Exercise Round 1

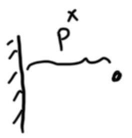
1)

ⓐ $\vec{y} = \vec{G}\,\vec{x} + \vec{b} + \vec{r}$

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix}_{6\times 1}}_{\vec{y}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}_{6\times 3}}_{\vec{G}} \underbrace{\begin{bmatrix} p^x \\ p^y \\ p^z \end{bmatrix}_{3\times 1}}_{\vec{x}} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ L \\ L \\ L \end{bmatrix}}_{\vec{b}} + \underbrace{\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{bmatrix}}_{\vec{r}}$$

b) $\vec{x} = \begin{bmatrix} p^x \\ p^y \\ p^z \end{bmatrix}$ $\qquad \underline{3} \rightarrow y_1, y_2, y_3$

or y_4, y_5, y_6

c) 

$$p^x \approx \frac{y_1 + (L - y_4)}{2}$$

$$p^y \approx \frac{y_2 + (L - y_5)}{2}$$

$$p^z \approx \frac{y_3 + (L - y_6)}{2}$$

2) a) $\vec{x} = \begin{bmatrix} p^x \\ p^y \\ p^z \end{bmatrix}$

$(0, 0, 0) \rightarrow \quad y_1 = \sqrt{(p^x)^2 + (p^y)^2 + (p^z)^2} + r_1$

$(0, L, 0) \rightarrow \quad y_2 = \sqrt{(p^x)^2 + (L-p^y)^2 + (p^z)^2} + r_2$

$(0, 0, L) \rightarrow \quad y_3 = \sqrt{(p^x)^2 + (p^y)^2 + (L-p^z)^2} + r_3$

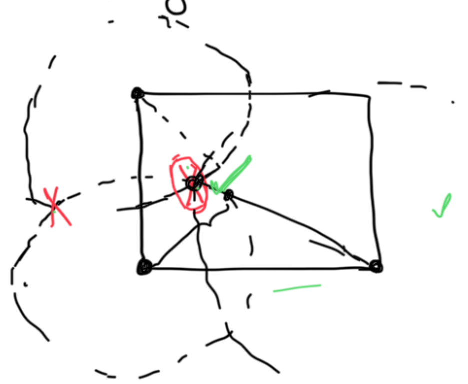
$\vdots$

$(L, L, L) \rightarrow \quad y_8 = \sqrt{(L-p^x)^2 + (L-p^y)^2 + (L-p^z)^2} + r_8$

b) $\vec{y} = g(\vec{x}) + \vec{r} \qquad \vec{x} = \begin{bmatrix} p^x \\ p^y \\ p^z \end{bmatrix}$

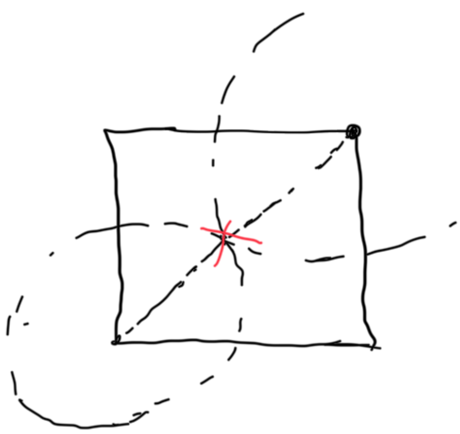
$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_8 \end{bmatrix}}_{\vec{y}} = \underbrace{\begin{bmatrix} \sqrt{(p^x)^2 + (p^y)^2 + (p^z)^2} \\ \vdots \\ \\ \sqrt{(L-p^x)^2 + (L-p^y)^2 + (L-p^z)^2} \end{bmatrix}}_{g(\vec{x})} + \underbrace{\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ \\ r_8 \end{bmatrix}}_{\vec{r}}$$

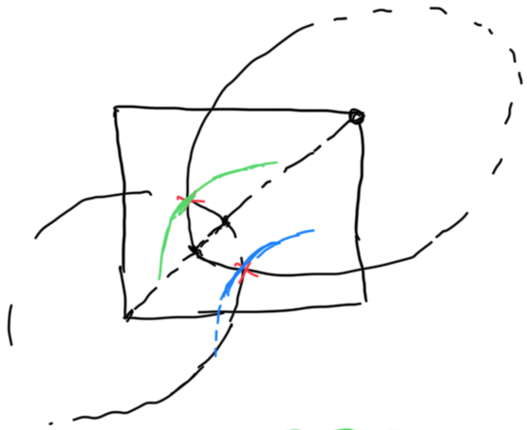
c) Imaginary



Prior: Inside + 2

3 measurement





3)

$$\vec{x}_n = \begin{bmatrix} p_n^x \\ p_n^y \\ p_n^z \\ v_n^x \\ v_n^y \\ v_n^z \end{bmatrix} = \begin{bmatrix} x_{1,n} \\ x_{2,n} \\ x_{3,n} \\ x_{4,n} \\ x_{5,n} \\ x_{6,n} \end{bmatrix}$$

$$x_{1,n} = x_{1,n-1} + x_{4,n-1}\,\Delta t + q_{1,n}$$

$$x_{2,n} = x_{2,n-1} + x_{5,n-1}\,\Delta t + q_{2,n}$$

$$x_{3,n} = x_{3,n-1} + x_{6,n-1}\,\Delta t + q_{3,n}$$

$$x_{4,n} = x_{4,n-1} + q_{4,n}$$

$$x_{5,n} = x_{5,n-1} + q_{5,n}$$

$$x_{6,n} = x_{6,n-1} + q_{6,n}$$

Vector form:

$$\underbrace{\begin{bmatrix} x_{1,n} \\ x_{2,n} \\ x_{3,n} \\ x_{4,n} \\ x_{5,n} \\ x_{6,n} \end{bmatrix}_{6\times 1}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}} \underbrace{\begin{bmatrix} x_{1,n-1} \\ x_{2,n-1} \\ x_{3,n-1} \\ x_{4,n-1} \\ x_{5,n-1} \\ x_{6,n-1} \end{bmatrix}} + \underbrace{\begin{bmatrix} q_{1,n} \\ q_{2,n} \\ q_{3,n} \\ q_{4,n} \\ q_{5,n} \\ q_{6,n} \end{bmatrix}}$$

$\dot{x}_n$ F $\dot{x}_{n-1}$ q_n