A model to Problem Statement 3

Based on Nash-Equilibrium of a multiplayer game

Gautam Sharma

Indian Institute of Technology, Guwahati

gautam.sharma@iitg.ac.in

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Problem Statement & Model

The problem statement loosely translates to making a strategy for a market in which I and some other organization (my competitor) control 20% of the total trading volume each of a particular commodity stock. I need to minimize my risk while maximizing the profits because of the volatility of the stock being decided majorly by both of us and due to this the stock price can really fluctuate.

The model is nothing but a collection of strategies (based on the Nash Equilibrium) that are made in various situations which can arise in the scenario described by the problem statement. To implement this model, one can analyze the trading volume data of the market or the main competitor (if it is available) to predict and find some patterns which could indicate a particular situation and act accordingly.

Assumptions to the model

The model has some general assumptions, but the model might have situation-dependent assumptions also.

- There are only two major traders in this particular scenario, what this implies is that the rest of the 60% trading volume is due to transactions involving small and retail investors. No third party can challenge the trade volume of the organizations in the statement.
- Both the major traders in the above market do not form an alliance/cooperation to control and monopolize the market.
- It is assumed that everyone has perfect knowledge about the market, i.e. market-demand curve, sentiment of the market using sentiment analysis etc.
- It is assumed that everyone acts rationally, wanting to maximize their profits.

Nash-Equilibrium

Nash equilibrium is a concept in game theory that refers to a situation in which all participants in a game or interaction choose a strategy that is the best response to the strategies chosen by the other participants. It is used commonly for describing non-cooperative games such as the one given in the problem statement. In the context of the stock market, the Nash equilibrium represents a point where all investors have chosen a strategy that maximizes their own profits, given the strategies of other investors in the market.

Model description using an example

Let's assume that the stock in question is currently priced at $50 per share. Investor A (me) and Investor B (my competitor) are trying to decide whether to buy or sell the stock, and their payoffs (in terms of profit or loss) will depend on their decisions and the decision of the other investor.

If both investors buy the stock and the price of the stock goes up to $60 per share, both investors will make a profit of $10 per share. If one investor buys the stock and the other sells the stock, the buyer will make a profit of $10 per share (if the price goes up to $60 per share) while the seller will make a loss of $10 per share (if the price goes down to $40 per share). If both investors sell the stock and the price goes down to $40 per share, both investors will make a loss of $10 per share.

Calculations

Using the information given in the previous slide, we can create a payoff matrix for Investor A (me) and Investor B (my competitor), where the rows represent Investor A's decision (to buy or sell) and the columns represent Investor B's decision:

	Investor B buys	**Investor B sells**
Investor A buys	\$10, \$10	-\$10, \$20
Investor A sells	\$20, -\$10	-\$10, -\$10

Table: Buying/Selling of stocks

In this matrix, the first number in each cell represents Investor A's profit (in dollars) and the second number represents Investor B's profit (in dollars).

Conclusion: Buy the stock

To find a Nash equilibrium, we need to find the strategy that maximizes each investor’s payoff, given the strategy of the other investor. In this case, we can see that the only Nash equilibrium is for both investors to choose the strategy of buying the stock. This is because if either investor chooses to sell the stock, the best response for the other investor is to buy the stock, which will result in a higher payoff for both investors if they both buy the stock. Therefore, the Nash equilibrium in this scenario is for both investors to **buy the stock**.

So, through this example, we describe our model: Under any circumstance, it is best to buy the stock, but this model is far from reality, as one should know we to stop due to the risk-minimizing constraints and the aggregate demand-supply curve of the stock. There are many ways to minimize our risk, but I present two mathematically sound models to minimize one’s risk in such a market in the next slides.

Risk Minimization - Portfolio Theory

Portfolio theory suggests that an investor can minimize risk by holding a portfolio of assets that are not perfectly correlated with each other. In the context of the simplified scenario of two investors deciding whether to buy or sell a stock, one way to minimize risk mathematically would be to **consider the correlation between the stock and other assets in the investor's portfolio**. For example, if the investor already holds a mix of stocks from different industries and sectors, they may be able to reduce their risk by choosing a strategy for the stock that is negatively correlated with their other assets. This would reduce the overall volatility of the investor's portfolio and help to minimize risk. By constructing a portfolio that includes a mix of assets with different expected returns and volatilities, and that are not perfectly correlated with each other, an investor can achieve a lower level of overall risk than if they invested in a single asset. **A numerical analysis of risk minimization is provided with the report with this presentation.**

Risk Minimization - Monte-Carlo Simulations

Monte Carlo simulation is a technique that uses computer models to simulate a large number of possible scenarios and outcomes for a given portfolio. By using Monte Carlo simulation, investors can analyze the potential risks and returns of their portfolio under different market conditions and assumptions. This can help investors identify potential risks and adjust their portfolios to minimize those risks.

For example, we could assume that the final value of the stock is normally distributed around the current value, with a certain level of volatility or uncertainty. We could also vary the assumptions for the estimates of Investor A and Investor B, such as their level of confidence in their estimates, their risk tolerance, or the correlation between their estimates. By calculating the probability of each scenario, we can change the parameters of the portfolio accordingly, to minimize the risk taken by the investor. **A numerical analysis of risk minimization is provided with the report with this presentation.**

Future extensions and Applications

The model proposed here is quite simple and misses various realistic situations faced in the market. We can use Cournot's Theory with the model proposed (known as **Cournot-Nash Equilibrium**) to further analyze the problem. We can also use the **Correlated Equilibrium** to analyze the situation (here the investors randomize their strategies over their positions, correlated equilibrium is a probability distribution over joint strategies, such that no player has an incentive to unilaterally deviate from their strategy, given the distribution)

The model described is quite practical, provided sufficient data analysis is done. We need to know the trading patterns of our competitor and simulate those patterns using monte-carlo simulations, we can assume normal distribution of the price of the stock to aid us in the risk calculation.

SWOT Analysis

Strength

The model is mathematically sound and includes randomization, risk tolerance as well as the sentiments of the market (using sentiment analysis), so can be able to simulate a real market situation.

Weaknesses

The model is not adaptive/self-correcting, so it doesn't learn from it's experiences with the market.

Oppurtunities

The correlations can be made more accurate by using better regression and machine learning algorithms.

Threats

Stock is assumed to follow normal distribution, this might not be.

The End