Kalman Filter and Proportional Navigation Based Missile Guidance System

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***Abstract*— Missile Guidance involves guiding the missile to its target. The target's accuracy has a huge impact on its effectiveness. In this paper, the Kalman Filter and the Proportional Navigation (PN) algorithm are used for missile guidance. It is a guidance, navigation, and control (GNC) system for a missile. A fully adaptive Kalman filter that processes a measurement every n milli-second is designed. The Kalman filter estimates the guidance signals for an augmented PN, the rate of the line of sight (LOS), and the target's normal acceleration. A comparative study for missile guidance was done by using different navigation algorithms and filters.**

Keywords— Kalman Filter; Proportional Navigation; Missile Guidance; Line of Sight; Nonlinear Control; Stability.

I. INTRODUCTION

Missile guidance encompasses a wide range of techniques for directing a missile or guided bomb to a target location. Kalman Filtering is a novel approach for missile guidance. Extended versions of kalman filter provides nonlinear control over objects which are very efficient and accurate. GNC deals with the design of control systems for the movement of aircraft, automobiles, ships, etc. In GNC, guidance implies determining the desired path from the initial location to the final location, navigation implies determining the location, velocity, and attitude of the vehicle, control implies manipulating the forces using steering controls, thrusters, etc. [2,3] In this paper, a missile guidance system was designed using the Kalman Filter and the PN Algorithm. The Kalman filter, a.k.a. The Linear Quadratic Estimator is a recursive filter algorithm that uses a sequence of measurements observed gradually over time that includes noise and other additional disturbances, to produce estimates of unknown variables [4,5]. PN a.k.a. Pro-Nav is a guidance law used by most missiles. It is implemented in Python, by writing the functions for PN, Kalman Filter, updating, measuring positions, and defining the parameters and initial conditions. In the comparative study, the Kalman filter is compared with the Extended Kalman Filter (EKF) and the Unscented Kalman Filter (UKF). PN guidance is compared with pursuit guidance (PG) and Q guidance.

The paper is organized as follows: The details of some of the related works are discussed in Section two. In section three, the concepts of the Kalman filter and proportional navigation are explained briefly. Implementation details and results are discussed in sections four and five, respectively. A comparative study of different estimators and guidance laws is being discussed in sections six and seven. Conclusion and Future Scope are discussed in the last section.

II. RELATED WORKS

In 2016, T S Gokul Nath et. al., detailly discussed different missile guidance laws and, the estimation of the Target acceleration using EKF, and Augmented Proportional Navigation Guidance is done to intercept the missile's target [1- 3]. States are estimated for a target measurement using Kalman Filter in a homing loop for the missile which has shown a great improvement in terms of performance and given an optimal estimation [4]. Missile guide estimation is done using EKF, an unknown input without direct feed through the method to reduce the time of interception and expenditure of energy [5]. In [6-7], missile guidance was implemented on the basis of target maneuver's Kalman filter estimation, a fourth-order extended Kalman filter was developed. In [8], a comparative study between the Kalman filter, EKF, and UKF was done for harmonic analysis of dynamic signals. In [9], the EKF was discussed verbosely. Different image classification has been done based on the output image [10,12]. A spacecraft's automated guidance, navigation, and control (GNC) system is its critical component. Its functions include identifying the spacecraft's position and orbital parameters, leading the spacecraft to proceed here on specified orbit or towards its desired target [11]. In [13] Kalman filter is being used to estimate the charging in the battery storage system. In this paper, a missile guidance system was designed using the Kalman Filter and the PN Algorithm. A comparative study was conducted between the Kalman Filter, EKF, UKF, and between PN, PG, and Q-Guidance.

III. Concepts

A. Kalman Filter

In control theory, the Kalman filter is a recursive filtering algorithm that estimates the internal states i.e., unknown variables, of a linear and dynamic system from a sequence of measurements with noises (White Gaussian) and other additional disturbances. It solves the linear quadratic gaussian (LQG) control problem by combining it with the linear quadratic regulator (LQR).

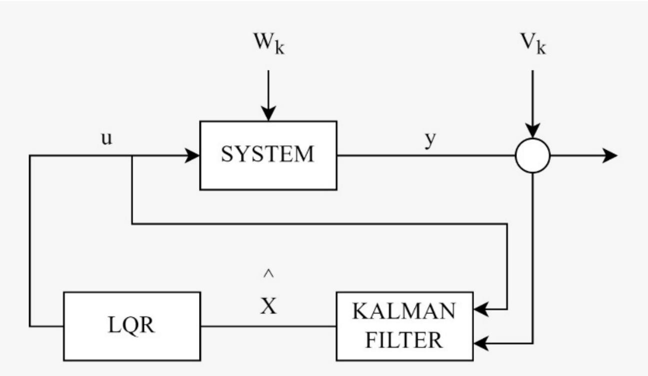

Fig. 1. LQG control problem

Sequential weighted least square:

System dynamical equation:

$$x_k = Ax_{k-1} + Au_{k-1} + w_k$$

Measurement equation:

$$z_k = Hx_k + v_k \quad k = 1,2,3, \ldots . N$$

Prediction step:

Predicted state estimate

$$\hat{x}_k = Ax_{k-1} + Bu_{k-1}$$

Predicted co-variance estimate:

$$\hat{p}_k = AP_{k-1}A^T + Q$$

Update Step:

Kalman Gain

$$K_k = \hat{p}_k H^T (H\hat{p}_k H^T + R)^{-1}$$

Measurement Residual

$$y_k = z - H\hat{x}_k$$

Updated state estimate

$$x_k = \hat{x}_k + K_k y_k$$

Updated co-variance estimate

$$P_k = (I - K_k H)\, \hat{p}_k$$

Where, A is state transition function,
B is control input function,
Q is co-variance of process noise,
R is co-variance of observation noise,
u_k is control vector,
x_k is state,
w_k is process noise,
v_k is observation noise,
P_k is Predicted co-variance estimate,
H is observation function,
K_k is optimal Kalman gain.

B. Proportional Navigation

It is the widely employed guidance law that states, "When the direct LOS of two vehicles does not alter as they get closer, then they are on a collision course." i.e., the velocity vector of the missile should be rotating at a rate that is proportional to the rate of rotation of LOS in the same direction.

$$a_n = N\dot{\lambda}V$$

PN can also obtained as

$$\vec{a} = N\vec{V_r} * \vec{\Omega}$$

Ω is LOS's of rotation vector,

$$\vec{\Omega} = \frac{\vec{R} \times \vec{V_r}}{\vec{R} \cdot \vec{V_r}}$$

V_r is target velocity corresponding to the missile,

$$\vec{V_r} = \vec{V_t} - \vec{V_m}$$

R is the range from the missile to the target,

$$\vec{R} = \vec{R_t} - \vec{R_m}$$

N is used as a gain and it is the unit less constant.

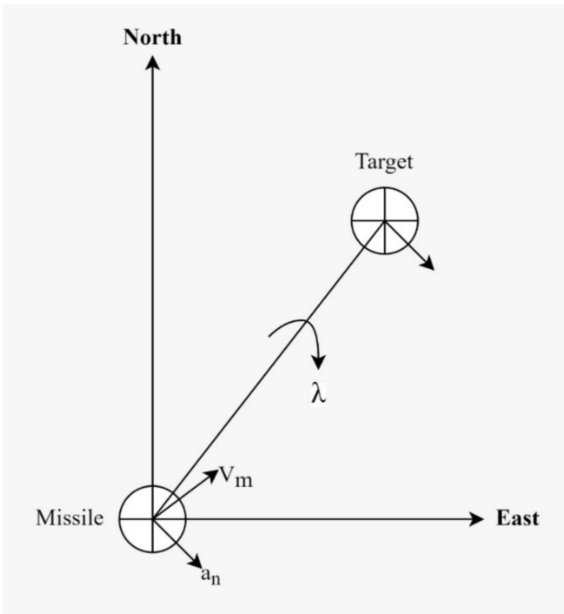

Fig. 2. Missile intercepting a target

In Fig. 2, the LOS slowly rotates from north direction to east direction so, missile should turn rightwards by a certain factor a.k.a. gain which is quicker than the LOS rate. This factor/gain is called as "N" [9].

IV. IMPLEMENTATION

It is implemented in Python. The modules used are Numpy, Pandas, Plotly, and Cufflinks. The specifications used are from the MIM-104 Patriot PAC1 missile and a Russian SU-27 at cruise speed. MIM-104 Patriot is the main missile defense system of the U.S Army. Russian SU-27 is a fighter jet that is very fast in terms of speed and was used in wars. The missile is launched at the origin and the plane is travelling in the Y direction offset by X = 10000m and Z = 10000m.

The initial conditions and parameters for the missile are taken as below

$$A = \begin{bmatrix} 1 & 0 & 0 & dt & 0 & 0 \\ 0 & 1 & 0 & 0 & dt & 0 \\ 0 & 0 & 1 & 0 & 0 & dt \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.5 * dt^2 & 0 & 0 \\ 0 & 0.5 * dt^2 & 0 \\ 0 & 0 & 0.5 * dt^2 \\ dt & 0 & 0 \\ 0 & dt & 0 \\ 0 & 0 & dt \end{bmatrix}$$

$$P = \begin{bmatrix} \sigma_x^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_y^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_z^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{V_x}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{V_y}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{V_z}^2 \end{bmatrix}$$

$$Q = \begin{bmatrix} .25(dt)^4 & 0 & 0 & .5(dt)^3 & 0 & 0 \\ 0 & .25(dt)^4 & 0 & 0 & .5(dt)^3 & 0 \\ 0 & 0 & .25(dt)^4 & 0 & 0 & .5(dt)^3 \\ .5(dt)^3 & 0 & 0 & (dt)^2 & 0 & 0 \\ 0 & .5(dt)^3 & 0 & 0 & (dt)^2 & 0 \\ 0 & 0 & .5(dt)^3 & 0 & 0 & (dt)^2 \end{bmatrix} * \sigma_\alpha^2$$

$$R = \begin{bmatrix} \sigma_{sens_x}^2 & 0 & 0 \\ 0 & \sigma_{sens_y}^2 & 0 \\ 0 & 0 & \sigma_{sens_z}^2 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 500\text{m}$

$\sigma_{V_x}^2 = \sigma_{V_y}^2 = \sigma_{V_z}^2 = 53553.98\text{m}^2\text{s}^{-2}$

dt = 10Hz

Missile Velocity = 960.4ms^{-1}

$\sigma_{sens_x}^2 = \sigma_{sens_y}^2 = \sigma_{sens_z}^2 = 25\text{m}^2$

$\sigma_\alpha^2 = 20\text{ms}^{-2}$

N = 3 (PN)

V. RESULTS

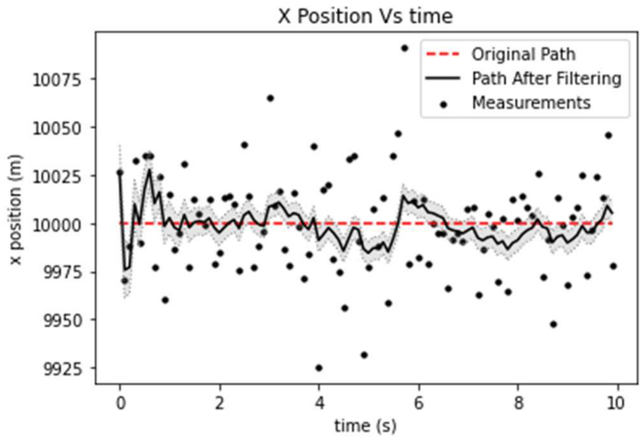

Fig. 3. Convergence between original and filter path w.r.t X axis

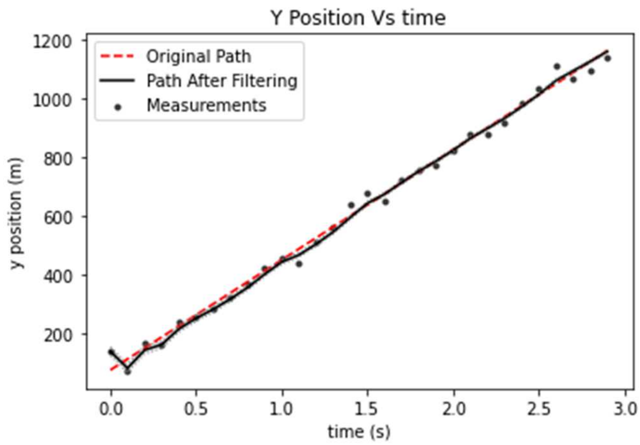

Fig. 4. Convergence between original and filter path w.r.t Y axis

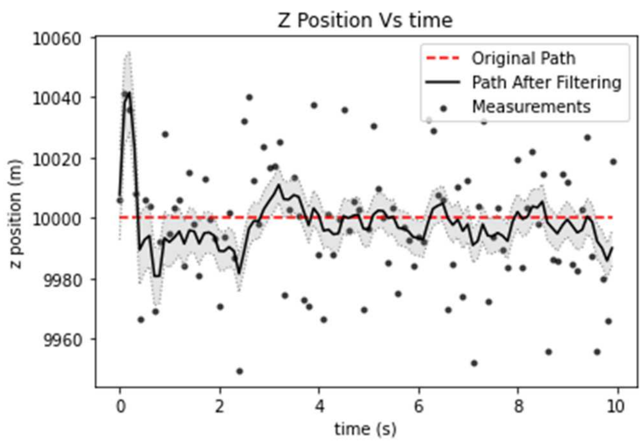

Fig. 5. Convergence between original and filter path w.r.t Z axis

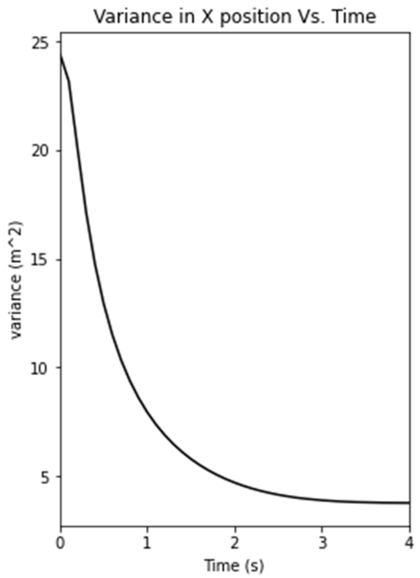

Fig. 6. Variance in position w.r.t X axis over a time

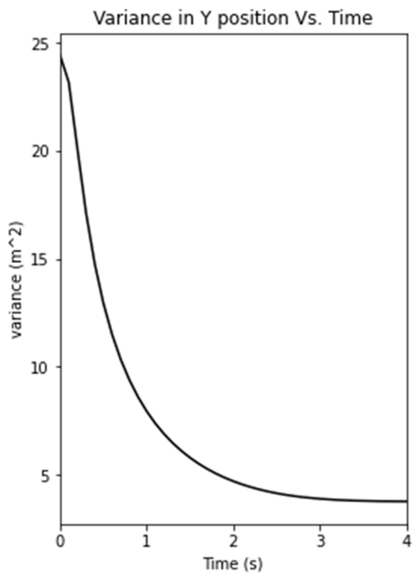

Fig. 7. Variance in position w.r.t Y axis over a time

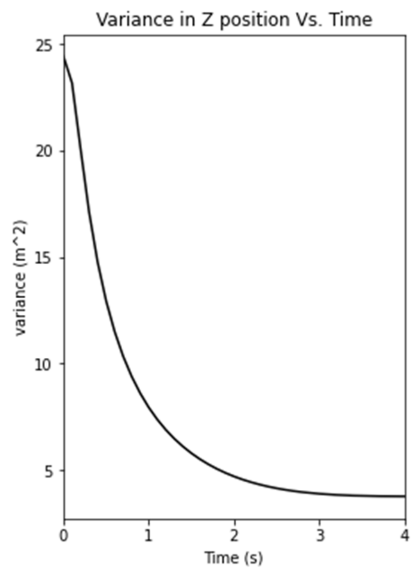

Fig. 8. Variance in position w.r.t Z axis over a time

Here, in Fig. 6,7,8 the variance of X, Y, Z positions of the missile over time can be seen. It can be observed that the system is reaching stability after some time.

VI. COMPARATIVE STUDY OF ESTIMATORS

A. Kalman Filter

Kalman Filter is the optimal solution for the LQG control problem. It uses linear transformations and has a prediction step where the next state is predicted using previous measurements and an update step where the current state is estimated from the measurement at the same step. It estimates the states by using a system that looks like a feedback control. It can be explained as a process whose parameters are estimated by the filter at a point in time and the feedback is measured with the noise (White Gaussian). It shows the best performance among the linear filters. [13]

B. Extended Kalman Filter(EKF)

Most real systems are in the form of non-linear functions, hence there is a need for an extended version of the Kalman filter which deals with the non-linear functions [10]. It is more or less the same as the Kalman filter, except at each time step it evaluates the jacobian with the latest predicted states, this can be utilized in the Kalman filter equations which linearize the non-linear function of the latest estimate a.k.a. covariance and mean. It determines the Gaussian Random Variable (GRV) by approximating the state distribution and then linearizing it with first-order differential equations. It provides first-order approximations for optimal predictions and gain, but these might not always be helpful because as the value of non-linearity increases, it can lead to huge errors and also a divergence of the filter. These can be avoided by using an unscented Kalman filter. [1,8]

C. Unscented Kalman Filter(UKF)

The main difference between EKF and UKF is: UKF does not use the linearization process. Instead of linearizing the jacobian matrices, UKF does a foreordained sampling approach i.e., the state distribution is once more approximated by a GRV, but this time it is represented with the help of a nominal set of 2L+1 chosen sigma points (these points are chosen such that the mean, covariance, and other high order moments will match the GRV), where L represents a dimension of its state. These sigma points completely determine the mean and covariance of GRV. It will be propagated utilising a non-linear system that accurately determines the posterior mean and the posterior covariance for any nonlinearity to second-order derivative, whereas in EKF only first-order accuracy was achieved. UKF with the same complexity as EKF improves the convergence and accuracy of the estimation. [7-9]

TABLE-I COMPARISON BETWEEN DIFFERENT FILTERS

	Kalman Filter	**EKF**	**UKF**
Linear Control	✓	✓	✓
Non Linear Control	X	✓	✓
Linearization of State Distribution	X	✓	X
Foreordained sampling approach	X	X	✓

VII. COMPARATIVE STUDY OF GUIDANCE LAWS

A. Pursuit Guidance(PG) Law

In PG, the velocity vector of the missile will always be directed towards its target and the turning rate of the missile will always be equal to the turning rate of LOS, as an outcome the missile constantly moves along LOS from the initial location to the target location. Since it only moves along LOS, the path will be highly curved which needs higher acceleration to reach further points that the missile might not afford [6].

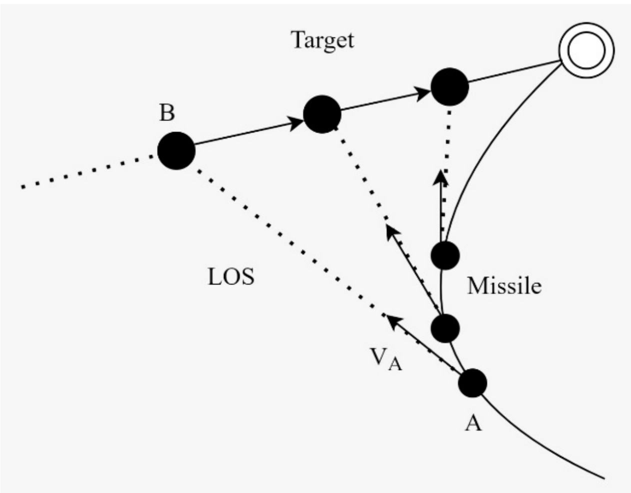

Fig. 9. Missile intercepting a target

B. Proportional Navigation (PN) Law

PN implements parallel navigation where the missile's velocity vector will rotate proportionally to the rotation rate of LOS. Unlike PG, it does not move along LOS, which leads to a less curved path, and it estimates the future position by trying to cut its way constantly. It is the most used guidance law [6].

C. Q Guidance Law

It is used for missile trajectories with a short boot phase. It drives the missile along a 2- or 3-dimensional trajectory to reach its target. It uses the optimal velocity vector which minimizes the amount of impulse required for the 2-impulse transfer from the present state to the target. It is the most complex guidance law and is preferably used for satellites [6].

TABLE-II COMPARISON BETWEEN DIFFERENT GUIDANCE LAWS

	Pursuit Guidance	Proportional Navigation	Q Guidance
Moves Along LOS	✓	X	X
Less Curved Path	X	✓	✓

VIII. CONCLUSION

A missile guidance system is implemented in Python using Kalman Filter and Proportional Navigation along with an understanding of the concepts and the results were observed to show that the missile was reaching stability, the original path and the filtered path were converging after a certain time. The Missile Guidance System can further be implemented using the Unscented Kalman Filter since it gives the optimal estimation even for non-linear systems.

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