Ch3. Summary

1 Least Square Estimators.

平稳，差分会算

① LSE (unbiased)

- ① OLS → 仅 $\mu=0$, $\sigma^2=\sigma^2 I$
- ② GLS → 仅 $\mu=0$. $var=\sigma^2 V$ V-一定要知
- ③ WLS → var↑, less important $w_{ii}=\frac{1}{\sigma_i^2}$
- ④ DLS → 越旧的越less important

condition: $E[\varepsilon_t]=0$

- ① $V=I \Rightarrow$ Constant variance & uncorrelated
- ② V is known. , σ^2 can be unknown. (non-constant variance & correlated)
- ③ weight $\propto \frac{1}{\text{variance}} \Rightarrow$ $\sigma^2 V$ is known ↑ non-constant variance & uncorrelated.
- ④ weight $\propto \frac{1}{\text{Time lag}} \Rightarrow$ non-constant variance & uncorrelated

只告诉平稳

independent of T

先去季节，后去趋势

② OLS: $$L=\sum_{i=1}^{n}\varepsilon_i^2=\sum_{i=1}^{n}\Big(y_i-\beta_0-\sum_{j=1}^{k}\beta_j x_j\Big)^2$$

Least squares normal equations:

$$\frac{\partial L}{\partial \beta_0}=0 \quad \frac{\partial L}{\partial \beta_1}=0 \quad \cdots \quad \frac{\partial L}{\partial \beta_k}=0.$$

$$\hat\beta_{OLS}=(X'X)^{-1}X'y.$$

- $E[\hat\beta_{OLS}]=\beta$
- $var(\hat\beta_{OLS})=\sigma^2(X'X)^{-1}$.

2) WLS: $$L=(y-X\beta)'W(y-X\beta)$$
$$=\sum_{i=1}^{n} w_{ii}\Big(y_i-\beta_0-\sum_{j=1}^{k}\beta_j x_j\Big)^2.$$

To find $\hat\beta_{WLS}$, we can use normal equations or matrix calculation

$$\hat\beta_{WLS}=(X'WX)^{-1}X'Wy.$$

- $E[\hat\beta_{WLS}]=\beta$
- $var(\hat\beta_{WLS})=(X'WX)^{-1}$ 前面无σ^2

3) GLS: Transform original model.

$$V^{-\frac12}y=V^{-\frac12}X\beta+\boxed{V^{-\frac12}\varepsilon}$$ → satisfies OLS requirement

如何 transform

$$\hat\beta_{GLS}=(X'V^{-1}X)^{-1}X'V^{-1}y$$

- $E[\hat\beta_{GLS}]=\beta$
- $var(\hat\beta_{GLS})=\sigma^2(X'V^{-1}X)^{-1}$ 有σ^2

③ Underfitting & Overfitting

⇓

Check unbiased / biased estimate

If biased, find the bias.

[Idea: substitute the small matrix into the large one]
(X or β).

④. Some Conclusions.

1>. GLS estimator is the most efficient / smallest variance of the model coefficients β unbiased linear estimator.

(Proof is required).

2>. When the errors are positively autocorrelated, the residual mean square by OLS may seriously underestimate the error variance σ^2.

3>. In the above case, the $se(\hat{\beta}_j)$ may be too small

2. Utility Tests & CI & PI

① $SST = SS_R + SS_E$

$\sum_{i=1}^{n}(y_i-\bar{y})^2 \qquad \sum_{i=1}^{n}(\hat{y}_i-\bar{y})^2 \qquad \sum_{i=1}^{n}(y_i-\hat{y}_i)^2$

$SST \xrightarrow{DoF} n-1$

$SSR \xrightarrow{DoF} p-1$

$SSE \xrightarrow{DoF} n-p$

partial：选部分变量检验显著性

F: significant

② T-test: Test on individual regression coefficients.

F-test: Test on groups of coefficients.

③. 1> CI on regression coefficients β_j:

$$\hat{\beta}_j \pm t_{\alpha/2, n-p}\, se(\hat{\beta}_j)$$ 公式要会写

where $se(\hat{\beta}_j) = \sqrt{\hat{\sigma}^2 C_{jj}}$.

where $\hat{\sigma}^2 = \frac{SSE}{n-p}$. C_{jj} is the j-th diagonal of $(X'X)^{-1}$.

2> CI on mean response $\mu_{y|x_0}$

$$\hat{y}(x_0) \pm t_{\alpha/2, n-p}\sqrt{\hat{\sigma}^2 x_0'(X'X)^{-1}x_0}$$

where $\hat{\sigma}^2 = \frac{SSE}{n-p}$

3> PI on the new observations. $y(x_0)$.

$$\hat{y}(x_0) \pm t_{\alpha/2, n-p}\sqrt{\hat{\sigma}^2[1 + x_0'(X'X)^{-1}x_0]}, \quad \hat{\sigma}^2 = \frac{SSE}{n-p}$$

3. First-order autoregressive process

① $y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$, $\varepsilon_t = \phi\varepsilon_{t-1} + a_t$

where $|\phi| < 1$, $a_t \sim NID(0, \sigma_a^2)$ } 记住

② If $\{\varepsilon_t\}$ are positively autocorrelated.

⇒ If we still apply OLS, the consequences are

a> Underestimate the error variance
b> Underestimate standard errors of coefficients.
c> T-test & F-test no longer convincible.
d> CI & PI are narrower than the actual

因为 se 更小了

③ Solution.

a) Adding missing variable 加回变量 (too hard to implement)
b> GLS (Require V), 或 WLS
c> Use a method that accounts for autocorrelation { Cochrane-Orcutt, MLE }

④ D-W test $\xrightarrow{\text{Objective}}$ check the first-order autocorrelation

D-W test

a> Test statistic: $d \approx 2(1-r_1)$

会看对应表

(表中 k 不含 β_0)

$H_0: \phi = 0$

$H_1: \phi > 0$

D的值在 interval 中

inconclusive

b> If $d < 2 \Rightarrow$ Testing 1st-order positive autocorrelation.

$d < d_L$, reject H_0, positively autocorrelated.

$d > d_U$, accept H_0, uncorrelated

$d_L \le d \le d_U$: test is inconclusive.

Test d^*

If $d > 2 \Rightarrow$ Testing 1st-order negative autocorrelation.

$d^* = 4 - d$

⑤ For $\varepsilon_t = \phi\varepsilon_{t-1} + a_t$, $a_t \sim NID(0, \sigma_a^2)$,

we have the following results (☆ Proof is required)

a> $\varepsilon_t = \sum_{j=0}^{\infty} \phi^j a_{t-j}$

b> $E[\varepsilon_t] = 0$

c> $var(\varepsilon_t) = \sigma^2 = \sigma_a^2 \left(\frac{1}{1-\phi^2}\right)$

d> $cov(\varepsilon_t, \varepsilon_{t\pm j}) = \phi^j \frac{\sigma_a^2}{1-\phi^2}$

e> $\rho_k = \phi^k$

especially $\rho_1 = \phi$

⑥ C-O Method.

$$y_t' = y_t - \phi y_{t-1} = \beta_0(1-\phi) + \beta_1(x_t - \phi x_{t-1}) + \varepsilon_t - \phi\varepsilon_{t-1}$$

$$\Rightarrow y_t' = \beta_0' + \beta_1 x_t' + a_t$$

1> Estimate $\hat{\phi}$ by using residuals of OLS.

$$\hat{\phi} = r_1 = \sum_{t=2}^{T} e_t e_{t-1} \Big/ \sum_{t=1}^{T} e_t^2.$$

lag-one sample correlation

2> Apply the OLS to obtain $\hat{\beta} = \begin{bmatrix} \hat{\beta}_0' \\ \hat{\beta}_1 \end{bmatrix}$

PI/CI > OLS's PI/CI

β_1 X

如何用 β_1 算 β_0

7 Maximum - Likelihood Method

a). Can write out the likelihood / log-likelihood function.

b). Should be able to how to obtain the estimate.

Eg. $\frac{\partial \ln L(y; \phi, \beta_0, \beta_1)}{\partial \phi} = 0.$

$\frac{\partial \ln L(y; \phi, \beta_0, \beta_1)}{\partial \beta_0} = 0$ $\frac{\partial \ln L(y; \phi, \beta_0, \beta_1)}{\partial \beta_1} = 0.$

c). If $E[\varepsilon_t] = 0$ & ε_t are normal and independent.
the MLE = LSE (无法说 linear 还是 non...