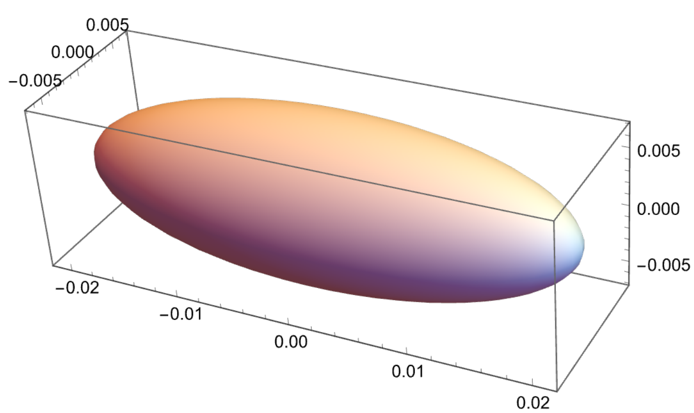
Sizzling Magnets [v5] - Theorie

§1. Vector Potential *A* at z_0**k**

In[1]:=
```
R = 2.1 * 10^-2 (* R = length of major axis -- along x-direction *)
```
Out[1]= 0.021

In[2]:=
```
r = 0.69 * 10^-2 (* r = length of minor axis -- along y- and z- directions *)
```
Out[2]= 0.0069

In[3]:=
```
Show[Graphics3D[Ellipsoid[{0, 0, 0}, {R, r, r}]], Axes → True, AxesStyle → Black]
```
Out[3]=



In[4]:=
```
b = R / r
```
Out[4]= 3.04348

In[5]:=
```
g[x1_, x2_, x3_, y1_, y2_, y3_] := Norm[{x1, x2, x3} - {y1, y2, y3}]^3
```

In[6]:=
```
z0 = 8 * 10^-2
```
Out[6]= $\frac{2}{25}$

In[7]:=
$$f[x\_, y\_, z\_] := \frac{z0 - z}{g[0, 0, z0, x, y, z]}$$

In[8]:=
```
NIntegrate[f[x, y, z], {x, y, z} ∈ Ellipsoid[{0, 0, 0}, {R, r, r}]]
(* this is the integral Iz -- then A = mu0 M / 4pi x Iz k *)
```
Out[8]= 0.00064269

In[9]:= $$\frac{4\pi b}{z0}\int_0^r \frac{x^2}{\sqrt{z0^2+(b^2-1)\,x^2}}\,dx$$ (* value of the above volume integral -- should give Iz at z0 **k** *)

Out[9]= 0.00064269

In[10]:= $$\text{JZaxis}[z\_] := \frac{2\pi b}{(b^2-1)}\left(r\sqrt{(b^2-1)\left(\frac{r}{z}\right)^2+1}-\frac{z}{\sqrt{b^2-1}}\,\text{ArcSinh}\left[\frac{r\sqrt{b^2-1}}{z}\right]\right)$$

(* closed form of above integral for b > 1 -- if b = 1 we end up with Luca's result for spherical magnets *)

In[11]:= JZaxis[z0]

Out[11]= 0.00064269

§2. Vector Potential *A* Over $\mathbf{R}^3$

In[12]:= $$\text{II}[x\_, y\_, z\_] := \text{NIntegrate}\left[\frac{\{x, y, z\} - \{x1, y1, z1\}}{g[x, y, z, x1, y1, z1]},\ \{x1, y1, z1\} \in \text{Ellipsoid}[\{0, 0, 0\}, \{R, r, r\}],\ \text{AccuracyGoal} \to 20\right]$$

(* this is the integral **I** -- then **A** = mu0 M / 4pi x **I** *)

In[13]:= II[0, 0, z0]

Out[13]= {0., 0., 0.00064269}

In[14]:= $$\text{II}\left[0,\ z0 * \text{Sin}\left[\frac{\pi}{3}\right],\ z0 * \text{Cos}\left[\frac{\pi}{3}\right]\right]$$

Out[14]= {0., 0.000556585, 0.000321345}

In[51]:= $$\text{JZaxis}[z0] * \text{Sin}\left[\frac{\pi}{3}\right]$$

Out[51]= 0.000556585

In[52]:= II[0, 57, 29]

Out[52]= $\{0.,\ 9.12633\times 10^{-10},\ 4.64322\times 10^{-10}\}$

In[53]:= $$\left\{0,\ \text{JZaxis}\left[\sqrt{57^2+29^2}\right] * \frac{57}{\sqrt{57^2+29^2}},\ \text{JZaxis}\left[\sqrt{57^2+29^2}\right] * \frac{29}{\sqrt{57^2+29^2}}\right\}$$

Out[53]= $\{0,\ 9.12633\times 10^{-10},\ 4.64322\times 10^{-10}\}$

In[15]:= Norm[II[0, z0 * Sin[$\frac{\pi}{3}$], z0 * Cos[$\frac{\pi}{3}$]]]

Out[15]=

0.00064269

In[16]:= M = {0, 0, 1} (* magnetization; scalable units *)

Out[16]=

{0, 0, 1}

In[17]:= II[12, 8, 5]

Out[17]=

$\{1.41304\times10^{-8},\ 9.42025\times10^{-9},\ 5.88766\times10^{-9}\}$

In[18]:= A[x_, y_, z_] :=
Cross[M, II[x, y, z]] * 10^{-7} (* vector potential **A** up to a factor of M *)

In[19]:= yy1 = 12 * 10^{-2}

Out[19]=

$\frac{3}{25}$

In[20]:= zz1 = 17 * 10^{-2} (* (0, yy1, zz1) is the test
location for verifying that **A** is theoretically correct *)

Out[20]=

$\frac{17}{100}$

In[21]:= A[0, yy1, zz1]

Out[21]=

$\{-5.56258\times10^{-12},\ 0.,\ 0.\}$

In[22]:= JZaxis[$\sqrt{yy1^2 + zz1^2}$] * (-Norm[M]) * $\left(\frac{yy1}{\sqrt{yy1^2 + zz1^2}}\right)$

(* this is the theoretical value for **A** in the yz-plane *)

Out[22]=

-0.0000556258

In[23]:= dds = 10^{-3}

Out[23]=

$\frac{1}{1000}$

In[24]:= N[dds]

Out[24]=

0.001

In[25]:= Ax[x_, y_, z_] := A[x, y, z].{1, 0, 0}

In[26]:= Ay[x_, y_, z_] := A[x, y, z].{0, 1, 0}

In[27]:=
```
Az[x_, y_, z_] := A[x, y, z].{0, 0, 1}
```

In[28]:=
$$B[x\_, y\_, z\_] := \left\{\frac{Az[x, y + dds, z] - Az[x, y, z]}{dds} - \frac{Ay[x, y, z + dds] - Ay[x, y, z]}{dds}, \frac{Ax[x, y, z + dds] - Ax[x, y, z]}{dds} - \frac{Az[x + dds, y, z] - Az[x, y, z]}{dds}, \frac{Ay[x + dds, y, z] - Ay[x, y, z]}{dds} - \frac{Ax[x, y + dds, z] - Ax[x, y, z]}{dds}\right\}$$

```
(* numerical approximation of B = curl A,
using interval size of dds = 1e(-3) *)
```

IMPORTANT!!! -- B is the magnetic field ***IF M* = 1 !!!!!**

In[29]:=
```
B[12, 12, 12]
```

Out[29]=
$\{4.6641\times10^{-17}, 4.66411\times10^{-17}, -5.16217\times10^{-21}\}$

In[30]:=
```
BYZplane[x_, r_] := Norm[B[0, r*Cos[x], r*Sin[x]]]
```

In[31]:=
```
B[0, 14.34*Cos[0.5], 14.34*Sin[0.5]]
```

Out[31]=
$\{0., 1.7926\times10^{-16}, -4.40804\times10^{-17}\}$

In[32]:=
```
B[0, 0, 14.34]
```

Out[32]=
$\{0., 0., 2.84046\times10^{-16}\}$

In[33]:=
```
1 / Out[42]
```

Out[33]=
$\frac{1}{\%42}$

In[34]:=
```
Tan[0.5]
```

Out[34]=
0.546302

In[35]:=
$BYZplane[0, 14.34*10^{-2}]$

Out[35]=
1.38345×10^{-10}

In[36]:=
$BYZplane[0.3, 14.34*10^{-2}]$

Out[36]=
1.5758×10^{-10}

In[37]:= $B5p0 = \text{Table}\left[\left\{\frac{\pi * x}{20}, \text{BYZplane}\left[\frac{\pi * x}{20}, 5.0 * 10^{-2}\right]\right\}, \{x, 0, 39\}\right]$

Out[37]=

$$\left\{\{0, 3.03726\times10^{-9}\}, \left\{\frac{\pi}{20}, 3.20503\times10^{-9}\right\}, \left\{\frac{\pi}{10}, 3.54882\times10^{-9}\right\}, \left\{\frac{3\pi}{20}, 3.9659\times10^{-9}\right\}, \left\{\frac{\pi}{5}, 4.39068\times10^{-9}\right\}, \left\{\frac{\pi}{4}, 4.79574\times10^{-9}\right\}, \left\{\frac{3\pi}{10}, 5.17342\times10^{-9}\right\}, \left\{\frac{7\pi}{20}, 5.51732\times10^{-9}\right\}, \left\{\frac{2\pi}{5}, 5.81117\times10^{-9}\right\}, \left\{\frac{9\pi}{20}, 6.02763\times10^{-9}\right\}, \left\{\frac{\pi}{2}, 6.13534\times10^{-9}\right\}, \left\{\frac{11\pi}{20}, 6.10955\times10^{-9}\right\}, \left\{\frac{3\pi}{5}, 5.94176\times10^{-9}\right\}, \left\{\frac{13\pi}{20}, 5.64518\times10^{-9}\right\}, \left\{\frac{7\pi}{10}, 5.25436\times10^{-9}\right\}, \left\{\frac{3\pi}{4}, 4.81868\times10^{-9}\right\}, \left\{\frac{4\pi}{5}, 4.39053\times10^{-9}\right\}, \left\{\frac{17\pi}{20}, 4.01193\times10^{-9}\right\}, \left\{\frac{9\pi}{10}, 3.70762\times10^{-9}\right\}, \left\{\frac{19\pi}{20}, 3.49188\times10^{-9}\right\}, \{\pi, 3.38562\times10^{-9}\}, \left\{\frac{21\pi}{20}, 3.42409\times10^{-9}\right\}, \left\{\frac{11\pi}{10}, 3.6365\times10^{-9}\right\}, \left\{\frac{23\pi}{20}, 4.01225\times10^{-9}\right\}, \left\{\frac{6\pi}{5}, 4.4933\times10^{-9}\right\}, \left\{\frac{5\pi}{4}, 4.99872\times10^{-9}\right\}, \left\{\frac{13\pi}{10}, 5.4535\times10^{-9}\right\}, \left\{\frac{27\pi}{20}, 5.80562\times10^{-9}\right\}, \left\{\frac{7\pi}{5}, 6.03195\times10^{-9}\right\}, \left\{\frac{29\pi}{20}, 6.13541\times10^{-9}\right\}, \left\{\frac{3\pi}{2}, 6.13534\times10^{-9}\right\}, \left\{\frac{31\pi}{20}, 6.05385\times10^{-9}\right\}, \left\{\frac{8\pi}{5}, 5.90335\times10^{-9}\right\}, \left\{\frac{33\pi}{20}, 5.68137\times10^{-9}\right\}, \left\{\frac{17\pi}{10}, 5.37556\times10^{-9}\right\}, \left\{\frac{7\pi}{4}, 4.97661\times10^{-9}\right\}, \left\{\frac{9\pi}{5}, 4.49345\times10^{-9}\right\}, \left\{\frac{37\pi}{20}, 3.96622\times10^{-9}\right\}, \left\{\frac{19\pi}{10}, 3.47445\times10^{-9}\right\}, \left\{\frac{39\pi}{20}, 3.13104\times10^{-9}\right\}\right\}$$

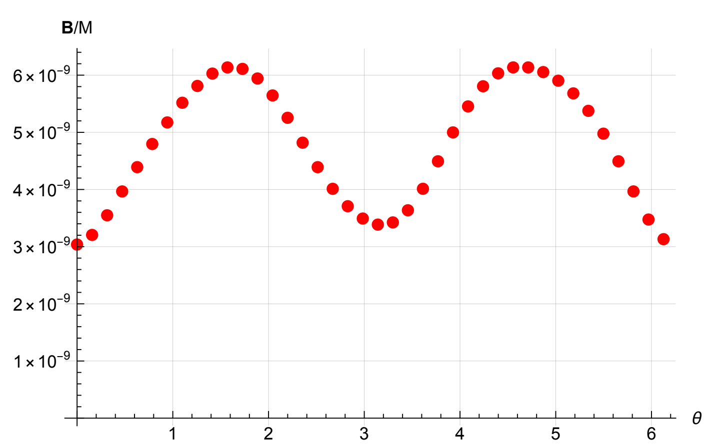
In[38]:= B9p5 = Table$\left[\left\{\frac{\pi * x}{20}, \text{BYZplane}\left[\frac{\pi * x}{20}, 9.5 * 10^{-2}\right]\right\}, \{x, 0, 39\}\right]$

Out[38]=

$\left\{\left\{0, 4.67775\times10^{-10}\right\}, \left\{\frac{\pi}{20}, 4.90022\times10^{-10}\right\}, \left\{\frac{\pi}{10}, 5.41226\times10^{-10}\right\},\right.$
$\left\{\frac{3\pi}{20}, 6.07424\times10^{-10}\right\}, \left\{\frac{\pi}{5}, 6.77903\times10^{-10}\right\}, \left\{\frac{\pi}{4}, 7.4642\times10^{-10}\right\}, \left\{\frac{3\pi}{10}, 8.09502\times10^{-10}\right\},$
$\left\{\frac{7\pi}{20}, 8.64573\times10^{-10}\right\}, \left\{\frac{2\pi}{5}, 9.0884\times10^{-10}\right\}, \left\{\frac{9\pi}{20}, 9.39068\times10^{-10}\right\},$
$\left\{\frac{\pi}{2}, 9.52094\times10^{-10}\right\}, \left\{\frac{11\pi}{20}, 9.45684\times10^{-10}\right\}, \left\{\frac{3\pi}{5}, 9.19352\times10^{-10}\right\},$
$\left\{\frac{13\pi}{20}, 8.74809\times10^{-10}\right\}, \left\{\frac{7\pi}{10}, 8.1592\times10^{-10}\right\}, \left\{\frac{3\pi}{4}, 7.4817\times10^{-10}\right\},$
$\left\{\frac{4\pi}{5}, 6.77844\times10^{-10}\right\}, \left\{\frac{17\pi}{20}, 6.11283\times10^{-10}\right\}, \left\{\frac{9\pi}{10}, 5.54669\times10^{-10}\right\},$
$\left\{\frac{19\pi}{20}, 5.14423\times10^{-10}\right\}, \left\{\pi, 4.97345\times10^{-10}\right\}, \left\{\frac{21\pi}{20}, 5.08748\times10^{-10}\right\},$
$\left\{\frac{11\pi}{10}, 5.48893\times10^{-10}\right\}, \left\{\frac{23\pi}{20}, 6.11561\times10^{-10}\right\}, \left\{\frac{6\pi}{5}, 6.86545\times10^{-10}\right\},$
$\left\{\frac{5\pi}{4}, 7.63157\times10^{-10}\right\}, \left\{\frac{13\pi}{10}, 8.32417\times10^{-10}\right\}, \left\{\frac{27\pi}{20}, 8.88065\times10^{-10}\right\},$
$\left\{\frac{7\pi}{5}, 9.26782\times10^{-10}\right\}, \left\{\frac{29\pi}{20}, 9.47807\times10^{-10}\right\}, \left\{\frac{3\pi}{2}, 9.52094\times10^{-10}\right\},$
$\left\{\frac{31\pi}{20}, 9.41206\times10^{-10}\right\}, \left\{\frac{8\pi}{5}, 9.16356\times10^{-10}\right\}, \left\{\frac{33\pi}{20}, 8.77984\times10^{-10}\right\},$
$\left\{\frac{17\pi}{10}, 8.26126\times10^{-10}\right\}, \left\{\frac{7\pi}{4}, 7.61441\times10^{-10}\right\}, \left\{\frac{9\pi}{5}, 6.86604\times10^{-10}\right\},$
$\left.\left\{\frac{37\pi}{20}, 6.07704\times10^{-10}\right\}, \left\{\frac{19\pi}{10}, 5.35305\times10^{-10}\right\}, \left\{\frac{39\pi}{20}, 4.84061\times10^{-10}\right\}\right\}$

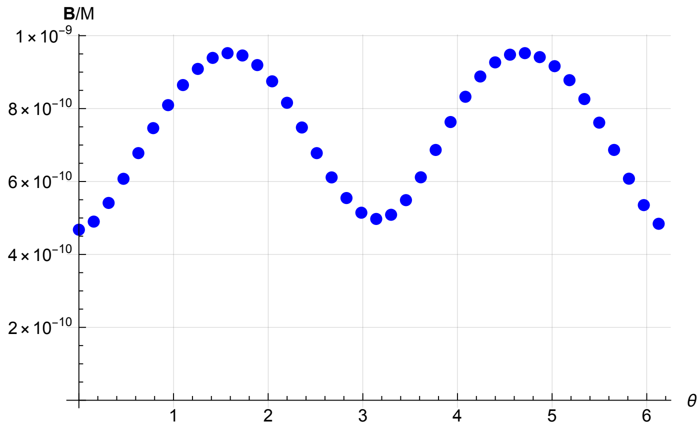
In[39]:=
```
ListPlot[B5p0, PlotStyle → {Red, PointSize[0.02]},
 AxesLabel → {"θ", "B/M"}, PlotRange → All, GridLines → Automatic]
```

Out[39]=



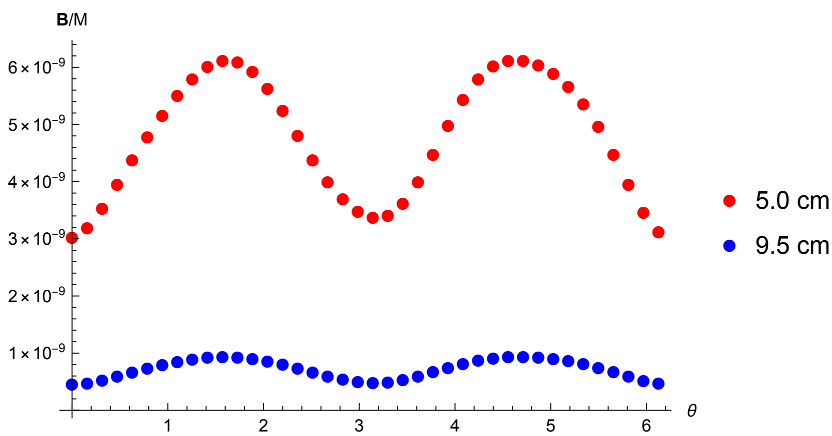
In[40]:=
```
ListPlot[B9p5, PlotStyle → {Blue, PointSize[0.02]},
 AxesLabel → {"θ", "B/M"}, PlotRange → All, GridLines → Automatic]
```

Out[40]=



In[41]:=
```
ListPlot[{B5p0, B9p5}, PlotStyle → {Red, Blue}, PlotMarkers → {"●", 10},
 AxesLabel → {"θ", "B/M"}, PlotLegends → {"5.0 cm", "9.5 cm"}]
```

Out[41]=



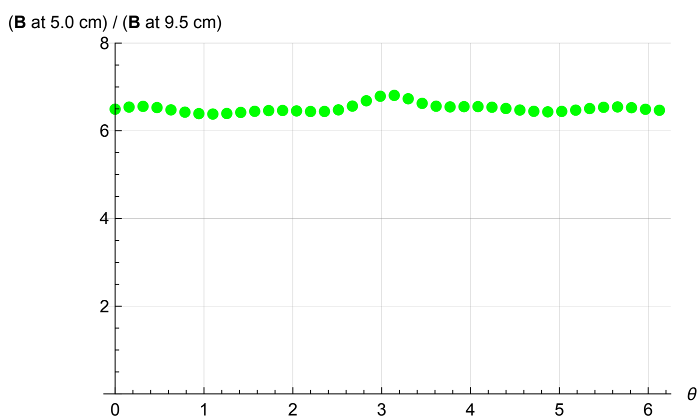
In[42]:=
```
BRto = Table[{x, Total[Select[B5p0, #[[1]] == x &][[All, 2]]] /
    Total[Select[B9p5, #[[1]] == x &][[All, 2]]]}, {x, Union[B5p0[[All, 1]]]}]
```

Out[42]=

$$\left\{\{0, 6.49299\}, \left\{\frac{\pi}{20}, 6.54059\right\}, \left\{\frac{\pi}{10}, 6.55701\right\}, \left\{\frac{3\pi}{20}, 6.52905\right\}, \left\{\frac{\pi}{5}, 6.47685\right\}, \left\{\frac{\pi}{4}, 6.425\right\}, \left\{\frac{3\pi}{10}, 6.39087\right\}, \left\{\frac{7\pi}{20}, 6.38155\right\}, \left\{\frac{2\pi}{5}, 6.39405\right\}, \left\{\frac{9\pi}{20}, 6.41874\right\}, \left\{\frac{\pi}{2}, 6.44405\right\}, \left\{\frac{11\pi}{20}, 6.46045\right\}, \left\{\frac{3\pi}{5}, 6.46299\right\}, \left\{\frac{13\pi}{20}, 6.45304\right\}, \left\{\frac{7\pi}{10}, 6.43979\right\}, \left\{\frac{3\pi}{4}, 6.44063\right\}, \left\{\frac{4\pi}{5}, 6.47719\right\}, \left\{\frac{17\pi}{20}, 6.56314\right\}, \left\{\frac{9\pi}{10}, 6.68439\right\}, \left\{\frac{19\pi}{20}, 6.78795\right\}, \{\pi, 6.8074\}, \left\{\frac{21\pi}{20}, 6.73042\right\}, \left\{\frac{11\pi}{10}, 6.62516\right\}, \left\{\frac{23\pi}{20}, 6.56066\right\}, \left\{\frac{6\pi}{5}, 6.5448\right\}, \left\{\frac{5\pi}{4}, 6.55006\right\}, \left\{\frac{13\pi}{10}, 6.5514\right\}, \left\{\frac{27\pi}{20}, 6.53738\right\}, \left\{\frac{7\pi}{5}, 6.50849\right\}, \left\{\frac{29\pi}{20}, 6.47327\right\}, \left\{\frac{3\pi}{2}, 6.44405\right\}, \left\{\frac{31\pi}{20}, 6.43201\right\}, \left\{\frac{8\pi}{5}, 6.44221\right\}, \left\{\frac{33\pi}{20}, 6.47093\right\}, \left\{\frac{17\pi}{10}, 6.50695\right\}, \left\{\frac{7\pi}{4}, 6.53578\right\}, \left\{\frac{9\pi}{5}, 6.54445\right\}, \left\{\frac{37\pi}{20}, 6.52656\right\}, \left\{\frac{19\pi}{10}, 6.49061\right\}, \left\{\frac{39\pi}{20}, 6.46828\right\}\right\}$$

In[43]:=
```
ListPlot[BRto, PlotStyle → {Green, PointSize[0.02]},
 AxesLabel → {"θ", "(B at 5.0 cm) / (B at 9.5 cm)"},
 PlotRange → {Automatic, {0, 8}}, GridLines → Automatic]
```

Out[43]=



In[44]:=
```
Mean[BRto].{0, 1}
```

Out[44]=

6.51428

It is important to say this here:

The datasets **B5p0** and **B9p5** are the **theoretical** values of the **B**-fields, taken to be measured at 5.0 cm and 9.5 cm respectively from the CM of the magnet (same setup as experimental).
Note in particular that **B5p0** and **B9p5** are NEVER equal to zero for all values of theta -- **THIS IS TO BE**

EXPECTED! However, we **normalized** the **B**-fields experimentally to get rid of the background fields, thus we must normalize the fields here as well. Thus the datasets **B5p0nm** and **B9p5nm** are the **normalized B**-fields -- we make the average of the points zero -- i.e. move all the data points down by some fixed amount.

So how to calculate **M**? Recall that these values all assume that **M = 1** -- well, we simply take ratios: [experimental amplitude] / [theoretical amplitude when M = 1] on the **normalized** data sets, which is fine
because normalization = shifting things down, the amplitude value is not changed at all. Also note that using either the 5.0 cm dataset or the 9.5 cm data set should give a similar value of **M** --
the ratio **B5p0 / B9p5** is roughly constant and equal to **6.6** and this matches experimental ratio of these values still.

ALSO a very important thing: average value of the ratio **B5p0 / B9p5** is about **6.51**, but the mean of the ratio **B5p0nm / B9p5nm** drops to **6.23**. The experimental mean of the same value is about **6.8**. The **6.51** value is CLOSER to the true average value. This is because there is less noise due to the sinusoidal wave -- especially because we are taking discrete measurements which would be a major source
of outliers the values are close to zero. In fact, this is exactly the reason why in the experimental data we threw out six outliers since they were TOO CLOSE to zero (ok think about it like this --
if x changes from 4 to 3.9 then 1/x doesn't change much, but when x changes from 1 to 0.9 then 1/x changes much more even though x changes by same amount. so the experimental error would be magnified by a huge amount which is bad). --> to get 6.8 avg.

In[45]:=
```
B5p0nm = (-{0, Mean[B5p0].{0, 1}} + B5p0ᵀ)ᵀ
```

Out[45]=

$$\left\{\left\{0, -1.75144\times10^{-9}\right\}, \left\{\frac{\pi}{20}, -1.58367\times10^{-9}\right\}, \left\{\frac{\pi}{10}, -1.23988\times10^{-9}\right\},\right.$$
$$\left\{\frac{3\pi}{20}, -8.228\times10^{-10}\right\}, \left\{\frac{\pi}{5}, -3.98025\times10^{-10}\right\}, \left\{\frac{\pi}{4}, 7.04236\times10^{-12}\right\},$$
$$\left\{\frac{3\pi}{10}, 3.84724\times10^{-10}\right\}, \left\{\frac{7\pi}{20}, 7.28621\times10^{-10}\right\}, \left\{\frac{2\pi}{5}, 1.02247\times10^{-9}\right\},$$
$$\left\{\frac{9\pi}{20}, 1.23893\times10^{-9}\right\}, \left\{\frac{\pi}{2}, 1.34664\times10^{-9}\right\}, \left\{\frac{11\pi}{20}, 1.32085\times10^{-9}\right\}, \left\{\frac{3\pi}{5}, 1.15306\times10^{-9}\right\},$$
$$\left\{\frac{13\pi}{20}, 8.56481\times10^{-10}\right\}, \left\{\frac{7\pi}{10}, 4.65659\times10^{-10}\right\}, \left\{\frac{3\pi}{4}, 2.99833\times10^{-11}\right\},$$
$$\left\{\frac{4\pi}{5}, -3.98172\times10^{-10}\right\}, \left\{\frac{17\pi}{20}, -7.76766\times10^{-10}\right\}, \left\{\frac{9\pi}{10}, -1.08108\times10^{-9}\right\},$$
$$\left\{\frac{19\pi}{20}, -1.29682\times10^{-9}\right\}, \left\{\pi, -1.40308\times10^{-9}\right\}, \left\{\frac{21\pi}{20}, -1.36461\times10^{-9}\right\},$$
$$\left\{\frac{11\pi}{10}, -1.1522\times10^{-9}\right\}, \left\{\frac{23\pi}{20}, -7.76452\times10^{-10}\right\}, \left\{\frac{6\pi}{5}, -2.95398\times10^{-10}\right\},$$
$$\left\{\frac{5\pi}{4}, 2.10023\times10^{-10}\right\}, \left\{\frac{13\pi}{10}, 6.64795\times10^{-10}\right\}, \left\{\frac{27\pi}{20}, 1.01692\times10^{-9}\right\},$$
$$\left\{\frac{7\pi}{5}, 1.24325\times10^{-9}\right\}, \left\{\frac{29\pi}{20}, 1.34671\times10^{-9}\right\}, \left\{\frac{3\pi}{2}, 1.34664\times10^{-9}\right\},$$
$$\left\{\frac{31\pi}{20}, 1.26515\times10^{-9}\right\}, \left\{\frac{8\pi}{5}, 1.11465\times10^{-9}\right\}, \left\{\frac{33\pi}{20}, 8.9267\times10^{-10}\right\},$$
$$\left\{\frac{17\pi}{10}, 5.86858\times10^{-10}\right\}, \left\{\frac{7\pi}{4}, 1.87912\times10^{-10}\right\}, \left\{\frac{9\pi}{5}, -2.95255\times10^{-10}\right\},$$
$$\left.\left\{\frac{37\pi}{20}, -8.22483\times10^{-10}\right\}, \left\{\frac{19\pi}{10}, -1.31425\times10^{-9}\right\}, \left\{\frac{39\pi}{20}, -1.65766\times10^{-9}\right\}\right\}$$

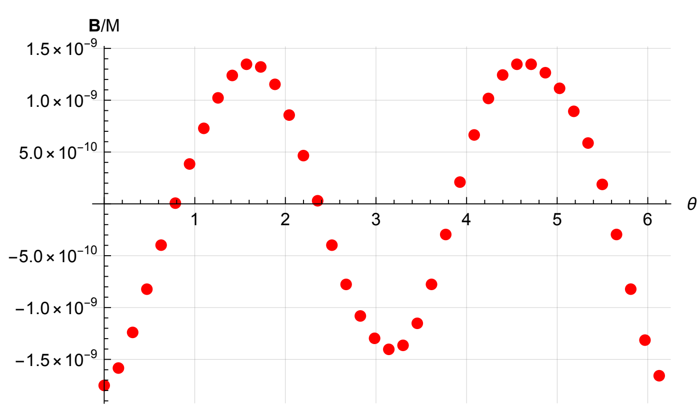
In[46]:=
```
B9p5nm = (-{0, Mean[B9p5].{0, 1}} + B9p5ᵀ)ᵀ
```

Out[46]=

$$\left\{\left\{0, -2.68905\times10^{-10}\right\}, \left\{\frac{\pi}{20}, -2.46658\times10^{-10}\right\},\right.$$
$$\left\{\frac{\pi}{10}, -1.95454\times10^{-10}\right\}, \left\{\frac{3\pi}{20}, -1.29256\times10^{-10}\right\}, \left\{\frac{\pi}{5}, -5.87767\times10^{-11}\right\},$$
$$\left\{\frac{\pi}{4}, 9.73953\times10^{-12}\right\}, \left\{\frac{3\pi}{10}, 7.28219\times10^{-11}\right\}, \left\{\frac{7\pi}{20}, 1.27893\times10^{-10}\right\},$$
$$\left\{\frac{2\pi}{5}, 1.7216\times10^{-10}\right\}, \left\{\frac{9\pi}{20}, 2.02388\times10^{-10}\right\}, \left\{\frac{\pi}{2}, 2.15414\times10^{-10}\right\},$$
$$\left\{\frac{11\pi}{20}, 2.09004\times10^{-10}\right\}, \left\{\frac{3\pi}{5}, 1.82672\times10^{-10}\right\}, \left\{\frac{13\pi}{20}, 1.38129\times10^{-10}\right\},$$
$$\left\{\frac{7\pi}{10}, 7.92403\times10^{-11}\right\}, \left\{\frac{3\pi}{4}, 1.149\times10^{-11}\right\}, \left\{\frac{4\pi}{5}, -5.88357\times10^{-11}\right\},$$
$$\left\{\frac{17\pi}{20}, -1.25397\times10^{-10}\right\}, \left\{\frac{9\pi}{10}, -1.82011\times10^{-10}\right\}, \left\{\frac{19\pi}{20}, -2.22257\times10^{-10}\right\},$$
$$\left\{\pi, -2.39335\times10^{-10}\right\}, \left\{\frac{21\pi}{20}, -2.27932\times10^{-10}\right\}, \left\{\frac{11\pi}{10}, -1.87788\times10^{-10}\right\},$$
$$\left\{\frac{23\pi}{20}, -1.25119\times10^{-10}\right\}, \left\{\frac{6\pi}{5}, -5.01346\times10^{-11}\right\}, \left\{\frac{5\pi}{4}, 2.64767\times10^{-11}\right\},$$
$$\left\{\frac{13\pi}{10}, 9.57364\times10^{-11}\right\}, \left\{\frac{27\pi}{20}, 1.51385\times10^{-10}\right\}, \left\{\frac{7\pi}{5}, 1.90102\times10^{-10}\right\},$$
$$\left\{\frac{29\pi}{20}, 2.11127\times10^{-10}\right\}, \left\{\frac{3\pi}{2}, 2.15414\times10^{-10}\right\}, \left\{\frac{31\pi}{20}, 2.04526\times10^{-10}\right\},$$
$$\left\{\frac{8\pi}{5}, 1.79676\times10^{-10}\right\}, \left\{\frac{33\pi}{20}, 1.41304\times10^{-10}\right\}, \left\{\frac{17\pi}{10}, 8.94461\times10^{-11}\right\},$$
$$\left\{\frac{7\pi}{4}, 2.47607\times10^{-11}\right\}, \left\{\frac{9\pi}{5}, -5.00764\times10^{-11}\right\}, \left\{\frac{37\pi}{20}, -1.28976\times10^{-10}\right\},$$
$$\left.\left\{\frac{19\pi}{10}, -2.01375\times10^{-10}\right\}, \left\{\frac{39\pi}{20}, -2.52619\times10^{-10}\right\}\right\}$$

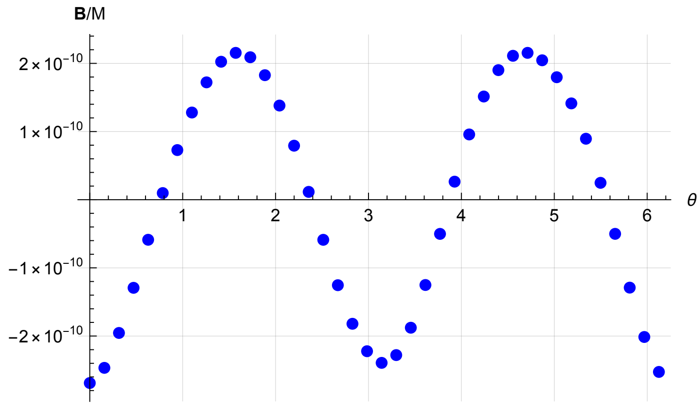
In[47]:=
```
ListPlot[B5p0nm, PlotStyle → {Red, PointSize[0.02]},
  AxesLabel → {"θ", "B/M"}, PlotRange → All, GridLines → Automatic]
```

Out[47]=



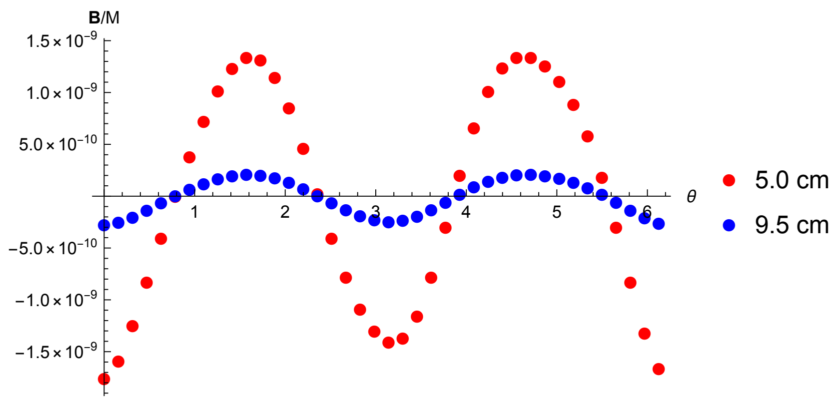
In[48]:=
```
ListPlot[B9p5nm, PlotStyle → {Blue, PointSize[0.02]},
 AxesLabel → {"θ", "B/M"}, PlotRange → All, GridLines → Automatic]
```

Out[48]=



In[49]:=
```
ListPlot[{B5p0nm, B9p5nm}, PlotStyle → {Red, Blue}, PlotMarkers → {"●", 10},
 AxesLabel → {"θ", "B/M"}, PlotLegends → {"5.0 cm", "9.5 cm"}]
```

Out[49]=



In[50]:=
```
MAGNETIZATION = Min[0.89264 * 10^-3 / Max[B5p0nm.{0, 1}], 0.13604 * 10^-3 / Max[B9p5nm.{0, 1}]]
 (* the numerators are the magnetic field obtained
 experimentally (by Govind & Michael), these are in mT
 hence the 10^-3 factor. the denominators are the maxima of the B5p0nm and
 B9p5nm datasets which determine the amplitude -- this is THE only use of the
 B5p0nm and B9p5nm datasets *)
```

Out[50]=

631528.

§3. Period of *Out-of-Plane* Oscillations