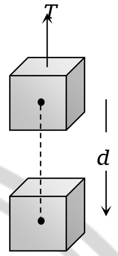
Work Done by Variable Force

1. (b) $W \int_0^{x_1} F.dx = \int_0^{x_1} Cx\ dx = C\left[\frac{x^2}{2}\right]_0^{x_1} = \frac{1}{2}Cx_1^2$

2. (c) When the block moves vertically downward with acceleration $\frac{g}{4}$ then tension in the cord



$T = M\left(g - \frac{g}{4}\right) = \frac{3}{4}Mg$

Work done by the cord = $\vec{F}\,.\,\vec{s} = Fs\cos\theta$

$= Td\cos(180°) = -\left(\frac{3Mg}{4}\right) \times d = -\frac{3}{4}Mgd$

3. (c) $W = \frac{F^2}{2k}$

If both springs are stretched by same force then $W \propto \frac{1}{k}$

As $k_1 > k_2$ therefore $W_1 < W_2$

i.e. more work is done in case of second spring.

4. (a) ΔP.E.$= \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2} \times 10[(0.25)^2 - (0.20)^2]$

$= 5 \times 0.45 \times 0.05 = 0.1J$

5. (a) $\frac{1}{2}kS^2 = 10\ J$ (given in the problem)

$\frac{1}{2}k[(2S)^2 - (S)^2] = 3 \times \frac{1}{2}kS^2 = 3 \times 10 = 30\ J$

6. (c) $U = \frac{F^2}{2k} \Rightarrow \frac{U_1}{U_2} = \frac{k_2}{k_1}$ (if force are same)

$\therefore \frac{U_1}{U_2} = \frac{3000}{1500} = \frac{2}{1}$

7. (d) Here $k = \frac{F}{x} = \frac{10}{1\times10^{-3}} = 10^4 N/m$

$W = \frac{1}{2}kx^2 = \frac{1}{2}\times 10^4 \times (40\times10^{-3})^2 = 8J$

8. (d) $W = \int_0^5 F dx = \int_0^5 (7 - 2x + 3x^2)\ dx = [7x - x^2 + x^3]^5$

= 35 – 25 + 125 = 135 J

9. (d) $S = \frac{t^3}{3} \therefore\ dS = t^2 dt$

$a = \frac{d^2S}{dt^2} = \frac{d^2}{dt^2}\left[\frac{t^3}{3}\right] = 2t\ m \rightleftarrows/\rightleftarrows s^2$

Now work done by the force $W = \int_0^2 F.dS = \int_0^2 ma.dS$

$\int_0^2 3\times 2t\times t^2 dt = \int_0^2 6t^3 dt = \frac{3}{2}[t^4]_0^2 = 24\ J$

10. (b) $W = \frac{1}{2}kx^2$

If both wires are stretched through same distance then $W \propto k$. As $k_2 = 2k_1$ so $W_2 = 2W_1$

11. (b) $\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow x = v\sqrt{\frac{m}{k}} = 10\sqrt{\frac{0.1}{1000}} = 0.1m$

12. (c) Force constant of a spring

$k = \frac{F}{x} = \frac{mg}{x} = \frac{1\times10}{2\times10^{-2}} \Rightarrow k = 500N \rightleftarrows/\rightleftarrows m$

Increment in the length = 60 – 50 = 10 cm

$U = \frac{1}{2}kx^2 = \frac{1}{2}500(10\times10^{-2})^2 = 2.5J$

13. (b) $W = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2} \times 800 \times (15^2 - 5^2) \times 10^{-4} = 8J$

14. (c) $100 = \frac{1}{2}kx^2$ (given)

$W = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}k[(2x)^2 - x^2]$

$= 3 \times \left(\frac{1}{2}kx^2\right) = 3 \times 100 = 300J$

15. (d) $U = \frac{1}{2}kx^2$ if x becomes 5 times then energy will become 25 times *i.e.* $4 \times 25 = 100J$

16. (c) $W = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2} \times 5 \times 10^3(10^2 - 5^2) \times 10^{-4} = 18.75J$

17. (a) The kinetic energy of mass is converted into potential energy of a spring

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow x = \sqrt{\frac{mv^2}{k}} = \sqrt{\frac{0.5\times(1.5)^2}{50}} = 0.15m$

18. (a) This condition is applicable for simple harmonic motion. As particle moves from mean position to extreme position its potential energy increases according to expression $U = \frac{1}{2}kx^2$ and accordingly kinetic energy decreases.

19. (c) Potential energy $U = \frac{1}{2}kx^2$

$\therefore U \propto x^2$ [if k = constant]

If elongation made 4 times then potential energy will become 16 times.

20. (b)

21. (d) $U \propto x^2 \Rightarrow \frac{U_2}{U_1} = \left(\frac{x_2}{x_1}\right)^2 = \left(\frac{0.1}{0.02}\right)^2 = 25 \therefore U_2 = 25U$

22. (a) If x is the extension produced in spring.

$$F = kx \Rightarrow x = \frac{F}{k} = \frac{mg}{k} = \frac{20\times 9.8}{4000} = 4.9cm$$

23. (a) $U = \frac{F^2}{2k} = \frac{T^2}{2k}$

24. (b) $U = A - Bx^2 \Rightarrow F = -\frac{dU}{dx} = 2Bx \Rightarrow F \propto x$

25. (d) Condition for stable equilibrium $F = -\frac{dU}{dx} = 0$

$$\Rightarrow -\frac{d}{dx}\left[\frac{a}{x^{12}} - \frac{b}{x^6}\right] = 0 \Rightarrow -12ax^{-13} + 6bx^{-7} = 0$$

$$\Rightarrow \frac{12a}{x^{13}} = \frac{6b}{x^7} \Rightarrow \frac{2a}{b} = x^6 \Rightarrow x = \sqrt[6]{\frac{2a}{b}}$$

26. (d)Friction is a non-conservative force.