DSP Assigment - 1

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Q1

(a) $h[n] = \delta[n-d]$ → delayed input

condition if $h[n] = 0$ for $n<0$ - casual.

and is $\sum |h[n]| < \infty$ - ~~unsta~~ Not stable

Casuality

$\delta[n-d]$

for $n<0$

if $d \geq 0$ so $n \neq d$ for any value of n

So the system $h[n]=0$ for all values of n

hence the system is casual

if $d<0$

when $n = d$. then the system is non-casual.

Stability

$$\sum_{n=-\infty}^{\infty} |\delta(n-d)| = 1 \quad \text{as} \quad \delta[n-d] = \begin{cases} 1 & n=d \\ 0 & \text{otherwise} \end{cases}$$

~~.~~ ∴ the system is stable ②

the system is casual for $\alpha \geq 0$ when $n < 0$

(b) $h[n] = u[n] - u[n-N]$

Casuality

when $n < 0$ then the equation is reduced to

$h[n] = -u[n-N]$

Now when ~~a~~ $N > 0$

~~similar con~~ , then $h[n] \neq 0$ as $n \neq N$ for any value of n hence the system is casual

when $N < 0$

$h[n] = 0$ for $n = N$ hence the system is ~~no~~ Non casual.

Stability

~~a~~ $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

$\Rightarrow \sum_{n-\infty}^{\infty} |u[n] - u[n-N]|$

$= \sum_{n=-\infty}^{0} u[n-N] + \sum_{1}^{\infty} |u[n] - u[n-N]|$

$= \sum 0$

$$\sum_{n=-\infty}^{\infty} | u[n] - u[n-N] |$$

(3)

$$h[n] = \begin{cases} 1 & 0 \leq n \leq N \\ 0 & n \geq N. \\ 0 & n < 0 \end{cases}$$

$$\sum_{n=\infty}^{n=-1} | u[n-N] | + \sum_{n=0}^{N} | u[n] | + \sum_{n=N+1}^{\infty} | u[n] - u[n-N] |$$

$\Rightarrow \quad 0 + 1 + 0 = 1$

the system is stable
and the system is causal for $N \in \mathbb{Z}^{+}$

(c) $h[n] = \frac{1}{2N+1} \sum_{k=-N}^{N} \delta[n-k]$

causaility

for $n < 0$

$$h[n] = \frac{1}{2N+1} [\delta[n+\overset{k=N}{N}] + \delta[n+(N-1)] + \delta[n+(N-2)] \cdots$$
$$+ \delta[n] + \delta[n-1] + \delta[n-2] \cdots$$
$$\cdots + \delta[n-(N-2)] + \delta[n-(N-1)]$$
$$+ \delta[n-N]$$

$$h[n] = \frac{1}{2N+1} [\delta[n+N] + \delta[n+(N-1)] + \delta[n+N-2] + \cdots \delta[n-1]]$$

so $h[n] \neq 0$ when $n = k$.

$\therefore$ The system is non- casual.

Stability.

$\sum |h[n]|$

~~h[n] =~~

$$\Rightarrow \sum_{n=-\infty}^{\infty} \frac{1}{2N+1} \sum_{k=-N}^{N} \delta[n-k]$$

$$\frac{1}{2N+1} \sum_{k=-N}^{N} \sum_{n=-\infty}^{\infty} \delta[n-k]$$

$$\frac{1}{2N+1} \sum_{k=-N}^{N} 1 \quad \text{as} \quad \delta[n-k] = \begin{cases} 1 & n=k \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{1}{2N+1} \; 2N+1 = 1$$

The system is stable.

(d) $h[n] = a^n u[-n]$

causality

~~the system is non casual for~~ n

when $n < 0$

$h[n] = 1$ $\therefore$ The system is non-casual.

(a) Stability

$h[n] = a^n u[-n]$

$\sum |h[n]| < \infty$

$\Rightarrow \sum_{n=-\infty}^{\infty} |a^n u[-n]|$

$\Rightarrow \sum_{n=-\infty}^{\infty} |a^n| |u[-n]|$

$= \sum_{n=-\infty}^{0} |a^n|$

~~when $a \to 0$~~

$\sum_{n=-\infty}^{0} |a^n| = |1| + \left|\frac{1}{a}\right| + \left|\frac{1}{a^2}\right| + \left|\frac{1}{a^3}\right| + \cdots + \left|\frac{1}{a^\infty}\right|$

Now when $a > 0$

$\sum_{n=-\infty}^{0} a^n = 1 + \left|\frac{1}{a}\right| + \left|\frac{1}{a^2}\right| + \left|\frac{1}{a^3}\right| + \cdots + 0$

~~as~~ T

when $0 < a < 1$

$\sum_{n=-\infty}^{0} a^n = 1 + \left|\frac{1}{a}\right| + \left(\frac{1}{a^2}\right) + \left|\frac{1}{a^3}\right| + \cdots + \infty$

when $-1 < a < 0$

$\sum_{n=-\infty}^{0} a^n = |1| + \left|\frac{1}{a}\right| + \left(\frac{1}{a^2}\right) + \left|\frac{1}{a^3}\right| + \cdots + \infty$

~~when~~ $\therefore$ the system is unstable for $-1 < a < 1$
and the system is stable for ~~a~~ $|a| > 1$

e) $h[n] = \sum_{k=-\infty}^{n} \delta[k]$

the system is casual as it ~~its only~~ de
$h[n] = 0$ for $n < 0$

stability.

$$\sum |h[n]| < \infty = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{n} \delta[k]$$

$$\Rightarrow \sum_{n=0}^{\infty} \sum_{k=-\infty}^{n} \delta[k] = \infty$$

the system is unstable.

(2)

$$x[n] \xrightarrow{i/p} \boxed{h[n]} \xrightarrow{o/p} y[n]$$

stability :- bounded input ~~gives~~; then the ~~other~~ system should give a bonded output.

$-|x[n]| \leq B_x < \infty$

$|y[n]| \leq B_y < \infty$

$y[n] = x[n] * h[n]$

$y[n] = \sum_{k} h[k]\, x[n-k]$

$$|y[n]| \leq \sum |h[k]\, x[n-k]|$$

$$|y[n]| \leq \sum |h[k]|\, |x[n-k]|$$

$$|y[n]| \leq B_x \sum_{k} |h[k]| < \infty$$

$\therefore$ there fore ~~if~~ if the impulse response of the system has a finite sum then the system will give a bounded output for and bonded input.

hence it is a sufficient condition..

(3)

(a) $x[n] = [4, 2, \underline{2}, 3, 1, 6]$

$h[n] = [\underline{1}, 0, 3, 1, 5]$

$y[n] = x[n] * h[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

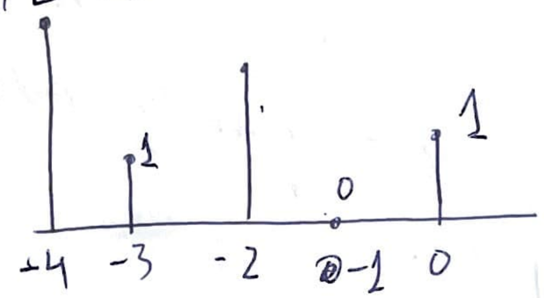
~~for~~ $n = 0$

$$y[0] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[-k]$$

$= x$

$x[k]$

$h[-k]$



$$y[0] = 4(3) + 2(0) + 2(1)$$
$$= 12 + 2 = 14.$$

$$y[1] = \sum_{k=-\infty}^{\infty} x[k]\, h[1-k]$$

$$= \cancel{x[0]h}$$

$$= x[-2]\,h[3] + x[-1]\,h[2] + x[0]\,h[1]$$
$$+ x[1]\,h[0] + \cancel{x[2]\,h[-1]}$$

$$= 4\times(1) + 2\times 3 + 2\times 1 + 3\times 1.$$

$$= 4 + 6 + 2 + 3$$

$$= 15$$

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$y[2] = \sum x[k]\, h[2-k]$

$= x[-2]h[4] + x[-1]h[3] + x[0]h[2]$
$+ x[1]h[1] + x[2]h[0]$

$= 4 \times 5 + 2 \times 1 + 2 \times 3 + 3 \times 0 + 1 \times 1$

$= 20 + 2 + 6 + 0 + 1$

$= \boxed{29}$

$y[3] = \sum x[k]\, h[3-k]$

$= 5 \times 2 + 2 \times 1 + 3 \times 3 + 6 \times 1$

$= \boxed{27}$

$y[4] = \sum_{k=-\infty}^{\infty} x[k]\, h[4-k]$

$= 5 \times 2 + 3 \times 1 + 3 \times 1 + 6 \times 0$

$= 16$

$y[5] = \sum_{k=-\infty}^{\infty} x[k]\, h[5-k]$

$= 5 \times 3 + 1(1) + 3 \times 6 = 34$

$y[6] = \sum_{k=-\infty}^{\infty} x[k]\, h[6-k]$

$= 5(1) + 6(1) = 11$

(8)

$$y[7] = \sum_{k=-\infty}^{\infty} x[k]\, h[7-k]$$

$$= 6(5) = 30$$

$y[8] = 0$

$y[9] = 0$ .... $y[10] = ..$

$$y[-1] = \sum_{k=-\infty}^{\infty} x[k]\, h[-1-k]$$

$$= 2(1) + 4(0) = 2.$$

$$= 2$$

$$y[-2] = \sum_{k=-\infty}^{\infty} x[k]\, h[-2-k]$$

$$= (4)(1)$$

$$= (4)$$

$y[-3] = 0$

⋮

Hence, the o/p $y[n]$ is

$y[n] = (4, 2, \underline{15}, 13, 29, 27, 16, 34, 11, 30)$

(b) $x[n] = [-1, -1, 0, 0, 1, 1, 0, 0, -1, -1]$ (9)

$h[n] = \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]$

$y[n] = \sum x[k]\, h[n-k]$

$y[0] = \sum x[k]\, h[-k]$

$= \frac{1}{3} \times (-1) = -\frac{1}{3}$

$y[1] = \sum_{k=-\infty}^{\infty} x[k]\, h[1-k]$

$= \frac{1}{3}(-1) + \frac{1}{3}(-1) = -\frac{2}{3}$

$y[2] = \sum x[k]\, h[2-k]$

$= \cancel{\frac{1}{3} \quad \frac{1}{3}}$

$= (-1) \times \frac{1}{3} + (-1) \times \frac{1}{3}$

$= -\frac{2}{3}$

$y[3] = \sum x[k]\, h[3-k]$

$= (-1)\frac{1}{3} = -\frac{1}{3}$

$y(4) = h[0]\, x[4] = \frac{1}{3}$

$y[5] = h[1]\, x[4] + x[5]\, h[0] = \frac{2}{3}$

$y[6] = h[2]x[4] + h[1]x[5]$

$= \frac{2}{3}$

$y[7] = h[2]x[5]$

$= \frac{1}{3}$

$y[8] = h[0]x[8]$

$= -\frac{1}{3}$

$y[9] = h[1]x[8] + h[0]x[9]$

$= \frac{-1}{3} - \frac{1}{3}$

$= -\frac{2}{3}$

$y[10] = h[1] \times x[9] + h[2]x[8]$

$= -\frac{2}{3}$

$y[11] = h[2]x[9] = \frac{-1}{3}$

$y[n] = \left[\frac{-1}{3} \ \frac{-2}{3} \ \frac{-2}{3} \ \frac{-1}{3} \ \frac{1}{3} \ \frac{2}{3} \ \frac{2}{3} \ \frac{1}{3} \ \frac{-1}{3} \ \frac{-2}{3} \ \frac{-2}{3} \ \frac{-1}{3} \right]$

(4)

(11)

(a) $y_1[n] = x[n] * h[n]$

$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$y_1[n] = \sum_{k=0}^{\infty} a^k u[k] \cdot [u[n-k] - u[n-M-k]]$$

$$= \sum_{k=-\infty}^{\infty} (u[k] - u[k-M])(a^{n-k} u[n-k])$$

$$= \sum_{k=-\infty}^{\infty} u[k] \cdot (a^{n-k} u[n-k]) - \sum (u[k-M])(a^{n-k}) u[n-k]$$

$$y[n] = \sum_{k=0}^{\infty} u[k] (a^{n-k} u[n-k]) - \sum_{k=M}^{\infty} u[k-M] (a^{n-k} u[n-k])$$

$$= \sum_{k=0}^{\infty} a^{n-k} u[n-k] - \sum_{k=M}^{\infty} a^{n-k} u[n-k]$$

$$= \sum_{k=0}^{M-1} a^{n-k} u[n-k] + \sum_{k=M}^{\infty} a^{n-k} u[n-k]$$

$$- \sum_{k=M}^{\infty} a^{n-k} u[n-k]$$

$$\boxed{y_1[n] = \sum_{k=0}^{M-1} a^{n-k} u[n-k]}$$

(12)

(b) $x_2[n] = a^n u[n]$.

$h_2[n] = u[n+M] - u[n]$

$y_2[n] = x_1[n] * h_1[n]$

$= \sum x[k]\, h[n-k]$

$= \sum h[k]\, x[n-k]$

$= \sum (u[k+M] - u[k])\, a^{n-k}\, u(n-k)$

$= \sum u[k+M]\, a^{n-k}\, u[n-k] - \sum u[k]\, a^{n-k}\, u[n-k]$

$$y_2[n] = \sum_{k=-M}^{\infty} a^{n-k}\, u[n-k] - \sum_{k=0}^{\infty} a^{n-k}\, u[n-k]$$

$$y_2[n] = \sum_{k=-M}^{-1} a^{n-k}\, u[n-k]$$

(c) $x_3[n] = a^n u[n]$

$h_3[n] = u[n+M] - u[n-M]$

$$y_3[n] = \sum_{k=-\infty}^{\infty} h_3[k]\, x_3[n-k]$$

$$= \sum_{k=-\infty}^{\infty} (u[k+M] - u[k-M])\, a^{n-k}\, u[n-k]$$

$$= \sum_{k=-\infty}^{\infty} u[k+M]\, a^{n-k}\, u[n-k] - \sum_{k=-\infty}^{\infty} u[k-M]\, a^{n-k}\, u[n$$

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$$= \sum_{k=-M}^{\infty} a^{n-k} u[n-k] - \sum_{k=M}^{\infty} a^{n-k} u[n-k]$$

$$= \sum_{k=-M}^{M-1} a^{n-k} u[n-k]$$

Hence,

$$y_3[n] = \sum_{k=-M}^{M-1} a^{n-k} u[n-k]$$

It is clearly visible that y_3 is the linear sum of $y_1[n]$ and $y_2[n]$

$$\therefore y_3[n] = y_1[n] + y_2[n]$$

$$\sum_{k=-M}^{M-1} a^{n-k} u[n-k] = \sum_{k=-M}^{-1} a^{n-k} u[n-k] + \sum_{k=0}^{M-1} a^{n-k} u[n-k]$$

5 eigen function, when given as input to an LTI system, the output is also the same but is multiplied with extra weight, this weight multiplied is an internal property of the system

$$x[n] = e^{sn} \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$y[n] = \sum h[k]\, e^{s(n-k)}$$

$$= \sum h[k]\, e^{s(-k)}\, e^{sn}$$

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$y[n] = e^{sn} \underbrace{\sum h[k] e^{-ks}}_{H(S)}$ (14)

$y[n] = e^{sn} \cdot H(S)$

$= x[n] \cdot H(S)$.

Input $x[n] = e^{sn}$ and output $= e^{sn} \times H(S)$
This $H(S)$ is the extra weight, hence e^{sn} is a eigen function of LTI system

$\boxed{\text{eigen values} = H(S) = \lambda = \sum_{k=-\infty}^{\infty} h[k] e^{-sk}}$

(6)

(a) $h[n] = e^{j\omega_0 n}$

$H(j\omega) = \sum_{k=-\infty}^{\infty} h[k] e^{-j\omega k}$ → frequency response

$= \sum_{k=-\infty}^{\infty} e^{j(\omega_0 - \omega) k}$.

using duality method.

~~$\delta[n]$~~

$\mathcal{F}(\delta[n]) = 1$

$u[n-k]$

$\mathcal{F}\{1\} = 2\pi\,\delta(\omega)$

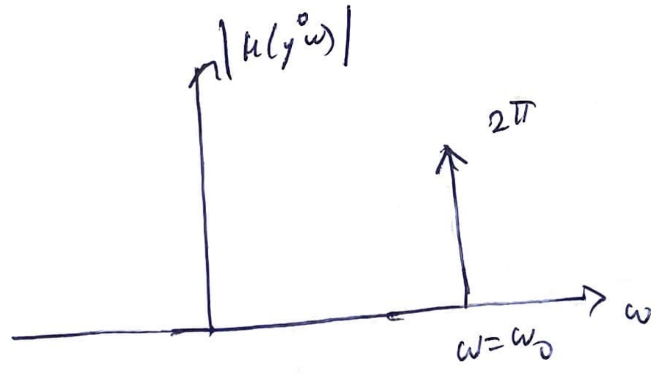
$\mathcal{F}[1\cdot e^{j\omega_0 n}] = 2\pi\,\delta(\omega-\omega_0)$

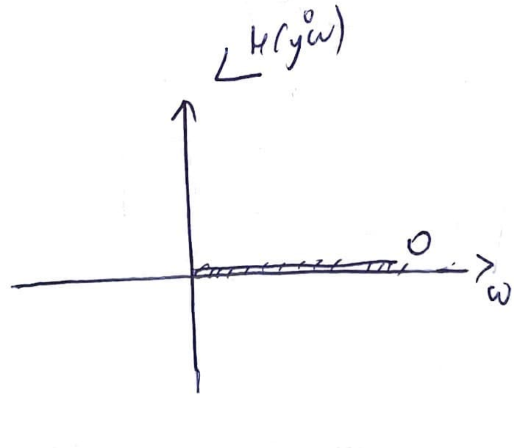
Multiplication in discrete domain leads to shift in fourier domain.

$$\boxed{H(j\omega) = \mathcal{F}[e^{j\omega_0 n}] = 2\pi(\omega-\omega_0)}$$

$|H(j\omega)| = 2\pi\,\delta(\omega-\omega_0)$

$\angle H(j\omega) = 0$





(16)

(b) $h[n] = a^n u[n-10] \quad |a| < 1$

$$H(j\omega) = \sum_{k=-\infty}^{\infty} h[k] e^{-j\omega k}$$

$$= \sum_{k=-\infty}^{\infty} a^k u[k-10] e^{-j\omega k}$$

$$= \sum_{k=10}^{\infty} a^k e^{-j\omega k}$$

$$= a^{10} e^{-j\omega \times 10} + a^{11} e^{-j\omega \times 11} + a^{12} e^{-j\omega \times 12} \quad \Rightarrow$$

G.P.S

. . .

$$H(j\omega) = \frac{(a e^{-j\omega})^{10}}{1 - (a e^{-j\omega})}$$

$$|H(j\omega)| = \frac{|a^{10} e^{-10 j\omega}|}{|1 - (a e^{-j\omega})|}$$

$$= \frac{a^{10}}{|1 - a[\cos(\omega) - j \sin(\omega)]|}$$

$$= \frac{a^{10}}{|(1 - a\cos\omega) + j a \sin\omega|}$$

$u[n-k]$

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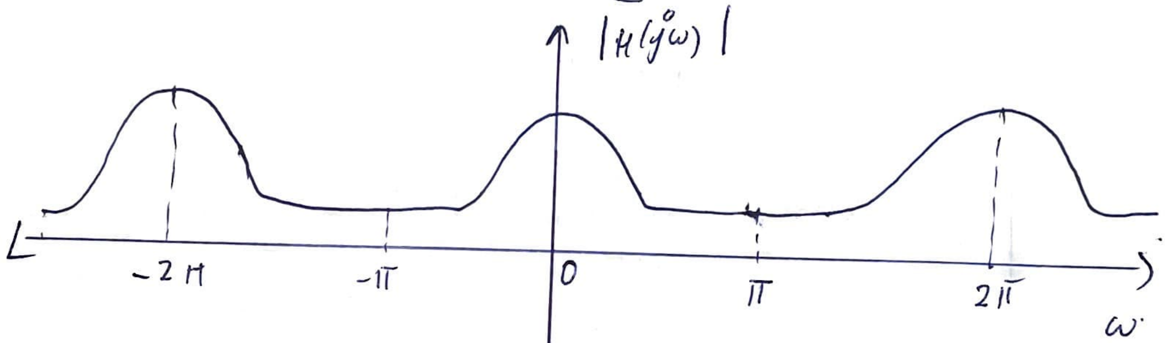
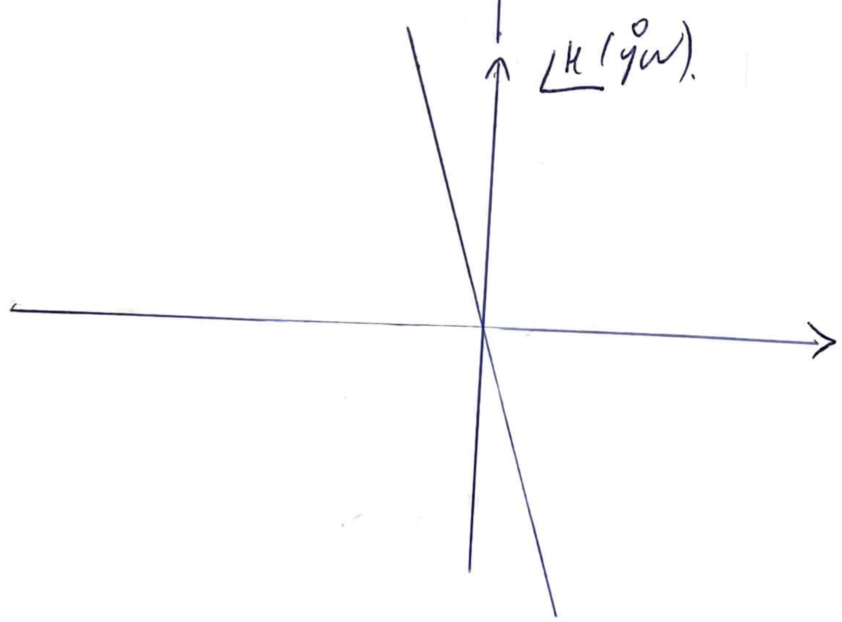
$$|H(j\omega)| = \frac{a^{10}}{\sqrt{(1-a\cos\omega)^2 + (a\sin\omega)^2}}$$

$$= \frac{a^{10}}{\sqrt{a^2+1-2a\cos\omega}}$$

$$\angle H(j\omega) = \angle a^{10}e^{-10j\omega} - \angle 1-ae^{-j\omega}$$

$$\angle H(j\omega = -10\omega - \left[\tan^{-1}\left[\frac{a\sin\omega}{1-a\cos\omega}\right)\right.$$





Given

$$h[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{else} \end{cases}$$

$$H(j\omega) = \sum_{k=-\infty}^{\infty} h[k]\, e^{-j\omega k}$$

$$= \sum_{k=0}^{M} h[k]\, e^{-j\omega k}$$

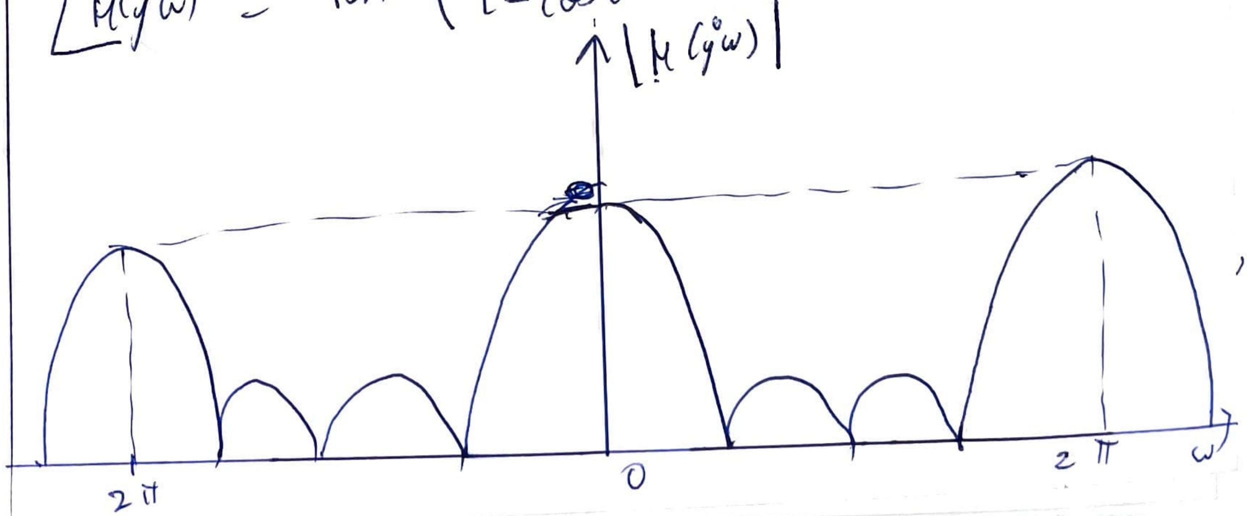
$$= \sum_{k=0}^{M} e^{-j\omega k} \rightarrow \text{G.P sum}$$

$$= \frac{1-(e^{-j\omega})^{M+1}}{1-e^{-j\omega}} = \frac{1-e^{j[M+1]\omega}}{1-e^{-j\omega}}$$

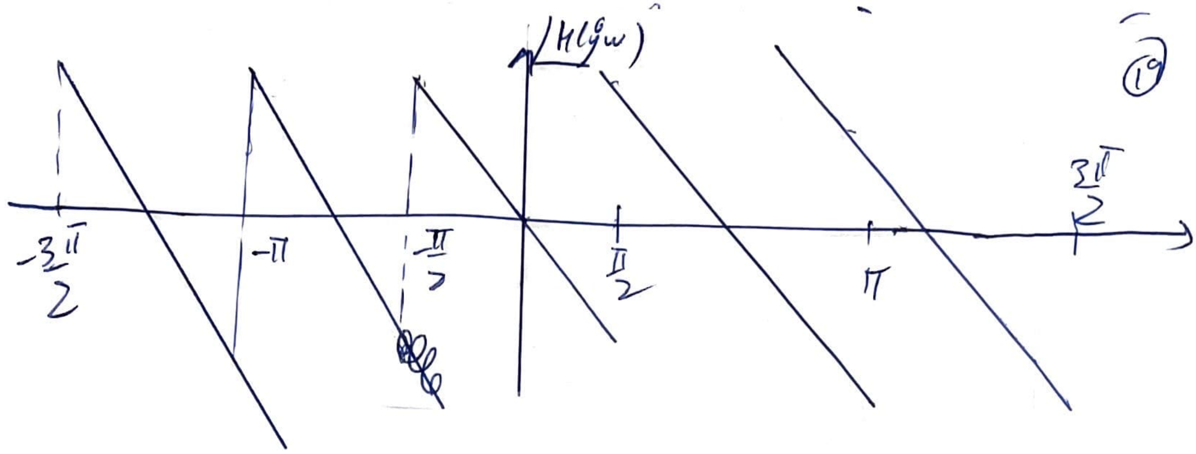
$$|H(j\omega)| = \frac{\sqrt{(1-\cos[(M+1)\omega])^2 + (\sin((M+1)\omega))^2}}{\sqrt{(1-\cos\omega)^2 + (\sin\omega)^2}}$$

$$|H(j\omega)| = \left| \frac{\sin\frac{(M+1)\omega}{2}}{\sin\frac{\omega}{2}} \right|$$

$$\angle H(j\omega) = \tan^{-1}\left(\frac{\sin(M+1)\omega}{1-\cos(M+1)\omega}\right) - \tan^{-1}\left[\frac{\sin\omega}{1-\cos\omega}\right]$$



,[n-
k]



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(d) $h[n] = \dfrac{\sin(\omega_c n)}{\pi n}$

$$H(j\omega) = \sum h[k] e^{-j\omega k}$$

$$= \sum_{k=-\infty}^{\infty} \frac{\sin(\omega_c k)}{\pi k} e^{-j\omega k}$$

~~let~~

let a function $X(j\omega) = \begin{cases} 1 & -\omega_c \le \omega \le \omega_c \\ 0 & \text{otherwis} \end{cases}$

$X(j\omega)$ is a rectangular wave

so

Now finding the invers fourier transform

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(j\omega) e^{j\omega n} d\omega \quad [-\omega_c, \omega_c] \subset [-\pi, \pi]$$

one periode of sinc in time domain

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{j\omega n} d\omega =$$

$$= \frac{1}{2\pi} \left[\frac{e^{j\omega n}}{jn} \right]_{-\omega_c}^{\omega_c}$$

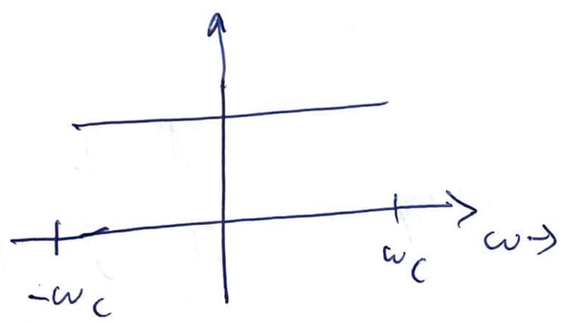
$$= \frac{1}{2\pi j n}\left[e^{j\omega_c n} - e^{-j\omega_c n}\right]$$

$$\Rightarrow \frac{1}{\pi \cancel{j} n}\left[\cancel{j}\sin(\omega_c n)\right]$$

$$x[n] = \frac{\sin \omega_c n}{\pi n}.$$

$$\therefore X(j\omega) = \begin{cases} 1 & -\omega_c \le \omega \le \omega_c \\ 0 & \text{otherwise} \end{cases}$$

$|H(j\omega)|$



$\angle H(j\omega)$

(7) $x[n] = -2 + 3\cos\left(\frac{\pi n}{4} + \frac{\pi}{3}\right) + 10\cos\left(\frac{3\pi n}{4} - \frac{\pi}{5}\right)$

$$h[n] = 3\delta[n] + 2\delta[n-1] + \delta[n-3]$$

$$H(j\omega) = \sum_{k=-\infty}^{\infty} h[k]\, e^{-j\omega k}$$

$$= \sum_{k=-\infty}^{\infty} \left(3\,\delta[k] + 2\delta[k-1] + \delta[k-3]\right) e^{-j\omega k}$$

$$= 3e^{-j\omega(0)} + 2\times e^{-j\omega(1)} + e^{-j\omega(3)}$$

$$H(j\omega) = 3 + 2e^{-j\omega} + e^{-3j\omega}.$$

(b) $|H(j\omega)| = |3 + 2e^{-j\omega} + e^{-3j\omega}|$ (27)

$= |3 + 2(\cos\omega - j\sin\omega) + \cos 3\omega - j\sin 3\omega)|$

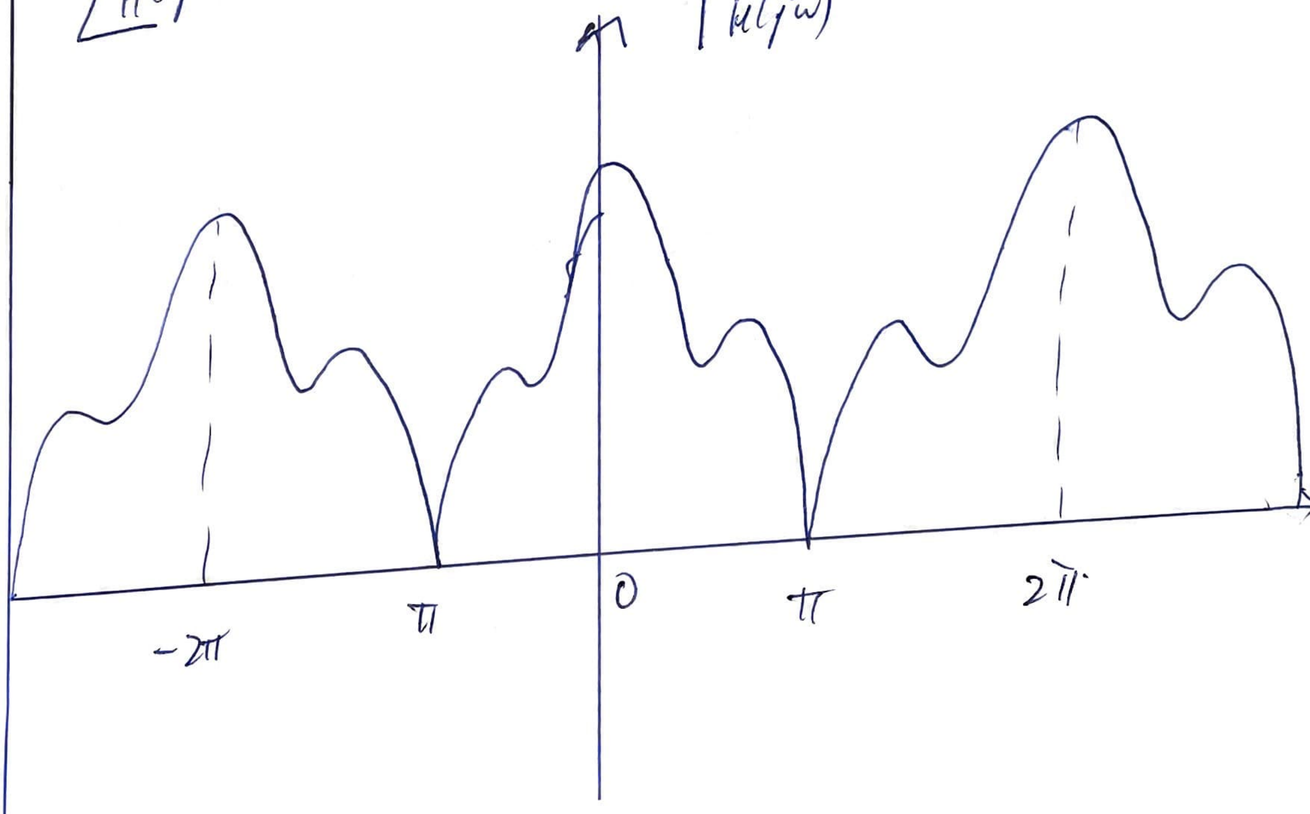
$= |(3 + 2\cos\omega + \cos 3\omega) + j(-2\sin\omega \mp \sin\omega)|$

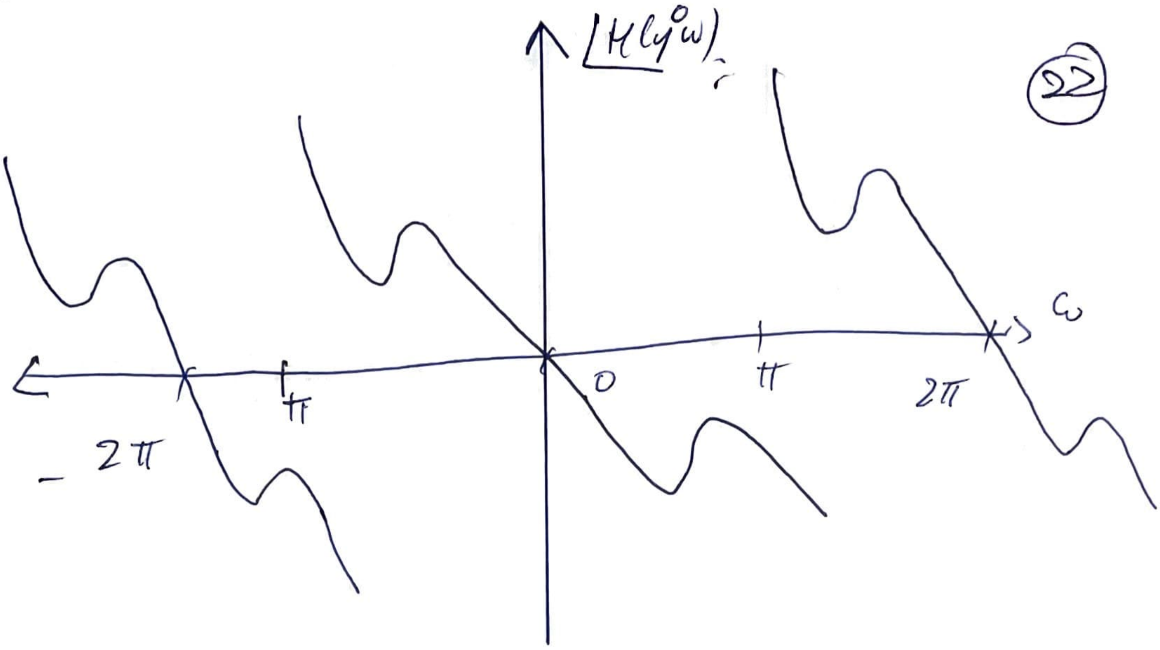
$= \sqrt{(3 + 2\cos\omega + \cos 3\omega)^2 + (2\sin\omega + \sin 3\omega)^2}$

$= \boxed{\sqrt{14 + 12\cos\omega + 4\cos 2\omega + 6\cos 3\omega}}$

$= \cancel{\sqrt{14}}$

$\angle H(j\omega) = \tan^{-1}\left(\dfrac{-2\sin\omega - \sin 3\omega}{3 + 2\cos\omega + \cos 3\omega}\right).$





(c) $y[n] = x[n] * h[n]$

$$= \sum_{k=-\infty}^{\infty} \left(3\delta[k] + 2\delta[k-1] + \delta[k-3]\right)\left(-2 + 3\cos\left(\frac{\pi(n-k)}{4} + \frac{\pi}{3}\right) + 10\cos\left(\frac{3\pi(n-k)}{4} - \frac{\pi}{5}\right)\right)$$

$$y[n] = -13 + 3\left[3\cos\left(\frac{\pi n}{4} + \frac{\pi}{3}\right) + 2\cos\left(\frac{\pi n}{4} + \frac{\pi}{12}\right) + \sin\left(\frac{\pi n}{4} + \frac{\pi}{12}\right)\right]$$

$$+ 10\left[3\cos\left(\frac{3\pi n}{4} - \frac{\pi}{5}\right) - 2\cos\left(\frac{3\pi n}{4} + \frac{\pi}{20}\right) + \sin\left(\frac{3\pi n}{4} + \frac{\pi}{20}\right)\right]$$