that has the (knows, worksFor).

The key advantage of the eager strategy over the lazy strategy is that it allows us to advance KBS from the kernel-search phase to the kernel-BFS phase. This can make KBS more efficient because generating all label sequences of length $2k$ from a source vertex is more expensive than the case of paths of length k, especially on a dense graph. In addition, as kernel candidates can be used to guide the search in the phase of kernel-BFS, vertices can be marked to avoid repeated traversal, which is not possible in the kernel-search phase aiming at exploring all possible label sequences.

5 RLC INDEX

To evaluate an RLC query (s, t, L^+), we need to check: *(i)* whether there exists a path p from s to t, and *(ii)* whether the path p can satisfy L^+. The main idea is to check the existence of a vertex u, *s.t.*, $s \rightsquigarrow u$ and $u \rightsquigarrow t$, and whether these two sub-paths can satisfy L^+. In this section, we present the RLC index, and also the corresponding query and indexing algorithm.

5.1 Overarching Idea

Given an RLC query (s, t, L^+), $|L| \leq k$, the idea is to check whether there exists a path $(s, ..., u, ..., t)$ whose label sequence satisfies the label constraint L^+, where u is an intermediate vertex in p. In other words, the query is answered by concatenating two MRs of the sub-paths of p, *i.e.*, $MR(\Lambda(s,u))$ and $MR(\Lambda(u,t))$.

Definition 5.1 (RLC Index). Let $G = (V, E, \mathbb{L})$ be an edge-labeled graph and k be a positive integer. The RLC index of G assigns to each vertex $v \in V$ two sets: $\mathcal{L}_{in}(v) = \{(u, L') | u \rightsquigarrow v, L' \in S^k(u,v))\}$, and $\mathcal{L}_{out}(v) = \{(w, L'') | v \rightsquigarrow w, L'' \in S^k(v,w)\}$. Therefore, there is a path $p(s,t)$ satisfying an arbitrary constraint L^+, $|L| \leq k$, if and only if one of the following cases is satisfied,

- Case 1: $\exists (x, L') \in \mathcal{L}_{out}(s)$ and $\exists (x, L'') \in \mathcal{L}_{in}(t)$, such that $L' = L'' = L$;
- Case 2: $\exists (t, L''') \in \mathcal{L}_{out}(s)$ or $\exists (s, L''') \in \mathcal{L}_{in}(t)$, such that $L''' = L$.

Example 5.2 (Running Example of the RLC Index). Consider the graph G shown in Fig. 2. The RLC index with $k = 2$ for G is presented in Table 2. We have $Q_1(v_3, v_6, (l_2, l_1)^+) = true$ because $\exists (v_1, (l_2, l_1)) \in \mathcal{L}_{out}(v_3)$ and $\exists (v_1, (l_2, l_1)) \in \mathcal{L}_{in}(v_6)$. Indeed, there exists the path $(v_3, l_2, v_4, l_1, v_1, l_2, v_3, l_1, v_6)$ from v_3 to v_6 in the graph in Fig. 2. For $Q_2(v_1, v_2, (l_2, l_1)^+)$, the answer is $true$ because $\exists (v_1, (l_2, l_1)) \in \mathcal{L}_{in}(v_2)$. Given $Q_3(v_1, v_3, (l_1)^+)$, the answer is $false$. Although v_1 can reach v_3, *e.g.*, $\exists (v_1, l_2) \in \mathcal{L}_{in}(v_3)$, the constraint $(l_1)^+$ of Q_3 cannot be satisfied.

Our indexing scheme is reminiscent of the 2-hop labeling framework [19], a canonical state-of-the-art framework for designing plain reachability indexes. However, the latter is not usable for RLC queries, thus leading to the need of a novel indexing algorithm.

In order to save storage space, when building an RLC index, redundant index entries have to be removed as much as possible.

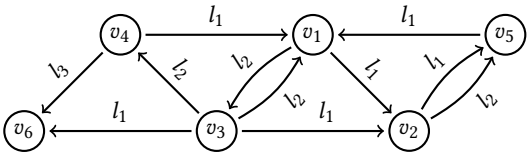

Figure 2: Running example.

Table 2: The RLC index for the graph in Fig. 2.

V	$\mathcal{L}_{in}(v)$	$\mathcal{L}_{out}(v)$
v_1	-	(v_1, l_2), (v_1, l_1), $(v_1, (l_2, l_1))$
v_2	(v_1, l_1), $(v_1, (l_2, l_1))$	$(v_1, (l_2, l_1))$, (v_1, l_1)
v_3	(v_1, l_2), $(v_1, (l_1, l_2))$	(v_1, l_2), $(v_1, (l_2, l_1))$, (v_1, l_1), $(v_3, (l_1, l_2))$
v_4	(v_1, l_2)	(v_1, l_1), $(v_3, (l_1, l_2))$
v_5	$(v_1, (l_1, l_2))$, (v_1, l_1),$(v_3, (l_1, l_2))$, (v_2, l_2)	(v_1, l_1), $(v_3, (l_1, l_2))$
v_6	$(v_1, (l_2, l_1))$, (v_3, l_1), $(v_3, (l_2, l_3))$, (v_4, l_3)	-

The underlying idea is that if there exists a path p such that $u \overset{L^+}{\rightsquigarrow} v$, then the RLC index only records the reachability information of the path p once, *i.e.*, either through Case 1 or Case 2 in Definition 5.1. This leads to the following definition.

Definition 5.3 (Condensed RLC Index). The RLC index is condensed, if for every index entry $(s, L) \in \mathcal{L}_{in}(t)$ (or $(t, L) \in \mathcal{L}_{out}(s)$), there do not exist index entries $(u, L') \in \mathcal{L}_{out}(s)$ and $(u, L'') \in \mathcal{L}_{in}(t)$ such that $L = L' = L''$.

We focus on designing an indexing algorithm that can build a correct (sound and complete) and condensed RLC index.

5.2 Query and Indexing Algorithm

The query algorithm is presented in Algorithm 1, where we use I to denote an index entry. Each index entry I has the schema (vid, mr), where vid represents vertex id and mr recorded minimal repeat. Given an RLC query (s, t, L^+), to efficiently find $(u, L') \in \mathcal{L}_{out}(s)$ and $(u, L'') \in \mathcal{L}_{in}(t)$, we execute a merge join over $\mathcal{L}_{out}(s)$ and $\mathcal{L}_{in}(t)$, shown at line 4 in Algorithm 1. The output of the merge join is a set of index entry pairs (I', I''), s.t. $I'.vid = I''.vid$. Case 1 of the RLC index (see Definition 5.1) is checked at line 5. Case 2 is checked at line 3. If one of these cases can be satisfied, the answer *true* will be returned immediately. Otherwise, index entries in $\mathcal{L}_{out}(s)$ and $\mathcal{L}_{in}(t)$ are exhaustively merged, and the answer *false* will be returned at last.

In the remainder of this subsection, we present an indexing algorithm (Algorithm 2) to build the RLC index which is sound, complete and condensed. We use v_i to denote a vertex with id i. Given a graph $G(V, E, \mathbb{L})$, the indexing algorithm mainly performs backward and forward KBS from each vertex in V to create index entries, and *pruning rules* are applied to accelerate index building as well as remove redundant index entries.

Indexing using KBS. We explain below how the backward KBS creates $\mathcal{L}_{out}$-entries. The forward KBS follows the same procedure, except that $\mathcal{L}_{in}$-entries will be created. Suppose that the backward KBS from vertex v_i is visiting v. If $|MR(\Lambda(v, v_i))| \leq k$, then we add $(v_i, MR(\Lambda(v, v_i)))$ into $\mathcal{L}_{out}(v)$. Although there may be cycles in a graph, the KBS will not go on forever, because when the depth of the search reaches k, the KBS will be transformed from its kernel-search phase into its kernel-BFS phase that is

Algorithm 1: Query Algorithm

```
1 procedure Query(s, t, L+)
2     if ∃(t, L) ∈ L_out(s) or ∃(s, L) ∈ L_in(t) then
3         return true;
4     for (I', I'') ∈ mergeJoin(L_out(s), L_in(t)) do
5         if I'.mr = L and I''.mr = L then
6             return true;
7     return false;
```

then guided by the Kleene plus of a kernel candidate, such that the KBS can terminate if any invalid label (or state) transition is met, or a vertex being visited with a label l_i of the kernel has been visited with a label l_j of the kernel, s.t. $i = j$.

KBSs are executed from each vertex in V and the execution follows a specific order. The idea is to start with vertices that have more connections to other vertices, allowing such vertices to be intermediate hops to remove redundancy. In the RLC index, we leverage the IN-OUT strategy, *i.e.*, sorting vertices according to $(|out(v)|+1)\times(|in(v)|+1)$ in descending order, which is known as an efficient and effective strategy for various reachability indexes based on the 2-hop labeling framework. The id of vertex v in the sorted list is referred to as the access id of v, denoted as $aid(v)$ starting from 1, *e.g.*, for the graph in Fig. 2, the sorted list is $(v_1, v_3, v_2, v_4, v_5, v_6)$, where $aid(v_3) = 2$.

Example 5.4 (Running Example of Indexing). Consider the graph in Fig. 2 and the RLC index in Table 2 with $k = 2$. The KBSs are executed from each vertex in the order of $(v_1, v_3, v_2, v_4, v_5, v_6)$. We explain below the backward KBS from v_1, which is the first search of the indexing algorithm. The traversal of depth 1 of this backward KBS visits v_4 and creates (v_1, l_1) in $\mathcal{L}_{out}(v_4)$, visits v_3 and creates (v_1, l_2) in $\mathcal{L}_{out}(v_3)$, and visits v_5 and creates $(v_1, l_1) \in \mathcal{L}_{out}(v_5)$. The traversal of depth 2 creates $(v_1, (l_2, l_1))$ in $\mathcal{L}_{out}(v_3)$, (v_1, l_2) in $\mathcal{L}_{out}(v_1)$, $(v_1, l_1) \in \mathcal{L}_{out}(v_2)$, and $(v_1, (l_2, l_1)) \in \mathcal{L}_{out}(v_2)$. Then the kernel-search phase of this KBS terminates because the depth of the search reaches 2, which generates kernel candidate l_1 with a set of frontier vertices $\{v_4, v_5, v_2\}$, kernel candidate l_2 with a set of frontier vertices $\{v_3, v_1\}$, and kernel candidate (l_2, l_1) with a set of frontier vertices (v_3, v_2). After this, this KBS is turned into three kernel-BFSs guided by $(l_1)^+$, $(l_2)^+$, and $(l_2, l_1)^+$ with the corresponding frontier vertices. The kernel-BFS terminates under the case of an invalid label transition or a repeated visiting. For example, the label of the incoming edge of v_3 is l_2, which is an invalid state transition of $(l_2, l_1)^+$ in the backward kernel-BFS from v_1, such that the kernel-BFS guided by $(l_2, l_1)^+$ terminates at v_3. For another example, index entry $(v_1, l_1) \in \mathcal{L}_{out}(v_1)$ is created when v_1 is visited for the first time by the kernel-BFS from v_1 guided by $(l_1)^+$, but this kernel-BFS will not continue when it visits v_5 that has already been visited with the label l_1.

Pruning Rules. To accelerate index construction as well as remove redundant index entries, we apply pruning rules during KBSs. For ease of presentation, we present the pruning rules for backward KBSs, and the same rules apply for forward ones.

- ***PR1:*** *If the k-MR of an index entry that needs to be recorded can be acquired from the current snapshot of the* RLC *index, then the index entry can be skipped.*
- ***PR2:*** *If vertex v_i is visited by the backward KBS performed from vertex $v_{i'}$ s.t. $aid(v_{i'}) > aid(v_i)$, then the corresponding index entry can be skipped.*

- ***PR3:*** *If vertex v_i is visited by the kernel-BFS phase of a backward KBS performed from vertex $v_{i'}$, and PR1 (or PR2) is triggered, then vertex v_i and all the vertices in $in(v_i)$ are skipped.*

Note that if PR2 is triggered then PR1 must be triggered because a path p can be visited by either the forward KBS from the source of p or the backward KBS from the target of p. However, checking for PR2 only requires the access id of vertices instead of evaluating a query using a snapshot of the RLC index. This is why PR2 is extracted from PR1. The correctness of Algorithm 2 with pruning rules is guaranteed by Theorem 5.10 presented in Section 5.3.

Example 5.5 (Running Example of Pruning Rules). Consider the forward KBS from v_3 for the graph in Fig. 2. It can visit v_2 through label sequence (l_2, l_1), such that it tries to create $(v_3, (l_2, l_1))$ in $\mathcal{L}_{in}(v_2)$. However, there already exist $(v_1, (l_2, l_1)) \in \mathcal{L}_{out}(v_3)$ and $(v_1, (l_2, l_1)) \in \mathcal{L}_{in}(v_2)$, such that $Q(v_3, v_2, (l_2, l_1)^+) = true$ with the current snapshot of the RLC index, *i.e.*, the reachability information has already been recorded. Thus, the index entry $(v_3, (l_2, l_1))$ in $\mathcal{L}_{in}(v_2)$ is pruned according to PR1. As an example of PR2, consider the backward KBS from v_2. It can visit v_1 through path $(v_1, l_2, v_3, l_1, v_2)$, such that it tries to create $(v_2, (l_2, l_1))$ in $\mathcal{L}_{out}(v_1)$. Given $aid(v_2) > aid(v_1)$, such that the index entry can be pruned by PR2. Consider the forward KBS from v_2 for an example of PR3. It visits v_2 through path $(v_2, l_2, v_5, l_1, v_1, l_2, v_3, l_1, v_2)$, where at the edge (v_5, l_1, v_1) the KBS is transformed from the kernel-search phase to the kernel-BFS phase that is then guided by $(l_2, l_1)^+$. When v_2 visits itself for the first time, the KBS tries to create index entry $(v_2, (l_2, l_1)) \in \mathcal{L}_{in}(v_2)$. However, this index entry can be pruned by PR1 because of $(v_1, (l_2, l_1)) \in \mathcal{L}_{out}(v_2)$ and $(v_1, (l_2, l_1)) \in (v_2)$. In this case, PR3 is triggered, which means v_2 and all the vertices in $out(v_2)$ are skipped by this kernel-BFS.

The indexing algorithm is presented in Algorithm 2. For ease of presentation, each procedure focuses on the backward case, and the forward case can be obtained by trivial modifications, *e.g.*, replacing in-coming edges with out-going edges. We use the KMP algorithm [31] to compute the minimum repeat of a label sequence, *i.e.*, `MR()` at line 13 in Algorithm 2. The indexing algorithm performs backward and forward KBS from each vertex. The KBS from a vertex v consists of two phases: kernel-search (line 6 to line 17) and kernel-BFS (line 24 to line 37). The kernel-search returns for each vertex v all kernel candidates and a set of frontier vertices $vSet$. The kernel-BFS is performed for each kernel candidate, *i.e.*, a BFS with vertices in $vSet$ as frontier vertices guided by a kernel candidate. PR1 and PR2 are included at line 19, which can be triggered by both kernel-search and kernel-BFS. PR3 implemented at line 35, on the other hand, can only be triggered by kernel-BFS.

Remark. An alternative version of the RLC index allowed to concatenate different minimum repeats to answer an RLC query, *i.e.*, in Case 1 of Definition 5.1, L' can be different from L'' in the alternative version. However, such a design will prevent the use of PR3 that can prune vertices and avoid redundant traversals. Consequently, the indexing time of the alternative version is much longer than the version introduced in this paper, *e.g.*, even for the smallest graph used in our experiments (the AD graph presented in Section 6), the indexing time of the alternative version is 32x slower than the current one. Thus, we focus on concatenating the same minimum repeats.

5.3 Analysis of RLC Index

We present in Theorem 5.9 that pruning rules can guarantee the condensed property of the RLC index, and in Theorem 5.10 that

Algorithm 2: Indexing Algorithm.

```
1  procedure kernelBasedSearch(v, k)
2      for (L, vSet) ∈ backwardKernelSearch(v, k) do
3          backwardKernelBFS(v, vSet, L);
4      for (L, vSet) ∈ forwardKernelSearch(v, k) do
5          forwardKernelBFS(v, vSet, L);
6  procedure backwardKernelSearch(v, k)
7      q ← an empty queue of (vertex, label sequence);
8      q.enqueue(v, ϵ);
9      map ← a map of (kernel candidates, vertex set);
10     while q is not empty do
11         (x, seq) ← q.dequeue();
12         for in-coming edge e(y, x) to x do
13             seq' ← λ(e(y, x)) ∘ seq; L ← MR(seq');
14             insert (y, v, L); map.get(L).add(x);
15             if |seq'| < k then
16                 q.enqueue(y, seq');
17     return map;
18 procedure insert(s, t, L)
19     if aid(t) > aid(s) or Query(s, t, L+) then
20         return false;
21     else
22         add (t, L) into L_out(s);
23         return true;
24 procedure backwardKernelBFS(v, vSet, L)
25     q ← an empty queue of (vertex, integer);
26     for x ∈ vSet do
27         mark x as visited by state 1, q.enqueue(x, |L|);
28     while q is not empty do
29         (x, i) ← q.dequeue(), i ← i − 1;
30         if i = 0 then i = |L| ;
31         label l ← L.get(i);
32         for in-coming edge e(y, x) to x do
33             if l ≠ λ(e(y, x)) or y was visited by state i
                then
34                 continue;
35             if i = 1 and insert(y, v, L) then
36                 continue;
37             q.enqueue(y, i); mark y visited by state i;
```

the RLC index constructed by Algorithm 2 is correct, *i.e.*, sound and complete. Before proceeding further, we first present the following lemmas that will be used to prove the two theorems.

Lemma 5.6. *Given a path $p(s, t)$ having a k-MR L. If the KBS from s can visit t (or the KBS from t can visit s), then the k-MR L of $p(s, t)$ must be recorded in the index.*

Proof. If the KBS from s can visit t, then regardless of whether PR1 or PR2 is applied, the k-MR L of $p(s, t)$ must be recorded. □

Lemma 5.7. *Given paths $p(s, u)$ and $p(u, t)$ with k-MR L, where $aid(u) < aid(s)$ and $aid(u) < aid(t)$. We have: if $aid(u) \leq i$, then the k-MR of the path from s to t through vertex u is recorded by Algorithm 2 in the i-th snapshot of the* RLC *index that is computed after performing KBS from a vertex with access id i.*

Proof. The proof is based on induction. It is trivial to prove the initial case $i = 1$. We assume the case $i = n$ is true and prove the case $i = n + 1$ below. We only need to show the case $aid(u) = n + 1$. Let $p(s, t) = (s, ..., u, ..., t)$.

Assuming the backward KBS from u does not visit s. Then PR3 is triggered, such that there exists vertex w, $aid(w) < aid(u)$, and $p(s, w)$ and $p(w, u)$ have the k-MR L. This case can be reduced to the case $i = n$, because the k-MR of path $(w, ..., u, ..., t)$ is L and $aid(w) < aid(u) < n + 1$. In the same way, we can also prove the case if the forward KBS from u does not visit t.

We consider the case that both the backward KBS and the forward KBS from u can visit s and t. For $p(s, u)$, we have the following two cases: Case (1) $\exists(u, L) \in \mathcal{L}_{out}(s)$; Case (2) $\exists(v, L) \in \mathcal{L}_{out}(s)$ and $\exists(v, L) \in \mathcal{L}_{in}(u)$, $aid(v) < aid(u)$. For Case (2), both $p(s, v)$ and $p(v, t)$ have the k-MR L, such that this case can be reduced to the case $i = n$ as $aid(v) < aid(u) = n + 1$. Then we only need to consider Case (1), *i.e.*, $\exists(u, L) \in \mathcal{L}_{out}(s)$. In the same way, for $p(u, t)$, we only need to consider the case $\exists(u, L) \in \mathcal{L}_{in}(s)$. Given this, the k-MR of the path from s to t is recorded by having $(u, L) \in \mathcal{L}_{out}(s)$ and $(u, L) \in \mathcal{L}_{in}(t)$. □

Lemma 5.8. *Given a path p from s to t with a k-MR L. If the index entry $(t, L) \in \mathcal{L}_{out}(s)$ (or $(s, L) \in \mathcal{L}_{in}(t)$) is pruned because of PR3, then we have one of the following two cases:*

- *$\exists(s, L) \in \mathcal{L}_{in}(t)$ (or $\exists(t, L) \in \mathcal{L}_{out}(s)$);*
- *$\exists(v, L) \in \mathcal{L}_{out}(s)$ and $\exists(v, L) \in \mathcal{L}_{in}(t)$, such that $aid(v) < aid(t)$ (or $aid(v) < aid(s)$).*

Proof. There exists two cases: $aid(t) \leq aid(s)$ or $aid(t) > aid(s)$. We prove the case of $aid(t) \leq aid(s)$. The proof for the other case follows the same sketch. Let $p(s, t) = (s, ..., u, ..., t)$, such that PR3 can be triggered. W.l.o.g. let $aid(u) < aid(t)$ (if $aid(u) \geq aid(t)$ and PR3 is triggered, then there exists vertex $w, aid(w) < aid(u)$, which can be reduced to the case of $aid(u) < aid(t)$). Given this, we have path $p(s, u)$ and $p(u, t)$ have the same k-MR L according to the definition of PR3. Then we have three cases: Case (1) $aid(s) > aid(u)$; Case (2) $aid(s) = aid(u)$; Case (3) $aid(s) < aid(u)$. Case (1) can be proved by Lemma 5.7, because $aid(s) > aid(u)$, $aid(t) > aid(u)$, and both $p(s, u)$ and $p(u, t)$ have k-MR L. Case (2) can be proved by Lemma 5.6 because the backward KBS from t can visit u, *i.e.*, $aid(s) = aid(u)$. Case (3) can also by proved by 5.6 if the forward KBS from s can visit t. The only case left is that $aid(s) < aid(u)$ and the forward KBS from s cannot visit t because of PR3. In this case, there must exist vertex $v, aid(v) < aid(s) < aid(u) = n + 1$, and the k-MR of $p(s, v)$ and $p(v, u)$ is L. Then we have $aid(v) < aid(s)$ and $aid(v) < aid(t)$, and both path $p(s, v)$ and $(v, ..., u, ..., t)$ have the k-MR L, which can be proved by Lemma 5.7. □

Theorem 5.9. *With pruning rules, the* RLC *index is condensed.*

Proof. Assuming $\exists(t, L) \in \mathcal{L}_{out}(s)$, and $\exists(u, L) \in \mathcal{L}_{out}(s)$ and $\exists(u, L) \in \mathcal{L}_{in}(t)$ in the RLC index. Then we have $aid(u) \geq aid(t)$, otherwise $(t, L) \in \mathcal{L}_{out}(s)$ can be pruned. Given this, $(u, L) \in \mathcal{L}_{in}(t)$ cannot exist because the backward KBS from t is performed earlier than the forward KBS from u, which means we have either $(t, L) \in \mathcal{L}_{out}(u)$, or $(v, L) \in \mathcal{L}_{out}(u)$ and $(v, L) \in \mathcal{L}_{in}(t)$, such that $(u, L) \in \mathcal{L}_{in}(t)$ is pruned. The proof follows the same sketch if $(s, L) \in \mathcal{L}_{in}(t)$ is considered. □

Theorem 5.10. *Given an edge-labeled graph G and the* RLC *index of G with a positive integer k built by Algorithm 2, there exists a path from vertex s to vertex t in G which satisfies a label constraint $L^+, |L| \leq k$, if and only if one of the following condition is satisfied*

(1) $\exists(x, L) \in \mathcal{L}_{out}(s)$ and $\exists(x, L) \in \mathcal{L}_{in}(t)$;
(2) $\exists(t, L) \in \mathcal{L}_{out}(s)$, or $\exists(s, L) \in \mathcal{L}_{in}(t)$.

Proof. (Sufficiency) It is straightforward.

(Necessity) Let p be the path from s to t with the k-MR L. W.l.o.g. let the backward KBS from t be performed first. We have two cases: the backward KBS from t can visit or cannot visit s. In the first case, the k-MR L of path p must be recorded according to Lemma 5.6. In the second case, PR3 must be triggered. According to Lemma 5.8, we have the k-MR of p is also recorded. □

We analyze the complexity of the RLC index below.

Query time. Given a query $Q(s, t, L^+)$, the time complexity of using Algorithm 1 to answer the query is $O(|\mathcal{L}_{out}(s)|+|\mathcal{L}_{in}(t)|)$, because we only need to take $O(|\mathcal{L}_{out}(s)| + |\mathcal{L}_{in}(t)|)$ time to apply the merge join to find $(x, L) \in \mathcal{L}_{out}(s)$ and $(x, L) \in \mathcal{L}_{in}(t)$. Note that index entries in $\mathcal{L}_{out}(s)$ and $\mathcal{L}_{in}(t)$ have already been sorted according to the access id of vertices, such that we do not need to sort index entries when applying the merge join.

Index size. The index size can be $O(|V|^2|\mathbb{L}|^k)$ in the worst case, since each $\mathcal{L}_{in}(v)$ or $\mathcal{L}_{out}(v)$ can contain $O(|V|C)$ index entries, where $C = O(|\mathbb{L}|^k)$ is the number of distinct minimum repeats for all label sequences derived from $|\mathbb{L}|$ of length up to k. C can be computed as follows, $C = \sum_{i=1}^{k} F(i)$, where $F(i) = |\mathbb{L}|^i - (\sum_{j \in f(i), j \neq i} F(j))$ with $F(1) = |\mathbb{L}|$ and $f(i)$ the set of factors of integer i.

Indexing time. In Algorithm 2, we perform a KBS from each vertex, and each KBS consists of two phases: the kernel-search phase and the kernel-BFS phase. Performing a kernel-search of depth k from a vertex requires $O(|\mathbb{L}|^k|V|^k)$ time, and generates $O(|L|^k)$ kernel candidates as discussed in the index size analysis. Each kernel candidate requires a kernel-BFS taking $O(|E|k)$ time. Hence the time complexity for performing a KBS is $O(|\mathbb{L}|^k|V|^k + |L|^k|E|k)$. The total index time is $O(|V|^{k+1}|\mathbb{L}|^k + |L|^k|V||E|k)$ in the worst case.

Note that these complexities resemble the complexity classes of previous label-constraint reachability indexes, *e.g.*, the landmark index [48] and the LC 2-hop labeling [42] for LCR queries.

6 EXPERIMENTAL EVALUATION

In this section, we study the performances of the RLC index. We first used real-world graphs to evaluate the indexing time, index size, and query time of the RLC index, where we focus on practical graphs and workloads. Then, we used synthetic graphs to conduct a comprehensive study of the impact of different characteristics on the RLC index, including label set size, average degree, the number of vertices, and also parameter k. Finally, we compared the query time of the RLC index with existing systems, where we additionally consider more types of reachability queries from real-world query logs in order to demonstrate the generality of our approach.

Baselines. To the best of our knowledge, the RLC index is the first indexing technique designed for processing recursive label-concatenated reachability queries, and indices for other types of reachability queries are not usable in our context because of specific path constraints defined in the RLC queries. Thus, the chosen baselines for the RLC index are BFS (breadth-first search) and BiBFS (bidirectional BFS), which we used to understand in

Table 3: Overview of real-world graphs.

Dataset	$\|V\|$	$\|E\|$	$\|\mathbb{L}\|$	Synthetic Labels	Loop Count	Triangle Count
Advogato (AD)	6K	51K	3		4K	98K
Soc-Epinions (EP)	75K	508K	8	√	0	1.6M
Twitter-ICWSM (TW)	465K	834K	8	√	0	38K
Web-NotreDame(WN)	325K	1.4M	8	√	27K	8.9M
Web-Stanford (WS)	281K	2M	8	√	0	11M
Web-Google (WG)	875K	5M	8	√	0	13M
Wiki-Talk (WT)	2.3M	5M	8	√	0	9M
Web-BerkStan (WB)	685K	7M	8	√	0	64M
Wiki-hyperlink (WH)	1.7M	28.5M	8	√	4K	52M
Pokec (PR)	1.6M	30.6M	8	√	0	32M
StackOverflow (SO)	2.6M	63.4M	3		15M	114M
LiveJournal (LJ)	4.8M	68.9M	50	√	0	285M
Wiki-link-fr (WF)	3.3M	123.7M	25	√	19K	30B

Table 4: Indexing time (IT) and index size (IS).

Dataset	RLC Index		ETC	
	IT (s)	IS (MB)	IT (s)	IS (MB)
AD	0.7	1.9	2216.1	2798.7
EP	22.6	29.3	-	-
TW	8.1	93.5	-	-
WN	33.1	122.6	-	-
WS	53.5	173.9	-	-
WG	101.3	403.6	-	-
WT	812.9	607.1	-	-
WB	167.1	474.2	-	-
WH	3707.2	1319.1	-	-
PR	3104.1	1212.6	-	-
SO	57072.5	844.2	-	-
LJ	18240.9	6248.1	-	-
WF	51338.7	6467.9	-	-

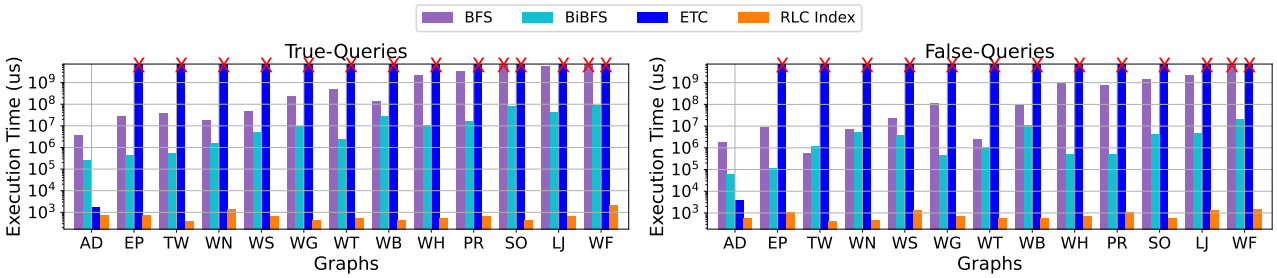

Figure 3: Execution time of 1000 true-queries or 1000 false-queries on real-world graphs

terms of query time how much improvement our index can provide against a full online traversal. In addition, we also include an extended transitive closure as a baseline, referred to as ETC. The indexing algorithm of ETC performs a forward KBS from each vertex without pruning rules, and records for every reachable pair of vertices (u, v) any k-MR of any path $p(u, v)$. In ETC, we use a hashmap to store reachable pairs of vertices and the corresponding set of k-MRs. There are two differences between the indexing algorithm of ETC and the one of the RLC index: (1) only forward KBS is used for building ETC, instead of forward and backward KBS for the RLC index, and (2) none of the pruning rules is applied for building ETC.

Datasets. We use real-world datasets and synthetic graphs in our experiments. We present the statistics of real-world datasets in Table 3 (sorted according to $|E|$), which are from either the SNAP [36] or the KNOECT [34] project. We also include for each graph the loop count (cycles of length 1) and the triangle count (cycles of length 3) shown in the last two columns in Table 3. We generate synthetic labels for graphs that do not have labels on their edges, indicated by the column of synthetic labels in Table 3. The edge labels have been generated according to the Zipfian distribution [9] with exponent 2. The synthetic graphs used in our experiments follows two different modes, namely the *Erdős–Rényi* (ER) model and the *Barabási-Albert* (BA) model. ER-graphs with $|V|$ nodes and $|E|$ edges are generated by uniformly choosing a graph from all possible graphs with $|V|$ nodes and $|E|$ edges. BA-graphs with $|V|$ nodes and $|E|$ edges are generated through first having a complete graph of $\frac{|V|}{2000}$ nodes and then continuously adding new nodes that are connected to previously generated $\frac{|E|}{|V|}$ nodes, *e.g.*, a generated BA-graph with 1M nodes and 5M edges has a complete sub-graph of 500 nodes. Consequently, ER-graphs have an almost uniform degree-distribution while BA-graphs have a skew in it. We use JGraphT [38] to generate the ER- and BA-graphs. The method to assign labels to edges in synthetic graphs is the same as the one used for real-world graphs.

Query generation. As a common practice to evaluate a reachability index in the literature, *e.g.*, [17, 42, 48], we generate for each real-word graph two comprehensive query sets, each of which contains 1000 true-queries and 1000 false-queries, respectively. Each query in the generated query sets has the expression of $(a \circ b)^+$, where a and b represent two distinct edge labels, and the concatenation of a and b is under the Kleene plus. We explain the method for query generation as follows. We uniformly select a source vertex s and a target vertex t, and also uniformly choose a label constraint L^+. Then a bidirectional breadth-first search is conducted to test whether s reaches t under the constraint of L^+. If the test returns *true*, we add $(s, t, L^+, true)$ to the true-query set, otherwise we add it into the false-query set. After that, we generate another (s, t, L^+), and repeat the above procedure until the completion of the two query sets.

Implementation and Setting. Our open-source implementation has been done in Java 11, spanning baseline solutions and the RLC index. We run experiments on a machine with 8 virtual CPUs of Intel(R) Xeon(R) 2.40GHz, and 128GB main memory, where the heap size of JVM is configured to be 120GB.

6.1 Performance on Real-World Graphs

In this section, we analyze the performance of the RLC index on real-world graphs. We compare the RLC index with ETC in terms

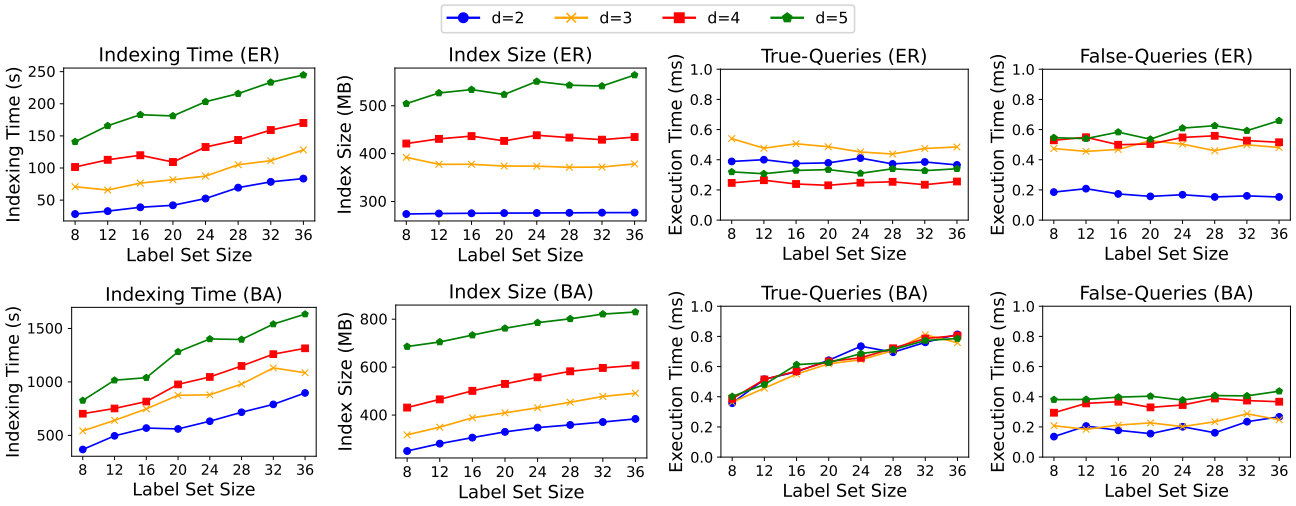

Figure 4: Indexing time, index size, and execution time for graphs with $|V|$ = 1M, varying d, and varying $|\mathbb{L}|$.

of indexing time and index size, and with BFS and BiBFS in terms of query time. The RLC index and ETC are built with $k = 2$ for each graph in this experiment. Notice that queries with $k = 2$ represents a large fraction (roughly 65%) of the property graph queries found in recent open-source query logs [14]. In addition, many of such queries timed out in the logs as they are complex to evaluate (NP-complete under the simple path semantics [10, 37]). More experiments with varying k are reported in Section 6.2.3.

Indexing time. Table 4 shows the indexing time of the RLC index and ETC on real-world graphs, where "-" indicates that the method timed out on the graph. Building ETC cannot be completed in 24 hours for real-world graphs (or it runs out of memory) except for the AD graph with the least number of edges. The RLC index for the AD graph can be built in 0.7s, leading to a four-orders-of-magnitude improvement over ETC. The indexing time improvement of the RLC index over ETC is mainly due to the pruning rules that skip vertices in graph traversals when building the RLC index. Thus, this experiment also shows the significant impact of pruning rules in terms of indexing time. The indexing time of the RLC index for the first 10 graphs is at most 1 hour. The last three graphs, *i.e.*, the SO graph, the LJ graph, and the WF graph are more challenging than the others, not only because they have more vertices and edges, but also because they have a larger number of loops and triangles, as shown in Table 3. The SO graph has the longest indexing time due to its highly dense and cyclic character, *i.e.*, it has 15M loops and 114M triangles. Although the WF graph has much fewer loops than the SO graph, it contains 30B triangles. Consequently, the indexing time of the WF graph is at the same order of magnitude as the one of the SO graph. While it has more vertices, triangles, and edge labels than the SO graph, the LJ graph requires a lower indexing time.

Index size. Table 4 shows the size of the RLC index for real-world graphs. The size of the RLC index is much smaller than the size of ETC for the AD graph (that is the only graph ETC can be built within 24 hours). The index size improvement is because of not only the indexing schema of the RLC index but also the application of pruning rules that can avoid recording redundant index entries. The effectiveness of pruning rules can also be observed between the PR graph and the SO graph. Although the SO graph is larger than the PR graph in terms of the number of vertices and the number of edges, the index size of the SO graph is smaller than the index size of the PR graph. Thus, this experiment also demonstrates the significant impact of pruning rules in terms of index size.

Query time. Fig. 3 shows the execution time of the true-query set and the false-query set. In general, the execution time of a query set of 1000 queries using the RLC index is around 1 millisecond for all the graphs in Table 3, except for the WF graph (that has the largest number of edges) for which around 2 milliseconds. Query execution using BFS times out for the true-queries on both the SO graph and the WF graph, and for the false queries on the WF graph. In addition, we only report the query time of ETC for the AD graph as ETC cannot be built for all the other graphs. As shown in Fig. 3, the RLC index shows an up to six-orders-of-magnitude improvement over BFS and four-orders-of-magnitude improvement over BiBFS. The query time using the RLC index is slightly faster than ETC, because the larger number of reachable pairs of vertices recorded in ETC leads to a slight overhead of checking for the reachability between a source s and a target t and also the existence of a minimum repeat of a path from s to t.

6.2 Impact of Graph Characteristics

In this section, we focus on analyzing the performance of the RLC index on synthetic graphs with different characteristics, namely average degree, label set size, the number of vertices, and input parameter k. The synthetic graphs included in these experiments are ER-graphs and BA-graphs. We generate for each graph a query set of 1000 true-queries and a query set of 1000 false-queries, which are referred to as *ER.T* and *ER.F* for an EA-graph, and *BA.T* and *BA.F* for a BA-graph. The input parameter k is set to two for all experiments but one in Section 6.2.3, where we analyze the RLC index with different k.

6.2.1 Impact of label set size and average degree. In this experiment, we use BA-graphs and EA-graphs with 1M vertices, and we vary the average degree d in $(2, 3, 4, 5)$, and label set size $|\mathbb{L}|$ in $(8, 12, 16, 20, 24, 28, 32, 36)$, *e.g.*, a graph with $d = 5$ and $|\mathbb{L}| = 16$ has 1M vertices, 5M edges, and 16 distinct edge labels. We aim at analyzing indexing time, index size, and query time of the RLC index as the increase of average degree and label set size. The experimental results for ER-graphs and BA-graphs are reported in Fig. 4. We discuss the results below.

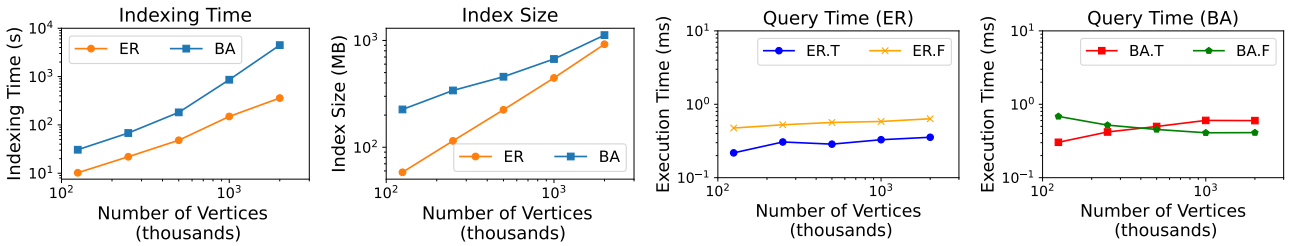

Figure 5: Indexing time, index size, and query execution time for graphs with $d = 5$, $|\mathbb{L}| = 16$, and varying $|V|$.

Indexing time. We observe in Fig. 4 that the indexing time for the used ER-graphs and BA-graphs with a fixed d shows a linear increase (for most cases) as $|\mathbb{L}|$ increases. This can be understood as follows. When $|\mathbb{L}|$ increases, the number of possible minimum repeats increases, requiring more time for the kernel-search phase of a KBS in the indexing algorithm to traverse the graph and generate potential kernel candidates, resulting in more kernel-BFS executions. The total number of possible minimum repeats can be quadratic in $|\mathbb{L}|$ in the worst case when the input parameter k of the RLC index is two. Furthermore, because there are more edges to traverse, the indexing time for both ER-graphs and BA-graphs with a fixed $|\mathbb{L}|$ increases linearly as d increases.

Index size. As illustrated in Fig. 4, an increase in average degree d can result in a larger index size for both ER-graphs and BA-graphs. The reason is that a vertex s can reach a vertex t through more paths, leading to a higher number of minimum repeats being recorded. As the size of the label set grows, the corresponding impact on ER-graphs is different from the one on BA-graphs. Specifically, the increase is negligible for ER-graphs with a small d, *e.g.*, 2, and becomes more noticeable for ER-graphs with a large d, *e.g.*, 5. For any d, however, we see a clear linear increase in index size with the growth in $|\mathbb{L}|$ for BA-graphs. This is because a BA-graph comprises a complete sub-graph, and vertices inside the complete sub-graph have higher degrees. Therefore, the KBSs executed from such vertices can create more index entries as $|\mathbb{L}|$ grows, as it can reach other vertices through paths with more distinct minimum repeats. However, because of the uniform degree distribution, the increase in the number of minimum repeats of paths from a vertex s to a vertex t due to an increase in $|\mathbb{L}|$ is not significant for ER-graphs when d is small, but the corresponding impact becomes stronger when d is larger.

Query time. As shown in Fig. 4 the growth of $|\mathbb{L}|$ has a different impact on query time. More precisely, when $|\mathbb{L}|$ rises, the execution time of both true- and false-queries for ER-graphs remains steady. When it comes to BA-graphs, increasing $|\mathbb{L}|$ can lead to a minor boost in true-query execution time but has almost no impact on false query execution time. This can be explained by the fact that the vertices in the complete sub-graph of a BA-graph can reach much more vertices than the vertices outside the complete sub-graph in the BA-graph, leading to a skew in the distribution of vertices in index entries, *i.e.*, many index entries have the same vertex. Furthermore, when $|\mathbb{L}|$ grows, the number of minimum repeats also increases, which makes the skew higher. As a result, processing true-queries will encounter situations where the query algorithm searches for a particular minimum repeat in a significant number of index entries with the same vertex. However, for false-queries, the query result can be returned instantly if there are no index entries with the same vertex. Fig. 4 also shows that d has a negligible impact on the execution time of the true-queries on the BA graph. A possible explanation for this might be that the number of index entries for some vertices in the BA graph does not increase significantly w.r.t the increase of d because of the skew in the degree distribution and a fixed $|\mathbb{L}|$.

6.2.2 Scalability. In this experiment, we use BA-graphs and EA-graphs with average degree 5, 16 edge labels, and vary the number of vertices in (125K, 250K, 500K, 1M, 2M). The goal is to analyze the scalability of the RLC index in terms of $|V|$. The results of indexing time, index size, and query time for both ER-graphs and BA-graphs are reported in Fig. 5.

Indexing time and index size. Fig. 5 shows that indexing time and index size grows with the increase of the number of vertices. The main reason is that graphs with more vertices require more KBS iterations, which increases indexing time and also the number of index entries. We also observe that the increase of V has a stronger impact on indexing time than on index size. The underlying reason is that the number of minimal repeats does not increase as quickly as the growth of $|V|$ since it is mostly influenced by the average degree and label set size. It is also worth noting that indexing BA-graphs is more expensive than indexing ER-graphs, because the presence of a complete sub-graph makes BA-graphs more challenging to process.

Query time. Fig. 5 shows that query time on the ER-graphs and true-query time on the BA-graphs slightly increase as the number of vertices grows, as expected on larger graphs. In addition, the false-query time is higher than the true-query time on ER-graphs, and the true-query time is higher than the false-query time on BA-graphs. We interpret the results as follows. Given a query (s, t, L^+) on a graph has a uniform distribution in vertex degree, the index entries (v, mr) in both $\mathcal{L}_{out}(s)$ and $\mathcal{L}_{in}(t)$ also have a uniform distribution in terms of v. Thus, the query algorithm (based on merge join) tends to perform an exhaustive search in $\mathcal{L}_{out}(s)$ and $\mathcal{L}_{in}(t)$ to find index entries with the same vertex v, which results in false-queries taking longer time to execute than true-queries. However, when the distribution of vertex degree is skewed, the index entries in $\mathcal{L}_{out}(s)$ and $\mathcal{L}_{in}(t)$ can be dominated by several vertices, *e.g.*, a vertex u in the complete sub-graph of a BA-graph. Furthermore, as there might exist more paths from u to s or u to t, the number of index entries with u can be relatively large because of distinct minimum repeats. Given this, the query algorithm can perform a faster search for false-queries than true-queries, because the number of distinct vertices in both $\mathcal{L}_{out}(s)$ and $\mathcal{L}_{in}(t)$ is not large. The query algorithm, on the other hand, needs to select a specific mr among index entries with vertex u of a high degree to process true-queries, which takes more time.

6.2.3 Impact of k. In this experiment, we aim at analyzing the impact of k on the index. We use a BA-graph and an EA-graph with 125K vertices, average degree 5, 16 edge labels. We build

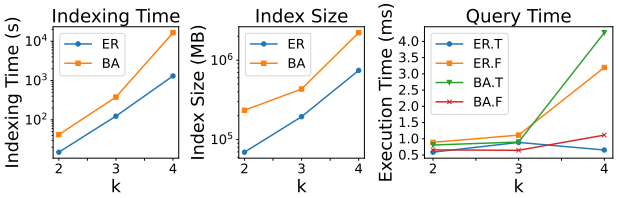

Figure 6: Evaluation of the RLC index with varying k.

RLC indexes for the BA-graph and the EA-graph with k in $(2, 3, 4)$. For each graph, we evaluate a true-query set of 1000 queries and a false-query set of 1000 queries using the three different indexes built with the three different k values. The results of indexing time, index size, and query time are reported in Fig. 6.

Overall results. Fig. 6 shows that the indexing time and index size of both types of synthetic graphs rise exponentially as k grows. The fundamental reason is that as k increases, the number of possible minimum repeats that have to be considered in graph traversals during indexing increases exponentially, which is an inherent step in building a complete reachability index for RLC queries. The exponential increase of minimum repeats w.r.t k also has an impact on query time, particularly on the true-queries of BA-graphs and the false-queries on ER-graphs. This is mainly due to larger index size, and the true-queries on BA-graphs and the false-queries on ER-graphs are more expensive to process than their opposites, respectively.

6.3 Comparison with existing systems

In this section, we focus on evaluating how much improvement the RLC index can provide over mainstream graph processing systems. We recall that many current graph query engines fail to evaluate RLC queries, thus we focused on three systems that are able to evaluate these queries on property graphs and RDF graphs. In order to show how the index performs with varying types of queries, such as longer concatenations or path queries frequently occurring in real-world query logs, we consider the following queries: **Q1** being a single label under the Kleene plus a^+; **Q2** consisting of a concatenation of length 2 $(a \circ b)^+$; **Q3** having the expression $(a \circ b \circ c)^+$ thus a concatenation of length 3. In this case, we build the index with $k = 3$ to support all the three RLC queries, especially Q3 having the longest concatenation. For the sake of completeness, an extended reachability query with the constraint $a^+ \circ b^+$ is also included in this experiment, which is referred to as **Q4**. We have employed Q4 to study the generality and applicability of the RLC index to a wide range of regular path queries in real-world graph query logs [14]. To deal with this query, we use the RLC index in combination with an online traversal to continuously check whether intermediately visited vertices can satisfy the path constraint.

Three graph engines, including commercial and open-sourced ones, used in the experiments have been selected as representatives of the few available graph engines capable of evaluating RLC queries. We do not reveal the identity of all the systems as some are proprietary and we cannot release performance data. Anonymized engines are referred to as Sys1 and Sys2 in the results, and the third one is Virtuoso Open-Source Edition(v7.2.6.3233). For the systems that support multi-threaded query evaluation, we set the system to single-threaded to ensure a fair comparison with our approach, which is single-threaded only. For Virtuoso, we disabled the transaction logging to avoid additional overheads, and configured it to work entirely in memory by setting the maximum amount of memory for transitive queries to the available memory of the server. Note that these systems have their own indexes by default, which we leave the configuration unchanged, *e.g.*, Virtuoso 7 has column-wise indexes by default. These indexes are not suitable for RLC queries, as confirmed by our comparative analysis.

Table 5: Speed-ups (SU) and workload size break-even points (BEP) of the RLC index over graph engines.

Sys.	RLC Query						Extended Query	
	Q1		Q2		Q3		Q4	
	SU	BEP	SU	BEP	SU	BEP	SU	BEP
Sys1	1200x	84100	10400x	34000	18400x	9400	34000x	300
Sys2	3000x	34900	202000x	1700	1300000x	130	104000x	98
Virtuoso	597x	180000	4900x	71700	38100000x	5	-	-

We use the WN graph as a representative of real-world graphs, which has a moderate number of vertices and edges, along with a sufficient number of cycles to evaluate. We build one RLC index with the parameter $k = 3$ for the WN graph and use it to process all four queries. In this case, the RLC index of the WN graph can be built in 5.9 minutes and takes up 821 megabytes. We run each query using each system within the 10-minutes time limit, and we repeat the execution 20 times and report the median of the query execution time. We use two metrics to evaluate the improvement of the RLC index, namely speed-ups (SU) and the workload size break-even points (BEP). SU shows the query time improvement of the RLC index over an included graph system. BEP indicates the number of queries that make the indexing time of the RLC index pay off. Table 5 shows the results, where "-" indicates that the query execution of this system timed out. The RLC index shows that using a single index lookup can gain significant speed-ups over included systems for processing Q3, which has the longest concatenation under the Kleene plus. We use the BEP value to understand the amortized cost of using the RLC index to accelerate Q3 processing. In particular, executing Q3 130 times on Sys2 is equivalent to the time it takes to build and query the RLC index the same number of times. The RLC index can also significantly improve the execution time for Q1, Q2, and Q4. To summarize, given a workload, an RLC index should be built with k being the length of the longest concatenation under the Kleene plus in that workload. The RLC index provide significant speedup even for queries where the concatenation length under the Kleene plus is less than k.

7 CONCLUSION

In this paper, we introduced RLC queries, reachability queries with a path constraint based on the Kleene plus over a concatenation of edge labels. RLC queries are becoming relevant in real-world applications, such as social networks, financial transactions and SPARQL endpoints. In order to efficiently process RLC queries online, we propose the RLC index, the first reachability index suitable for these queries. We design an indexing algorithm with pruning rules to not only accelerate the index construction but also remove redundant index entries. Our comprehensive experimental study demonstrates that the RLC index allows to strike a balance between online processing (full online traversals) and offline computation (a materialized transitive closure). Last but not least, our open-source implementation of the RLC index can significantly speed up the processing of RLC queries in current mainstream graph engines.

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A PROOF OF THEOREM 4.3

A proof of Theorem 4.3 is provided in this section. Before that, we first show the kernel of a label sequence is unique in Lemma A.1 that will be used in the proof of Theorem 4.3.

Lemma A.1. *If L has a kernel, then the kernel is unique.*

Proof. The proof is based on induction. The statement is if a label sequence L of length n has a kernel, then the kernel is unique. It is trivial to prove the initial case $|L| = 2$. Assuming the case n is true, then we show the case $n+1$ is also true. Let $|L| = n+1$. We use $\bar{L}$ to denote the label sequence obtained by removing the last label of L, *i.e.*, $|\bar{L}| = n$. We prove the case $n+1$ below.

Assuming L can have two different kernels L_1 and L_2, and $L_1 \neq L_2$, then $L = (L_1)^{h_1} \circ L_1', h_1 \geq 2$ and $L = (L_2)^{h_2} \circ L_2', h_2 \geq 2$. If $|L_1'| = 0$ and $|L_2'| = 0$, then L has two MRs, which is contradictory (Lemma 3.1). If $|L_1'| \neq 0$ and $|L_2'| \neq 0$, then $\bar{L}$ has kernels L_1 and L_2, which contradicts to the case n. The remaining cases are that only one of L_1' and L_2' has a length of 0. W.l.o.g. consider $|L_1'| = 0$ and $|L_2'| \neq 0$. Given this, if $h_1 > 2$, then $\bar{L}$ also has kernels L_1 and L_2, which is contradictory. Then we have $h_1 = 2$ and $|L_1'| = 0$, *i.e.*,

$$L = L_1 \circ L_1. \tag{1}$$

In addition, we have

$$L = (L_2)^{h_2} \circ L_2', h_2 \geq 2, |L_2'| \neq 0. \tag{2}$$

Let $L = (l_1, ..., l_{2|L_1|})$ and $|L_1| = a|L_2| + b, 1 \leq a, b < |L_2|$. According to Equ. (1), we have $l_i = l_{i+|L_1|}, 1 \leq i \leq |L_1|$ and $l_{i'} = l_{i'-|L_1|}, |L_1| < i' \leq 2|L_1|$, which means $l_i = l_{i+a|L_2|+b}$ and $l_{i'} = l_{i'-a|L_2|-b}$. Based on Equ. (2), we have $l_i = l_{i+b}$ and $l_{i'} = l_{i'-b}$. Given this, consider the following two cases: case (i) if $2|L_1| \bmod b = 0$, then $|MR(L_1 \circ L_1)| = b \neq |\bar{L}|$ that contradicts to the fact that L_1 is the unique MR of L; case (ii) if $2|L_1| \bmod b \neq 0$, then $L = (L_3)^{h_3} \circ L_3'$, where either $|L_3| = b, h_3 \geq 2$, and $|L_3'| \neq 0$, or $|L_3| < b$ and $h_3 > 2$. Note that, in the two sub-cases of case (ii), L_3' is ϵ, or a proper prefix of L_3. Therefore, $\bar{L}$ has a kernel L_3, $|L_3| \leq b$. However, $\bar{L}$ also has a kernel L_2 and $|L_2| > b \geq |L_3|$, which is also a contradiction. □

Based on Lemma A.1, we prove Theorem 4.3 below.

Proof of Theorem 4.3. It is not difficult to prove Case 1 and Case 2. We focus on Case 3 below. For ease of presentation, let $\Lambda(u, x) = \Lambda(p(u, x))$ and $\Lambda(x, v) = \Lambda(p(x, v))$.

(Sufficiency) Because $\Lambda(u, x)$ has the kernel L' and the tail L'', such that $\Lambda(u, x) = (L')^h \circ L'', h \geq 2$. Thus, we have $MR(\Lambda(u, x) \circ \Lambda(x, v)) = MR((L')^h \circ L'' \circ \Lambda(x, v)) = L'$, otherwise $MR(L'' \circ \Lambda(x, v)) \neq L'$. In addition, we have $|L'| \leq k$ because $|\Lambda(u, x)| = 2k$. Thus, L' is the k-MR of p.

(Necessity) We show that p does not have a non-empty k-MR in the following two cases.

- Case (i): $\Lambda(u, x)$ does not have a kernel and a tail. Assume that p can have a non-empty k-MR L''' in this case. Because $|L'''| \leq k$ and $|\Lambda(u, x)| = 2k$, then $\Lambda(u, x)$ has a kernel and a tail, which is contradiction to the case definition.
- Case (ii): $\Lambda(u, x)$ has a kernel L' and a tail L'', but $MR(L'' \circ \Lambda(x, v)) \neq L'$. Assume that p has a non-empty k-MR L''' in this case. Knowing that $|L'''| \leq k$ and $|\Lambda(u, x)| = 2k$, we have $L''' = L'$ because the kernel of $\Lambda(u, x)$ is unique (Lemma A.1). Therefore, $MR((L')^h \circ L'' \circ \Lambda(x, v)) = L', h \geq 2$, which means $MR(L'' \circ \Lambda(x, v)) = L'$, that is also a contradiction.

□