Exercise 02.

$$\overrightarrow{O'p} \cdot (\overrightarrow{OO'} \times \overrightarrow{O'p'}) = 0$$

where, $\overrightarrow{O'p}$: the vector from the 2nd camera center O' to the point p,
$\overrightarrow{O'p'}$: " p',
$\overrightarrow{OO'} = t$: the vector from the 1st camera center O to 2nd camera center O'.

This equation states that the three vectors are coplanar.

$x'^T E x = 0$, where x and x' are homogeneous coordinates of the p and p'.
↳ Essential matrix

3D point in 1st camera's coordinate system : $x = (x, y, 1)^T$
3D point in 2nd camera's coordinate system : $x' = (x', y', 1)^T$
The projection matrices for two cameras:

$P = [I|0]$, $P' = [R|t]$, where R is rotation matrix, t is translation vector

$\overrightarrow{OO'} \times \overrightarrow{O'p'} = [t]_\times R x$, where $t = [t_1, t_2, t_3]^T$
↳ skew-symmetric matrix

$$[t]_\times = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}$$

using this, Essential matrix is defined :

$$E = [t]_\times R$$

Substitute E into the coplanarity condition :

$$x'^T E x = x'^T ([t]_\times R) x$$

∴ E encapsulate the relationship between the two cameras and selfies the epipolar constraint: $x'^T E x = 0$.
This constraint ensures that the projections x and x' of a 3D point X lie on their respective epipolar lines.

Exercise 03.

a) $d = 1cm$, $b = 6cm$, $f = 1cm$.

$$\text{depth } z_p = \frac{b \cdot f}{d} \xrightarrow[\text{to values}]{\text{Substitute}} \frac{6 \cdot 1}{1} = 6cm$$

$\therefore$ the depth point P is $z_p = 6cm$

b) $d_{min} = 1$ pixel
$w = 0.01$ mm/pixel
$b = 6cm$
$f = 1cm$

The real disparity for 1 pixel : $d_{min} = 1\text{pixel} \times 0.01\text{mm/pixel}$
$= 0.01\text{mm}$
$= 0.001\text{cm}$

$$z = \frac{b \cdot f}{d} \longrightarrow z_{max} = \frac{b \cdot f}{d_{min}}$$

$$= \frac{6 \cdot 1}{0.001} = 6000cm.$$

$\therefore$ The depth is $z > 6000cm$ for disparities below 1 pixel.

c) left camera matrix : $P_l = [I|0]$
right camera matrix : $P_r = [I|t]$, $t = (-6, 0, 0)^T$
$Q = (3, 0, 3)$

The homogeneous coordinates of Q in the left camera :

$$x_l = P_l \cdot Q = [I|0] \cdot \begin{bmatrix} 3 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} \xrightarrow[\text{non-homogeneous}]{\text{convert to}} x_l = \left(\frac{3}{3}, \frac{0}{3}\right) = (1, 0)$$

projection of Q in the left camera is $(1, 0)$

$$E = [t]_\times R \text{ , where } t = (-6, 0, 0)^T \Rightarrow [t]_\times = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & -6 & 0 \end{bmatrix}$$

Since $R = I$: $E = [t]_\times$

$$l_r = E \cdot X_l = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & -6 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 18 \\ 0 \end{bmatrix}$$

exercise12

November 26, 2024

[1]:
```
# This cell is used for creating a button that hides/unhides code cells to␣
 ↪quickly look only the results.
# Works only with Jupyter Notebooks.

from IPython.display import HTML

HTML('''<script>
code_show=true;
function code_toggle() {
if (code_show){
$('div.input').hide();
} else {
$('div.input').show();
}
code_show = !code_show
}
$( document ).ready(code_toggle);
</script>
<form action="javascript:code_toggle()"><input type="submit" value="Click here␣
 ↪to toggle on/off the raw code."></form>''')
```

[1]: <IPython.core.display.HTML object>

[2]:
```
# Description:
#   Exercise12 notebook.
#
# Copyright (C) 2018 Santiago Cortes, Juha Ylioinas
#
# This software is distributed under the GNU General Public
# Licence (version 2 or later); please refer to the file
# Licence.txt, included with the software, for details.

# Preparations
import os
import numpy as np
import matplotlib.pyplot as plt
import cv2
```

```python
# Select data directory
if os.path.isdir('/coursedata'):
    # JupyterHub
    course_data_dir = '/coursedata'
elif os.path.isdir('../../../coursedata'):
    # Local installation
    course_data_dir = '../../../coursedata'
else:
    # Docker
    course_data_dir = '/home/jovyan/work/coursedata/'

print('The data directory is %s' % course_data_dir)
data_dir = os.path.join(course_data_dir, 'exercise-12-data')
print('Data stored in %s' % data_dir)
```

```
The data directory is /coursedata
Data stored in /coursedata/exercise-12-data
```

Fill your name and student number below.

0.0.1 Name: Jeewon Han

0.0.2 Student number: 102562190

1 CS-E4850 Computer Vision Exercise Round 12

The problems should be solved before the exercise session and solutions returned via MyCourses. Upload to MyCourses both: this Jupyter Notebook (.ipynb) file containing your solutions to the programming tasks and the exported pdf version of this Notebook file. If there are both programming and pen & paper tasks kindly combine the two pdf files (your scanned/LaTeX solutions and the exported Notebook) into a single pdf and submit that with the Notebook (.ipynb) file. Note that (1) you are not supposed to change anything in the utils.py and (2) you should be sure that everything that you need to implement should work with the pictures specified by the assignments of this exercise round.

1.0.1 Make sure to complete the pen and paper exercices in the PDF attached.

1.1 Fundamental matrix estimation.

a) Implement the eight-point algorithm as explained on slide 28 of Lecture 11. Note the skeleton function and follow the input output structure

b) Implement the normalized eight-point algorithm as explained on slide 31 of Lecture 11 (Algorithm 11.1. in Hartley & Zisserman).

The epipolar lines obtained with both F-matrix estimates should be close to those visualized by the example script.

[9]:
```
def estimateF(x1,x2):
    # Return the fundamental matrix F (3 by 3), based on two sets of␣
 ↪homogeneous 2D points x1 and x2.
    # Input: x1,x2 numpy ndarray (3 by N) containing matching 2D homogeneous␣
 ↪points.
    # Output: F numpy ndarray (3 by 3) containing the fundamental matrix.
    # Number of points
    N = x1.shape[1]

    # Construct matrix A for the linear system
    A = np.zeros((N, 9))
    for i in range(N):
        A[i] = [
            x2[0, i] * x1[0, i], x2[0, i] * x1[1, i], x2[0, i],
            x2[1, i] * x1[0, i], x2[1, i] * x1[1, i], x2[1, i],
            x1[0, i], x1[1, i], 1
        ]

    # Solve the homogeneous linear system using SVD
    _, _, Vt = np.linalg.svd(A)
    F = Vt[-1].reshape(3, 3)

    # Enforce rank-2 constraint on F
    U, S, Vt = np.linalg.svd(F)
    S[2] = 0  # Set the smallest singular value to zero
    F = U @ np.diag(S) @ Vt
    return F

def estimateFnorm(x1,x2):
    # Return the fundamental matrix F (3 by 3), based on two sets of␣
 ↪homogeneous 2D points x1 and x2.
    # Input: x1,x2 numpy ndarray (3 by N) containing matching 2D homogeneous␣
 ↪points.
    # Output: F numpy ndarray (3 by 3) containing the fundamental matrix based␣
 ↪on normalized homogeneous points.
    F=np.eye(3)

    def normalize_points(points):
        """Normalize the points so that the mean distance to the origin is␣
 ↪sqrt(2)."""
        mean = np.mean(points[:2], axis=1)
        std = np.std(points[:2])
        T = np.array([
            [1/std, 0, -mean[0]/std],
            [0, 1/std, -mean[1]/std],
            [0, 0, 1]
        ])
```

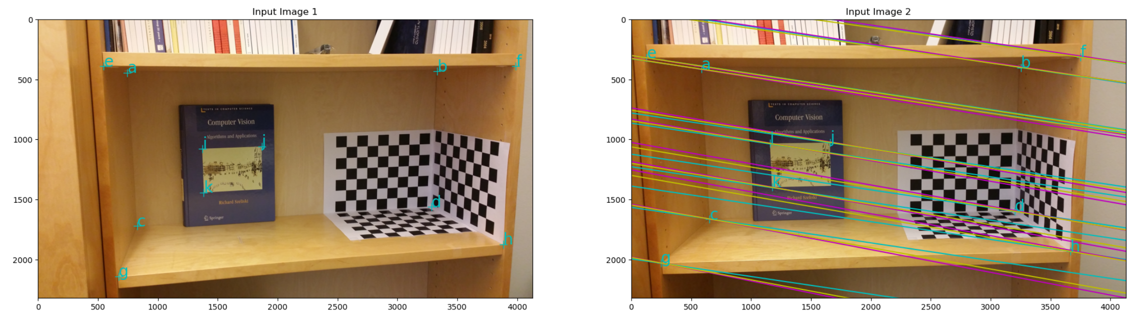
```
        normalized_points = T @ points
        return normalized_points, T

    # Normalize the points
    x1_norm, T1 = normalize_points(x1)
    x2_norm, T2 = normalize_points(x2)

    # Compute F using the normalized points
    F_norm = estimateF(x1_norm, x2_norm)

    # Denormalize the fundamental matrix
    F = T2.T @ F_norm @ T1  # Overwrite F with the computed value

    return F

def vgg_F_from_P(P1,P2):
    # Return the fundamental matrix F (3 by 3), based on two camera parameter␣
 ↪arrays.
    # Input: P1, P2 numpy ndarray (3 by 4) containing intrinsic and extrinsic␣
 ↪parameters.
    # Output: F numpy ndarray (3 by 3) containing the fundamental matrix.
    X=[]
    Y=[]
    X.append(P1[[1,2],:])
    X.append(P1[[2,0],:])
    X.append(P1[[0,1],:])
    Y.append(P2[[1,2],:])
    Y.append(P2[[2,0],:])
    Y.append(P2[[0,1],:])
    F=np.zeros([3,3])

    for i in range(3):
        for j in range(3):
            M=np.concatenate([X[j],Y[i]])
            F[i,j]=np.linalg.det(M)
    return F
```

[14]:

```
# Point locations
x1 = 1.0e+03*np.array([0.7435,3.3315,0.8275,3.2835,0.5475,3.9875,0.6715,3.
 ↪8835,1.3715,1.8675,1.3835])
y1 = 1.0e+03*np.array([0.4455,0.4335,1.7215,1.5615,0.3895,0.3895,2.1415,1.
 ↪8735,1.0775,1.0575,1.4415])
x2 = 1.0e+03*np.array([0.5835,3.2515,0.6515,3.1995,0.1275,3.7475,0.2475,3.
 ↪6635,1.1555,1.6595,1.1755])
y2 = 1.0e+03*np.array([0.4135,0.4015,1.6655,1.5975,0.3215,0.3135,2.0295,1.
 ↪9335,1.0335,1.0255,1.3975])
```

```
# Camera parameters
P1= np.row_stack([[-0.001162918366053,0.000102986385133,-0.000344703214391,0.995200644722518],\
                  [-0.000019974831639,0.001106889654747,-0.000150591916681,0.097841118173777],\
                  [-0.000000053632777,0.000000044849673,-0.000000270734766,0.000249501614496]])

P2= np.row_stack([[-0.001272880601540, 0.000093061493378,-0.000574486218854, 0.996457618133488],\
                  [-0.000002971652037, 0.001271207503106,-0.000200323351541, 0.084074548573989],\
                  [-0.000000020226464, 0.000000043518811,-0.000000316928290, 0.000265554210072]])

# Make homogenous representations of points
pts1=np.row_stack([x1,y1,np.ones_like(x1)])
pts2=np.row_stack([x2,y2,np.ones_like(x2)])

# Read images
# Read images
im1 = cv2.imread(data_dir+'/im1.jpg')
im2 = cv2.imread(data_dir+'/im2.jpg')

im1 = cv2.cvtColor(im1, cv2.COLOR_BGR2RGB)
im2 = cv2.cvtColor(im2, cv2.COLOR_BGR2RGB)


# Labels
labels = ['a','b','c','d','e','f','g','h','i','j','k']

# Create figure
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(25,25))
ax = axes.ravel()
ax[0].imshow(im1)
ax[0].plot(x1, y1, 'c+', markersize=10)

# Put labels
for i in range(len(x1)):
    ax[0].annotate(labels[i], (x1[i], y1[i]), color='c', fontsize=20)
ax[0].set_title("Input Image 1")
ax[1].imshow(im2)
ax[1].plot(x2, y2, 'c+', markersize=10)
for i in range(len(x2)):
    ax[1].annotate(labels[i], (x2[i], y2[i]), color='c', fontsize=20)
```

```python
ax[1].set_title("Input Image 2")

# Get ground truth fundamental matrix
F=vgg_F_from_P(P1,P2)

# Create lines
#eplinesA=F@pts1
#eplinesB=F@pts2
eplinesA=np.dot(F,pts1)
eplinesB=np.dot(F,pts2)

#  Plot lines
px=np.array([0,np.shape(im2)[1]])
for i in range(np.shape(pts1)[1]):
    py=(-eplinesA[0,i]*px-eplinesA[2,i])/eplinesA[1,i]
    ax[1].plot(px,py,'c-');

# Get fundamental matrix and draw epipolar lines
F=estimateF(pts1,pts2)
#eplinesA=F@pts1
#eplinesB=F@pts2
eplinesA=np.dot(F,pts1)
eplinesB=np.dot(F,pts2)
for i in range(np.shape(pts1)[1]):
    py=(-eplinesA[0,i]*px-eplinesA[2,i])/eplinesA[1,i]
    ax[1].plot(px,py,'m-');

# Get fundamental matrix from normalized algorithm and draw epipolar lines
F=estimateFnorm(pts1,pts2)
#eplinesA=F@pts1
#eplinesB=F@pts2
eplinesA=np.dot(F,pts1)
eplinesB=np.dot(F,pts2)
for i in range(np.shape(pts1)[1]):
    py=(-eplinesA[0,i]*px-eplinesA[2,i])/eplinesA[1,i]
    ax[1].plot(px,py,'y-');

ax[1].axes.set_xlim([0,np.shape(im2)[1]])
ax[1].axes.set_ylim([np.shape(im2)[0],0])
plt.show()
```



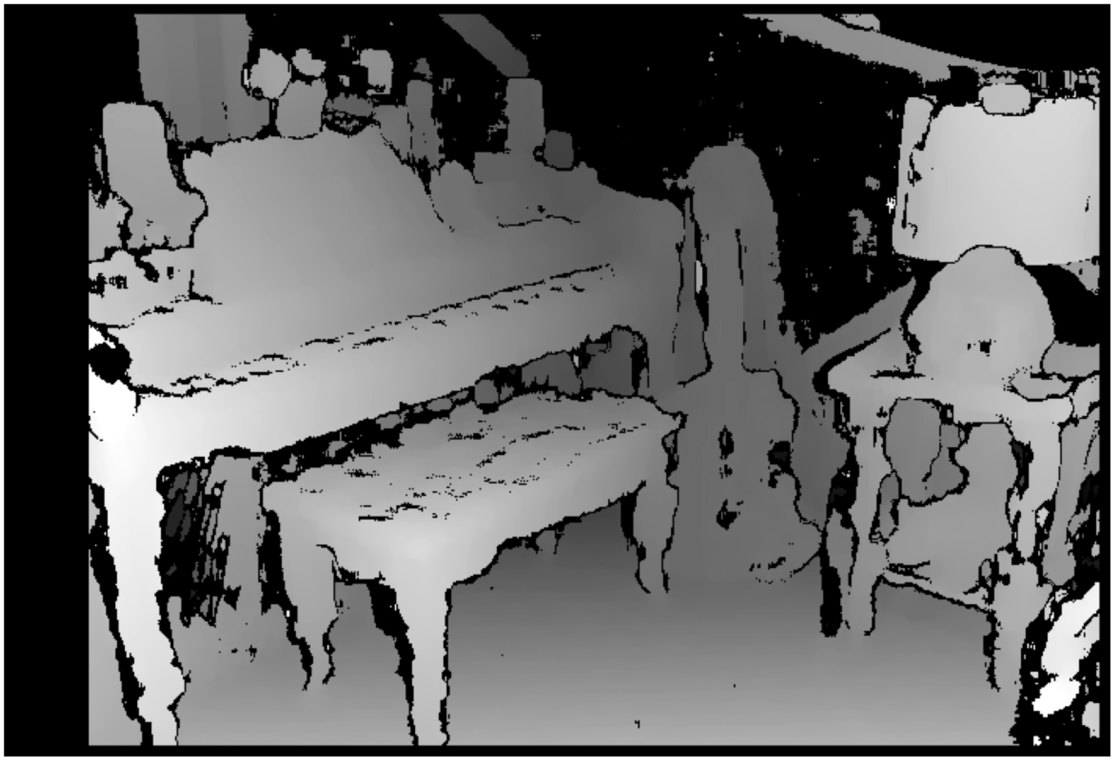
1.2 Demo. Stereo disparity computation. (Just a demo, no points given)

Run and study the opencv stereo disparity and depth estimation.

[11]:
```
# Import images
sc=0.25

imgL = cv2.resize(cv2.imread(data_dir+'/im0.png',0), (0,0), fx=sc, fy=sc)
imgR = cv2.resize(cv2.imread(data_dir+'/im1.png',0), (0,0), fx=sc, fy=sc)
imgL_col = cv2.resize(cv2.imread(data_dir+'/im0.png'), (0,0), fx=sc, fy=sc)
imgR_col = cv2.resize(cv2.imread(data_dir+'/im1.png'), (0,0), fx=sc, fy=sc)

# Show images
plt.figure(figsize=[15,15])
plt.subplot(121)
plt.imshow(imgL_col[:,:,[2,1,0]])
plt.axis('off')
plt.subplot(122)
plt.imshow(imgR_col[:,:,[2,1,0]])
plt.axis('off')

# Compute disparity
stereo = cv2.StereoBM_create(numDisparities=16*3, blockSize=15)
disparity = stereo.compute(imgL,imgR)

# Show disparity
plt.figure(figsize=[15,15])
plt.imshow(disparity,'gray')
plt.axis('off')
plt.title('Disparity')
#ndistp=cv2.guidedFilter(imgL, disparity, 9, 4,0.1)

# Calibration data
baseline=17.8089 #cm
f_length=2826.171*sc #pixels
```

```python
c_point=np.array([1415.97,965.806])*sc # pixels

# Get depth from disparity
point=np.zeros([np.count_nonzero(disparity>1),6])
ind=0
for i in range(np.shape(disparity)[0]):
    for j in range(np.shape(disparity)[1]):
        if disparity[i,j]>1:
            # Save point information into point cloud
            # [pixel_x,pixel_y,disparity,color]
            point[ind,0:3]=j,i,disparity[i,j]
            point[ind,3:6]=imgL_col[i,j]/255.0
            ind+=1
# Z=baseline*focal/disparity
# openCV disparity is (16*actual_disparity). This depends on the algorithm.
# It is in order to use signed shorts and keep good subpixel accuracy.
point[:,2]=baseline*f_length/(point[:,2]/16.0)
#X=Z*(pixel_u-center_u)/focal
point[:,0]=point[:,2]*(point[:,0]-c_point[0])/f_length
#Y=Z*(pixel_v-center_v)/focal
point[:,1]=-point[:,2]*(point[:,1]-c_point[1])/f_length

# Delete points on the far background
inl=(point[:,2]<2000)
point=point[inl,:]

plt.show()
```

Disparity



[12]:
```
def visualize_points(pts,R,img,f=1000,cp=[400,300]):
    #visualize colored points given a rotation matrix
    # rotate around the mean of the point cloud
    c=np.mean(point[:,0:3],0)
    #r_point=((point[:,0:3]-c)@R_y)+c
    r_point=np.dot((point[:,0:3]-c),R_y)+c

    #Project back to the same camera model
    K=np.float32([[f,0,cp[0]],[0,f,cp[1]],[0,0,1]])

    # Sort by depth (painter's algorithm)
    ind=np.argsort(r_point[:,2])
    r_point=r_point[np.flip(ind,0),:]

    #Project
    #uvk=K@r_point.T
    uvk=(np.dot(K,r_point.T))
    color=point[:,[5,4,3]]
    color=color[np.flip(ind,0),:]

    # Normalize homogeneous  coordinates
    uv=uvk[0:2,:]/(uvk[2,:])
```

```python
    # Draw projected points
    plt.scatter(uv[0,:],uv[1,:],marker='.',s=10,c=color)
    plt.xlim([0,np.shape(imgL)[1]])
    plt.ylim([0,np.shape(imgL)[0]])
    plt.axis('off')



# Visualize points from two different angles
plt.figure(figsize=[30,15])
plt.subplot(121)
# Rotate around y axis to visualize
ang_y=-20.0
ang_y=ang_y/180.0*3.14
R_y=np.float32([[np.cos(ang_y),0,np.sin(ang_y)],[0,1,0],[-np.sin(ang_y),0,np.
    ↪cos(ang_y)]])
visualize_points(point,R_y,imgL,f_length,c_point)
plt.subplot(122)
# Rotate around y axis to visualize
ang_y=20.0
ang_y=ang_y/180.0*3.14
R_y=np.float32([[np.cos(ang_y),0,np.sin(ang_y)],[0,1,0],[-np.sin(ang_y),0,np.
    ↪cos(ang_y)]])
visualize_points(point,R_y,imgL,f_length,c_point)
plt.show()
```