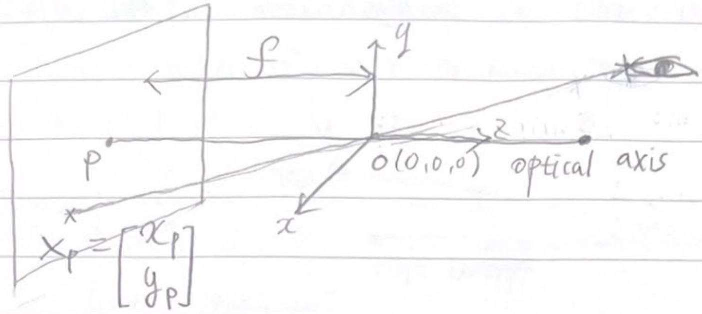
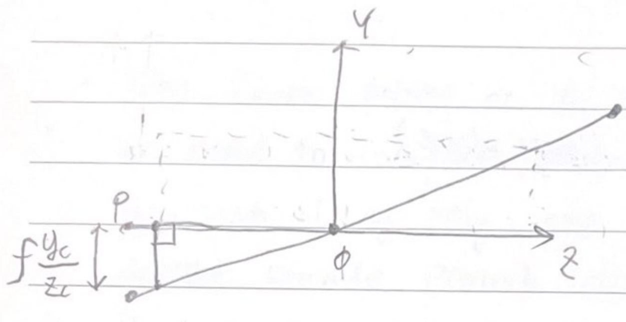
CV #2

1 Pinhole camera (3D point in the world is projected onto 2D image plane)



$$X_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

$$X_p = \begin{bmatrix} x_p \\ y_p \end{bmatrix}$$



$Z \propto f$

The Coordinates of the intersection on the image plane

$$= \begin{bmatrix} f\frac{x_c}{z_c} \\ f\frac{y_c}{z_c} \\ f \end{bmatrix} \xrightarrow{\text{ignore } f} \begin{bmatrix} f\frac{x_c}{z_c} \\ f\frac{y_c}{z_c} \end{bmatrix} : 2D$$

Or Using Homogeneous Coordinates

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} \longrightarrow \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}, \quad \begin{bmatrix} x_p \\ y_p \end{bmatrix} \longrightarrow \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix}$$

$x = P \cdot X$, $P = diag(f, f, 1) \cdot [I \mid 0]$

$$P = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \Rightarrow x = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} fX_c \\ fy_c \\ z_c \end{bmatrix}$$

$$\text{i.e. } \begin{bmatrix} f\frac{x_c}{z_c} \\ f\frac{y_c}{z_c} \end{bmatrix}$$

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2. Image Coordinates = $\begin{bmatrix} x_p \\ y_p \end{bmatrix}$

Pixel Coordinates = $\begin{bmatrix} u \\ v \end{bmatrix}$

a) u-axis // x-axis && v-axis // y-axis

= (They are alligned)

$u = u_0 + m_u \cdot x_p$

$v = v_0 + m_v \cdot y_p$

b) u-axis // x-axis , The angle between u-axis and v-axis is θ

= (v-axis is rotated by θ relative to y-axis

$u = u_0 + m_u \cdot x_p$

$v = v_0 + m_v \cdot (x_p \sin\theta + y_p \cos\theta)$

3. $\tilde{p} = K \cdot X_c$, where $\tilde{p} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$, $X_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$

$$x_p = f\frac{x_c}{z_c}, \quad y_p = f\frac{y_c}{z_c}$$

$$u = u_0 + m_u \cdot x_p = u_0 + m_u \cdot f\frac{x_c}{z_c}$$
$$v = v_0 + m_v \cdot y_p = v_0 + m_v \cdot f\frac{y_c}{z_c}$$

$$K = \begin{bmatrix} m_u \cdot f & 0 & u_0 \\ 0 & m_v \cdot f & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\tilde{p} = K \cdot \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} \Rightarrow \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} m_u \cdot f & 0 & u_0 \\ 0 & m_v \cdot f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

4.

The world coordinates $X_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$, $X_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$

$$X_c = R \cdot X_w + t$$

In Homogeneous Coordinates,

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

Camera Coordinates to pixel Coordinates

$\Rightarrow \tilde{p} = K \cdot \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$ Substitute $X_c = R \cdot X_w + t$

$$\Rightarrow \tilde{p} = K \cdot (R \cdot X_w + t)$$

R and t are part of 3x4 projection matrix.

$\therefore P = K \cdot [R \ t]$

5.

a) Decompose $\vec{x}$ relative to axis u

$\rightarrow$ along u : $(u \cdot \vec{x})u$

$\rightarrow$ perpendicular to u : $\vec{x} - (u \cdot \vec{x})u$

The component of x perpendicular to u will rotate in the plane perpendicular to u. and its new direction can be obtained using the cross product.

: $\sin\theta(u \times x)$

$\therefore Rx = \cos\theta \cdot x + \sin\theta \cdot (u \cdot x) + (1-\cos\theta)(u \cdot x) \cdot u$

(b) $u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

* $u \times x = \begin{bmatrix} u_2x_3 - u_3x_2 \\ u_3x_1 - u_1x_3 \\ u_1x_2 - u_2x_1 \end{bmatrix}$: Cross product

* $u \cdot x = u_1x_1 + u_2x_2 + u_3 \cdot x_3$: dot product

* $R = I + \sin\theta \cdot K + (1-\cos\theta) \cdot K^2$

$$K = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix}$$

$$K^2 = \begin{bmatrix} -u_2^2 - u_3^2 & u_1u_2 & u_1u_3 \\ u_1u_2 & -u_1^2 - u_3^2 & u_2u_3 \\ u_1u_3 & u_2u_3 & -u_1^2 - u_2^2 \end{bmatrix}$$

$$\therefore R = \begin{bmatrix} \cos\theta + (1-\cos\theta)u_1^2 & (1-\cos\theta)u_1u_2 - \sin\theta u_3 & (1-\cos\theta)u_1u_3 + \sin\theta u_2 \\ (1-\cos\theta)u_1u_2 + \sin\theta u_3 & \cos\theta + (1-\cos\theta)u_2^2 & (1-\cos\theta)u_2u_3 + \sin\theta u_1 \\ (1-\cos\theta)u_1u_3 - \sin\theta u_2 & (1-\cos\theta)u_2u_3 + \sin\theta u_1 & \cos\theta + (1-\cos\theta)u_3^2 \end{bmatrix}$$