Solutions of the Sensor Fusion Exercise Round 1

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Exercise 1. (Drone positioning problem I)

(a) The measurements and quantities of interest are:

$$\begin{aligned}\mathbf{y} &= \begin{bmatrix} y_1 & y_2 & \dots & y_6 \end{bmatrix}^\top \\ \mathbf{x} &= \begin{bmatrix} p^x & p^y & p^z \end{bmatrix}^\top\end{aligned} \tag{1}$$

We need to rewrite the measurement model in this exercise in a vector notation with the following form:

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{b} + \mathbf{r} \tag{2}$$

In order to have $\mathbf{y}$ as the following form

$$\mathbf{y} = \begin{bmatrix} P^x + r_1 \\ P^y + r_2 \\ P^z + r_3 \\ L - P^x + r_4 \\ L - P^y + r_5 \\ L - P^z + r_6 \end{bmatrix} \tag{3}$$

$\mathbf{G}$, $\mathbf{b}$, and $\mathbf{r}$ should be:

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{4}$$

$$\mathbf{b} = \begin{bmatrix} 0 & 0 & 0 & L & L & L \end{bmatrix}^\top \tag{5}$$

$$\mathbf{r} = \begin{bmatrix} r_1 & r_2 & \dots & r_6 \end{bmatrix}^\top \tag{6}$$

(b) 3 independent sets. For example, one minimal subset can be y_1, y_2 and y_3.

(c)

$$
\begin{aligned}
p^x &\approx \frac{1}{2}(y_1 + (L - y_4)),\\
p^y &\approx \frac{1}{2}(y_2 + (L - y_5)),\\
p^z &\approx \frac{1}{2}(y_3 + (L - y_6)),
\end{aligned} \tag{7}
$$

Exercise 2. (Drone positioning problem II)

(a)

$$
\begin{aligned}
y_1 &= \sqrt{(p^x)^2 + (p^y)^2 + (p^z)^2} + r_1,\\
y_2 &= \sqrt{(L - p^x)^2 + (p^y)^2 + (p^z)^2} + r_2,\\
&\vdots\\
y_8 &= \sqrt{(L - p^x)^2 + (L - p^y)^2 + (L - p^z)^2} + r_8,
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
\mathbf{y} &= \begin{bmatrix} y_1 & y_2 & \dots & y_8 \end{bmatrix}^\top\\
\mathbf{x} &= \begin{bmatrix} p^x & p^y & p^z \end{bmatrix}^\top = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^\top\\
\mathbf{r} &= \begin{bmatrix} r_1 & r_2 & \dots & r_8 \end{bmatrix}^\top
\end{aligned} \tag{9}
$$

b) With these definition we can easily rewrite in the form of:

$$
\mathbf{y} = \mathbf{g}(\mathbf{x}) + \mathbf{r} \tag{10}
$$

$$
\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_8 \end{bmatrix} = \begin{bmatrix} \sqrt{(p^x)^2 + (p^y)^2 + (p^z)^2} \\ \sqrt{(L - p^x)^2 + (p^y)^2 + (p^z)^2} \\ \vdots \\ \sqrt{(L - p^x)^2 + (L - p^y)^2 + (L - p^z)^2} \end{bmatrix} + \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_8 \end{bmatrix} \tag{11}
$$

Exercise 3. (Dynamic model)

The 3D model is:

$$
\begin{aligned}
p_n^x &= p_{n-1}^x + v_{n-1}^x \Delta t + q_{1,n},\\
p_n^y &= p_{n-1}^y + v_{n-1}^y \Delta t + q_{2,n},\\
p_n^z &= p_{n-1}^z + v_{n-1}^z \Delta t + q_{3,n},\\
v_n^x &= v_{n-1}^x + q_{4,n},\\
v_n^y &= v_{n-1}^y + q_{5,n},\\
v_n^z &= v_{n-1}^z + q_{6,n}.
\end{aligned} \tag{12}
$$

$$\mathbf{X_n} = \begin{bmatrix} p^x & p^y & p^z & v^x & v^y & v^z \end{bmatrix}^\top = \begin{bmatrix} x_{1,n} & x_{2,n} & x_{3,n} & x_{4,n} & x_{5,n} & x_{6,n} \end{bmatrix}^\top$$
$$\mathbf{q}_n = \begin{bmatrix} q_{1,n} & q_{2,n} & \dots & q_{6,n} \end{bmatrix}^\top \tag{13}$$

$$\begin{aligned} x_{1,n} &= x_{1,n-1} + x_{4,n-1}\Delta t + q_{1,n}, \\ x_{2,n} &= x_{2,n-1} + x_{5,n-1}\Delta t + q_{2,n}, \\ x_{3,n} &= x_{3,n-1} + x_{6,n-1}\Delta t + q_{3,n}, \\ x_{4,n} &= x_{4,n-1} + q_{4,n}, \\ x_{5,n} &= x_{5,n-1} + q_{5,n}, \\ x_{6,n} &= x_{6,n-1} + q_{6,n}. \end{aligned} \tag{14}$$

which we can then write as a compact form:

$$\mathbf{X_n} = \mathbf{F}\mathbf{X_{n-1}} + \mathbf{q}_n \tag{15}$$

where

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$