

$$\Delta p = H^{-1} \sum_x \left[ \nabla I \frac{\partial w}{\partial p} \right]^T [T(x) - I(W(x; p))]$$

, where  $H$  is Hessian Matrix,  $\nabla I$  is image ingredient.

In simple translation (where  $w(x; p) = x + p$ ), this  $\frac{\partial w}{\partial p}$  becomes a constant, simplifying the iterative update process.

The Lucas-Kanade algorithm is a linear least-squares problem

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

In both cases, the summation and matrix setup capture the intensity gradients and displacements, allowing us to solve for  $u$  and  $v$  (the translation parameters). Thus, both solutions are equivalent in the case of translation.

$$\Delta p = \begin{bmatrix} u \\ v \end{bmatrix}, \quad \frac{\partial w}{\partial p} = \begin{bmatrix} \frac{\partial w_x}{\partial u} & \frac{\partial w_x}{\partial v} \\ \frac{\partial w_y}{\partial u} & \frac{\partial w_y}{\partial v} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{aligned} H &= \sum_x \left[ \nabla I \frac{\partial w}{\partial p} \right]^T \left[ \nabla I \frac{\partial w}{\partial p} \right] = \sum_x \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_y I_x & \sum I_y I_y \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_y I_x & \sum I_y^2 \end{bmatrix}^{-1} \sum_x \begin{bmatrix} I_x \\ I_y \end{bmatrix} [T(x) - I(W(x; p))]$$

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_y I_x & \sum I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$