(a)

$$y_n = ax_n + b + r_n$$

$$J(a,b) = \sum_{n=1}^{N} (y_n - ax_n - b)^2$$

partial derivative with respect to a : $\dfrac{\partial J(a,b)}{\partial a} = \sum_{n=1}^{N} -2x_n(y_n - ax_n - b) = 0$

$$\Rightarrow \sum_{n=1}^{N} x_n y_n = a\sum_{n=1}^{N} x_n^2 + b\sum_{n=1}^{N} x_n$$

partial derivative with respect to b : $\dfrac{\partial J(a,b)}{\partial b} = \sum_{n=1}^{N} -2(y_n - ax_n - b) = 0$

$$\Rightarrow \sum_{n=1}^{N} y_n = a\sum_{n=1}^{N} x_n + N\cdot b$$

$$\therefore \quad a = \frac{N\sum_{n=1}^{N} x_n y_n - \sum_{n=1}^{N} x_n \cdot \sum_{n=1}^{N} y_n}{N\sum_{n=1}^{N} x_n^2 - \left(\sum_{n=1}^{N} x_n\right)^2}$$

$$b = \frac{\sum_{n=1}^{N} y_n - a\sum_{n=1}^{N} x_n}{N}$$

(b)

$$G = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}$$

$$J(x) = (y - Gx)^T (y - Gx), \text{ where } x = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\frac{\partial J(x)}{\partial x} = -2G^T(y - Gx) = 0$$

$$\Rightarrow G^T G x = G^T y$$

$$\therefore x = (G^T G)^{-1} G^T y$$

$G^T G$ is 2x2 matrix :

$$G^T G = \begin{bmatrix} \sum_{n=1}^{N} x_n^2 & \sum_{n=1}^{N} x_n \\ \sum_{n=1}^{N} x_n & N \end{bmatrix}$$

$G^T y$ is 2x1 matrix :

$$G^T y = \begin{bmatrix} \sum_{n=1}^{N} x_n y_n \\ \sum_{n=1}^{N} y_n \end{bmatrix}$$

$$\therefore a = \frac{N\sum_{n=1}^{N} x_n y_n - \sum_{n=1}^{N} x_n \cdot \sum_{n=1}^{N} y_n}{N\sum_{n=1}^{N} x_n^2 - \left(\sum_{n=1}^{N} x_n\right)^2}$$

$$\therefore b = \frac{\sum_{n=1}^{N} y_n - a\sum_{n=1}^{N} x_n}{N}$$