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A First-Order, Worldwide, Ionospheric, Time-Delay Algorithm

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A first-order model algorithm designed to reduce ionospheric time-delay errors by approximately 50% rms on a world-wide basis for single frequency users of the Global Positioning System/NAVSTAR is described. The algorithm was designed for greatest accuracy during times of day when ionospheric time-delay errors are expected to be largest. The algorithm is available in several options from one which requires only a monthly mean solar-flux value to one which requires 9 daily or monthly update coefficients. Several approximations

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20. Abstract (continued)

also are made in the geometry calculations to further reduce operational user computer storage and running time requirements.

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A First-Order, Worldwide, Ionospheric, Time-Delay Algorithm

1. INTRODUCTION

The development of an advanced satellite navigation system having worldwide coverage and positional accuracy of a few feet has been undertaken by the Defense Department as a Tri-Service project with management responsibilities assigned to the Air Force's Space and Missile Systems Organization (SAMSO). This advanced development, called the Global Positioning System (GPS) or NAVSTAR, will use timing signals at L band emitted from satellites in 12-hour, 60-degree inclination, retrograde orbits to determine the user location. Accuracies of a few meters rms will be available to the primary class of users. The time delay due to the group path retardation of the earth's ionosphere can be as high as 100 meters at L band, requiring this class of users to utilize a two-frequency receiving system. An operational two-frequency system with two L-band signals separated by approximately 350 to 400 MHz, and with modest receiver signal-to-noise ratios, is expected to reduce the ionospheric error to an acceptable value.

There will be another class of users of the GPS who will not require the accuracies attainable by the two-frequency system. However, they will still require a first-order correction for the time delay of the earth's ionosphere which should reduce the ionospheric time delay error by approximately 50% or more, on an rms basis. Due to the user's limited computer memory and computation

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capability, the model algorithm must be of minimum size and complexity. Such a first-order, time-delay correction algorithm is the subject of this report.

2. CHARACTERISTICS OF IONOSPHERIC TIME DELAY

The ionospheric time delay is directly proportional to the total electron content along the path between the satellite and the user. Total Electron Content (TEC) is a function of many variables; among them are local time, season, geographic location, and state of solar and magnetic activity. The TEC is highest in most regions of the world within a few hours of local noon and is greatest at regions located approximately plus and minus 20 degrees away from the geomagnetic equator. This equatorial anomaly region, as it is called, is the region of the world where single-frequency GPS users will encounter the greatest errors. Statistics of daily values of TEC from individual stations ranging from auroral to equatorial locations show that the rms difference from the monthly mean values during the midday hours is approximately 20 to 30 percent. Thus, an accurate specification of the monthly mean values alone will represent an elimination of 70 to 80 percent (rms) of the ionospheric error. The problem, then, is how to best represent the monthly mean behavior of ionospheric time delay, particularly during those times of the day and in those regions of the world when the time delay is greatest, using the minimum number of coefficients and computation time.

3. DESIGN OF THE ALGORITHM

The model algorithm was designed to meet several criteria. First, it was designed to fit best during those times of the day when the ionospheric delay is the largest, that is, during the afternoon period. Second, it was designed to have versatility in the coefficient array so that trade offs can be made in the number of coefficients used to represent the latitude dependence and the coefficients can be weighted for the latitudes of greatest interest, such as the Continental United States (CONUS). Alternatively, the coefficients can be weighted for best fit in those geographic locations where the time delays are greatest, namely the regions located approximately plus and minus 20 degrees either side of the geomagnetic equator. Finally, the model was simplified by making approximations to geometry, wherever possible, to make its computer running time shorter and storage-space requirements smaller.

The available experimental values of TEC from which time-delay values are obtained are not sufficient, by themselves, to form the basis of a worldwide time-delay model. However, there are sufficient TEC data to yield important information on the statistical behavior of time delay for locations representative of several regions of the world. These statistics will be presented in a separate report. The available data also show, quite well, the diurnal behavior of TEC for various seasons and solar-flux values. These data have been used to determine the form of the diurnal algorithm and they have been supplemented with the Bent[1] model representation for those regions where actual data were not available, particularly for the shape of the latitude gradient.

4. MODEL ALGORITHM

The average monthly diurnal behavior of time delay at any location, as a function of time of day, has been represented by a simple positive cosine wave dependence, with an additional constant offset which we shall call the DC term. In Figure 1 two examples of actual ionospheric time-delay (T_g) data are shown with a best-fit cosine-type curve representation of each set of actual data included. While the cosine curve is, of course, not a perfect fit to the actual data, it can be

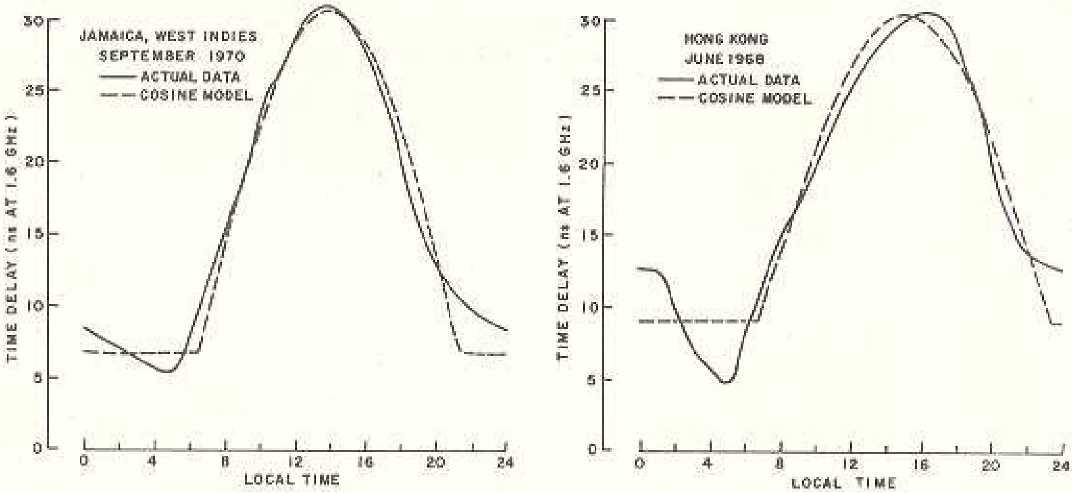

Figure 1. Two Examples of Actual Monthly Average Time-Delay Data

1. Llewellyn, S. K., and Bent, R. B. (1973) Documentation and Description of the Bent Ionospheric Model, AFCRL-TR-73-0657 and SAMSO-TR-73-252.

made to fit reasonably well during the maximum of the day when the absolute T_g values are greatest. The cosine function used is:

$$T_g = DC + A \cos \frac{(t - \phi)2\pi}{P} \quad , \tag{1}$$

where DC, A, ϕ, and P are the four parameters required to obtain a complete diurnal representation of T_g at any location. Only the positive half of the period of the cosine curve is used in this simple representation; thus, when the absolute value of the quantity $(t - \phi)2\pi/P$ becomes greater than 90 degrees, T_g is represented only by the DC term.

The simple positive cosine representation of the actual T_g is designed to fit best near the hours of maximum T_g. The fit near sunrise and well after sunset is expected to be poor; however, during these times the absolute value of T_g is much smaller than near the peak of the diurnal curve. The lower, later nighttime values are represented only by the constant DC term.

A check of the cosine diurnal fit, as compared with the actual monthly mean values, for a few months representative data from eight different TEC observing stations, revealed that the cosine algorithm was within approximately 10 percent of the actual 24-hr rms error.

As a further computation efficiency, since the cosine diurnal dependence is used only for the positive half of the waveform, the first two terms of the cosine expansion are used instead of the actual cosine function itself. Thus, the algorithm becomes:

$$T_g = DC + A \left[1 - \frac{x^2}{2} + \frac{x^4}{24}\right] \quad , \tag{2}$$

where

$$x = \frac{(t - \phi)2\pi}{P} \quad ,$$

and t is local time at the ionospheric point.

The error between the expansion and the actual cosine reaches a maximum of 0.02A at the points where it just intersects the DC value. Since the cosine fit is too low at these points, the series approximation to the cosine is actually slightly better than the cosine.

Other forms of diurnal algorithms, such as a Gaussian suggested by Antoniadis and daRosa (private communication), and a statistical F distribution have been considered. The 2-term expansion of the cosine was chosen as a reasonable compromise of accuracy versus algorithm difficulty.

5. GEOMETRY CALCULATION REQUIREMENTS

In determining the TEC, ionospheric time delay, from a user location to a satellite, several geometrical calculations must be made. The TEC must be found at the geographic point where the ray path intersects the mean ionospheric height, rather than at the user location. This point is taken here at a mean height of 350 kilometers. For a satellite at the lowest elevation considered here, 5 degrees, this point is approximately 14 degrees of earth angle away from the user location. The obliquity, or slant factor, also must be calculated for the mean ionospheric location. Finally, since the TEC is best ordered in geomagnetic, rather than geodetic coordinates, the assumption is made that the TEC is only a function of geomagnetic latitude and local time. Any two points on the earth having the same geomagnetic latitude are assumed to have the same TEC at the same local time. Thus, a conversion to geomagnetic latitude is required, and a calculation is made of local time at the geodetic longitude of the sub-ionospheric location. If the exact form of all the necessary geometry calculations is used, the computer time used would be greater than that required for the TEC algorithm itself. Therefore, in the derivation of these geometrical functions, simplifying assumptions were made in many cases to reduce the calculation complexity. A discussion of each of the steps in the geometry and the simplifying assumptions follows.

5.1 Earth Angle

The ionospheric time delay is computed at the point where the ray from the satellite intersects the ionosphere, rather than at the observer. For an observer located near the earth's surface looking at a satellite at 5 degrees elevation, the ionospheric intersection location is approximately 14 degrees of earth-centered angle from the observer. If the satellite is located along the observer's meridian at 5 degrees elevation angle, the 14 degree latitude difference can represent a significant gradient, particularly near the equatorial anomaly. If the 14 degree earth angle is all in longitude, at 40 degrees latitude, this represents a time difference of 1.2 hours, during which time the ionospheric time delay could have changed considerably.

The exact earth angle representation is:

$$A = 90 - el - Z \quad , \tag{3}$$

where

$$Z = \sin^{-1} (.948 \cos el) \quad ,$$

for a mean ionospheric height of 350 km. A useful approximation[2] is:

$$A = \frac{445}{el + 20} - 4 \quad . \tag{4}$$

In Figure 2 both the actual and the approximate earth angle calculations versus satellite elevation angle are given. This approximation is less than 0.2 degrees in error for all elevations greater than 10 degrees. It is off by only 0.4 degrees and 0.3 degrees at elevations of 5 and 0 degrees, respectively.

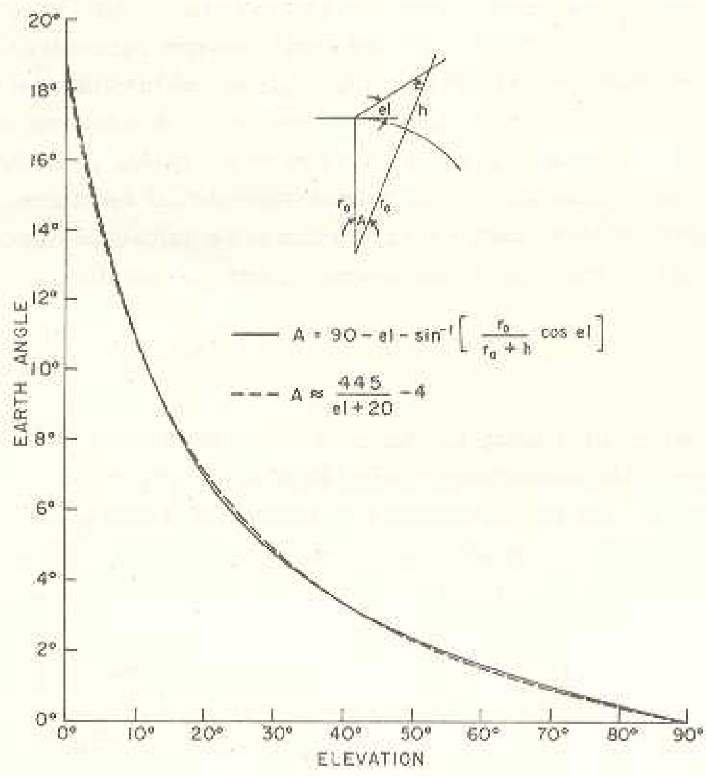

Figure 2. Approximation to Earth Angle, A

2. R.S. Allen assisted in developing this useful approximate form of earth angle.

5.2 Ionospheric Geodetic Latitude and Longitude

Given the station coordinates, elevation angle, and earth angle to the ionospheric point, its coordinates may be found by the following:

$$\phi_I = \sin^{-1}\{\sin\phi_o \cos A + \cos\phi_o \sin A \cos az\} \quad , \tag{5}$$

$$\lambda_I = \lambda_o + \sin^{-1}\left\{\frac{\sin A \sin az}{\cos\phi_I}\right\} \quad , \tag{6}$$

where ϕ_o is the station latitude, λ_o is the station longitude. If a flat earth approximation is used the following simplification can be made to the above two equations:

$$\phi_I = \phi_o + A \cos az \quad , \tag{7}$$

$$\lambda_I = \lambda_o + \frac{A \sin az}{\cos\phi_I} \quad . \tag{8}$$

In Figure 3 the geometry of the ionospheric location calculations is illustrated. At latitudes of 40 degrees and equatorward, the maximum error of these approximations is approximately 1 degree. At 60 degrees latitude, errors in determining ionospheric latitude reach a maximum of -3 degrees at 5 degrees elevation angle

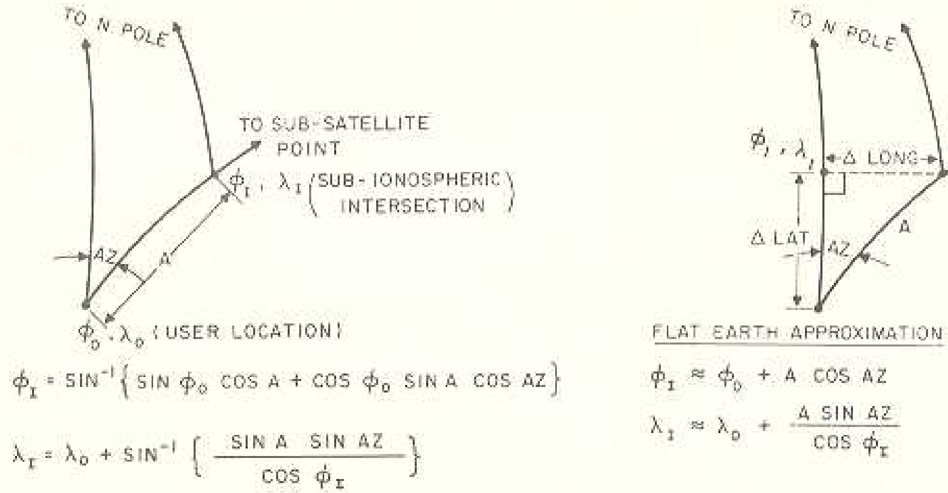

Figure 3. Approximations to Finding Sub-Ionospheric Coordinates

for a few azimuths. Ionospheric longitude errors reach only 2 degrees at the same location and direction. At high latitudes above 75 degrees an ionospheric latitude equal to the observer location is recommended, regardless of observer's elevation angle. Time delays at these latitudes are generally much lower and more highly variable than in the equatorial and mid-latitude regions. Thus, the time delay errors introduced by neglecting the conversion from user position to mean ionospheric position above 75 degrees will not significantly increase the overall worldwide rms errors of the model.

5.3 Conversion from Geodetic to Geomagnetic Latitude

Ionospheric time delay is, to an approximation, best ordered by geomagnetic, rather than geographic latitude. Thus, it is necessary to perform a transformation from geodetic to geomagnetic latitude. In this representation there is no true longitude dependence of ionospheric time delay, only that which comes about due to the differences between geomagnetic and geodetic latitudes as a function of longitude. The transformation from geodetic to geomagnetic latitude, assuming that the earth's magnetic field can be represented by an earth centered dipole, is given by:[3]

$$\sin \Phi = \sin \phi \sin \phi_P + \cos \phi \cos \phi_P \cos (\lambda - \lambda_P) \quad , \tag{9}$$

where

$$\phi_P = 78.3^\circ N \quad ,$$

$$\lambda_P = 291.0^\circ E \quad .$$

The approximation

$$\Phi = \phi + 11.6^\circ \cos (\lambda - 291) \tag{10}$$

represents the exact form to within 1 degree at all geomagnetic latitudes equatorward of ±40 degrees geomagnetic latitude. At geomagnetic latitude up to ±65 degrees, it is within ±2 degrees. Figure 4 is a numeric representation of the error of the approximation to geomagnetic latitude. Over the entire CONUS region the error in the approximate form is less than 1 degree.

5.4 Finding Local Time

Given the universal time and the approximate longitude of the ionospheric point, λ_I, the local time at the ionospheric point is simply:

3. Davies, K. (1966) Ionospheric Radio Propagation, Dover Publications, New York.

GEOCENTRIC DIPOLE MINUS APPROXIMATION
GEOM LAT = GEOG LAT + 11.6(COS(GEOG LONG - 291))
VALUES LESS THAN ONE DEGREE NOT SHOWN

GEOG LONG (EAST)

30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	
-4	-3	-2	-1					-1	-2	-3	-4	-4	-4	-4	-3	-2			-2	-3	-4	75
-3	-2	-1							-2	-2	-3	-3	-3	-3	-2				-1	-2	-3	70
-2	-2	-1							-1	-2	-2	-3	-2	-2	-1					-1	-2	65
-2	-2	-1							-1	-2	-2	-2	-2	-2						-1	-2	60
-2	-1									-1	-2	-2	-2	-1							-1	55
-1	-1									-1	-1	-1	-1								-1	50
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2	2	1							1	2	2	2	1									-55
2	2	2						1	2	2	2	2	2	1							1	-60
3	2	2	1					1	2	2	3	2	2	1							1	-65
3	3	3	2				1	2	3	3	3	3	2	1							2	-70
4	4	4	3	2			2	3	4	4	4	4	3	2	1					1	2	-75
30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	

Figure 4. A Numeric Representation of the Error of the Approximation to Geomagnetic Latitude

$$t = \frac{\lambda_I}{15} + UT \quad . \qquad \text{(hours)} \tag{11}$$

If t is greater than 24, make t = t - 24 hours.

5.5 The Obliquity Factor

The vertical time delay at the sub-ionospheric point must be multiplied by an obliquity factory defined as the secant of the zenith angle at the mean ionospheric height. An average ionospheric height of 350 km is assumed. The exact obliquity or slant factor is:

$$SF = \sec \{\sin^{-1}[.948 \cos el]\} \quad . \tag{12}$$

An approximation which is within 2 percent of the exact value for elevation angles from 5° to 90° is:

$$SF = 1 + 2 \left[\frac{96 - el}{90}\right]^3 \quad . \tag{13}$$

The approximation saves 3 trigonometric functions at the expense of finding the cube of a number. Both the exact and the approximate slant factors are shown in Figure 5.

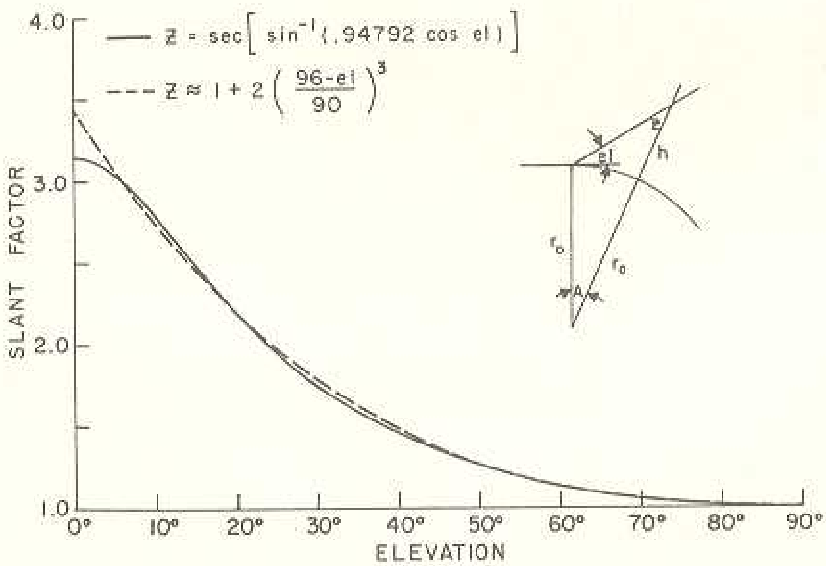

Figure 5. Approximation to Slant Factor, Z

6. ALGORITHM LATITUDE COEFFICIENTS

The four coefficients used in the time delay algorithm are all functions of latitude, season, solar flux, and magnetic activity. A complete representation of all these variables in each of the coefficients would be complex indeed, if even possible. What has been done here is to address only the monthly average latitude dependence of each of these four coefficients for various seasons of a solar maximum year when the highest ionospheric time delays are expected. Possible methods of determining the daily variation of the algorithm coefficients, or of simply including a daily update factor, are discussed in a later section of this report.

6.1 The DC Term

The DC term in the model algorithm represents the nighttime behavior of the ionosphere. Recent evidence from combined Faraday and group-delay measurements from signals transmitted from the geostationary satellite, ATS-6, shows that the Faraday data, which is really representative only of TEC up to approximately 2000 km of vertical height, is considerably in error in the nighttime. The Bent model also was integrated in vertical height only up to 2000 km. The new group-delay data available thus far show a nearly constant additional TEC above the TEC normally measured by the Faraday effect. Thus, with this model representation it is a simple matter to increase the DC term to account for the additional electrons not measurable by the Faraday effect.

The magnitude of the DC term versus latitude is shown in Figures 6 and 7. Since the DC term is generally much smaller than the amplitude term, A, it may be sufficient to represent its latitude dependence by a simple linear dependence with geomagnetic latitude either side of a reference latitude, or by a single constant for all latitudes. Alternatively, it can be represented by a second-order polynomial in geomagnetic latitude covering the entire latitude range. The largest errors will occur at the equatorial anomaly locations with a second-degree polynomial fit because of the inability of a second-degree equation to fit the complex shape of the actual DC term versus latitude.

In the actual finding of the coefficients for the latitude dependence of the DC and the amplitude terms it is important to make certain that they do not yield negative values at certain latitudes. A least squares polynomial fit to a set of experimental points can yield unexpected values in portions of the data set. The responsibility for ensuring that the latitude coefficients remain positive can be done well in advance, so that thoroughly tested coefficients will be loaded into the user computer.

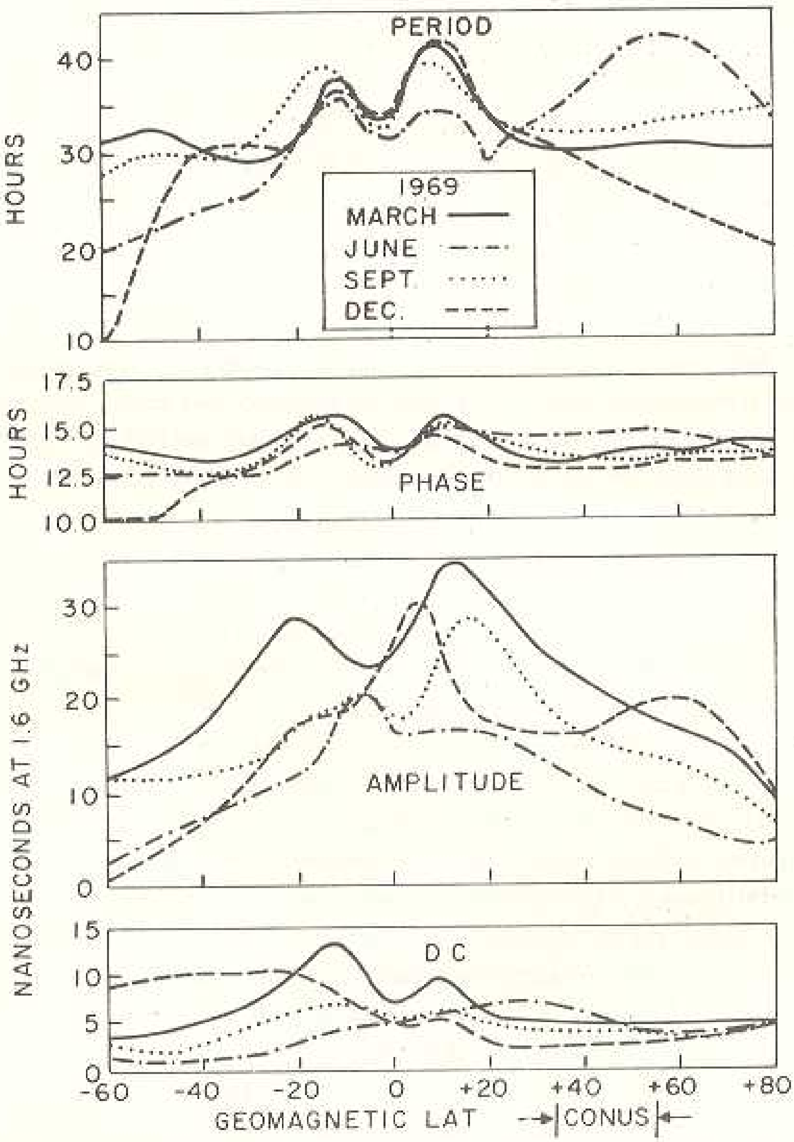

Figure 6. Cosine Model Coefficients (Bent Model) vs Geomagnetic Latitude for Four Seasons, 1969

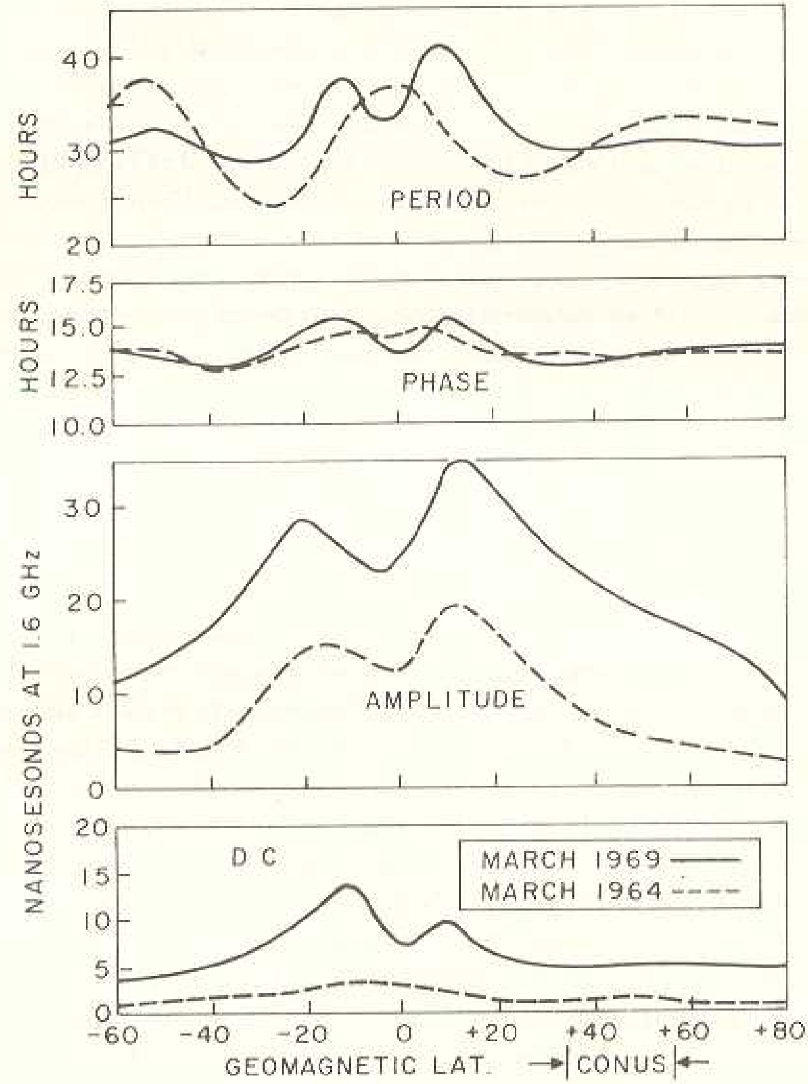

Figure 7. Cosine Model Coefficients (Bent Model) vs Geomagnetic Latitude for the Month of March for the Solar Minimum Year 1964 and the Solar Maximum Year 1969

6.2 The Amplitude Term

The amplitude of the cosine fit is the most important term in the algorithm, and is the most highly variable with latitude, season, and solar flux. It is the parameter that should be changed on a daily update basis, if this is deemed necessary and practical with the operational satellite system. To closely represent the amplitude term as a function of geomagnetic latitude several coefficients are required. From Figures 6 and 7 the amplitude dependence with geomagnetic latitude can be seen. Note that the CONUS region extends approximately from 35°N to 55°N geomagnetic latitude. It is obvious from the curves in Figures 6 and 7 that no single, fixed shape will match the latitude dependence of the amplitude term. Therefore, it is necessary to use some form of polynomial representation for the latitude dependence of amplitude. Because of the numerous inflection points in the curve for some seasons a polynomial of degree 5 or 6 is necessary to obtain a close fit. However, if the equatorial minimum in the amplitude term, which is particularly obvious during the equinoxes, is smoothed over, a second-degree polynomial can do a fair job of representing the amplitude behavior versus geomagnetic latitude. The same caution regarding negative values at high latitudes must be observed for the testing of the amplitude coefficients as was given for the DC term latitude coefficients.

6.3 The Phase of the Diurnal Variation

Happily, the phase term in the ionospheric time-delay algorithm is not a highly variable function. A plot of the phase term for a northern midlatitude station, for all the monthly averages of a six-year period covering from solar maximum to well down towards the solar minimum, revealed that the phase of the diurnal maximum rarely changed more than plus and minus one hour from a mean of 14 hours local time. Results from the Bent model, in Figures 6 and 7, and from actual monthly mean TEC data from several stations, show that the phase lies between 12.5 and 15.5 for most of the latitude range covering the mid and equatorial latitudes. Therefore a constant value of 14 hours local time was taken for the phase of the diurnal variation. If a particular sub-model is required for a limited geographic region, perhaps some improvement can be made by making the phase a function of time of the year; but, in this attempt at a worldwide representation, a constant value for all latitudes and seasons is deemed adequate.

6.4 The Diurnal Period

The period of the diurnal cosine representation is, of course, not fixed at 24 hours. In fact it is nearly always significantly higher than 24 hours and is a function of geomagnetic latitude and season as shown in Figures 6 and 7. As is the case with the amplitude and DC terms, a second-degree polynomial with geomagnetic latitude is recommended as the function to represent the latitude dependence of the cosine period term. Again, the equatorial anomaly region will not be perfectly represented by a second degree polynomial; but, the fit should be adequate.

7. OPTIONS FOR MODEL INCORPORATION INTO THE SYSTEM USER COMPUTER

There are several optional methods by which the operational user computer and satellite data-link combination can determine an ionospheric correction. Four options are suggested here.

7.1 Option 1

The most sophisticated, and also the most costly in terms of coefficients to be transmitted down the satellite to user link, is to transmit, and update once every 24 hours, 3, second-degree polynomial coefficients for each of the three algorithm coefficients, DC, amplitude, and period, making a total of 9 coefficients transmitted and updated each day. Eight-bit accuracy for each coefficient is more than adequate, making a total of 72 bits of ionospheric information in each message.

7.2 Option 2

One step removed in complexity is to update only the three amplitude coefficients on a daily basis and to have the user computer store all the coefficients for the seasonal and solar-flux dependence of the DC and the period terms. If these coefficients are changed monthly and are given for two values of solar flux the user computer must store 12 times 2 times 6 coefficients or 144, eight-bit numbers on a permanent basis. The user computer would interpolate between the two sets of stored coefficients depending on the actual solar flux. In addition, three amplitude-coefficient numbers plus a solar flux must be transmitted down the satellite to user link. The link will carry only 4, eight-bit numbers, or 32 bits of ionospheric information in each message. Using this option, the user still has the capability of obtaining updated amplitude coefficients and the master station computer has the option of weighting the daily values of amplitude coefficients for best accuracy over any particular region, such as the CONUS.

7.3 Option 3

A third option would have <u>all</u> the coefficients <u>permanently</u> stored in the user computer and would have only a daily value of solar activity transmitted down the satellite to user link. With this option, the data-line requirements are minimal, but the user computer must permanently store 12 times 9 times 2, or 216, eight-bit numbers. With the current improvements in Read Only Memories, ROM's, a 256 $\times$ 8 chip could conveniently hold all the necessary coefficients.

7.4 Option 4

An extremely simple representation of the DC, amplitude, and period functions of geomagnetic latitude, which would not require an ROM, could be used. In the case of DC, this would be a constant which would vary only with solar flux, not with season. For amplitude, only linear gradients from the geomagnetic equator would be used. For the period term, a constant for all seasons and solar-flux values would be used. The update information required would be only solar flux. Thus, only one number would be specified, that is, the solar flux, or some similar number which would be a multiplicative factor on the amplitude. With this option, the shape of the latitude gradient would remain fixed, or would change in a simple manner with seasonal solar flux, and only one dimensional variations would be allowed, as only one parameter, namely the solar flux, is specified.

With options 1 and 2 above, a network of ionospheric monitoring stations could be used to determine the latitude coefficients of the algorithm coefficients. With options 3 and 4, only a daily solar-flux value, easily obtained from the Air Weather Service's Global Weather Central, would be required.

There are other options, for example, transmitting two sets of amplitude coefficients, one for users in the CONUS region, and one for users in the equatorial region, or separate coefficients for each hemisphere, but these have not been considered in any detail.

Table 1. Summary of Options Considered

	Variables Transmitted	User Storage Requirements	Bits Transmitted Per Message (8 Bits Per Number Assumed)
Option 1	DC (3), Amp. (3), Period (3). Total = 9	Minimal	72
Option 2	Amp. (3), Solar Flux. Total = 4	144 (8-bit numbers)	32
Option 3	Solar Flux Only Total = 1	216 (8-bit numbers)	8
Option 4	Solar Flux Only Total = 1	Minimal	8

8. DETERMINING ALGORITHM LATITUDE COEFFICIENTS OPERATIONALLY

If consideration is given to an option which would require ionospheric measurements to determine updated coefficients, this can be done with as few as six monitoring stations located along the American eastern-longitude meridian. A single station, looking at satellites at 5 degrees of elevation angle, can make measurements of the equivalent vertical time delay at latitudes up to plus and minus 14 degrees from the station latitude. Thus, 3 stations, spaced at approximately 30 degree latitude intervals, can be used to monitor the latitude gradients of time delay over the entire Northern Hemisphere along one longitude sector. A total of 6 stations could monitor the entire latitude gradient for both hemispheres along one meridian.

Since no true longitude dependence is included in the model, measurements along one meridian are sufficient to specify the entire global representation. Stations in the northern hemisphere might include one under the equatorial anomaly at the Panama Canal Zone, one in the mid-latitudes near Washington, D.C., and one near the auroral latitudes located at either Goose Bay or Great Whale River, Canada. In the Southern hemisphere, stations might be located at the geomagnetic equator at Huancayo, Peru, near the Southern equatorial anomaly peak at the latitude of Santiago, Chile and perhaps in the Falkland Islands.

The data taken at these monitoring stations could be used to make daily values of coefficients of latitude gradients of the parameters which go into the algorithm, namely A, DC, phase, and period. The most important coefficient to attempt to determine on a daily basis is the amplitude, A. The amplitude coefficient is the most important and also probably the most highly variable one on a day to day basis, during most seasons. If any daily updated latitude coefficients are to be computed, the amplitude representation should be considered first. If only a single, general factor, perhaps dependent upon the solar ionizing flux or state of geomagnetic activity, is to be transmitted, it can be used to alter the amplitudes at all latitudes equally on a percentage basis. In this case no ionospheric monitoring stations will be required. Provision is made for this factor in the algorithm.

9. SUMMARY OF STEPS IN MODEL ALGORITHM

Given: (ϕ_o, λ_o, az, el, UT, Julian date)

Find earth angle:

$$A = \frac{445}{el + 20} - 4 \ .$$

Find sub-ionospheric latitude:

$$\phi_I = \phi_o + A \cos (az) \quad .$$

If ϕ_o is greater than 75^o, let $\phi_I = 75^o$; if ϕ_o is less than -75^o, let $\phi_I = -75^o$.

Find sub-ionospheric longitude:

$$\lambda_I = \lambda_o + \frac{A \sin (az)}{\cos \phi_I} \quad .$$

Find geomagnetic latitude:

$$\Phi_I = \phi_I + 11.6^o \cos (\lambda_I - 291) \quad .$$

Find local time:

$$t = \frac{\lambda_I}{15} + UT \quad .$$

Find slant factor:

$$SF = 1 + 2 \left[\frac{96 - el}{90}\right]^3 .$$

Also given: solar flux, coefficients of latitude dependence of algorithm variables.

Calculate $DC = D_o + D_1 \Phi_I + D_2 \Phi_I^2 + D_3 \Phi_I^3$.

$$Ampl. = A_o + A_1 \Phi_I + A_2 \Phi_I^2 + A_3 \Phi_I^3 \quad .$$

$$Period = P_o + P_1 \Phi_I + P_2 \Phi_I^2 + P_3 \Phi_I^3 \quad .$$

Note: In option 4 these polynomials are replaced by the following:

$$DC = 2 + 3^* \left[\frac{F_{10.7} - 60}{90}\right] \quad .$$

$$Ampl. = 19 - \frac{|\Phi|}{4} + \left[\frac{F_{10.7} - 60}{90}\right] \left[13 + 5 \cos \left(\frac{m - 3}{3}\right) \pi\right]$$

m = month of the year ,

Period = 32 hours .

Next, calculate:

$$x = \left(\frac{t - 14}{\text{Period}} \right) 2\pi \quad .$$

Test: If absolute x is greater than 1.57,

$$T_g = SF * DC \quad .$$

End.

If absolute x is less than 1.57,

$$T_g = SF* \left\{ DC + \text{Ampl.} \left[1 - \frac{x^2}{2} + \frac{x^4}{24} \right] \right\} .$$

End.

10. MODEL ACCURACY

This is the toughest part. How much increased accuracy do you buy for adding additional coefficients? Particularly, how much greater accuracy is obtained by actually setting up a six-station ionospheric monitoring network and sending down various numbers of coefficients to the users via the satellite to user data link? Also, since these coefficients are probably 24 hours old, how much improvement do you attain over monthly mean corrections? Finally, even if it is decided to send just one number down the satellite to user data link, and store all the other coefficients in a user computer ROM, how big does the ROM have to be to attain what correction? Is there a "knee in the curve" of number of coefficients versus accuracy?

Unfortunately, the answers to the above questions are largely unknown. Limited tests of the algorithm, using second-degree polynomials for the geomagnetic dependence for the DC, amplitude, and the period terms, have been made. These values were compared with the monthly average actual behavior of TEC at a few, selection stations and a better than 50 percent rms average correction capability was achieved. More testing remains to be done.

The problem in determining the polynomial coefficients is that the best present set of them is probably available from the Bent Ionospheric Model: but, little actual TEC data are currently available that show the accuracy of the Bent model in the important equatorial anomaly regions. As the GPS becomes operational, better coefficients could be found from a network of ionospheric monitoring stations and these new coefficients could be put, in retrofit fashion, into each user computer to replace the original ones that can be determined from the Bent model.

11. CONCLUSIONS AND RECOMMENDATIONS

Various trade offs in number of model coefficients have been suggested.

The diurnal behavior of ionospheric time delay at any location can be represented adequately by a positive half of a cosine wave with a DC constant term for the nighttime behavior. The amplitude and period of the cosine term are functions of geomagnetic latitude, as is the DC term. Since the geomagnetic dependence of these parameters changes with season, they are represented as polynomial functions with different coefficients for each season. The simplest form of latitude representation of the algorithm parameters is a constant for the DC term, changing only with solar flux, with constants for the period and phase terms. The amplitude term is the big unknown at this time due to the unknown shape of the latitude gradient and its changes with season and solar flux. Therefore, some form of model representation, which allows the latitudinal shape to be changed as a function of time, at least seasonally, is recommended. A second-degree polynomial of amplitude, with geomagnetic latitude, is recommended as the coefficients can be weighted for a particular latitude range of interest, such as the CONUS.

More testing of the suggested models against the monthly average TEC data base should yield a better idea of the rms improvement versus model complexity.