Exercise 02.

$$\vec{O'p} \cdot (\vec{OO'} \times \vec{O'p'}) = 0$$

where, $\vec{O'p}$: the vector from the 2nd camera center O' to the point p,
$\vec{O'p'}$: " p',
$\vec{OO'} = t$: the vector from the 1st camera center O to 2nd camera center O'.

This equation states that the three vectors are coplanar.

$$x'^T E x = 0,$$ where x and x' are homogeneous coordinates of the p and p'.

↳ Essential Matrix

3D point in 1st camera's coordinate system : $x = (x, y, 1)^T$
3D point in 2nd camera's coordinate system : $x' = (x', y', 1)^T$
The projection matrices for two cameras:

$P = [I|0]$, $P' = [R|t]$, where R is rotation matrix, t is translation vector

$$\vec{OO'} \times \vec{O'p'} = [t]_\times R x, \quad \text{where } t = [t_1, t_2, t_3]^T$$

↳ skew-symmetric matrix

$$[t]_\times = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}$$

using this, Essential matrix is defined :

$$E = [t]_\times R$$

Substitute E into the coplanarity condition :

$$x'^T E x = x'^T ([t]_\times R) x$$

∴ E encapsulate the relationship between the two cameras and selfies the epipolar constraint: $x'^T E x = 0$.
This constraint ensures that the projections x and x' of a 3D point X lie on their respective epipolar lines.

Exercise 03.

a) $d = 1cm$, $b = 6cm$, $f = 1cm$.

$$\text{depth } z_p = \frac{b \cdot f}{d} \xrightarrow[\text{to values}]{\text{Substitute}} \frac{6 \cdot 1}{1} = 6cm$$

$\therefore$ the depth point P is $z_p = 6cm$

b) $d_{min} = 1$ pixel
$w = 0.01$ mm / pixel
$b = 6cm$
$f = 1cm$

The real disparity for 1 pixel : $d_{min} = 1\text{pixel} \times 0.01\text{mm/pixel}$
$= 0.01mm$
$= 0.001cm$

$$z = \frac{b \cdot f}{d} \longrightarrow z_{max} = \frac{b \cdot f}{d_{min}}$$

$$= \frac{6 \cdot 1}{0.001} = 6000cm.$$

$\therefore$ The depth is $z > 6000cm$ for disparities below 1 pixel.

c) left camera matrix : $P_l = [I|0]$
right camera matrix : $P_r = [I|t]$, $t = (-6, 0, 0)^T$
$Q = (3, 0, 3)$

The homogeneous coordinates of Q in the left camera :

$$x_l = P_l \cdot Q = [I|0] \cdot \begin{bmatrix} 3 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} \xrightarrow[\text{non-homogeneous}]{\text{Convert to}} x_l = \left(\frac{3}{3}, \frac{0}{3}\right) = (1, 0)$$

. projection of Q in the left camera is $(1, 0)$

$E = [t]_\times R$, where $t = (-6, 0, 0)^T \Rightarrow [t]_\times = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & -6 & 0 \end{bmatrix}$

Since $R = I$: $E = [t]_\times$

$$l_r = E \cdot X_l = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & -6 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 18 \\ 0 \end{bmatrix}$$