

## Exercise and Homework Round 6

These exercises (except for the last) will be gone through on **Friday, October 25, 12:15–14:00** in the exercise session. The last exercise is a homework which you should return via mycourses by **Friday, November 1 at 12:00**.

### Exercise 1. (Ansatz solution of linear ODE)

Consider the spring model with zero force  $u(t) = 0$ :

$$ma(t) + kp(t) + \eta v(t) = 0.$$

- (a) Solve the equation with exponential ansatz  $p(t) = C \exp(\lambda t)$ .
- (b) Fix suitable initial conditions, visualize the solution, and discuss effects of the parameters  $k$  and  $\eta$  on the solution.

### Exercise 2. (Matrix exponential)

- (a) Rewrite the spring model in the previous exercise as a state-space model of the form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}_u u(t) \tag{1}$$

- (b) With  $u(t) = 0$ , solve the equation in terms of the matrix exponential  $\exp(\mathbf{A}t)$ .
- (c) Visualize the solution and compare it to the solution obtained in the previous exercise.
- (d) Generalize the solution to the case with  $u(t) \neq 0$ .

### Exercise 3. (Numerical solution of ODEs)

Consider the state-space ODE in the previous exercise, but now assume that the force is given by the following alternatives:

$$\begin{aligned}u(t) &= 1, \\u(t) &= \sin(t), \\u(t) &= t.\end{aligned}\tag{2}$$

- (a) Solve the state-space ODE with Euler's method.
- (b) Compare the solution to the solution obtained in Exercise 2d.
- (c) Use a builtin ODE solver to obtain a numerical solution to the ODE.

### Homework 6 (DL Friday, November 1 at 12:00)

Consider the scalar differential equation

$$\dot{x} = f x, \quad x(0) = x_0,\tag{3}$$

with  $f = -1/2$  and  $x_0 = 3$ .

- (a) Solve the equation analytically.
- (b) Use Euler's method to solve the equation numerically.
- (c) Use a builtin ODE solver (Python's `odeint`) to numerically solve the equation.
- (d) Visualize the solutions and compare them to each other.

#### Important note:

To complete this homework assignment, please log in to the Aalto Jupyter-Hub website at <https://jupyter.cs.aalto.fi>.

Next, use your Aalto account credentials to sign in. Once logged in, select the course titled "ELEC-E8740 - Basics of Sensor Fusion D (2024)."

For guidance on how to complete the assignment in JupyterLab, kindly refer to <https://scicomp.aalto.fi/aalto/jupyterhub/nbgrader-jupyterlab/>.