Summary

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Models for Discrete Distributions

Name	Remarks	Probability mass function	values of X	Parameter Space	Mean	Variance
Discrete Uniform	Outcomes that are equally likely (finite)	$f(x) = \frac{1}{N}$	$x = 1, 2, \dots, N$	$N=1,2,\dots$	$\frac{N+1}{2}$	$\frac{N^2-1}{12}$
Bernoulli Binomial	Bernoulli trial $X = \text{Number of}$ successes in n fixed trials	$f(x) = \theta^{x} (1 - \theta)^{1-x}$ $f_{X}(x) = \binom{n}{x} \theta^{x} (1 - \theta)^{n-x}$	x = 0, 1 $x = 0, 1, 2, \dots, n$	$0 \le \theta \le 1$ $0 \le \theta \le 1$; $n = 1, 2, 3, \dots$	$rac{ heta}{n heta}$	$\frac{\theta(1-\theta)}{n\theta(1-\theta)}$
Geometric	X = Number of failures before the first success	$f_X(x) = \theta(1-\theta)^x$	$x = 0, 1, 2, \dots$	$0 \le \theta \le 1$	$\frac{1-\theta}{\theta}$	$\frac{1-\theta}{\theta^2}$
Poisson	X = number of events in a given period of time, space, region or length	$f_X(x) = \frac{e^{-\lambda}(\lambda)^x}{x!}$	$x = 0, 1, 2, \dots$	$\lambda > 0$	λ	λ

Models for Continuous Distributions

Name	Probability density function	Values of X	Parameter Space	Mean	Variance
Uniform	$f_X(x) = \frac{1}{b-a}$	$a \le x \le b$	$-\infty < a < b < \infty$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Normal (Gaussian)	$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	$-\infty < x < \infty$	$-\infty < \mu < \infty; \ \sigma > 0$	μ	σ^2
Gamma	$f_X(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-x/\beta}$ $f_X(x) = \frac{1}{\beta} e^{-x/\beta}$	$0 < x < \infty$	$\alpha > 0; \beta > 0$	lphaeta	$lphaeta^2$
Exponential	$f_X(x) = \frac{1}{\beta} e^{-x/\beta}$	$0 < x < \infty$	$\beta > 0$	β	eta^2
Beta	$f_X(x) = \frac{1}{B(\alpha, \beta)} x^{\alpha - 1} (1 - x)^{\beta - 1}$	0 < x < 1	$\alpha > 0; \beta > 0$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)}$
Chi-square	$f_X(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1} (1-x)^{\beta-1}$ $f_X(x) = \frac{1}{2^{k/2} \Gamma(k/2)} x^{k/2-1} e^{-x/2}$	$x \in (0, \infty)$ if $k = 1$, otherwise $x \in [0, \infty)$	$k=1,2,\ldots$	k	Variance $ \frac{(b-a)^2}{12} $ $ \sigma^2 $ $ \alpha\beta^2 $ $ \beta^2 $ $ \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)} $ $ 2k $ $ \lambda^2 \left[\Gamma(\left(1+\frac{\beta}{2}\right)\right] $
Weibull	($x \in [0, \infty)$	scale- $\lambda \in (0, \infty)$, shape- $k \in (0, \infty)$	$\lambda\Gamma(1+1/k)$	$\lambda^2 \left[\Gamma(\left(1 + \frac{2}{3}\right) \right]$
	$f_X(x) = \begin{cases} f_X(x) = \frac{k}{\lambda} \left(\frac{k}{\lambda}\right)^{k-1} e^{-t} \\ 0 \end{cases}$	$0 \le x \ge 0$ $x < 0$			
	(4.8)				

4.9. SAMPLING DISTRIBUTIONCHAPTER 4. DISTRIBUTION THEORY