Exercise and Homework Round 2

These exercises (except for the last) will be gone through on **Friday, September 20, 12:15–14:00** in the exercise session. The last exercise is a homework which you should return via MyCourses by **Friday, September 27 at 12:00**.

Exercise 1. (1D Gaussian distribution)

Recall that the expected value of a random variable x with probability density function $p(x)$ is

$$\mathrm{E}\{x\} = \int_{-\infty}^{\infty} x\, p(x)\, \mathrm{d}x \tag{1}$$

and the variance is

$$\mathrm{var}\{x\} = \int_{-\infty}^{\infty} (x - \mathrm{E}\{x\})^2\, p(x)\, \mathrm{d}x. \tag{2}$$

Furthermore, recall that for any probably density we have $\int_{-\infty}^{\infty} p(x)\, \mathrm{d}x = 1$.

(a) Show that for any random variable we can also express the variance as

$$\mathrm{var}\{x\} = \mathrm{E}\{x^2\} - (\mathrm{E}\{x\})^2. \tag{3}$$

(b) Compute the mean of the Gaussian distribution by brute-force integration:

$$p(x) = \mathcal{N}(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2} (x - \mu)^2 \right). \tag{4}$$

Exercise 2. (Multivariate mean and covariance)

Show that if $\mathbf{z} = \mathbf{L}\,\mathbf{x}$, $\mathrm{E}\{\mathbf{z}\} = \mathbf{m}$, and $\mathrm{Cov}\{\mathbf{x}\} = \mathbf{P}$, then $\mathrm{E}\{\mathbf{z}\} = \mathbf{L}\,\mathbf{m}$ and $\mathrm{Cov}\{\mathbf{z}\} = \mathbf{L}\,\mathbf{P}\,\mathbf{L}^{\mathsf{T}}$.

Exercise 3. (Weighted vs. non-weighted cost functions)

Recall that $\sigma_n^2 = 1$ to corresponds non-weighted least squares.

(a) Show that if we have a weighed least squares problem corresponding to

$$y_n = g(x_n) + r_n, \qquad \operatorname{var}\{r_n\} = \sigma_n^2, \tag{5}$$

then if we define

$$\begin{aligned} \hat{y}_n &= \frac{y_n}{\sigma_n}, \\ \hat{g}(x_n) &= \frac{g(x_n)}{\sigma_n}, \\ \hat{r}_n &= \frac{r_n}{\sigma_n}, \end{aligned} \tag{6}$$

this transforms the problem to a non-weighted least squares problem

$$\hat{y}_n = \hat{g}(x_n) + \hat{r}_n, \qquad \operatorname{var}\{\hat{r}_n\} = 1. \tag{7}$$

(b) Show that the cost functions for this non-weighted and the original weighted problems are equivalent.

Homework 2 (DL Friday, September 27 at 12:00)

Consider a 2D Gaussian distribution with the probability density function

$$p(x_1, x_2) = \frac{1}{2\pi\,\sigma_1\,\sigma_2} \exp\left(-\frac{1}{2\sigma_1^2}(x_1 - \mu_1)^2 - \frac{1}{2\sigma_2^2}(x_2 - \mu_2)^2\right). \tag{8}$$

Derive the mean and covariance of this distribution by brute-force integration.