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CSC 222
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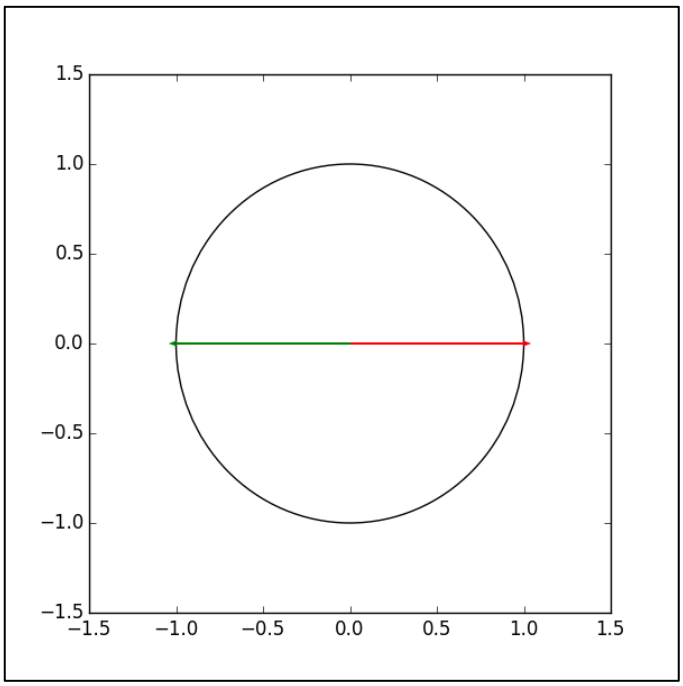
Lab #3 Report

1. Exploring the Roots of Unity

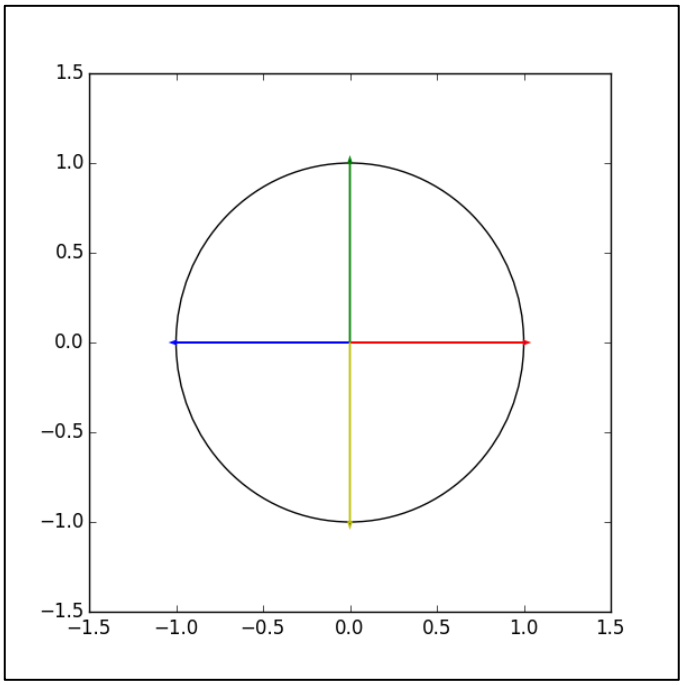
    a. *The Nth roots of unity lie in the complex unity circle.*

        i. After writing the Complex number class and the method to generate complex numbers for a given value of N, I also wrote a method that graphed the complex roots of unity via the equation wn = exp(i*2*pi/N) where the root is -1, resulting in an imaginary value . In order to see a pattern of how the *Nth* root of unity appeared, I tested several different values for N: 2, 4, 8, 16, and 32, and obtained the following graphs. We can see from the range of the graphs that they the roots of unity are bounded by (-1, 1), which shows that they lie on the complex unit circle.

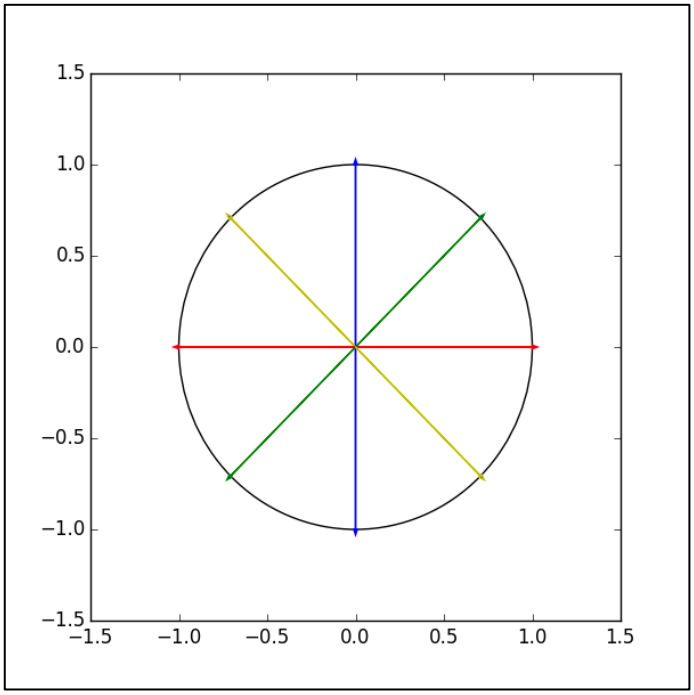
N = 2:



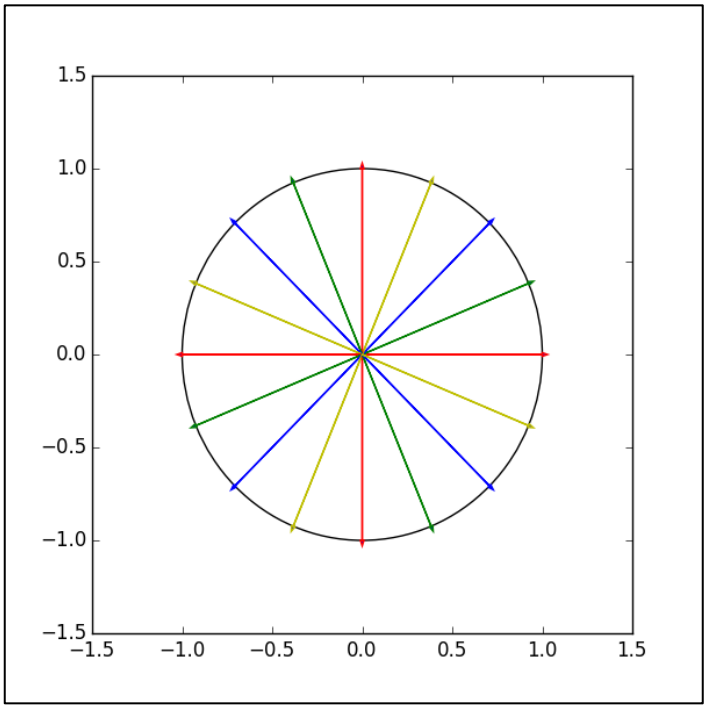
N = 4:


1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

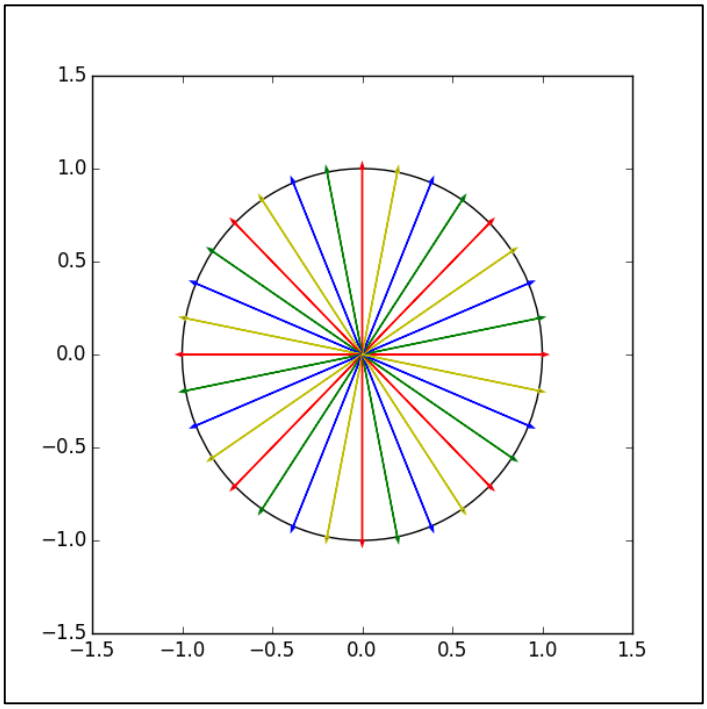
N = 8:


1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

N = 16:


1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

N = 32:


1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

b. We can see that the following root pairs rule provided in the worksheet is **not true**. The correct rule value should be (wN^0, wN^N-2), (wN^1, wN^N-3), etc… Moreover, for each complex root in each output (N=2, N=4, N=8, etc…), we can clearly see that there exists both a positive and negative value for the same root. Below, I have attached my data, proving these results. The green denotes all of the positive values and the yellow denotes the corresponding negative values.

Roots 2:

W0 = 1.00000 + 0.00000i
W1 = -1.00000 + 0.00000i

Roots 4:

W0 = 1.00000 + 0.00000i
W1 = 0.00000 + 1.00000i
W2 = -1.00000 + 0.00000i
W3= -0.00000 + -1.00000i

Roots 8:

W0 = 1.00000 + 0.00000i
W1 = 0.70711 + 0.70711i
W2 = 0.00000 + 1.00000i
W3 = -0.70711 + 0.70711i
W4 = -1.00000 + 0.00000i
W5 = -0.70711 + -0.70711i
W6 = -0.00000 + -1.00000i
W7 = 0.70711 + -0.70711i

etc…

c. Yes, the 32nd roots of unity can be generated from the 16th roots of unity, and the 16th roots of unity can be generated from the 8th roots of unity. We can see this exemplified in the following data:

Roots 16:

1.00000 + 0.00000i
0.92388 + 0.38268i
0.70711 + 0.70711i
0.38268 + 0.92388i
0.00000 + 1.00000i
-0.38268 + 0.92388i
-0.70711 + 0.70711i
-0.92388 + 0.38268i
-1.00000 + 0.00000i
-0.92388 + -0.38268i
-0.70711 + -0.70711i
-0.38268 + -0.92388i
-0.00000 + -1.00000i
0.38268 + -0.92388i
0.70711 + -0.70711i
0.92388 + -0.38268i

Roots 32:

1.00000 + 0.00000i
0.98079 + 0.19509i
0.92388 + 0.38268i
0.83147 + 0.55557i
0.70711 + 0.70711i
0.55557 + 0.83147i
0.38268 + 0.92388i
0.19509 + 0.98079i
0.00000 + 1.00000i
-0.19509 + 0.98079i
-0.38268 + 0.92388i
-0.55557 + 0.83147i
-0.70711 + 0.70711i
-0.83147 + 0.55557i
-0.92388 + 0.38268i
-0.98079 + 0.19509i
-1.00000 + 0.00000i
-0.98079 + -0.19509i
-0.92388 + -0.38268i
-0.83147 + -0.55557i
-0.70711 + -0.70711i
-0.55557 + -0.83147i
-0.38268 + -0.92388i
-0.19509 + -0.98079i
-0.00000 + -1.00000i
0.19509 + -0.98079i
0.38268 + -0.92388i
0.55557 + -0.83147i
0.70711 + -0.70711i
0.83147 + -0.55557i
0.92388 + -0.38268i
0.98079 + -0.19509i

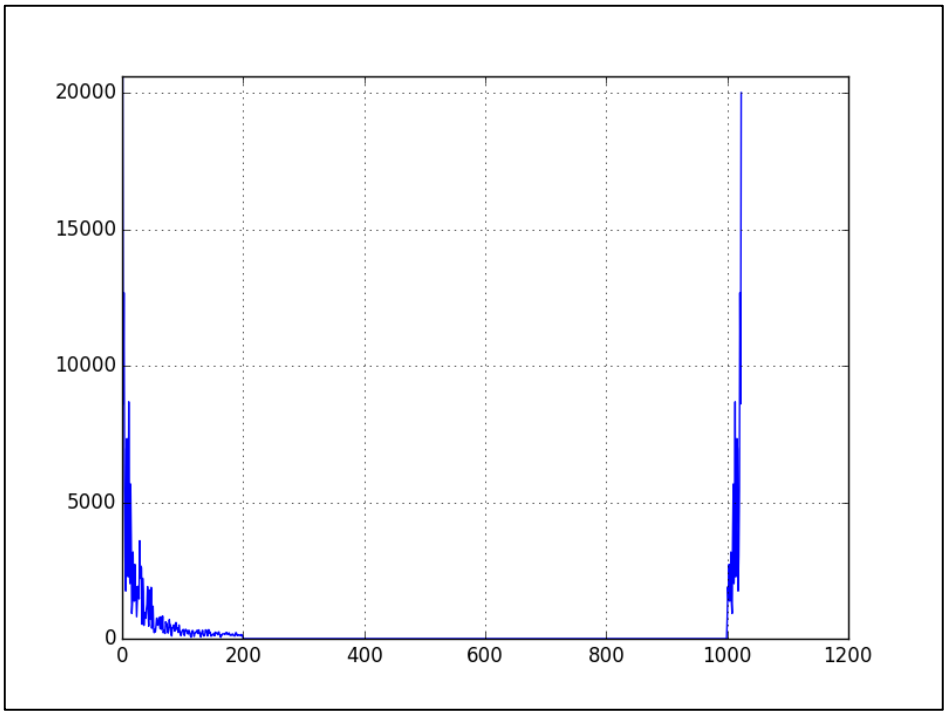
As we can see, the first 8 roots of unity are included in the 16 roots of unity, and this pattern will continue for all numbers that are multiples (2n) of each other.

2. Exploring the discrete Fourier transform:

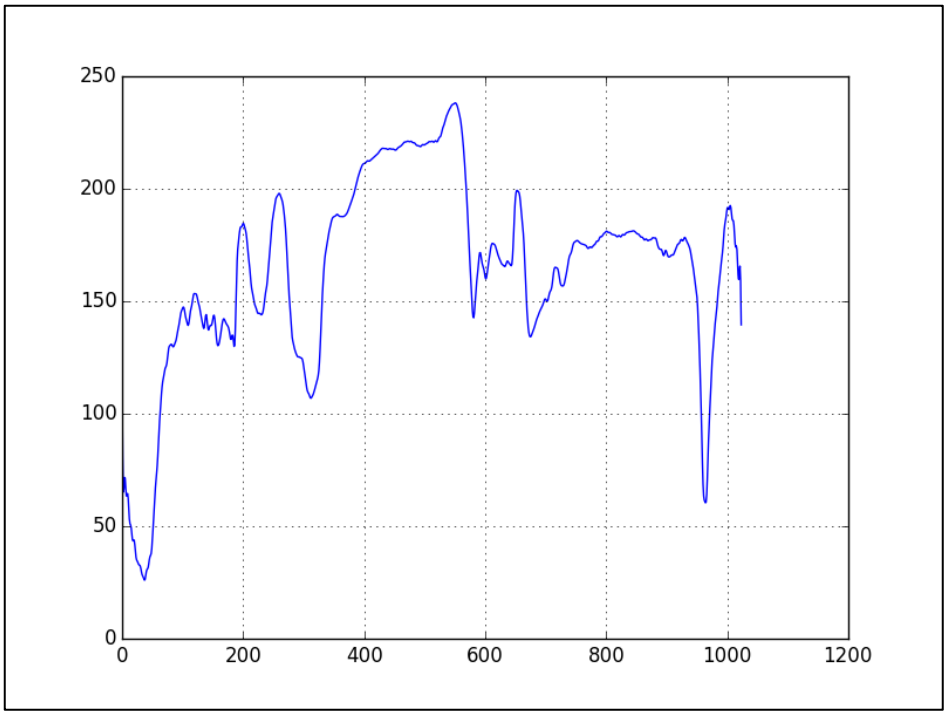
    a. When I ran my Fourier generation algorithm against inputs N=2, N=4, N=8, I obtained the following matrices. Although the formatting for the N=8 matrix is not as "pretty" as the others, this is because Python could not print it to the window size. However, the values are correct and were tested against octave.

```
F:2
[[ 1.+0.j  1.+0.j]
 [ 1.+0.j -1.+0.j]]


F:4
[[ 1.+0.j  1.+0.j  1.+0.j  1.+0.j]
 [ 1.+0.j  0.-1.j -1.+0.j  0.+1.j]
 [ 1.+0.j -1.+0.j  1.+0.j -1.+0.j]
 [ 1.+0.j  0.+1.j -1.+0.j  0.-1.j]]


F:8
[[  1.00000000e+00+0.j           1.00000000e+00+0.j           1.00000000e+00+0.j
    1.00000000e+00+0.j           1.00000000e+00+0.j           1.00000000e+00+0.j
    1.00000000e+00+0.j           1.00000000e+00+0.j         ]
 [  1.00000000e+00+0.j           7.07106781e-01-0.70710678j
    0.00000000e+00-1.j          -7.07106781e-01-0.70710678j
   -1.00000000e+00+0.j          -7.07106781e-01+0.70710678j
    0.00000000e+00+1.j           7.07106781e-01+0.70710678j]
 [  1.00000000e+00+0.j          -8.26946080e-16-1.j          -1.00000000e+00+0.j
    8.26946080e-16+1.j           1.00000000e+00+0.j          -8.26946080e-16-1.j
   -1.00000000e+00+0.j           8.26946080e-16+1.j         ]
 [  1.00000000e+00+0.j          -7.07106781e-01-0.70710678j
    0.00000000e+00+1.j           7.07106781e-01-0.70710678j
   -1.00000000e+00+0.j           7.07106781e-01+0.70710678j
    0.00000000e+00-1.j          -7.07106781e-01+0.70710678j]
 [  1.00000000e+00+0.j          -1.00000000e+00+0.j           1.00000000e+00+0.j
   -1.00000000e+00+0.j           1.00000000e+00+0.j          -1.00000000e+00+0.j
    1.00000000e+00+0.j          -1.00000000e+00+0.j         ]
 [  1.00000000e+00+0.j          -7.07106781e-01+0.70710678j
    0.00000000e+00-1.j           7.07106781e-01+0.70710678j
   -1.00000000e+00+0.j           7.07106781e-01-0.70710678j
    0.00000000e+00+1.j          -7.07106781e-01-0.70710678j]
 [  1.00000000e+00+0.j           8.26946080e-16+1.j          -1.00000000e+00+0.j
   -8.26946080e-16-1.j           1.00000000e+00+0.j           8.26946080e-16+1.j
   -1.00000000e+00+0.j          -8.26946080e-16-1.j         ]
 [  1.00000000e+00+0.j           7.07106781e-01+0.70710678j
    0.00000000e+00+1.j          -7.07106781e-01+0.70710678j
   -1.00000000e+00+0.j          -7.07106781e-01-0.70710678j
    0.00000000e+00-1.j           7.07106781e-01-0.70710678j]]
```

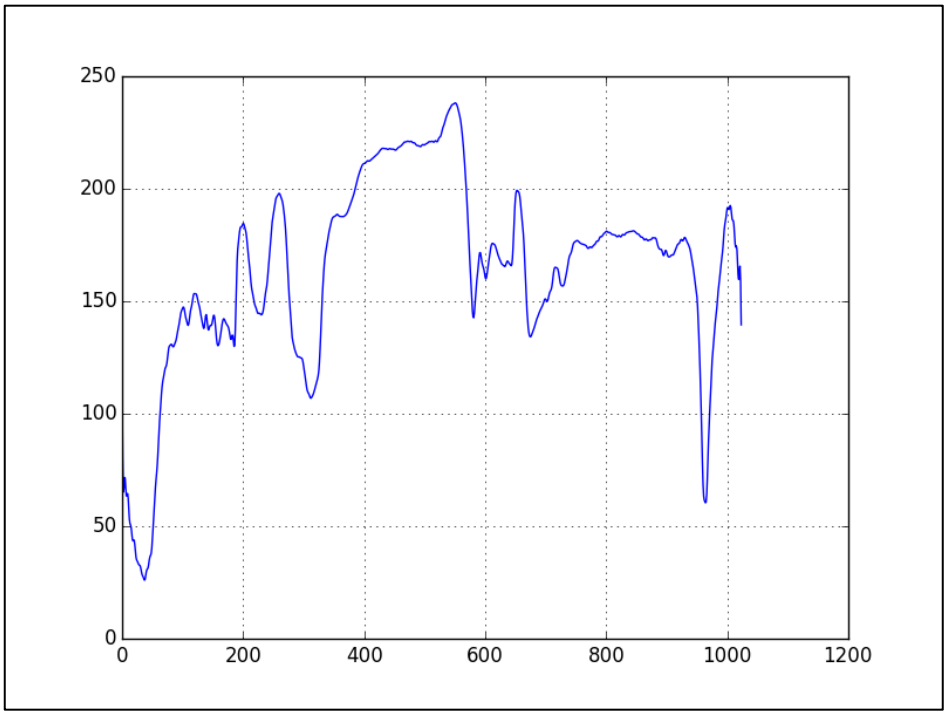
b. After finding the F matrices for different sizes of N, I computed a new square (1024 x 1024) matrix using the DFT and then multiplying this result by the 1D signal array (signals), read in from the external 1Dsignal.txt file. This signal matrix is defined as g^ ("g-hat").



c. The inverse of g^ (DFT) is displayed below. As expected, this graph results in the original signal.

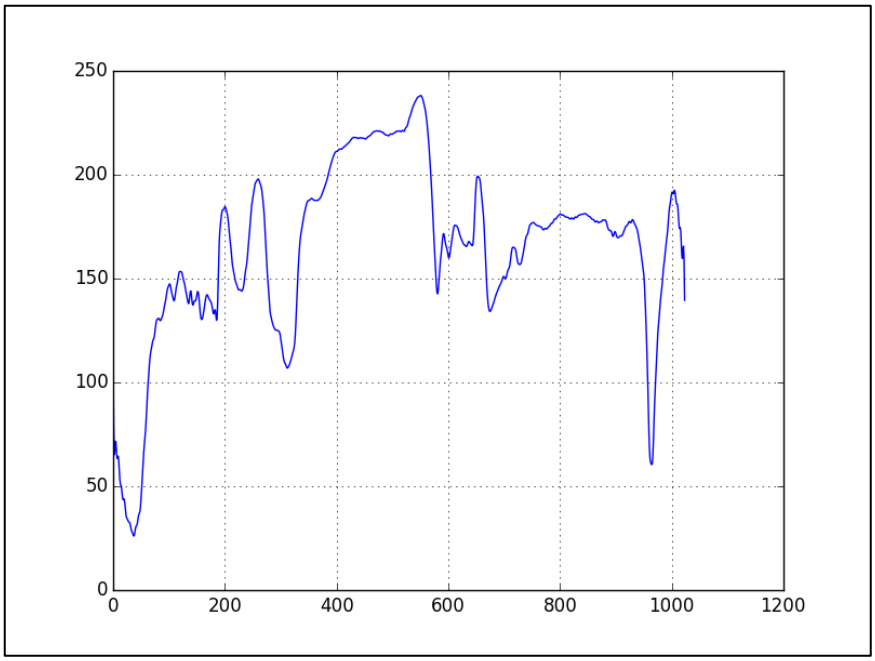


As we can see, this graph matches the origin signal, verifying that in fact, our DFT is correct.



Bonus Question:

Experimenting with the g^ values, I decided to make elements g200 through g1000 zero, keeping the outside values (the ones that are truly reflected on the original graph). After modifying the g^ array to contain zeroes in some values, I found the magnitude of it by taking it's absolute value for each element and graphed it. Below is the result. As we can see, the signal is marginally smoother.



Entire Bash Output:

```
(FourierTransformation)grantmcgovern@gMAC:~/Dropbox/Develop
er/Projects/CSC222/FourierTransformation(FourierTransformat
ion)grantmcgovern@gMAC:~/Dropbox/Developer/Projects/CSC222/
FourierTransformat$ python lab3.py

Roots 2:

 1.00000 + 0.00000i
 -1.00000 + 0.00000i

Roots 4:

 1.00000 + 0.00000i
 0.00000 + 1.00000i
 -1.00000 + 0.00000i
 -0.00000 + -1.00000i

Roots 8:

 1.00000 + 0.00000i
 0.70711 + 0.70711i
 0.00000 + 1.00000i
 -0.70711 + 0.70711i
 -1.00000 + 0.00000i
 -0.70711 + -0.70711i
 -0.00000 + -1.00000i
 0.70711 + -0.70711i

Roots 16:

 1.00000 + 0.00000i
 0.92388 + 0.38268i
 0.70711 + 0.70711i
 0.38268 + 0.92388i
 0.00000 + 1.00000i
 -0.38268 + 0.92388i
 -0.70711 + 0.70711i
 -0.92388 + 0.38268i
 -1.00000 + 0.00000i
 -0.92388 + -0.38268i
 -0.70711 + -0.70711i
 -0.38268 + -0.92388i
 -0.00000 + -1.00000i
 0.38268 + -0.92388i
```

```
 0.70711 + -0.70711i
 0.92388 + -0.38268i

Roots 32:

 1.00000 + 0.00000i
 0.98079 + 0.19509i
 0.92388 + 0.38268i
 0.83147 + 0.55557i
 0.70711 + 0.70711i
 0.55557 + 0.83147i
 0.38268 + 0.92388i
 0.19509 + 0.98079i
 0.00000 + 1.00000i
 -0.19509 + 0.98079i
 -0.38268 + 0.92388i
 -0.55557 + 0.83147i
 -0.70711 + 0.70711i
 -0.83147 + 0.55557i
 -0.92388 + 0.38268i
 -0.98079 + 0.19509i
 -1.00000 + 0.00000i
 -0.98079 + -0.19509i
 -0.92388 + -0.38268i
 -0.83147 + -0.55557i
 -0.70711 + -0.70711i
 -0.55557 + -0.83147i
 -0.38268 + -0.92388i
 -0.19509 + -0.98079i
 -0.00000 + -1.00000i
 0.19509 + -0.98079i
 0.38268 + -0.92388i
 0.55557 + -0.83147i
 0.70711 + -0.70711i
 0.83147 + -0.55557i
 0.92388 + -0.38268i
 0.98079 + -0.19509i


F:2
[[ 1.+0.j  1.+0.j]
 [ 1.+0.j -1.+0.j]]


F:4
[[ 1.+0.j  1.+0.j  1.+0.j  1.+0.j]
 [ 1.+0.j  0.-1.j -1.+0.j  0.+1.j]
 [ 1.+0.j -1.+0.j  1.+0.j -1.+0.j]
```

```
 [ 1.+0.j  0.+1.j -1.+0.j  0.-1.j]]


F:8
[[  1.00000000e+00+0.j           1.00000000e+00+0.j
1.00000000e+00+0.j
    1.00000000e+00+0.j           1.00000000e+00+0.j
1.00000000e+00+0.j
    1.00000000e+00+0.j           1.00000000e+00+0.j
]
 [  1.00000000e+00+0.j           7.07106781e-01-0.70710678j
    0.00000000e+00-1.j          -7.07106781e-01-0.70710678j
   -1.00000000e+00+0.j          -7.07106781e-01+0.70710678j
    0.00000000e+00+1.j           7.07106781e-
01+0.70710678j]
 [  1.00000000e+00+0.j          -8.26946080e-16-1.j
-1.00000000e+00+0.j
    8.26946080e-16+1.j           1.00000000e+00+0.j
-8.26946080e-16-1.j
   -1.00000000e+00+0.j           8.26946080e-16+1.j
]
 [  1.00000000e+00+0.j          -7.07106781e-01-0.70710678j
    0.00000000e+00+1.j           7.07106781e-01-0.70710678j
   -1.00000000e+00+0.j           7.07106781e-01+0.70710678j
    0.00000000e+00-1.j          -7.07106781e-
01+0.70710678j]
 [  1.00000000e+00+0.j          -1.00000000e+00+0.j
1.00000000e+00+0.j
   -1.00000000e+00+0.j           1.00000000e+00+0.j
-1.00000000e+00+0.j
    1.00000000e+00+0.j          -1.00000000e+00+0.j
]
 [  1.00000000e+00+0.j          -7.07106781e-01+0.70710678j
    0.00000000e+00-1.j           7.07106781e-01+0.70710678j
   -1.00000000e+00+0.j           7.07106781e-01-0.70710678j
    0.00000000e+00+1.j          -7.07106781e-01-
0.70710678j]
 [  1.00000000e+00+0.j           8.26946080e-16+1.j
-1.00000000e+00+0.j
   -8.26946080e-16-1.j           1.00000000e+00+0.j
8.26946080e-16+1.j
   -1.00000000e+00+0.j          -8.26946080e-16-1.j
]
 [  1.00000000e+00+0.j           7.07106781e-01+0.70710678j
    0.00000000e+00+1.j          -7.07106781e-01+0.70710678j
   -1.00000000e+00+0.j          -7.07106781e-01-0.70710678j
    0.00000000e+00-1.j           7.07106781e-01-
0.70710678j]]
```

```
F^-1:
[[  1.25000000e-01 +1.36487447e-18j   1.25000000e-01
+6.93889390e-17j
    1.25000000e-01 +2.77555756e-17j   1.25000000e-01
+5.55111512e-17j
    1.25000000e-01 +4.34913699e-17j   1.25000000e-01 -
1.38777878e-16j
    1.25000000e-01 -6.93889390e-17j   1.25000000e-01
+4.16333634e-17j]
 [  1.25000000e-01 -4.16333634e-17j   8.83883476e-02
+8.83883476e-02j
   -6.93889390e-17 +1.25000000e-01j  -8.83883476e-02
+8.83883476e-02j
   -1.25000000e-01 -1.38777878e-17j  -8.83883476e-02 -
8.83883476e-02j
    4.16333634e-17 -1.25000000e-01j   8.83883476e-02 -
8.83883476e-02j]
 [  1.25000000e-01 +9.81307787e-18j  -4.90653893e-18
+1.25000000e-01j
   -1.25000000e-01 -1.96261557e-17j  -7.35980840e-17 -
1.25000000e-01j
    1.25000000e-01 +4.90653893e-18j   4.90653893e-17
+1.25000000e-01j
   -1.25000000e-01 +9.81307787e-18j   4.90653893e-18 -
1.25000000e-01j]
 [  1.25000000e-01 +0.00000000e+00j  -8.83883476e-02
+8.83883476e-02j
    8.83177008e-17 -1.25000000e-01j   8.83883476e-02
+8.83883476e-02j
   -1.25000000e-01 +0.00000000e+00j   8.83883476e-02 -
8.83883476e-02j
   -1.47196168e-16 +1.25000000e-01j  -8.83883476e-02 -
8.83883476e-02j]
 [  1.25000000e-01 +1.96261557e-17j  -1.25000000e-01
+1.96261557e-17j
    1.25000000e-01 +3.92523115e-17j  -1.25000000e-01 -
9.81307787e-18j
    1.25000000e-01 -9.81307787e-18j  -1.25000000e-01 -
2.94392336e-17j
    1.25000000e-01 +1.96261557e-17j  -1.25000000e-01 -
0.00000000e+00j]
 [  1.25000000e-01 +1.67519718e-17j  -8.83883476e-02 -
8.83883476e-02j
   -1.25639788e-16 +1.25000000e-01j   8.83883476e-02 -
8.83883476e-02j
   -1.25000000e-01 -5.02559153e-17j   8.83883476e-02
```

```
+8.83883476e-02j
    1.08887817e-16 -1.25000000e-01j  -8.83883476e-02
+8.83883476e-02j]
 [  1.25000000e-01 -5.92271642e-18j  -2.36908657e-17 -
1.25000000e-01j
   -1.25000000e-01 +1.77681493e-17j   2.96135821e-17
+1.25000000e-01j
    1.25000000e-01 +5.92271642e-18j  -5.92271642e-18 -
1.25000000e-01j
   -1.25000000e-01 -1.18454328e-17j  -1.77681493e-17
+1.25000000e-01j]
 [  1.25000000e-01 +0.00000000e+00j   8.83883476e-02 -
8.83883476e-02j
    8.83177008e-17 -1.25000000e-01j  -8.83883476e-02 -
8.83883476e-02j
   -1.25000000e-01 +1.96261557e-17j  -8.83883476e-02
+8.83883476e-02j
   -1.07943857e-16 +1.25000000e-01j   8.83883476e-02
+8.83883476e-02j]]


Identity Matrix Check
[[  1.00000000e+00 +5.39260384e-33j  -1.38777878e-17
+2.77555756e-17j
    1.89287618e-17 +0.00000000e+00j   0.00000000e+00
+0.00000000e+00j
    0.00000000e+00 +0.00000000e+00j   1.38777878e-17
+0.00000000e+00j
    3.08340215e-17 +0.00000000e+00j   1.38777878e-17
+1.38777878e-17j]
 [  4.38400669e-17 -4.56681599e-17j   1.00000000e+00
+3.46944695e-17j
    1.25129133e-17 +6.93889390e-18j   1.17961196e-16
+2.63677968e-16j
    8.02181944e-17 +1.76172183e-16j  -2.08166817e-17 -
2.56739074e-16j
    5.48436707e-17 -2.08166817e-17j   2.22044605e-16
+9.02056208e-17j]
 [ -3.25354341e-17 +4.48562444e-17j   1.00486722e-16
+1.07318844e-16j
    1.00000000e+00 +2.70569826e-17j   1.15635167e-16
+1.48952207e-16j
    3.49164861e-17 -4.90653893e-18j  -1.53619885e-17 -
1.21196632e-16j
    1.11022302e-16 -3.99185929e-17j   1.08441793e-16
+7.30923976e-17j]
 [ -8.47760363e-17 +9.14559737e-18j   2.77555756e-17
+1.04083409e-16j
```

```
    1.36487447e-18 -4.85722573e-17j   1.00000000e+00 -
2.08166817e-17j
    5.44949737e-17 +2.75812271e-17j   1.66533454e-16 -
2.08166817e-17j
    4.23008439e-17 -2.08166817e-17j   4.16333634e-17
+4.16333634e-17j]
 [ -4.16333634e-17 +4.97627833e-17j   2.77555756e-17
+1.38777878e-16j
    5.04601773e-17 +1.38777878e-16j   0.00000000e+00 -
2.77555756e-17j
    1.00000000e+00 +8.90150948e-17j   2.77555756e-17 -
1.11022302e-16j
    2.18966159e-16 -1.11022302e-16j   1.24900090e-16
+2.77555756e-17j]
 [  2.92648851e-17 +3.69011730e-17j   1.73472348e-16
+3.46944695e-17j
   -2.63907011e-17 -6.93889390e-18j   0.00000000e+00
+5.55111512e-17j
    2.87717531e-17 -1.74348493e-19j   1.00000000e+00 -
8.32667268e-17j
    5.61786317e-17 -1.45716772e-16j   9.71445147e-17 -
3.46944695e-17j]
 [  3.25354341e-17 -1.06549069e-17j   5.79173120e-17
+8.69701854e-17j
    1.11022302e-16 +1.10674805e-16j   1.16571213e-16
+7.30923976e-17j
    1.87128119e-16 +5.06046123e-17j  -1.11570457e-16 -
4.61670456e-16j
    1.00000000e+00 -5.55422637e-17j   1.07505747e-16
+7.95632683e-17j]
 [  8.10600234e-17 -7.34237355e-17j   0.00000000e+00
+2.08166817e-17j
    1.09657428e-16 +4.85722573e-17j   0.00000000e+00 -
4.85722573e-17j
    5.85596837e-17 +9.63872937e-18j  -2.01227923e-16 -
7.63278329e-17j
   -2.84230561e-17 -2.01227923e-16j   1.00000000e+00
+8.32667268e-17j]]

[  1.67841000e+05   +0.j           -
1.96611299e+04+3761.75850266j
  -6.27441742e+01+8612.04949857j ...,  -1.26833947e+04
+156.98038805j
  -6.27441742e+01-8612.04949857j  -1.96611299e+04-
3761.75850266j]

Displaying Frequency Graph...
```

Result:

Matrices Equivalent: True