

Exercise and Homework Round 8

These exercises (except for the last) will be gone through on **Friday, November 8, 12:15–14:00** in the exercise session. The last exercise is a homework which you should return via mycourses by **Friday, November 15 at 12:00**.

Exercise 1. (Discretization of spring model)

- (a) Recall the state-space form of the spring model in the Exercise round 6:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t), \quad (1)$$

where we have put $u(t) = 0$.

- (b) Write down the solution in terms of the matrix exponential $\exp(\mathbf{A}t)$. Plot the solution by evaluating the solution for $t = 0, \dots, 1$ on a dense grid. Compute the matrix exponentials using numerical `expm` function.
- (c) Discretize the model with $\Delta t = 0.1$ to the form

$$\mathbf{x}_n = \mathbf{F}\mathbf{x}_{n-1}, \quad (2)$$

and compute \mathbf{F} numerically with `expm`.

- (d) Visualize the solution solution from (a) and the discretized solution from (b), and check that they match at the discretization points.

Exercise 2. (Wiener velocity model)

- (a) Recall the 1D Wiener velocity model from the lecture slides.
- (b) Write down the discretization of the model in form (with fixed Δt)

$$\mathbf{x}_n = \mathbf{F}\mathbf{x}_{n-1} + \mathbf{q}_n. \quad (3)$$

What does \mathbf{F} look like, what are the mean and covariance of \mathbf{q}_n ?

- (c) Simulate trajectories from the (discretized) model by using a suitable initial mean and covariance.
- (d) The mean and covariance matrix for each time step can be computed by the recursions

$$\begin{aligned} \mathbf{m}_n &= \mathbf{F}\mathbf{m}_{n-1}, \\ \mathbf{P}_n &= \mathbf{F}\mathbf{P}_{n-1}\mathbf{F}^\top + \mathbf{Q}_n. \end{aligned} \quad (4)$$

Check that the empirical mean and covariance match the theoretical ones.

Exercise 3. (Euler–Maruyama discretization of robot model)

Consider the following 2D dynamic model of a robot platform:

$$\begin{aligned} \dot{p}^x(t) &= v(t) \cos(\varphi(t)) + w_1(t), \\ \dot{p}^y(t) &= v(t) \sin(\varphi(t)) + w_2(t), \\ \dot{\varphi}(t) &= \omega_{\text{gyro}}(t) + w_3(t), \end{aligned}$$

where p^x, p^y is the position, φ is the orientation angle, v is the speed input, ω_{gyro} is the gyroscope reading, and w_1, w_2, w_3 are independent white noise processes with spectral densities q_1, q_2, q_3 .

- (a) Form Euler–Maruyama discretization of the model with a discretization step Δt . Simulate (random) trajectories from the model.
- (b) Form linearization-based discretization of the model. When does this coincide with the Euler–Maryuama discretization? Simulate (random) trajectories from the model.

Homework 8 (DL Friday, November 15 at 12:00)

Consider the scalar differential equation

$$\dot{x} = a x + u, \quad x(0) = x_0, \quad (5)$$

with $a = -1/2$ and $x_0 = 3$, where $u = u(t)$ is some given input function.

- (a) With discretization step Δt , form discretization of the model with zeroth-order-hold (ZOH) approximation in form

$$x_n = f_n x_{n-1} + l_n u_{n-1}. \quad (6)$$

- (b) By assuming that $u(t) = 1$, and $\Delta = 0.1$ simulate trajectory of length 100 steps from the discretized model.
- (c) Solve the equation using builtin ODE solver (e.g. Python's `odeint`) and check that the solution matches the above at the discretization points.

Important note:

To complete this homework assignment, please log in to the Aalto Jupyter-Hub website at <https://jupyter.cs.aalto.fi>.

Next, use your Aalto account credentials to sign in. Once logged in, select the course titled "ELEC-E8740 - Basics of Sensor Fusion D (2024)."

For guidance on how to complete the assignment in JupyterLab, kindly refer to <https://scicomp.aalto.fi/aalto/jupyterhub/nbgrader-jupyterlab/>.