



## Homework-2

### Note

- \* Plagiarism is strictly prohibited
- \* Late submissions will be penalized with a factor of 0.5 per each hour
- \* Deadline will not be extended under any circumstances

1. Comment on the causality and stability of the systems with following unit sample responses.

(a)  $h[n] = \delta[n - d]$  where  $d$  is an integer

(b)  $h[n] = u[n] - u[n - N]$

(c)  $h[n] = \sum_{k=-N}^{2N+1} \delta[n - k]$

$P_N$

(d)  $h[n] = a^n u[-n]$ , Derive condition on  $a \in \mathbb{R}$  for stability

(e)  $h[n] = \sum_{k=-\infty}^n \delta[k]$

$(N \in \mathbb{Z}^+)$

2. Recall the condition on the unit sample response for stability

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

$$|h[k]| < \infty$$

Show that the above is a sufficient and necessary condition for stability with appropriate examples.

3. Compute and sketch the LTI system output for the following input and unit sample response sequence pairs

(a)  $x[n] = [4, 2, 2, 3, 1, 6]$   $h[n] = [1, 0, 3, 1, 5]$

(b)  $x[n] = [-1, -1, 0, 0, 1, 1, 0, 0, -1, -1]$   $h[n] = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$

4. Compute the LTI system output ( $y_*[n]$ ) for the following input and unit sample response pairs.

(a)  $x_1[n] = a^n u[n]$ ,  $|a| < 1$ ;  $h_1[n] = u[n] - u[n - M]$

$$(b) x_2[n] = a^n u[n], |a| < 1; h_2[n] = u[n + M] - u[n]$$

$$(c) x_3[n] = a^n u[n], |a| < 1; h_3[n] = u[n + M] - u[n - M]$$

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How do you relate the outputs  $y_1[n]$ ,  $y_2[n]$  and  $y_3[n]$  ? ( $M \in \mathbb{Z}^+$ )

5. What is an eigenfunction ? Check whether  $e^{sn}$  is eigenfunction of an LTI system with unit sample response  $h[n]$  ? If so, derive the eigenvalues?
6. Compute and sketch the magnitude and phase response for the LTI systems with the following unit sample responses

$$(a) h[n] = e^{j\omega_0 n}$$

$$(b) h[n] = a^n u[n - 10], |a| < 1$$

$$( \quad \quad \quad \mathbb{Z}^+ ) \text{ 0 else}$$

$$(c) h[n] =$$

$$1 \quad 0 \leq n \leq M(\in$$

$$(d) h[n] = \frac{\sin(\omega_c n)}{\pi n}$$

7. For an LTI system with input and unit sample response  $x[n]$

$$= -2 + 3\cos\left(\frac{\pi n}{4} + \frac{\pi}{3}\right) + 10\cos(3\pi n$$

$$4 - \frac{\pi}{5})$$

$$h[n] = 3\delta[n] + 2\delta[n - 1] + \delta[n - 3]$$

- (a) Compute frequency response  $H(e^{j\omega})$
- (b) Sketch the magnitude ( $|H(e^{j\omega})|$ ) and phase ( $\angle H(e^{j\omega})$ ) response
- (c) Compute the system output  $y[n]$

