Homework 4

October 10, 2024

0.1 Homework 4 (DL Friday, October 11 at 12:00)

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

0.1.1 QUESTION: Implement the gradient descent algorithm to minimize $J(x) = (1.1 - sin(x))^2$. Also, empirically test the effect of the step size on the convergence speed.

0.1.2 Gradient decent formulation (check page 12 of houndout 4):

$$\hat{\mathbf{x}}^{(i+1)} = \hat{\mathbf{x}}^{(i)} - \gamma \frac{\partial J(\mathbf{x})}{\partial \mathbf{x}} |_{\mathbf{x}=\hat{x}^{(i)}}, \quad (1)$$

0.1.3 where $J(\mathbf{x})$ is the cost function.

[2]:
```
def J(x):
    return (1.1 - np.sin(x))**2
```

0.1.4 Part a (1 point): For the gradient descent implementation, you need to compute the gradient of the cost function. In the section below, implement the code for the gradient of the cost function $J(\mathbf{x})$.

[3]:
```
def gradient_of_J(x):
    """ Return the gradient of the cost function.
    """
    # YOUR CODE HERE
    return 2 * (1.1 - np.sin(x)) * (-np.cos(x))
    raise NotImplementedError()
```

[4]:
```
"""Check the result for several inputs"""
assert gradient_of_J(0.0) == -2.2
assert np.allclose(gradient_of_J(np.pi/2), 0.)
assert np.allclose(gradient_of_J(np.pi), 2.2)
assert np.allclose(gradient_of_J(3 * np.pi/2), 0.0)
```

0.1.5 Part b (2 points): Implement the gradient descent algorithm.

0.1.6 In the (partially) provided code below, the aim is to compute the outcomes of the gradient descent algorithm when minimizing the cost function J. The algorithm starts with a given initial value denoted as x_0. Our goal here is to obtain the results after running the algorithm for 5 iterations. Your task is to code the update rule for the gradient descent algorithm in the mentioned place.

[7]:
```
def gradient_descent_algorithm(x0, step_size, number_of_iterations = 5):
    x_results = np.zeros((number_of_iterations + 1,)) # To set the first result␣
 ↪as the initial value x0, we increase the count of x_results by one.
    x_results[0] = x0
    for i in range(number_of_iterations):
        #Implement the gradient descent update rule here
        # HINT: you should calculate x_results[i+1] from x_results[i]
        # and the code should call function gradient_of_J
        # YOUR CODE HERE
        x_results[i+1] = x_results[i] - step_size * gradient_of_J(x_results[i])
    return x_results
```

0.1.7 You can evaluate the correctness of your code by executing the following provided code with initial value $x_0 = 0$ and different step sizes $[1, 0.1, 0.01]$.

[ ]:
```

```

[8]:
```
"""Check the result for several inputs"""
assert np.allclose(gradient_descent_algorithm(0.0, 1, 5), np.array([0.0, 2.2, 1.
 ↪8569, 1.7775, 1.7277, 1.6926]), rtol=1e-03, atol=1e-04)
assert np.allclose(gradient_descent_algorithm(0.0, 0.1, 5), np.array([0.0, 0.
 ↪22, 0.3921, 0.5248, 0.6285, 0.7113]), rtol=1e-03, atol=1e-04)
assert np.allclose(gradient_descent_algorithm(0.0, 0.01, 5), np.array([0.0, 0.
 ↪022, 0.04355, 0.0647, 0.08533, 0.1055]), rtol=1e-03, atol=1e-04)
```

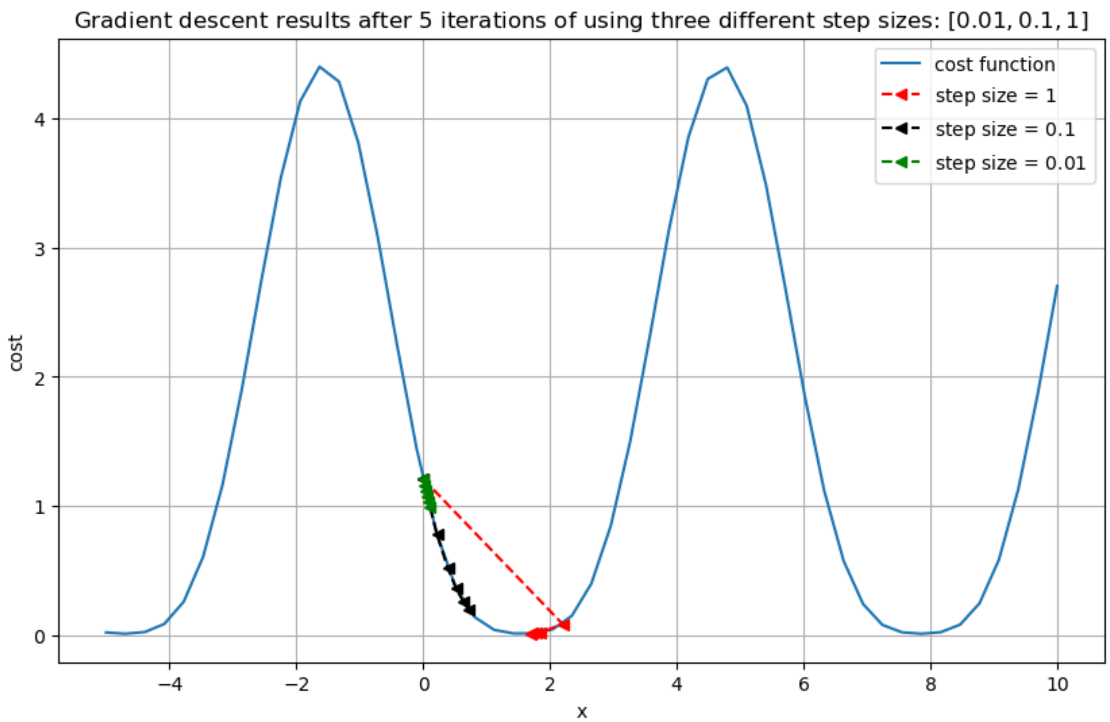
0.1.8 In the code below, you will observe the effect of the step size when running the gradient descent algorithm for 5 iterations.

[9]:
```
def plot_GD_stepsize(number_of_iteration):
    x = np.linspace(-5,10)
    x_results_1 = gradient_descent_algorithm(0.0, 1, number_of_iteration)
    x_results_2 = gradient_descent_algorithm(0.0, 0.1, number_of_iteration)
    x_results_3 = gradient_descent_algorithm(0.0, 0.01, number_of_iteration)

    mu = 0.1
    plt.figure(figsize=(10,6))
    plt.plot(x, J(x), label='cost function')
    plt.plot(x_results_1, J(x_results_1), 'r<--', label = 'step size = $1$')
    plt.plot(x_results_2, J(x_results_2), 'k<--', label = 'step size = $0.1$')
```

```
    plt.plot(x_results_3, J(x_results_3), 'g<--', label = 'step size = $0.01$')

    plt.title("Gradient descent results after {} iterations of using three␣
 ↪different step sizes: $[0.01, 0.1, 1]$".format(number_of_iteration))
    plt.xlabel('x')
    plt.ylabel('cost')
    plt.legend()
    plt.grid()
    return True
plot_GD_stepsize(5);
```



0.1.9 Part c) (1 point): Your task here is to run the "plot_GD_stepsize" function for several numbers of iterations, specifically $[10, 20, 50]$, and observe the results. (You do not need to code anything here; just run the code below and observe the effect of the step size on the convergence rate.)

[ ]:

```
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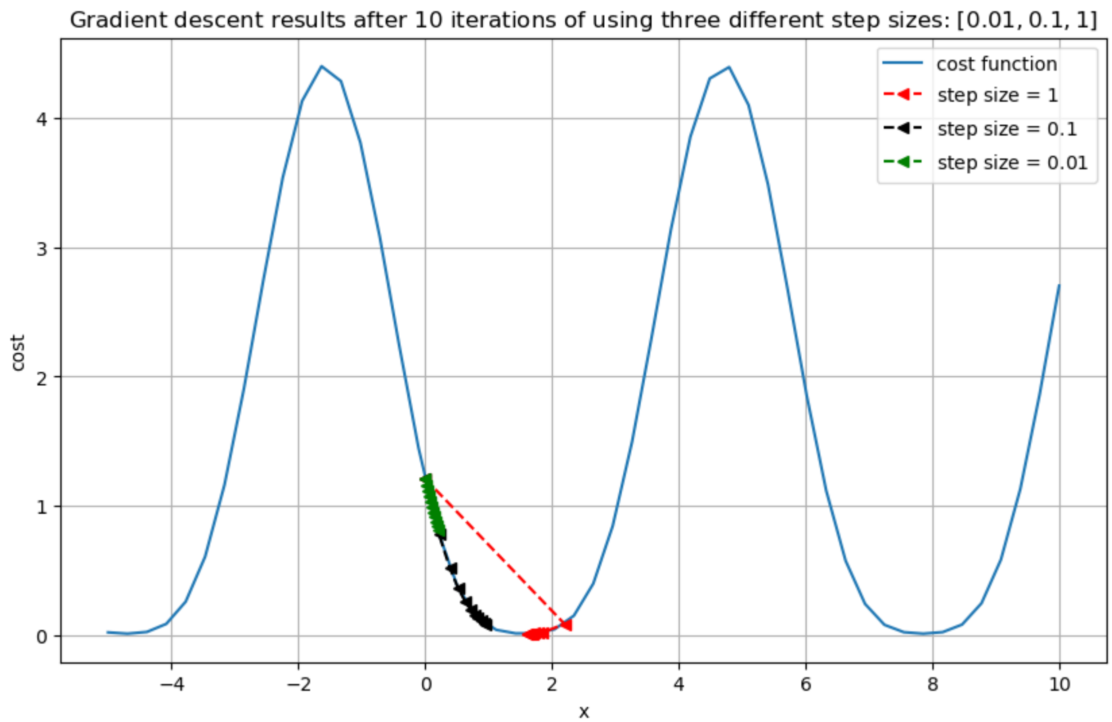
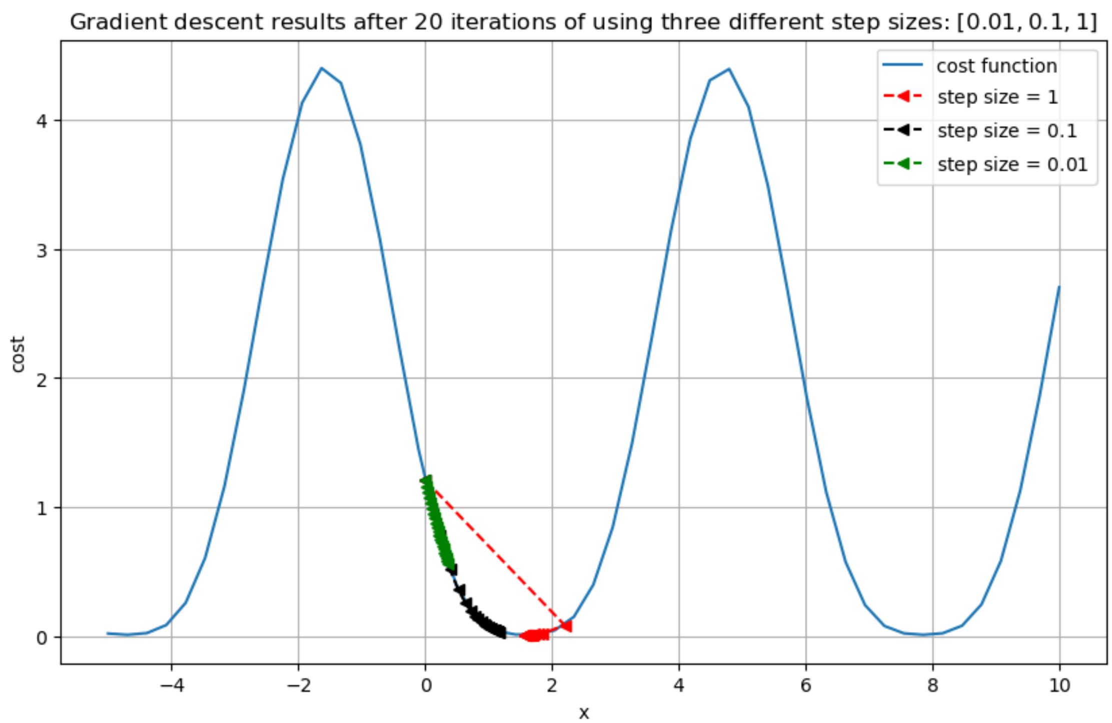
[10]:

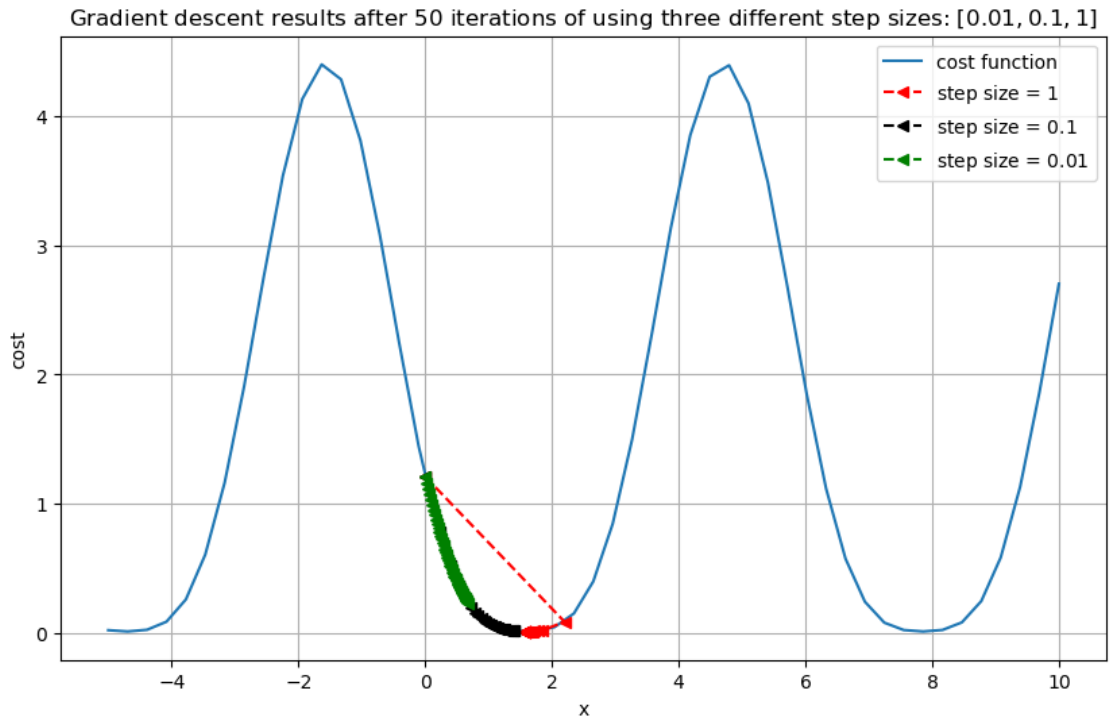
```
plot_GD_stepsize(10);
```



[11]:
```
plot_GD_stepsize(20);
```



[12]:
```
plot_GD_stepsize(50);
```



[ ]:
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