

① $S(t) = u(t)$.

The fourier transform of $S(t)$ is ~~$S(F)$~~ $S(F)$

~~$S(F)$~~ $S(F) = \int_{-\infty}^{\infty} S(t) e^{-j\omega t} dt.$

$$u(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$S(F) = \int_{-\infty}^{\infty} u(t) e^{-j\omega t} dt$$

$$= \int_0^1 u(t) e^{-j\omega t} dt.$$

$$= \int_0^1 (1) e^{-j\omega t} dt$$

$$= -\frac{1}{j\omega} [e^{-j\omega t}]_0^1$$

$$= -\frac{1}{j\omega} [e^{-j\omega} - e^{-j\omega(0)}]$$

$$S(F) = \frac{1}{j\omega} [1 - e^{-j\omega}]$$

$$S(F) = \frac{1 - e^{-j\omega}}{j\omega} = \cancel{1} \cancel{[\cos - j\sin]} = 1 - [\cos(\omega) - j\sin(\omega)]$$

$$S(F) = \frac{1 - \cos(\omega) + j\sin\omega}{j\omega}$$

$$= \frac{\sin\omega}{\omega} - \frac{j[1 - \cos\omega]}{\omega}$$

$$|S(F)| = \frac{1}{\omega} \sqrt{\sin^2\omega + (1 - \cos\omega)^2}$$

$$= \frac{1}{\omega} \sqrt{1 + 1 - 2\cos\omega}$$

$$= \frac{1}{\omega} \sqrt{2 - 2\cos\omega}$$

$$= \frac{1}{\omega} \sqrt{2 \times 2 \cos^2 \frac{\omega}{2}}$$

$$= \frac{1}{\omega} \sqrt{2 \times 2 \sin^2 \frac{\omega}{2}}$$

$$|S(F)| = \frac{2 \sin \frac{\omega}{2}}{\omega}$$