

# Basic features of fixed-income security



### Preface:

- 固定收益证券是某一市场主体的财务责任,该主体承 诺在未来特定日期支付特定金额的货币。
- 固定收益证券包括债券和优先股。
- ▶ 本质上就是一个借款合同(借条),合同双方为:
- 借入方: Issuer/Borrower/Seller
- 借出方: Investor/Lender/Buyer/Bondholder

# Basic features of fixed-income security



### Issuer/Borrower

- > Supranational Organizations, such as World Bank.
- > Sovereign (national) governments, such as China.
- Non-sovereign (local) governments, such as city of Edmonton.
- Quasi-government entities, such as agencies that are owned by government.
- Companies, such as corporate issuers.

# Basic features of fixed-income security



### Maturity date; tenor(time to maturity)

- Money market securities: one year or less;
- Capital market securities: more than one year;
- Perpetual bond: no stated maturity date.

### Par value/Face value/Maturity value

➤ The amount that the issuer agrees to repay the bondholders on the maturity date.

# Basic features of fixed-income security



### Coupon (票息)

- The annual amount of interest payments.
- Coupon = coupon rate × par value
  - Coupon rate/Nominal rate
  - The interest rate that the issuer agrees to pay each year until the maturity date.
  - Coupon frequency
    - Coupon payments may be made annually, semiannually, quarterly, or monthly, etc.
    - ✓ Generally semi-annually paid in U.S.



# Basic features of fixed-income security



### **Currency denomination**

- Dual-currency bonds: make coupon payments in one currency and pay the par value at maturity in another currency.
- Currency option bonds:
  - give bondholders the right to select one of two currencies for each payment in which they want to receive.
  - Can be viewed as a combination of a single-currency bond plus a foreign currency option.



# Example

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A bond has par value of \$100 and maturity of 3 years. The coupon rate 5% and coupon payment are made semiannually. please figure out its cash flow with a timeline.

### Answer:

There are 6 periodic payments of \$2.5 and the payment at maturity of \$100.

\$2.5	\$2.5	\$2.5	\$2.5	\$2.5	\$2.5	
Time(Yr.						
3	2.5	2	1.5	1	0.5	0

# Summary



- ➤ Importance: ☆☆
- Content:
  - Five basic features of fixed-income security: issuer, maturity, par value, coupon rate and frequency, currency denomination
- Cash flow of fixed income security: periodic coupon payments + par value at maturity
- Exam tips:
  - · 是Fixed income这门课的基础;
- 可能出Periodic coupon payment的计算题。







# **Bond indenture & other considerations**

### Tasks:

- Describe the components of bond indenture.
- Describe the legal, regulatory and tax consideration related to bond investment.

# **Bond indenture**



### Bond indenture (契约)/Trust deed

- The legal contract that describe the form of bond, the obligations of issuer, and the rights of bondholders.
  - · Legal identity of the bond issuer and its legal form
  - Source of repayment proceeds
  - · Asset or collateral backing
  - · Credit enhancements
  - Covenants



# **Bond indenture**



### Legal identity of the bond issuer and its legal form

- > The issuer is identified in the indenture by its legal name.
  - · Corporate legal entity: e.g. Wal-mart Store Inc.
  - Special purpose entity/vehicle/company(SPE/SPV/SPC)

# **Bond indenture**



### Source of repayment proceeds

- > How issuer service the debt and repay the principal
  - Supranational bond: repayment of previous loans, paid-in capital from its members.
  - Sovereign bonds: tax revenues, print money.
  - Non-sovereign government debt: general taxing authority, cash flows of the project funded, special taxes or fees.



# Bond indenture



### Source of repayment proceeds (Cont.)

- > How issuer service the debt and repay the principal.
  - Securitized bonds: cash flow from underlying financial assets.
  - · Corporate bonds: cash flow from its operations.

# **Bond indenture**



### **Asset or collateral Backing**

- A way to reduce/alleviate credit risk, and decreases yield.
  - Secured bonds: backed by assets or financial guarantees pledged to ensure debt repayment in case of default.
  - Unsecured bonds: have no collateral, bondholders have only a general claim on the issuer's assets and cash flows.
  - Debentures: are generally unsecured bonds in many jurisdictions, e.g. U.S.



# **Bond indenture**



### Types of collateral backing bonds

- Collateral trust bonds: secured by securities such as common shares, other bonds or other financial assets.
- Equipment trust certificates: secured by specific type of equipment or physical assets.
- Mortgage-backed securities(MBS): debt obligation that represent claims to cash flow from pools of mortgage loans.

# **Bond indenture**



### Credit Enhancement (信用增强)

- Variety of provisions used to reduce the credit risk and decrease the bond's yield.
  - · Internal credit enhancement
    - ✓ Subordination: waterfall structure, tranches.
    - Overcollateralization: post more collateral than is needed to obtain or secure financing.
    - Excess spread: more cash flow received from the assets used to secure the issue than the interest paid to investors.



# **Bond indenture**



### Credit Enhancement (Cont.)

- External credit enhancement
  - Surety bond: reimburse investors for any losses incurred if the issuer defaults, by a insurance company.
  - ✓ Bank guarantee: same as surety bond but by a bank.
  - ✓ Letter of credit: promise to lend money to issuing entity for any cash flow shortfalls.

# **Bond indenture**



### Covenants (条款)

- Legally enforceable rules that borrowers and lenders agree on at the time of a new bond issue.
- Negative covenants: prohibitions on borrower (can not)
- ✓ Restrictions on asset sales;
- ✓ Negative pledge of collateral;
- ✓ Restrictions on additional borrowings.
- Affirmative covenants: promises by borrower (should do)
- ✓ Maintain financial ratios;
- ✓ Timely payment of principal and interest.

# Legal and regulatory consideration



### National bond markets

- Includes all the bonds issued and traded in a specific country, and denominated in the currency of that country.
- Domestic bonds: issued by entities that are incorporated in that country.
- Foreign bonds: issued by entities that are incorporated in another country (Yankee/panda/Samurai/kangaroo bonds).

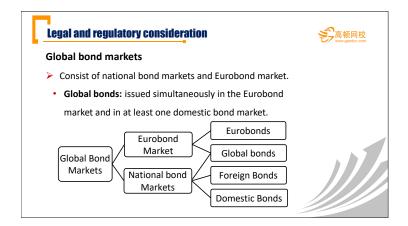
# Legal and regulatory consideration



### **Eurobond market**

- Issued outside the jurisdiction of any single country and can be denominated in any currency.
- Eurobonds: issued and traded on Eurobond market.





# Legal and regulatory consideration



### Bearer bonds (不记名债券)

- Does not keep records of who owns the bonds, only the clearing system knows who the bond owners are.
- Most Eurobonds are bearer bonds.

### Registered Bonds (记名债券)

- Ownership is recorded by either name or serial number.
- Most domestic and foreign bonds are registered bonds.
- Some investors may prefer bearer bonds to registered bonds, possibly for tax reasons.

# Tax consideration

### Interest

- The income portion (interest) is generally taxed at the ordinary income tax rate.
- Tax-exempt securities are exception.

### Capital gain or loss

- Usually treated differently from taxable income.
- In some countries, there is a different tax rate for longterm and short-term capital gains.





- Content:
- 5 components of bond indenture: legal identity of the bond issuer and its legal form, source of repayment proceeds, asset or collateral backing, credit enhancements, covenants.
- Other considerations: legal, regulatory, and tax.
- ➤ Exam tips: Covenants的类别的辨识是常见考点。





# **Structure of Cash Flow - Commitment**

### Tasks:

- > Describe the structure of principal repayment.
- Describe the structure of coupon payment, including fixed coupon and floating coupon.

# Structure of cash flows: principal repayment



### **Bullet bond**

The entire payment of principal occurs at maturity.

### Amortizing bond

- > Has a periodic repayment schedule of principal.
- Fully amortized bond: has a schedule that reduces outstanding principal to zero by the maturity date.
- Partially amortized bond: only a portion of the principal is repaid by the maturity date.
  - ✓ Balloon payment: amount required at maturity to retire the bond's outstanding principal.

# Structure of cash flows: principal repayment



Year	Cash Flow	Interest Payment	Principal repayment	Outstanding principal
0	-1000			1000
1	60	60	0	1000
2	60	60	0	1000
3	60	60	0	1000
4	60	60	0	1000
5	1060	60	1000	0

# Structure of cash flows: principal repayment

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# Example: fully amortized bond

	-			
Year	Cash Flow	Interest Payment	Principal repayment	Outstanding principal
0	-1000			
1	237.4	60	177.4	822.6
2	237.4	49.36	188.04	634.56
3	237.4	38.07	199.33	435.24
4	237.4	26.11	211.28	223.96
5	237.4	13.44	223.96	0

### Structure of cash flows: principal repayment



### Example: partially amortized bond

Year	Cash Flow	Interest Payment	Principal repayment	Outstanding principal
0	-1000	· ayee	repayment	principal
1	201.92	60	141.92	858.08
2	201.92	51.48	150.43	707.65
3	201.92	42.46	159.46	548.19
4	201.92	32.89	169.03	379.17
5	401.92	22.75	379.17	0

# Structure of cash flows: principal repayment



Sinking fund provision (偿债基金条款)

- Refers to an issuer's plans to set aside funds over time to retire the bond, retire amount is based on the provision.
- · Advantage: less credit risk;
- Disadvantage: more reinvestment risk.

# Structure of cash flows: coupon payment



### Fixed-rate coupon bonds

- pays a fixed periodic coupon over a specified time to maturity, e.g. plain vanilla bond/conventional bond.
- Zero-coupon (pure discount) bond: do not pay coupon.

### Step-up coupon bonds (梯升债券)

Coupon increases by specified margins at specified dates.

# Structure of cash flows: coupon payment



### Deferred coupon bonds/split coupon bonds (递延债券)

- Pays no coupons for its first few years but then pays a higher coupon for the remainder of its life.
  - Zero-coupon bond can be thought as an extreme form.

### Payment-in-Kind (PIK) coupon bonds (实物支付债券)

Allows the issuer to pay interest in the form of additional amounts of the bond issue rather than as a cash payment.



# Structure of cash flows: coupon payment



### Floating-rate notes (FRN)

- Do not have a fixed coupon, and coupon rate is linked to an external reference rate, such as LIBOR.
- Coupon rate = reference rate+ quoted margin (spread)
- Reference rate reset periodically, quoted margin is usually constant.
- ✓ Variable-rate note: the spread is not fixed.
- Coupon payments are in arrears: based on previous period's reference rate.

# Structure of cash flows: coupon payment



### Floating-rate notes (Cont.)

- > Almost all FRNs have quarterly coupons.
- FRNs may include a floor or a cap:
- Floor prevents coupon from falling below a specified minimum rate.
- Cap prevents coupon from rising above a specified maximum rate.
- Inverse or reverse FRN (inverse floater):
- Coupon rate = reference rate + quoted margin



# Structure of cash flows: coupon payment



### Index-linked bonds (指数挂钩债券)

- Coupon payments and/or principal repayment linked to a specific index.
- Inflation-linked bonds: coupon payments and/or principal repayment linked to a inflation index (CPI, RPI).
- Example: TIPS (TIPS-treasury inflation protection securities)
  - ✓ Principe protected (change with inflation)
  - ✓ Coupon rate does not change
  - ✓ Coupon will change (because principal change)

# Structure of cash flows: coupon payment



### Credit-linked coupon bonds (信用挂钩债券)

- Coupon changes when the bond's credit rating changes.
- Rating decrease leads to high coupon rate, which may result in further deterioration of the credit rating.

### Equity-linked notes (ELN, 股权挂钩债券)

- > Final payment based on the return of an equity index.
- > Typically principal protected if no default.
- Can be thought of as a zero-coupon bond with a return profile linked to the value of the equity index.



# Summary



- ➤ Importance: ☆☆☆
- Content:
  - · Cash flow structure of principal repayments:
  - ✓ Bullet bonds
  - Amortizing bonds: fully amortizing bond, partial amortizing bond.
  - ✓ Sinking fund provision

# **Summary (Cont.)**



- Content (Cont.):
  - · Cash flow structure of coupon payments:
  - Fixed coupon: fixed-rate coupon bonds, step-up coupon bonds, deferred coupon bonds, PIK coupon bonds
  - Floating coupon: floating-rate notes, index-linked bonds, credit-linked coupon bonds, equity-linked notes
- ➤ Exam tips: 主要是考Cash flow类别的辨识。





# **Structure of Cash Flow - Contingency**

### Tasks:

- Describe contingency provisions embedded in bond, including callable, puttable, and convertible bond.
- Identify the impacts of contingency provisions on borrower/lender, and on price or yield of the bond.

# Structure of cash flows: contingency provision



### Contingency provision (或有条款)

- Clause that allows for some action if the event or circumstance does occur.
  - Embedded option (内嵌期权): contingency provision in indenture for bonds, provide issuers or bondholders the right, but not the obligation, to take some action.
  - ✓ Callable bonds (可赎回债券)
  - ✓ Putable bonds (可回售债券)
  - ✓ Convertible bonds (可转债)

# Structure of cash flows: contingency provision



### Callable bond

- Gives the issuer the right to redeem all or part of the bond before the specified maturity date.
- Can protect issuers against decline in interest rate.
- · Call price: the price to redeem the bond;
- Call provisions are beneficial to the issuer;
- Investors face more reinvestment risk, and will ask higher yield, and pay lower price;
- Callable bond = non-callable bond call option

# Structure of cash flows: contingency provision



### **Puttable bonds:**

- Gives the bondholders the right to sell the bond back to the issuer at a pre-determined price on specified dates.
- Can protect investor against increase in interest rate.
  - · Put price: the price to sell back the bond;
  - · Put provisions are beneficial to the investors;
  - Investors will ask lower yield, and pay higher price;
  - Puttable bond = non-puttable bond + put option



# Structure of cash flows: contingency provision



### Convertible bond

- Gives bondholder the right to exchange the bond for a specified number of common shares in issuing company.
- Hybrid security with both debt and equity features.
- Conversion price: the share price at which convertible bond can be converted into shares.
- Conversion provisions are beneficial to bondholder.
- Investors will ask lower yield, and pay higher price.
- Convertible bond = straight bond + call option on equity

# Structure of cash flows: contingency provision



### Convertible bond (Cont.)

- Contingent convertible bonds: convert to common equity automatically if a specific event occurs.
- Warrant (认股权证): entitles the holder to buy the underlying stock of the issuing company at fixed exercise price until expiration date.
- · An attached option rather than an embedded option.

# Summary



- ► Importance: ☆☆☆
- Content:
  - Bonds with contingency provision: callable bonds, puttable bonds, convertible bonds.
    - Callable bonds: benefit the issuer (borrower), lower price and higher yield;
    - Puttable and convertible bonds: benefit the investor (lender), higher price and lower yield.
- Exam tips: 掌握三种或有条款的概念,最常考三个或有条款对谁有好处,怎么影响债券的price和yield。





# **Fixed Income Market**

### Tasks:

- Describe classifications of fixed-income markets.
- Describe primary markets and secondary markets.
- Describe government-related bonds and corporate bonds.



# **Classifications of fixed-income markets**



### Classification by type of issuer

- Government and government-related sector
- Corporate sector
- Structured finance sector

### Classification by credit quality

- Investment grade bond
- Non-investment grade bond

# **Classifications of fixed-income markets**



### Classification by maturity

- Money market security: original maturity of one year or less.
- Capital market security: original maturity longer than one year.

### Classification by type of coupon

- Fixed-rate bonds
- Floating-rate bonds



# Interbank offered rate



### London Interbank Offered Rate (LIBOR)

- The most widely used reference rate for floating-rate bonds.
  - Is a group of rates for different currencies and different borrowing periods ranging from overnight to one year.
- The rates are based on expected rates for unsecured loans from one bank to another in the interbank money market.

# Issuance and trading of bonds



### Primary bond market

- Markets in which issuers first sell bonds to investor.
- Public offering: any member of the public may buy the bonds;
- Private placement: only a selected group of investors may buy the bonds.

### Secondary bond market

markets in which existing bonds are subsequently traded among investors.



# **Issuance and trading of bonds**



### **Public Offering**

- Underwritten offering: the investment bank (underwriter) guarantees the sale of the bond issue at an offering price, and takes the risk associated with selling the bonds.
  - Syndicated offering: a group, or syndicate, of investment banks underwrite the issuance.
- Best efforts offering: the investment bank only serves as a broker, and sell the bond issue for a commission.

# Issuance and trading of bonds



### **Public Offering (Cont.)**

- Auction: an issuing mechanism that involves bidding and helpful for price discovery.
- · Commonly used by issuing government debts.
- Shelf registration(上架注册): issuer prepares a single, all-encompassing offering circular that describes a range of future bond issuances, all under the same document.
  - ✓ Issue bond over time when issuer needs to raise funds and save fee.



# Issuance and trading of bonds



### Secondary market

- Organized exchange: provides a place where buyers and sellers can meet to arrange their trades
- OTC markets: buy and sell orders initiated from various locations are matched through a communication network
  - Dealers will post bid and ask price

### Settlement

the process that the bonds are passed to the buyer and payment is received by the seller

# Government-related bonds



### Sovereign bonds

- Issued by national governments and backed by the taxing authority.
- On-the-run: most recently issued bonds, also referred to as a benchmark issue.
- > Can denominated in local currency or foreign currency.
  - It is common to observe a higher credit rating for sovereign bonds issued in local currency than for those issued in a foreign currency.



# **Government-related bonds**



### Sovereign bonds (Cont.)

- U.S Treasuries: sovereign bonds in the United States
- · T-Bills: original maturity is one year or shorter
- T-Notes: original maturity is longer than one year and up to 10 years
- T-Bonds: original maturity is longer than 10 years

# **Government-related bonds**



### Non-sovereign bonds

issued by government below the national level such as provinces, regions, states, and cities

### Quasi-government bonds/Agency bonds

Issued by entities created by national government, e.g. government-sponsored enterprises (Fannie Mae, Freddie Mac), usually rated very high

### Supranational bonds

issued by supranational agencies (multilateral agencies)

# Corporate debts



### **Bank loans**

- **Bilateral loan**: from a single lender to a single borrower
- Syndicated loan: from a group of lender to a single borrower

### **Commercial paper**

- Short term, unsecured promissory note.
- Traditionally, only the largest, most stable companies issued commercial paper

# Corporate debts



### Corporate notes and bonds

- > Often with original maturities on less than 1 year
  - Medium-term note (MTN): a misnomer, can have very long maturity

# Summary

### 

- Content:
  - Market classification: by issuer, by credit quality, by maturity, by coupon, etc.
  - Primary market:
  - Public offering: underwritten offering, best efforts offering, auction.
  - ✓ Private placement
  - Secondary market: organized exchange, OTC market.

# Summary



- Content (Cont.):
  - Government related bonds: sovereign bonds, nonsovereign bonds, quasi-government bonds, supranational bonds.
  - Corporate debt: bank loan, commercial paper, corporate notes and bonds (MTN).
- ➤ **Exam tips:** 掌握概念为主, public offering的三种发行机制 相对容易考到。





# Short-term funding to banks

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### **Retail Deposits**

- Demand deposits/Checking accounts: available to customers "on demand, and typically pay no interest
- Saving accounts: pay interest but do not offer the same transactional convenience as demand deposits
- Money market account: offer money market rates of return and depositors can access funds at short or no notice
  - An intermediate between demand deposit and saving accounts for depositor

# Short-term funding to banks



### Short-term wholesale funds

- Central bank funds: banks with reserve surplus loan money to banks with reserve shortage.
- Interbank funds: loans and deposits between banks.

# Short-term funding to banks



### Short-term wholesale funds (Cont.)

- Large-denomination negotiable certificates of deposit
   (CD)
  - CD represents a specified amount of funds on deposit for a specified maturity and interest rate
  - Negotiable CD allows any depositor (initial or subsequent) to sell the CD in the open market prior to the maturity date



# Repurchase agreements



### Repurchase agreement (Repo)

- The sale of a security with a simultaneous agreement by the seller to buy it back at an agree-on price (repurchase price) and future date (repurchase date).
- > In practical, a repo can be viewed as a collateralized loan.
  - · Overnight repo: the term is one day;
  - · Term repo: the term is more than one day;
  - · Repo to maturity: the term is until the final maturity date.

# **Repurchase agreements**



### Repurchase agreement (Cont.)

- > Repo rate: the interest rate implied by two prices.
- > The repo rate is higher when:
  - Repo term is longer;
  - · Credit quality of the collateral security is lower;
  - Collateral security is not delivered to the lender;
  - · Collateral security is in low demand or high supply;
  - Interest rate for alternative sources of funds are higher.

# **Repurchase agreements**



### Repurchase agreement (Cont.)

- Repo margin/haircut: the difference between the market value of the collateral security and the value of the loan.
- The rope margin is higher when:
- Repo term is longer;
- · Credit quality of the collateral security is lower;
- Credit quality of the borrower is lower;
- Collateral security is in low demand or high supply.

# Repurchase agreements



### Repurchase agreement (Cont.)

- Reverse repurchase agreement/Reverse repo: view repo through the cash lending counterparty.
  - · Repurchase agreement: borrowing money.



# Summary



- ▶ Importance: ☆☆
- > Content:
  - · Short-term funding to banks:
  - ✓ Retail Deposits
  - ✓ Short-term wholesale funds: central bank funds, interbank funds, large-denomination negotiable CD.

# Summary



- Content (Cont.):
  - Repo: overnight repot, term repo, repo to maturity.
  - ✓ Factors on repo rate & repo margin: repo term (↑), credit quality of collateral and borrower (↓), collateral delivery (↓), collateral demand (↓) or supply (↑), interest rate (↑).
- > Exam tips:
  - 掌握基本概念。
  - 常考点:影响Repo rate和Repo margin大小的因素和方向。

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# **Bond Pricing with YTM & Spot Rates**

### Tasks:

- > Calculate bond price given a market discount rate.
- Identify the relationships among a bond's price, coupon rate, maturity, and yield-to-maturity.
- Define spot rates and calculate bond price with spot rates.

# Bond price



# Pricing bond with a single discount rate

- Bond price is the present value (PV)of the promised cash flows.
  - Market discount rate: discount rate used in the PV calculation, the required rate of return (required yield) by investors given the risk of the bond investment.

$$P = \sum_{t=1}^{n} \frac{P M T_{t}}{(1+r)^{t}} + \frac{F}{(1+r)^{n}}$$

F: face value of the bond.

# Bond price



### Example 1:

➤ Suppose the coupon rate on a bond is 4% and the payment is made once a year. If the time-to-maturity is five years and the market discount rate is 6%, the price of the bond should be:

$$P = \frac{4}{(1.06)^1} + \frac{4}{(1.06)^2} + \frac{4}{(1.06)^3} + \frac{4}{(1.06)^4} + \frac{104}{(1.06)^5} = 91.575$$

When coupon rate < discount rate, the bond is priced at discount, or a discount bond.

# Bond price



### Example 2:

Suppose the coupon rate on a bond is 6% and the payment is made once a year. If the time-to-maturity is five years and the market discount rate is 6%, the price of the bond should be:

$$P = \frac{6}{(1.06)^1} + \frac{6}{(1.06)^2} + \frac{6}{(1.06)^3} + \frac{6}{(1.06)^4} + \frac{106}{(1.06)^5} = 100$$

When coupon rate = discount rate, the bond is priced at par, or a par bond.

# **Bond price**



### Example 3:

Suppose the coupon rate on a bond is 8% and the payment is made once a year. If the time-to-maturity is five years and the market discount rate is 6%, the price of the bond should be:

$$P = \frac{8}{(1.06)^1} + \frac{8}{(1.06)^2} + \frac{8}{(1.06)^3} + \frac{8}{(1.06)^4} + \frac{108}{(1.06)^5} = 108.425$$

When coupon rate > discount rate, the bond is priced at premium, or a premium bond.

# Practice



Suppose that a three-year, zero-coupon bond with par value of 100 has a required rate of return of 10%, what is the bond price?

- A. 81.03
- B. 74.62
- C. 69.58

### B is correct.

$$74.62 = \frac{100}{(1+5\%)^6}$$



# Bond yield

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### Yield-to-Maturity (YTM)

- The internal rate of return of on the cash flow.
- If the market price of a bond is known, the valuation equation can be used to calculate its YTM.

$$P = \sum_{t=1}^{n} \frac{P M T_{t}}{(1 + Y T M)^{t}} + \frac{F}{(1 + Y T M)^{n}}$$

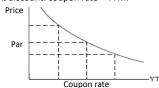
# Bond yield Yield-to-Maturity (Cont.) Three critical assumptions for YTM: The investor hold the bond until maturity; The issuer makes full and timely coupon and principal payments; The investor is able to reinvest coupon payments at YTM.

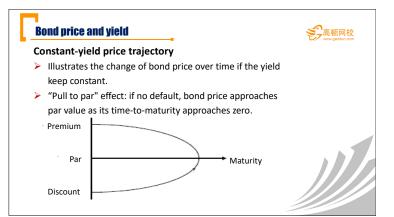
# **Bond price and yield**



### Relationship between bond price & YTM

- > The bond price is inversely related to the YTM.
- At premium: coupon rate > YTM;
- At par: coupon rate = YTM;
- At discount: coupon rate < YTM.





# Practice



Suppose that a four-year, 5% annual coupon payment bond is priced at 105 per 100 of par value. What is the yield-to-maturity?

- A. 3.63%
- B. 5.0%
- C. 6.72%

### A is correct.

$$105 = \frac{5}{(1+3.63\%)^1} + \frac{5}{(1+3.63\%)^2} + \frac{5}{(1+3.63\%)^3} + \frac{105}{(1+3.63\%)^3}$$

# Bond price



### Pricing bonds with spot rates

- Use a sequence of spot rates that correspond to the cash flow dates to calculate the bond price.
  - Spot rates/zero rates: YTM on zero-coupon bonds maturing at the date of each cash flow.
  - A more fundamental approach to pricing bond, and it is the "no-arbitrage price".

$$P = \sum_{t=1}^{n} \frac{P M T_{t}}{(1 + Z_{t})^{t}} + \frac{F}{(1 + Z_{n})^{n}}$$

Z<sub>n</sub>: spot rate for period n

# Bond price

### Example:

Suppose that the one-year spot rate is 2%, the two-year spot rate is 3%, and the three-year spot rate is 4%. Then, the price of a three-year bond that makes a 5% annual coupon payment is:

$$P = \frac{5}{(1.02)^{1}} + \frac{5}{(1.03)^{2}} + \frac{105}{(1.04)^{3}} = 102.96$$

# Summary



### ➤ Importance: ☆☆☆

- Content:
  - Bond price: PV of cash flow.
  - YTM: IRR of cash flow.
  - Spot rates: YTMs on zero-coupon bonds.
  - . Bond price Vs. YTM, and bond price Vs. time:
  - ✓ Bond price-yield curve: premium, at par, discount;
  - ✓ Constant-yield price trajectory: pull to par.
- Exam tips:
  - · 必须掌握bond price的计算,包括计算器的使用。
  - · 必须掌握bond price的两幅图所对应的关系。





# **Quote Conventions & Matrix Pricing**

### Tasks:

- Describe and calculate the flat price, accrued interest, and the full price of a bond.
- Describe matrix pricing.

# **Bond price and yield**



### Conventions for quote and calculation

> Bond dealers usually quote flat price while the full price will be paid, and there can be a difference between them. Full price = Flat price + Accrued interest (AI) Dirty price = Clean price + Accrued interest (AI)

Full price=
$$\left[\frac{PMT}{(1+r)^1} + \frac{PMT}{(1+r)^2} + \dots + \frac{PTM+F}{(1+r)^n}\right] \times (1+r)^{t/T}$$

# **Bond price and yield**



### Conventions for quote and calculation (Cont.)

> Accrued interest(AI): the proportional share of the next coupon payment.

coupon payment. 
$$AI = \frac{t}{T} \times PMT$$
 PMT PMT+

0 settlement 1 ··· ··· ·· n date

- · Conventions to count days: actual/actual and 30/360.
- ✓ Vary from market to market, but actual/actual is most common for government bonds and 30/360 is often used on corporate bonds.

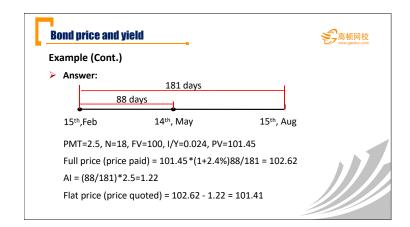
# **Bond price and yield**



### Example

> Consider a 5% semiannual coupon bond with par value of 100 and required yield of 4.8%. The bond will matures on 15 February 2024 and coupon are made on 15 February and 15 August of each year. The bond is to be priced for settlement on 14 May 2015, and that date is 88 days into the 181-day period. What should be the price paid and what should be the price quoted?





# Bond price

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### Matrix pricing

- Estimate the market discount rate and price based on the price of more frequently traded comparable bonds.
  - Comparable bonds: similar times-to-maturity, coupon rates, and credit quality (totally, similar risk).
  - Commonly used for bonds inactively-traded or not yet issued.

# **Bond price**

### Example:

- An analyst needs to value a 3-year, 4% semiannual coupon corporate bond, Bond X. It is not actively traded and not recently transacted. However, there are quoted prices for four corporate bonds with similar credit quality:
  - Bond A: 2-year, 3% semiannual, trading at \$98.5
  - Bond B: 2-year, 5% semiannual, trading at \$102.25
- Bond C: 5-year, 2% semiannual, trading at \$90.250
- Bond D: 5-year, 4% semiannual, trading at \$99.125

# **Bond price**



### Example (Cont.)

> Step 1: Base on the available information, we calculate the YTM for each bond and we get the following table:

	2%	3%	4%	5%
	Coupon	Coupon	Coupon	Coupon
2-Years		A: 98.5/ 3.786%		B: 102.25/ 3.821%
3-Years			Bond X	
4-Years				
5-Years	C: 90.25/ 4.181%		D: 99.125/ 4.196%	

# Bond price

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### Example (Cont.)

- > **Step 2**: calculates the average yield for each year:
- (0.03786 + 0.03821)/2= 0.038035 (2-year maturity)
- (0.04181 + 0.04196)/2= 0.041885 (5-year maturity)
- > Step 3: Calculates interpolated yield for bond X:
- 0.038035 +(3-2)/(5-2) x (0.041885 0.038035) =
   0.039318
- Step 4: use the interpolated yield to calculate the price of bond X
- N=6, YTM=1.9659, PMT=2, FV=100. PV=? (100.191)

# **Bond price**



### Matrix pricing (Cont.)

- Also used to estimate the required yield spread over the benchmark rate.
  - Benchmark rate: typically the yield-to-maturity on a government bond having the same, or close to the same, time-to-maturity.
  - The spread is the difference between the yield-tomaturity on the bond and the benchmark rate.

# Summary



### ➤ Importance: ☆☆

### Content:

- Flat price: price quoted; full price: price paid;
- Full price = flat price + AI, AI = PMT ×(t/T)
- Matrix price: estimate YTM based on frequently traded comparable bonds.
- ✓ Used for bonds inactively-traded or not yet issued.

### Exam tips:

- 掌握AI的计算, full price和flat price的定义, 及相互关系。
- 了解matrix pricing的用途(一般不考计算)。

# Bond Yield



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### Tasks:

- Calculate and interpret yield measures for bonds.
- Calculate spot rates from forward rates, forward rates from spot rates, and bond price using forward rates.

# Bond vield



### Yield measures for fixed-rate bonds

- Yield measures typically are annualized.
  - For bonds maturing in more than one year, yields typically are annualized and compounded.
- Yield on instruments maturing in one year or less typically are annualized but not compounded (simple interest basis).

# Bond yield



### Yield measures for fixed-rate bonds (Cont.)

- Street convention yield: the IRR on cash flow assuming the payments are made on the scheduled dates, and neglect weekends and holidays.
- > True yield: the IRR on the cash flows using the actual calendar of weekends and holidays.
  - · True yield never higher than street convention yield.

# Bond yield



### Yield measures for fixed-rate bonds (Cont.)

- Current yield: the sum of the coupon payments received over the year divided by the flat price.
  - Example: the current yield of a 2% semiannual coupon bond priced at 95 is: 2/95 = 2.11%.
- Simple yield: the sum of annul coupon payment plus straight-line amortization of discount or premium, divided by the flat price.

# **Bond yield**



### Yield measures for fixed-rate bonds (Cont.)

- Yield to call: the IRR on cash flows assuming the embedded call option is exercised.
  - Yield to first call: assuming the embedded call option is exercised at the first call date.
  - · Yield to second call ... ...
- Yield to worst: the lowest of the sequence of yields-tocall and the yield-to-maturity.



# Bond yield

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### Example:

- Suppose that a 7-year, 8% annual coupon bond is callable and priced at 105, the "call schedule" is that it is first callable at 102 in four years, callable at 101 in five years.
  - Yield to maturity:
     PV=-105, PMT=8, n=7, FV=100; I/Y=7.07%
  - Yield to first call:
     PV=-105, PMT=8, n=4, FV=102, I/Y=6.975%
  - Yield to second call:
     PV=-105, PMT=8, n=5, FV=101, I/Y=6.975%

# Bond yield



### Yield measures for floating-rate bonds

- Coupon rate = reference rate +/- quoted margin
- ➤ Quoted margin: the specified yield spread over the reference rate, that compensate investor for the difference in the credit risk of the issuer and that implied by the reference rate.
- Required margin: the yield spread over, or under, the reference rate such that the FRN is priced at par value on a rate reset date.



# **Bond yield**



### Yield measures for floating-rate bonds (Cont.)

- If quoted margin > required margin, FRNs will be priced at premium.
- If quoted margin = required margin, FRNs will be priced at par.
- If quoted margin < required margin, FRNs will be priced at discount.

# Bond yield



### Yield measures for money market instruments

- Discount rates or add-on rates are used for quoted money market rates.
  - Discount rates usually used for commercial paper, T-

bills: 
$$PV = FV \times \left(1 - \frac{Days}{Year} \times DR\right)$$

• Add-on rates usually used for bank CD, repos:

$$PV = \frac{FV}{\left(1 + \frac{Days}{Vear} \times AOR\right)}$$



# Bond yield

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### Forward market

- Agreement to the terms for the transaction is on the trade date, but delivery of the security and payment for it is deferred to a future date.
- Forward rate: the interest rate on a bond or money market instrument traded in a forward market.
- ✓ "2y5y" means "the two year into five-year rate".
- ✓ The first number (2) refers to when to start the forward contract and the second number (5) refers to the tenor.

# Bond vield



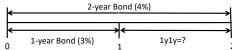
### Relationship between forward rates and spot rates

- Implied forward rates (IFR, forward yields): a break-even reinvestment rate that are calculated from spot rates:  $(1+z_a)^A \times (1+IFR_{a,B,a})^{B-A} = (1+z_B)^B$
- IFR<sub>A,B-A</sub> is a forward rate that starts in period A and ends in period B, its tenor is B - A periods.

# Bond yield

### Example:

Suppose the YTM on a 1-year, annual government bond is 3%, and YTM on a 2-year annual government bond is 4%, compute the "1y1y" implied forward rate.



Formula:

$$(1+0.03)\times(1+1y1y)=(1+0.04)^2 \rightarrow 1y1y=5.01\%$$

• Simplification: 1y1y = 4% + 4% - 3% = 5%

# Practice



The investor observes these prices and YTM on zero coupon government bonds, calculate the "2y1y" implied forward rate.

1-year, price: 97.50, YTM: 2.548%

2-year, price: 94.25, YTM: 2.983%

3-year, price: 91.75, YTM: 2.891%

$$(1 + \frac{0.02983}{2})^4 \times (1 + IFR_{4,2})^2 = (1 + \frac{0.02891}{2})^6$$

IFR<sub>42</sub> = 0.013536

 $2y1y = 0.013536 \times 2 = 0.0271$ 



# Bond price



### Pricing bonds with forward rates

- Forward rates can be used to value a bond in the same manner as spot rates because they are interconnected.
- Discount bond cash flows one period by one period with forward rates.

# **Bond price**



### Example:

> Suppose an analyst needs to value a four-year, 3.75% annual coupon payment bond, and he find the following forward rates: 0y1y at 1.88%, 1y1y at 2.77%, 2y1y at 3.54%, 3y1y at 4.12%. Calculate the bond price.

$$\begin{aligned} \text{Price} = & \frac{3.75}{1.0188} + \frac{3.75}{1.0188 \times 1.0277} + \frac{3.75}{1.0188 \times 1.0277 \times 1.0354} \\ & + \frac{103.75}{1.0188 \times 1.0277 \times 1.0354 \times 1.0412} = 102.637 \end{aligned}$$

# Summary



- ➤ Importance: ☆☆☆
- Content:
  - Yield measures for fixed-rate bonds: compounded Vs. not compounded, street convention yield Vs. true yield, current yield Vs. simple yield, yield to (1<sup>st</sup>, 2<sup>nd</sup> ...) call, yield to worst.
  - Yield measures for floating-rate bonds: quoted margin
     Vs. required margin.

# Summary



### Content (Cont.):

- Yield measures for money market instruments: discount rates Vs. add-on rates.
- · Spot rates Vs. forward rates.
- Pricing bond with forward rates.

### Exam tips:

- 非常重要的考点,包括概念的辨析和计算。
- 常考点: Spot rates和forward rates的相互换算。

