Quantitative Methods -- 2017

Instructor: Feng

Brief Introduction

Topic weight:

SS1	Ethics & Professional Standards	15%
SS2 - SS3	Quantitative Methods	12%
SS4 - SS5	Economics	10%
SS6 - SS9	Financial Reporting and Analysis	20%
SS10 - SS11	Corporate Finance	7%
SS12	Portfolio Management	7%
SS13 - SS14	Equity Investment	10%
SS15 - SS16	Fixed Income	10%
SS17	Derivatives	5%
SS18	Alternative Investments	4%
	Weights:	100%

Brief Introduction

Content:

- **Study session 2 (Quantitative Method: Basic Concepts)**
  - Reading 6: The Time Value of Money
  - Reading 7: Discounted Cash Flow Applications
  - Reading 8: Statistical Concepts and Market Returns
  - Reading 9: Probability Concepts

Brief Introduction

Content (Cont.):

- **Study session 3 (Quantitative Method: Application)**
  - Reading 10: Common Probability Distributions
  - Reading 11: Sampling and Estimation
  - Reading 12: Hypothesis Testing
  - Reading 13: Technical Analysis

Brief Introduction

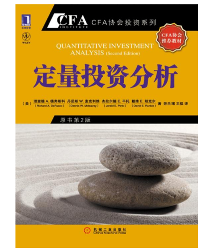
考纲对比:

- 与2016年相比，2017年的考纲没有变化。

Brief Introduction

推荐阅读:

- 定量投资分析
  - Richard A. DeFusco, Dennis W. Mcleavey, Jerald E. Pinto, David E. Runkle
  - ISBN: 978-7-111-38802-9
  - 机械工业出版社



Brief Introduction

学习建议:

- 本门课程难度不高，要有信心；
- 听课与做题相结合，但并不建议“刷题”；
- 最重要的，认真、仔细的听课。

胜负谁人定，兴衰岂无由；

古今多少事，尽在笑谈中！

Interest Rate

Tasks:

- Interpret interest rate and explain its components;
- Calculate and interpret effective annual rate, given stated annual interest rate and frequency of compounding.

Interest Rate

Time value of money

- Money available at the present time is worth more than the same amount in the future due to its potential earning capacity.
  - Provided money can earn interest, any amount of money is worth more the sooner it is received;
  - It concerns equivalence relationships between cash flows occurring on different dates.

Interest Rate

Cash flow additivity principle

- The amounts of money can only be added on if they are indexed at the same point in time.

Interest Rate

Interpretations of Interest Rate

- **Required rate of return:** minimum rate of return an investor must receive in order to accept the investment;
- **Discounted rate**: the rate at which we discount the future amounts to find their value today;
- **Opportunity cost:** the value that investors forgo by choosing a particular course of action.

Interest Rate

Components of interest rate

- Real risk-free interest rate
  - Single-period interest rate for risk-free security without inflation expected.
- Inflation premium
  - Compensating investors for expected inflation risk.

Interest Rate

Components of interest rate (Cont.)

- **Risk premium**
  - **Default risk premium**: compensating investors for the possibility that the borrower will fail to make the promised payments in time and in full amount;
  - **Liquidity premium**: compensating investors for the risk of loss relative to an investment's fair value if the investment needs to be converted to cash quickly.

Interest Rate

Components of interest rate (Cont.)

- **Risk premium**
  - **Maturity premium:** compensating investors for the increased sensitivity of the market value of debt to a change in market interest rates as maturity is extended.

Interest Rate

Components of interest rate

- Nominal interest rate = Real risk-free interest rate
  + Inflation premium
  + Default risk premium
  + Liquidity premium
  + Maturity premium
- Nominal risk-free interest rate = Real risk-free interest rate
  + Inflation premium

Interest Rate

Simple interest

- The annual interest rate times the principal.

Compounding interest

- The interest earned on interest is count in.

Interest Rate

Example

- If the annual interest rate is 10% and the principal is \$1000, what is the interest earned in 2 years under simple interest and compounding interest?

Answer:

- Under simple interest:
  Interest earned = \$1000 × 10% × 2 = \$200
- Under compounding interest:
  Interest earned = \$1000 × (1+10%) ×(1+10%) - \$1000
  = \$210

Interest Rate

Stated annual interest rate/Quoted interest rate (r_s)

- The annual interest rate that does not account for compounding within the year.

Compounding frequency (m)

- The number of compounding periods per year.
  - **Continuous compounding**: the number of compounding periods per year becomes infinite.

Periodic interest rate (r_s/m)

- Stated annual rate divided by the compounding frequency.

Interest Rate

Effective annual rate

- The rate by which a unit of currency will grow in a year with interest on interest included.

$$EAR=(1+\text{Periodic interest rate})^m-1=(1+\frac{r_s}{m})^m-1$$

- For continuous compounding:

$$EAR=e^{r_s}-1$$

Interest Rate

Example

- If the stated annual rate is 8%, compute the effective annual rate with quarterly compounding.

- **Answer:**
  - EAR = $(1 + 8\%/4)^4 - 1 = 1.0824 - 1 =$ **8.24%**

Summary

- **Importance:** ☆☆
- **Content:**
  - Interpretations of interest rate;
  - Components of interest rate;
  - Calculation of EAR.
- **Exam tips:**
  - 常考点：计算题，不同compounding frequency时名义利率和有效利率之间的换算。

Time Value of Money Problem

Tasks:

- Calculate and interpret future value (FV) and present value (PV) of different types of cash flows.

Present Value and Future Value

Relationship between PV and FV

- **Present value (PV)**: the value of an initial investment.
- **Future value (FV)**: the value of an initial investment would be worth n periods from today.
  - Present value and future value are equivalent measures separated in time.

$$FV = PV \times (1+r)^n \quad \text{or} \quad PV = \frac{FV}{(1+r)^n}$$

where: r = periodic rate, n = number of periods.

Present Value and Future Value

Relationships between PV and FV (Cont.)

- For a given interest rate, the FV increases with the number of periods;
- For a given number of periods, the FV increases with the interest rate;
- For a given interest rate, the farther in the future the amount to be received, the smaller that amount's PV;
- Holding time constant, the larger the interest rate, the smaller the PV of a future amount.

Present Value and Future Value

Example

- Suppose a $10,000 investment and a stated annual interest rate of 8%, compute the future value with monthly compounding and continuous compounding in one year.

Answer:

- For monthly compounding:

$$FV = PV \times \left(1+\frac{r_s}{m}\right)^m = 10{,}000 \times \left(1+\frac{0.08}{12}\right)^{12} = \$10{,}829.99$$

- For continuous compounding:

$$FV = PV \times e^{r_s} = 10{,}000 \times e^{0.08} = \$10{,}832.87$$

Present Value and Future Value

Future value of a single cash flow

- **Example**: what is the future value of $200 invested today in two years when the interest rate is 10%?

I = 10%　0 — 1 — 2
$200 → ?

- **Answer:** FV=200×(1+10%)×(1+10%)=200 ×(1.1^2)=242
  - Using financial calculator:
    N=2; I/Y=10; PV=200; PMT=0; CPT: FV= -242

Present Value and Future Value

Present value of a single cash flow

- **Example:** what is the present value of $200 to be received in two years when the interest rate is 10%?

I = 10%　0 — 1 — 2
? ← $200

- **Answer:** PV=200÷(1.1^2)=165.29
  - Using financial calculator:
    N=2; I/Y=10; FV=200; PMT=0; CPT: PV= -165.29

Present Value and Future Value

Annuity

- A finite set of constant sequential cash flows.
  - **Ordinary annuity**: all constant cash flows occurring at the end of each period;
  - **Annuity due**: all constant cash flows occurring at the beginning of each period.

Perpetuity

- A set of constant never-ending sequential cash flows occurring at the end of each period.

Present Value and Future Value

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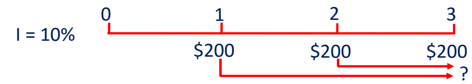
Future value of an ordinary annuity

- **Example:** what is the value in 3 years time of $200 to be received at the end of each year for three years when the interest rate is 10%?



- **Answer:** FV=200×(1.1²)+200×(1.1)+200=662
  - Using financial calculator:

    N=3; I/Y=10; PV=0; PMT=200; CPT: FV= -662

Present Value and Future Value

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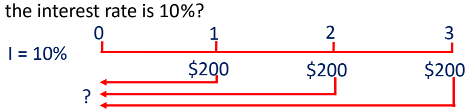
Present value of an ordinary annuity

- **Example:** what is the present value of $200 to be received at the end of each year for three years when the interest rate is 10%?



- **Answer:** PV=200÷(1.1)+200÷(1.1²)+200÷ (1.1³)=497.37
  - Using financial calculator:

    N=3; I/Y=10; FV=0; PMT=200; CPT: PV= -497.37

Present Value and Future Value

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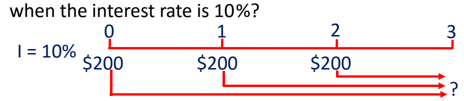
Future value of an annuity due

- **Example:** what is the value in 3 years time of $200 to be received at the beginning of each year for three years when the interest rate is 10%?



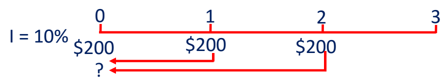
- **Answer:** FV = 200×(1.1)+200×(1.1²)+200×(1.1³) = 728.2
  - Using financial calculator (BGN Mode):

    N=3; I/Y=10; PV=0; PMT=200; CPT: FV= -728.2

Present Value and Future Value

Present value of an annuity due

- **Example:** what is the present value of \$200 to be received at the start of each year for three years when the interest rate is 10%?



- **Answer:** PV = 200+200÷(1.1)+200÷(1.1²) = 547.11
  - Using financial calculator (BGN Mode): N=3; I/Y=10; FV=0; PMT=200; CPT: FV= -547.11

Present Value and Future Value

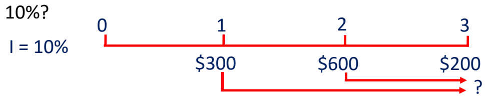
Present value of perpetuity

- $PV = \frac{A}{r}$
  - A = the periodic payment to be received forever
- **Example:** a preferred stock will pay \$8 per year forever and the rate of return is 10%. What is its value?
- **Answer:** PV = 8 ÷ 0.1 = 80

Present Value and Future Value

Future value of a series of unequal cash flow

- **Example:** what is the total value in 3 years time of \$300 received at the end of 1st year, \$600 at the end of 2nd year, and \$200 at the end of 3rd year when the interest rate is 10%?

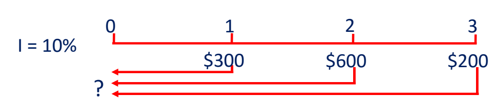


- **Answer:** FV = 200+600×(1.1)+300×(1.1²) = 1233

Present Value and Future Value

Present value of a series of unequal cash flow

- **Example:** what is the total present value of \$300 received at the end of 1st year, \$600 at the end of 2nd year, and \$200 at the end of 3rd year when the interest rate is 10%?



- **Answer:** PV = 300÷(1.1)+600÷(1.1²)+200÷(1.1³) = 918.86

Present Value and Future Value

Discount rate or growth rate

- **Example:** Elmer has won his $4 million state lottery and has been offered 20 annual payments of $200,000 each beginning today or a single payment of $2,267,000. What is the annual discount rate used to calculate the lump-sum pay-out amount?
- **Answer:** using financial calculator (BGN Mode): N=20; FV=0; PV=2,267,000; PMT=-200,000; CPT: I/Y= 7%.

Present Value and Future Value

Number of periods

- **Example:** Elmer has won his $4 million state lottery and has been offered 20 annual payments of $200,000 each beginning today or a single payment of $2,267,000. If Elmer can choose the amount of his annual pay-out, based on a 7% discount rate, how many payments of $232,631 could Elmer receive if his first payment were today?
- **Answer:** using financial calculator (BGN Mode): FV=0; PV=2,267,000; PMT=-232,631; I/Y= 7%; CPT: N=15.

Present Value and Future Value

Size of payment

- **Example:** what is the monthly payment on a $100K, 30-year home loan with stated rate of 6%?
- **Answer:** using financial calculator: N=30×12=360; I/Y=6/12=0.5; PV=100,000; FV=0; CPT: PMT= -599.55.

Summary

- **Importance:** ☆☆
- **Content:**
  - Calculation of PV and FV of single cash flows, annuity, perpetuity, unequal cash flows;
  - Calculation of discount rate, number of periods, size of payment.
- **Exam tips:**
  - 考计算题。

Evaluation of Cash Flow Streams

Tasks:

- Calculate and interpret net present value (NPV) and internal rate of return (IRR) of an investment;
- Contrast the NPV rule to the IRR rule, and identify problems associated with the IRR rule.

Evaluation of Cash Flow Streams

Net Present Value (NPV)

- The present value of its cash inflows(benefits) minus the present value of its cash outflows(costs).
- Calculation of NPV:
  - Identify all cash flows;
  - Determine the discount rate or opportunity cost (r);
  - Find the present value of each cash flow;
  - Sum up all present value to get NPV.

$$NPV = CF_0 + \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \ldots + \frac{CF_n}{(1+r)^n}$$

Evaluation of Cash Flow Streams

Net Present Value (Cont.)

- Apply the NPV rules:
  - If NPV > 0, undertake the project;
  - If NPV ≤ 0, should not undertake the project;
  - For mutually exclusive projects (can only invest in one), choose the one with higher positive NPV.

Evaluation of Cash Flow Streams

Example:

- A project requires an initial outlay of $2 million, cash flows at end of year 1, 2, 3 are $0.5 million, $0.75 million, $1.35 million, respectively. If the discount rate is 10% per year, calculate the net present value.

Answer:

$NPV = -2 + 0.5/(1.10) + 0.75/(1.10)^2 + 1.35/(1.10)^3$

$= \$0.089$ mil.

Evaluation of Cash Flow Streams

Internal rate of return (IRR):

- The discount rate that makes net present value equal to zero.

$$NPV = 0 = CF_0 + \frac{CF_1}{(1+IRR)^1} + \frac{CF_2}{(1+IRR)^2} + \ldots + \frac{CF_n}{(1+IRR)^n}$$

- Apply the IRR rules:
  - IRR > opportunity cost of capital, undertake the project
  - IRR ≤ opportunity cost of capital, should not undertake the project.

Evaluation of Cash Flow Streams

Example

- A project requires an initial outlay of $2 million, cash flows at end of year 1, 2, 3 are $0.5 million, $0.75 million, $1.35 million, respectively. If the discount rate is 10% per year, calculate the IRR.

Answer:

$0 = -2 + 0.5/(1+IRR) + 0.75/(1+IRR)^2 + 1.35/(1+IRR)^3$

IRR = 12.13%

Evaluation of Cash Flow Streams

Problems with IRR rules

- NPV and IRR rules give the same accept or reject decision when projects are independent, but may rank projects differently if projects are mutually exclusive when:
  - The size or scale of the projects differs;
  - The timing of the projects' cash flows differs.

Evaluation of Cash Flow Streams

Problems with IRR rules (Cont.)

- Stick to the NPV rule when NPV's and IRR's suggestions are conflict.
- When the signs of cash flows change more than once, there can be more than one IRR.

Summary

- **Importance:** ☆☆
- **Content:**
  - Calculation of NPV and IRR of an investment;
  - Apply the NPV and IRR rules;
  - Problems with IRR rules.
- **Exam tips:**
  - 考计算题。

Portfolio Return Measurement

Tasks:

- Calculate and compare the holding period return, money-weighted and time-weighted rates of return;
- Calculate and interpret, and convert among the bank discount yield, effective annual yield, and money market yield for money market instruments.

Portfolio Return Measurement

Holding period return

- The return that an investor earns over a specified holding period.

$$HPR = \frac{P_1 - P_0 + D_1}{P_0}$$

Example

- Stock purchased nine months ago for \$29 just paid a dividend of \$1.30 and is valued at \$30.50. Calculate the nine-month holding period return.

Answer: HPR = (30.50 + 1.30 -29)/29 = 9.66%

Portfolio Return Measurement

Time-weighted return (TWR)

- The compound return that \$1 initially invested in the portfolio over a stated measurement period.
- Calculation of TWR:
  - Break the overall evaluation period into sub-periods based on the dates of significant cash inflows and outflows;
  - Calculate the HPRs for each sub-periods;
  - Link or compound HPRs to obtain an annual rate of return.

Portfolio Return Measurement

Time-weighted return (Cont.)

$$TWR=\left[\left(\frac{\text{End Value}_1}{\text{Begin Value}_1}\right)\left(\frac{\text{End Value}_2}{\text{Begin Value}_2}\right)\cdots\cdots\left(\frac{\text{End Value}_n}{\text{Begin Value}_n}\right)\right]^{\frac{1}{N}}-1$$

Portfolio Return Measurement

Money-weighted return (MWR)

- MWR accounts for the timing and amount of all cash flows into and out of the portfolio.
  - If more funds to invest at an unfavorable time, MWR will tend to be depressed;
  - If more funds to invest at a favorable time, MWR will tend to be elevated.
- Calculation of MWR: similar to IRR.

$$CF_0+\frac{CF_1}{1+MWR}+\ldots+\frac{CF_N}{(1+MWR)^N}=0$$

Portfolio Return Measurement

TWR vs. MWR

- **Time weighted return:**
  - Not affected by cash withdrawals or additions;
  - Periods can be any length between significant cash flows.
- **Money weighted return:**
  - Assign more weights to the return of larger cash flows;
  - Affected by cash withdrawals or additions;
  - Periods must be equal length.
    - ✓ Use shortest period with no significant cash flows.

Portfolio Return Measurement

TWR vs. MWR (Cont.)

- **Example:** Eric invests $1,000 in an account. After one year, the value of his investment is $1,200 and Eric adds another $800 into the account. At the end of Year 2, the total value of the investment is $2,200. Calculate the annual TWR and MWR.

Answer:

TWR = $[(1.2)(1.1)]^{1/2} - 1$ = 14.89%; MWR = 13.623%.

- Using your calculator to calculate MWR:
  CF_0 = –1,000; CF_1 = –800; CF_2 = 2,200; CPT: IRR = 13.623%.

Money Market Yields

Holding period yield (HPY)

- HPY = (Ending Value/Beginning Value) - 1

Bank discount yield (BDY)

- BDY = (Discount/Face Value) × (360/Days to maturity)
  - Discount rate, simple interest, 360-day annualized.

Money Market Yield (MMY)

- **MMY** = (Discount/Price) × (360/Days to maturity)
  - Add-on rate, simple interest, 360-day annualized.

Money Market Yields

Bond Equivalent Yield (BEY)

- BEY = (Discount/Price) × (365/Days to maturity)
  - Add-on rate, simple interest, 365-day annualized;
  - Only for money market, not available for capital market.

Effective annual yield (EAY)

- EAY = $(1+HPY)^{365/Days}$ -1
  - Add-on rate, compound interest, 365-day annualized.

Money Market Yields

Example

- A 90-day T-bill is purchased for $997.40. What are the bank discount yield, holding period yield, money market yield, and the effective yield?

Answer:

Bank discount yield: [(1,000 – 997.40)/1,000] × 4 = 1.04%;

90-day holding period return: 1,000/997.4 – 1 = 0.2607%;

Money market yield: 0.2607 × (360/90) = 1.0428%;

Effective annual yield: $(1{,}000/997.4)^{365/90}$ – 1 = 1.0614%.

Summary

- **Importance:** ☆☆
- **Content:**
  - TWR vs. MWR of portfolios;
  - HPY, BDY, MMY, BEY, and EAY of money instrument.
- **Exam tips:**
  - 常考点：TWR和MWR的计算与大小关系比较。

Basic Concepts of Statistics

Tasks:

- Describe basic concepts of statistics, including population and parameter, sample and sample statistics, and frequency distribution;
- Distinguish among types of measurement scale.

Basic Concept of Statistics

Descriptive statistics

- Study of how data can be summarized effectively to describe the important aspects of large data sets.

Inferential statistics

- Involves making forecasts, estimates, or judgments about a larger group from the smaller group actually observed.

Basic Concept of Statistics

Population

- All members of a specified group.
  - **Parameter:** any descriptive measure of a population characteristic.

Sample

- A subset of a population.
  - **Sample statistic:** any descriptive measure of a sample characteristic .

Basic Concept of Statistics

Measurement scales

- Nominal Scale
  - Categorizing the data according to their characteristics but does not rank them;
  - Weakest level of measurement.
- Ordinal scale
  - Sorting data into categories that are ordered with respect to some characteristics.

Basic Concept of Statistics

Measurement scales (Cont.)

- Interval scale
  - Ranking the data with assurance that the differences between scale values are equal;
  - The scale values can be added or subtracted meaningfully.

Basic Concept of Statistics

Measurement scales (Cont.)

- Ratio scale
  - Having all the characteristics of interval measurement scales as well as a true zero point as the origin;
  - The scale values can be added, subtracted, multiplied or divided meaningfully;
  - The strongest level of measurement.

Presentation of Data

Frequency distribution

- Tabular display of data summarized into a relatively small number of intervals.
  - **Absolute frequency distribution:** the actual number of observations in a given interval;
  - **Relative frequency distribution:** the percentage of dividing absolute frequency of each interval by the total number of observations.

Presentation of Data

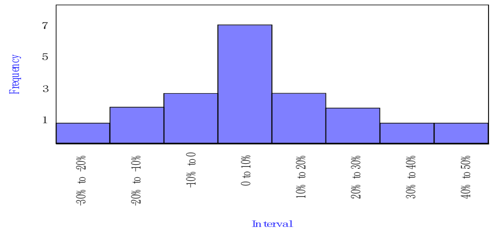
Frequency distribution (Cont.)

- **Cumulative Frequency Distribution**
  - Shows the percentage of observations less than the upper bound of each interval.

Presentation of Data

Histogram

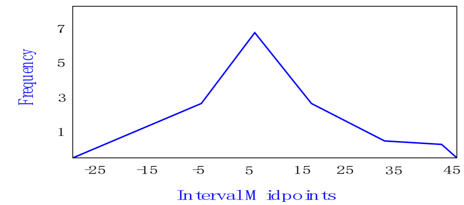
- A bar chart of data that have been grouped into a frequency distribution.



Presentation of Data

Frequency Polygon

- Constructed by plotting the midpoint of each interval on the x-axis and the absolute frequency of each interval on the y-axis, then connecting each point with a straight line.



Summary

- **Importance:** ☆☆
- **Content:**
  - Descriptive statistics vs. inferential statistics;
  - Population vs. sample;
  - Types of measurement scale;
  - Frequency and frequency distribution.
- **Exam tips:**
  - 常考点：types of measurement scale的辨析。

Measures of Central Tendency: Means

Tasks:

- Calculate and interpret population mean, sample mean, arithmetic mean, weighted average or mean, geometric mean, harmonic mean.

Quantitative Description of Distribution

Quantitative descriptions of return distribution

- **Central tendency:** where returns are centered;
- **Quantiles**: how return located (location);
- **Dispersion:** how far returns are dispersed from center;
- **Skewness:** whether the distribution of returns is symmetrically shaped;
- **Kurtosis:** whether extreme outcomes are likely or whether fatty tails exist.

Measures of Central Tendency

Central tendency

- Mean
  - Arithmetic mean
  - Geometric mean
  - Weighted mean
  - Harmonic mean
- Median
- Mode

Measures of Central Tendency

Arithmetic mean

- Equal to the sum of the observations divided by the number of the observations.
  - Population Mean

$$\mu = \frac{\sum_{i=1}^{N} X_i}{N}$$

  Sample Mean

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

Measures of Central Tendency

Arithmetic mean (Cont.)

- Arithmetic mean return focus on average single-period performance.
- Advantage:
  - Easy to work with mathematically;
  - Uses all the information about the size and magnitude of the observations.
- Disadvantage:
  - Sensitive to extreme values.

Measures of Central Tendency

Geometric mean

- The n^{th} root of a set of observations.

$$G = \sqrt[n]{X_1 X_2 X_3 \ldots X_n} \quad \text{with} \quad X_i \geq 0 \text{ for } i = 1,2,3,\ldots,n.$$

- Used to calculate average periodic compound rate of return on investment

$$\text{Periodic return}_{\text{compound}} = \sqrt[n]{(1+R_1)(1+R_2)\ldots\ldots(1+R_n)} - 1$$

✓ Geometric mean return focus on the profitability of an investment over a multi-period horizon

Measures of Central Tendency

Harmonic mean

- Calculation method:

$$\overline{X}_{\text{Harmonic}} = \frac{N}{\sum_{i=1}^{N} \frac{1}{X_i}}$$

where : N = number of purchases (equal $ amounts)
X_i = share price for each purchase

- Used to find the **average cost per share** of stock purchased over time in constant dollar amounts.

Measures of Central Tendency

Comparison among different means

- Harmonic Mean ≤ Geometric Mean ≤ Arithmetic Mean
  - The equal sign will only be valid given all the observations are same;
  - Greater variability of the different observation, the more the arithmetic mean will exceed the geometric mean and harmonic mean as well.

Measures of Central Tendency

Comparison among different means (Cont.)

- **Example:** please calculate the arithmetic mean, geometric mean and harmonic mean of 2, 3, 4.

$$\text{Arithmetic Mean} = \frac{2+3+4}{3} = 3 \text{ (largest)}$$

$$\text{Geometric Mean} = \sqrt[3]{2 \times 3 \times 4} = 2.88$$

$$\text{Harmonic Mean} = \frac{3}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} = 2.77 \text{ (smallest)}$$

Measures of Central Tendency

Weighted mean

- A mean in which different observations have different proportional influence on the mean.

$$\overline{X}_W = \sum_{i=1}^{n} w_i R_i = w_1 R_1 + w_2 R_2 + \ldots\ldots\ldots w_n R_n$$

Where:

- $R_1, R_2, \ldots\ldots R_n$ are the returns for assets 1,2,……,n;
- $w_1, w_2, \ldots\ldots, w_n$ are the portfolio weights.
  - ✓ $w_1 + w_2 + \ldots\ldots + w_n = 1$.

Measures of Central Tendency

Weighted mean (Cont.)

- Weighted mean are mostly used to calculate the portfolio return, or the *expected value* based on probabilities.
  - Arithmetic mean is a special case of weighted mean as the weight for each observation are equally assigned.

Measures of Central Tendency

Weighted mean (Cont.)

- **Example:** an investor has a $12,000 portfolio consisting of $7,000 in stock A with an expected return of 20% and $5,000 in stock B with an expected return of 10%. What is the investors expected return on the portfolio?
- **Answer:**

$$R_{portfolio} = \frac{7000}{12000} \times 20\% + \frac{5000}{12000} \times 10\% = 15.8\%$$

Summary

- **Importance:** ☆☆
- **Content:**
  - Measures of central tendency: arithmetic means, geometric means, weighted means, and harmonic means.
- **Exam tips:**
  - 常考点：考概念题，几种均值的适用场合和优缺点。

Median, Mode, and Quantiles

Tasks:

- Calculate and interpret median and mode;
- Calculate and interpret quartiles, quintiles, deciles, and percentiles.

Measures of Central Tendency

Median

- The value of the middle item of a set of items sorted into ascending or descending order.
  - Odd number of n items, median occupies the (n+1)/2 position; even number of n items, median is equal to the mean of the items occupying the n/2 and (n+2)/2 positions.

Measures of Central Tendency

Median (Cont.)

- The value of the middle item of a set of items sorted into ascending or descending order.
  - **Advantage**: not affected by extreme values (a.k.a., outliers) as arithmetic mean.
  - **Disadvantage**: only one or two numbers considered, rest is to be ignored.

Measures of Central Tendency

Median (Cont.)

- **Example:** please find out the median of following set of items, respectively.
  - 2, 5, 7, 11, 14

    Answer: median = 7.
  - 3, 7, 9, 10, 15, 20

    Answer: median = (9 + 10) / 2 = 9.5.

Measures of Central Tendency

Mode

- Most frequently occurring value of the distribution.
  - The distribution could have more than one mode, or even no mode (bimodal, trimodal, etc.);
  - Mostly used with nominal data.
- **Example**: please find out the mode of following set of items: 2, 4, 5, 5, 7, 8, 8, 8, 10, 12.

  Answer: mode = 8.

Measures of Location

Quantile

- A value at or below which a stated fraction of the data lies.
  - **Quartiles:** the distribution divided into quarters;
  - **Quintiles:** the distribution divided into the fifths;
  - **Deciles:** the distribution divided into the tenths
  - **Percentiles:** the distribution divided into the hundredths.
- Quantiles are often used to rank performance and investment research.

Measures of Location

Quantile (Cont.)

- Formula for Location of data in ascending order:

$$L_y=(n+1)\frac{y}{100}$$

Where: n=the number of data

y=the y^{th} percentile

Measures of Location

Quantile (Cont.)

- **Example:**

  For data with 17 observations, find out the location of 3rd quintile.

  Answer: $L_y = (17+1) \times 0.60 = 10.8$

  For ascending ordered observations, this is eight-tenths of the way from the 10^{th} observation to the 11^{th} observation.

Summary

- **Importance:** ☆☆
- **Content:**
  - Measures of central tendency: median, mode;
  - Measures of location: quantiles.
- **Exam tips:**
  - 常考点：quantiles的计算。

Measures of Dispersion

Tasks:

- Calculate and interpret range, mean absolute deviation, variance and standard deviation;
- Master the application of Chebyshev's inequality;
- Calculate and interpret coefficient of variation and the Sharpe ratio.

Measures of Dispersion

Dispersion

- Variability around the central tendency, usually used to address the risk.
- **Range**
  - Range = Maximum Value – Minimum Value
  - Easy for computation, but only use two numbers and tell nothing about the distribution of the data set.

Measures of Dispersion

Dispersion (Cont.)

- **Mean Absolute Deviation (MAD)**

$$MAD = \frac{\sum_{i=1}^{n}\left|X_i - \bar{X}\right|}{n}$$

Where : $\bar{X}$ is the sample mean, n is the number of observations.

Measures of Dispersion

Dispersion (Cont.)

- **Variance:** equal to average of the sum of squared deviations around the mean.

Population Variance: $\sigma^2 = \dfrac{\sum_{i=1}^{N}(X_i-\mu)^2}{N}$

Where: μ is the population mean, N is the size of population.

Sample Variance: $s^2 = \dfrac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}$

Where: $\bar{X}$ is the sample mean, n is the sample size.

Measures of Dispersion

Dispersion (Cont.)

- **Standard deviation:** positive squared root of variance.

Population Standard Deviation: $\sigma = \sqrt{\dfrac{\sum_{i=1}^{N}(X_i-\mu)^2}{N}}$

Where: μ is the population mean, N is the size of population.

Sample Standard Deviation: $s = \sqrt{\dfrac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}}$

Where: $\bar{X}$ is the sample mean, n is the sample size.

Measures of Dispersion

Chebyshev's inequality

- For any distribution with finite variance, the minimum percentage of observations that lie **within k standard deviations** of the mean would be $1-1/k^2$, given k>1.
- **Example:**

  According to Chebyshev's inequality, what is the minimum percentage of observations lie within 2 standard deviations of the mean?

  Answer: $1-1/2^2=75\%$.

Measures of Dispersion

Coefficient of variation (CV)

- The ratio of the standard deviation of a set of observations to their mean value.

  $CV = \dfrac{s}{\bar{X}}$

  - CV has no units of measurement, so permits direct comparisons of dispersions across different data sets;
  - A measure of risk per unit of mean return, thus the lower is better.

Measures of Dispersion

Sharpe ratio

- The ratio of the mean excess return on portfolio P to the standard deviation of the returns of portfolio P.

$$\text{Sharpe ratio} = \frac{\bar{R}_P - \bar{R}_F}{\sigma_P}$$

- No units of measurement, so permits direct comparisons of dispersions across different data sets.
- A measure of excess return per unit of risk, thus the higher is better (only valid for positive Sharpe ratio).

Summary

- **Importance:** ☆☆☆
- **Content:**
  - Measures of dispersion: range, MAD, variance and standard deviation;
  - Chebyshev's inequality;
  - CV & Sharpe ratio.
- **Exam tips:**
  - 常考点：Chebyshev' s inequality 和 CV & Sharpe ratio，可能出计算题。

Skewness & Kurtosis

Tasks:

- Explain measures of sample skewness and kurtosis;
- Describe the relative locations of the mean, median, and mode for a unimodal, nonsymmetrical distribution.

Skewness

Skewness (Sk)

- Indicating the degree of symmetry of return distributions.

$$\text{Sample skewness } (S_k) = \left[\frac{n}{(n-1)(n-2)}\right]\frac{\sum_{i=1}^{n}(X_i - \bar{X})^3}{s^3}$$

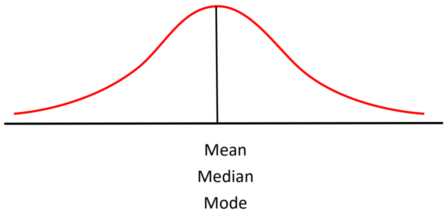
Where: n is the sample size;
s is the sample standard deviation.

- $S_k = 0 \rightarrow$ Symmetrical distribution;
- $S_k > 0 \rightarrow$ Positively (right) skewed distribution;
- $S_k < 0 \rightarrow$ Negatively (left) skewed distribution.

Skewness

Skewness (Cont.)

- Symmetrical distribution ($S_k = 0$)

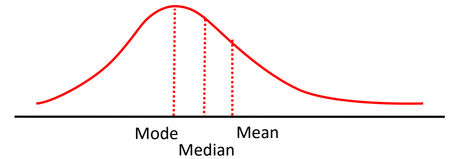


Mean = Median = Mode

Skewness

Skewness (Cont.)

- Positively (right) skewed distribution



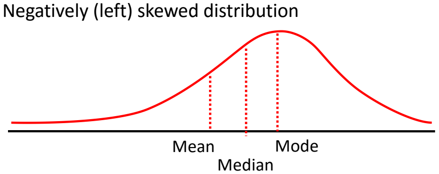
Mean > Median > Mode

- Frequent small losses and a few extreme gains (fatter/longer right tail).

Skewness

Skewness (Cont.)

- Negatively (left) skewed distribution



Mean < Median < Mode

- Frequent small gains and a few extreme losses (fatter/longer left tail).

Kurtosis

Kurtosis

- Measuring the degree to which the distribution is more or less peaked than normal distribution.
  - **Leptokurtic**
    - ✓ More peaked, fatter tailed than normal distribution;
    - ✓ Kurtosis > 3, excess kurtosis > 0.

Kurtosis

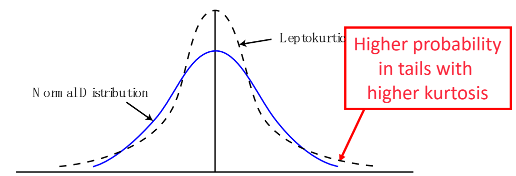
Kurtosis (Cont.)

- **Mesokurtic**
  - ✓ Identical to normal distribution;
  - ✓ Kurtosis = 3, excess kurtosis = 0.
- **Platykurtic**
  - ✓ Less peaked, thinner tailed than normal distribution;
  - ✓ Kurtosis < 3, excess kurtosis < 0.

Kurtosis

Kurtosis (Cont.)

➢ Leptokurtic vs. Normal distribution



Summary

➢ **Importance:** ☆☆

➢ **Content:**

- Skewness: positively & negatively, and the relative location of mean, median, and mode;
- Kurtosis: leptokurtic, mesokurtic, platykurtic.

➢ **Exam tips:**

- 常考点：正偏、负偏时mean、median、mode的相对位置。

Basic Concepts & Types of Probability

Tasks:

➢ Define basic concepts of probability, including random variable, outcome, and event;

➢ Distinguish different types of probability.