CV exercise

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a) $ax + by + c = 0$

$x^T = (x \ y \ 1) \qquad I = (a \ b \ c)^T$

Inner product : $x^T I = (x \ y \ 1)\begin{pmatrix} a \\ b \\ c \end{pmatrix} = ax + by + c = 0$

$\therefore x^T I = 0$

b) $I = (a \ b \ c)^T \qquad I' = (a' \ b' \ c')^T$

The intersection point x are on both lines

$\Rightarrow x^T I = 0$ and $x^T I' = 0$

$x = I \times I'$ is orthagonal to both I & I'

$\Rightarrow x$ is on both lines

$\therefore x = I \times I'$

c) Let $x = \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$ and $x' = \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}$

they represent the line passing through both points

$I = x \times x'$

$I = (a, b, c)$, So Any point P which is on the line

can Satisfy $P^T I = 0$. $\Rightarrow$ Homogeneous Coordinates

$\therefore I = x \times x'$

d) $y = \alpha x + (1-\alpha) x'$

Let's say y is on the line through x and x'

$\Rightarrow y^T I = 0$

$= (\alpha x + (1-\alpha) x')^T I = \alpha x^T I + (1-\alpha) x'^T I$

from c) $x^T I = 0$ and $x'^T I = 0$

$\therefore \alpha x^T I + (1-\alpha) x'^T I = 0$

$\Rightarrow y$ is on the line through x and x'

②

a) • Translation

$t_x \to x$, $t_y \to y$

$$T = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

• Euclidean Transformation (Rotation + Translation)

θ $\quad$ t_x, t_y

$$E = \begin{pmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

• Similarity Transformation (Scaling + Rotation + Translation)

s $\quad$ θ , t_x, t_y

$$S = \begin{pmatrix} s\cos\theta & -s\sin\theta & t_x \\ s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

• Affine Transformation (linear transformation + translation)

$a_{11}, a_{12}, a_{21}, a_{22}$ $\quad$ t_x, t_y

$$A = \begin{pmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{pmatrix}$$

• Projective Transformation

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}$$

b) (DoF of Translation) = 2 : t_x, t_y

(DoF of Euclidean-) = 3 : t_x, t_y, θ

(DoF of similarity -) = 4 : t_x, t_y, θ, s

(DoF of Affine -) = 6 : $a_{11}, a_{12}, a_{21}, a_{22}, t_x, t_y$

(DoF of projective -) = 8 $p_{11}, p_{12}, p_{13}, p_{21} \cdots , p_{32}$

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c) A projective transformation is represented by 3x3 matrix with 9 elements, but the num of DoF is only 8. It's because of Homogeneous Coordinates. It means only ratios of the matrix elements matter. One DoF is fixed due to the scaling factor.

③

a) $x' = Hx$

$I = (a\ b\ c)^T,\ I^T x = 0$

To determine I', check if x is on the line I, then x' should be on the line I'.

$I'^T x' = 0, \quad x' = Hx$

$\Rightarrow I'^T H x = 0$

$\Rightarrow (I'^T H) x = 0$

$\Rightarrow I'^T H = I^T$

$\Rightarrow I' = (H^{-T}) I$

H^{-T} means the inverse of H.

b) $I = \dfrac{(I_1^T x_1)(I_2^T x_2)}{(I_1^T x_2)(I_2^T x_1)}$

- Transformation of lines and points

$I_1' = H^{-T} I_1, \quad I_2' = H^{-T} I_2$

$x_1' = H x_1, \quad x_2' = H x_2$

- Invariant Calculation

$I' = \dfrac{(I_1'^T x_1')(I_2'^T x_2')}{(I_1'^T x_2')(I_2'^T x_1')}$

$I_1' = H^{-T} I_1, \quad I_2' = H^{-T} I_2, \quad x_1' = H x_1, \quad x_2' = H x_2$

$$l_1'^T x_1' = (H^{-T} l_1)^T (H x_1) = x_1^T H^{-1} H^{-T} l_1$$
$$= x_1^T (H^{-T})^T H^{-1} l_1 = l_1^T x_1$$
$$l_2'^T x_2' = (H^{-T} l_2)^T (H x_2) = l_2^T x_2$$
$$l_1'^T x_2' = (H^{-T} l_1)^T (H x_2) = l_1^T x_2$$
$$l_2'^T x_1' = (H^{-T} l_2)^T (H x_1) = l_2^T x_1$$
$$I' = \frac{(l_1^T x_1)(l_2^T x_2)}{(l_1^T x_2)(l_2^T x_1)} = I$$

$\therefore$ I is invariant.

- why fewer points or lines don't work?

we need to compare several elements for meaningful invariant. Using only one pair of points or lines doesn't provide enough information to ensure invariance. It cannot be easily normalized.