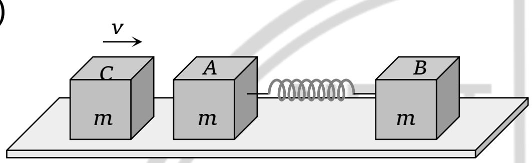
Conservation of Energy and Momentum

61. (b) Kinetic energy $E = \frac{P^2}{2m} = \frac{(Ft)^2}{2m} = \frac{F^2t^2}{2m}$[As $P = F\,t$]

62. (b) Potential energy of spring = $\frac{1}{2}Kx^2$

$\therefore\ PE \propto x^2 \Rightarrow PE \propto a^2$

63. (a)



Initial momentum of the system (block *C*) = *mv*

After striking with *A*, the block *C* comes to rest and now both block *A* and *B* moves with velocity *V*, when compression in spring is maximum.

By the law of conservation of linear momentum

$mv = (m + m)\,V \Rightarrow V = \frac{v}{2}$

By the law of conservation of energy

K.E. of block *C* = K.E. of system + P.E. of system

$\frac{1}{2}mv^2 = \frac{1}{2}(2m)V^2 + \frac{1}{2}kx^2$

$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx^2$

$\Rightarrow kx^2 = \frac{1}{2}mv^2$

$\Rightarrow x = v\sqrt{\frac{m}{2k}}$

64. (c) $p = \sqrt{2mE} \therefore P \propto \sqrt{m} \Rightarrow \frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{m}{4m}} = \frac{1}{2}$

65. (d) $E = \frac{P^2}{2m} \Rightarrow E \propto \frac{1}{m} \Rightarrow \frac{E_1}{E_2} = \frac{m_2}{m_1}$

66. (b) $E = \frac{P^2}{2m} = \frac{4}{2\times3} = \frac{2}{3}J$

67. (d) Both fragment will possess the equal linear momentum

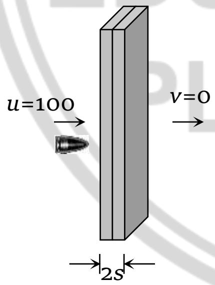
$m_1v_1 = m_2v_2 \Rightarrow 1 \times 80 = 2 \times v_2 \Rightarrow v_2 = 40\ m \rightleftarrows.\rightleftarrows/\rightleftarrows\ s$

$\therefore$ Total energy of system= $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$= \frac{1}{2} \times 1 \times (80)^2 + \frac{1}{2} \times 2 \times (40)^2$

= 4800 *J* = 4.8 *Kj*

68. (b)



Let the thickness of each plank is *s*. If the initial speed of bullet is 100 *m/s* then it stops by covering a distance 2*s*

By applying $v^2 = u^2 - 2as \Rightarrow 0 = u^2 - 2as$

$s = \frac{u^2}{2a}\ s \propto u^2$ [If retardation is constant]

If the speed of the bullet is double then bullet will cover four times distance before coming to rest *i.e.* $s_2 = 4(s_1) = 4(2s) \Rightarrow s_2 = 8s$

So number of planks required = 8

69. (a) $E = \frac{P^2}{2m}$ if *P* = constant then $E \propto \frac{1}{m}$

According to problem $m_1 > m_2 \therefore E_1 < E_2$

70. (c) Kinetic energy= $\frac{1}{2}mv^2$

As both balls are falling through same height therefore they possess same velocity.

but $KE \propto m$ (If *v* = constant)

$\therefore \frac{(KE)_1}{(KE)_2} = \frac{m_1}{m_2} = \frac{2}{4} = \frac{1}{2}$

71. (b) $E = \frac{P^2}{2m} \therefore E \propto \frac{1}{m}$ (If *P* = constant)

i.e. the lightest particle will possess maximum kinetic energy and in the given option mass of electron is minimum.

72. (a) $P = E \Rightarrow mv = \frac{1}{2}mv^2 \Rightarrow v = 2m/s$

73. (c) Initial kinetic energy $E = \frac{1}{2}mv^2$ …(i)

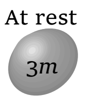
Final kinetic energy $2E = \frac{1}{2}m(v+2)^2$…(ii)

by solving equation (i) and (ii) we get

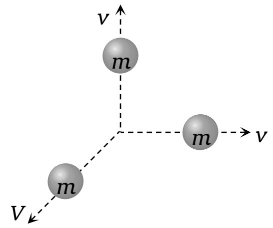
$$v = (2 + 2\sqrt{2})\ m/s$$

74. (c)

At rest



Before explosion



After explosion

Initial momentum of $3m$ mass = 0 …(i)

Due to explosion this mass splits into three fragments of equal masses.

Final momentum of system = $m\vec{V} + mv\hat{\imath} + mv\hat{\jmath}$ …(ii)

By the law of conservation of linear momentum

$m\vec{V} + mv\hat{\imath} + mv\hat{\jmath} = 0 \Rightarrow \vec{V} = -v(\hat{\imath} + \hat{\jmath})$

75. (c)

As the momentum of both fragments are equal therefore $\frac{E_1}{E_2} = \frac{m_2}{m_1} = \frac{3}{1}$ i.e. $E_1 = 3E_2$ …(i)

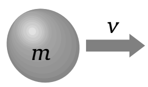
According to problem $E_1 + E_2 = 6.4 \times 10^4 J$ …(i)

By solving equation (i) and (ii) we get

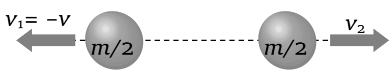
$E_1 = 4.8 \times 10^4 J$ and $E_2 = 1.6 \times 10^4 J$

76. (a)

77. (b)



Before explosion



After explosion

Let the initial mass of body = m

Initial linear momentum = mv …(i)

When it breaks into equal masses then one of the fragment retrace back with same velocity

$\therefore$ Final linear momentum = $\frac{m}{2}(-v)+\frac{m}{2}(v_2)$ ...(ii)

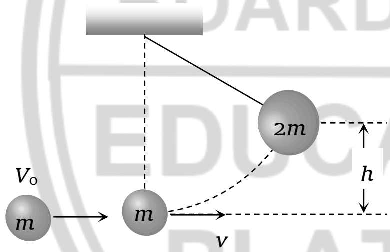
By the conservation of linear momentum

$\Rightarrow mv = \frac{-mv}{2}+\frac{mv_2}{2} \Rightarrow v_2 = 3v$

i.e. other fragment moves with velocity 3*v* in forward direction

78. (a)

79. (a)



Initial momentum of particle = mV_0

Final momentum of system (particle + pendulum) = 2*mv*

By the law of conservation of momentum

$\Rightarrow mV_0 = 2mv \Rightarrow$ Initial velocity of system $v = \frac{V_0}{2}$

$\therefore$ Initial K.E. of the system = $\frac{1}{2}(2m)v^2 = \frac{1}{2}(2m)\left(\frac{V_0}{2}\right)^2$

If the system rises up to height *h* then P.E. = $2mgh$

By the law of conservation of energy

$\frac{1}{2}(2m)\left(\frac{V_0}{2}\right)^2 = 2mgh \Rightarrow h = \frac{V_0^2}{8g}$

80. (d) $\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$

81. (d) Change in momentum = Force × time

$P_2 - P_1 = F \times t = 0.2 \times 10 = 2$

$\Rightarrow P_2 = 2 + P_1 = 2 + 10 = 12kg \rightleftarrows - \rightleftarrows m \rightleftarrows / \rightleftarrows s$

Increase in K.E. = $\frac{1}{2m}(P_2^2 - P_1^2) \ = \frac{1}{2\times5}[(12)^2 - (10)^2]$

$= \frac{44}{10} = 4.4J$

82. (b) $E \propto P^2$ (if *m* = constant)

Percentage increase in *E* = 2(Percentage increase in *P*)

= 2 × 0.01%=0.02%

83. (c) 1 *amu*= $1.66 \times 10^{-27} kg$

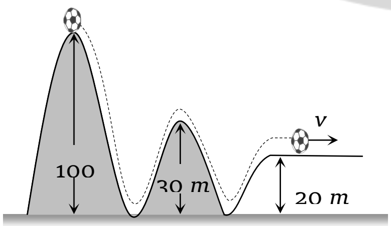
$E = mc^2 = 1.66 \times 10^{-27} \times (3 \times 10^8)^2 = 1.5 \times 10^{-10} J$

84. (b) Change in gravitational potential energy

= Elastic potential energy stored in compressed spring

$\Rightarrow mg(h + x) = \frac{1}{2}kx^2$

85. (c)



Ball starts from the top of a hill which is 100 *m* high and finally rolls down to a horizontal base which is 20 *m* above the ground so from the conservation of energy $mg(h_1 - h_2) = \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{2g(h_1 - h_2)} = \sqrt{2 \times 10 \times (100 - 20)}$

$= \sqrt{1600} = 40m/s.$

86. (c) When block of mass *M* collides with the spring its kinetic energy gets converted into elastic potential energy of the spring.

From the law of conservation of energy

$\frac{1}{2}Mv^2 = \frac{1}{2}KL^2 \therefore v = \sqrt{\frac{K}{M}}L$

Where *v* is the velocity of block by which it collides with spring. So, its maximum momentum

$P = Mv = M\sqrt{\frac{K}{M}}L = \sqrt{MKL}$

After collision the block will rebound with same linear momentum.

87. (b)

According to law of conservation of linear momentum

$m_A v_A = m_B v_B = 18 \times 6 = 12 \times v_B \Rightarrow v_B = 9\ m/s$

K.E. of mass 12 *kg*, $E_B = \frac{1}{2}m_B v_B^2$

$= \frac{1}{2} \times 12 \times (9)^2 = 486J$

88. (c) Force = Rate of change of momentum

Initial momentum $\vec{P}_1 = mv\sin\theta\ \hat{i} + mv\cos\theta\ \hat{j}$

Final momentum $\vec{P}_2 = -mv\sin\theta\ \hat{i} + mv\cos\theta\ \hat{j}$

$\therefore\ \vec{F} = \frac{\Delta\vec{P}}{\Delta t} = \frac{-2mv\sin\theta}{2\times10^{-3}}$

Substituting $m = 0.1\ kg$, $v = 5\ m/s$, $\theta = 60°$

Force on the ball $\vec{F} = -250\sqrt{3}N$

Negative sign indicates direction of the force