

3.1 (1) $P_{X,Y}(x,y)$ を求めよ

$$P_{X,Y}(x,y) = P_{Y|X}(y|x) \cdot P_X(x)$$

$$x=0, y=0 \quad P_{X,Y}(x,y) = 0.8 \times 0.7 = 0.56$$

$$x=0, y=1 \quad P_{X,Y}(x,y) = 0.2 \times 0.7 = 0.14$$

$$x=1, y=0 \quad P_{X,Y}(x,y) = 0.4 \times 0.3 = 0.12$$

$$x=1, y=1 \quad P_{X,Y}(x,y) = 0.6 \times 0.3 = 0.18$$

$P_{X,Y,Z}(x,y,z) = P_{Z|X,Y}(z|x,y) \cdot P_{X,Y}(x,y)$ を求めよ

$$x=0, y=0, z=0 \quad P_{X,Y,Z}(x,y,z) = 0.9 \times 0.56 = 0.504$$

$$x=0, y=0, z=1 \quad P_{X,Y,Z}(x,y,z) = 0.1 \times 0.56 = 0.056$$

$$x=0, y=1, z=0 \quad P_{X,Y,Z}(x,y,z) = 0.5 \times 0.14 = 0.070$$

$$x=0, y=1, z=1 \quad P_{X,Y,Z}(x,y,z) = 0.5 \times 0.14 = 0.070$$

$$x=1, y=0, z=0 \quad P_{X,Y,Z}(x,y,z) = 0.8 \times 0.12 = 0.096$$

$$x=1, y=0, z=1 \quad P_{X,Y,Z}(x,y,z) = 0.2 \times 0.12 = 0.024$$

$$x=1, y=1, z=0 \quad P_{X,Y,Z}(x,y,z) = 0.3 \times 0.18 = 0.054$$

$$x=1, y=1, z=1 \quad P_{X,Y,Z}(x,y,z) = 0.7 \times 0.18 = 0.126$$

x	y	z	$P_{X,Y,Z}(x,y,z)$
0	0	0	0.504
0	0	1	0.056
0	1	0	0.070
0	1	1	0.070
1	0	0	0.096
1	0	1	0.024
1	1	0	0.054
1	1	1	0.126

$$(2) P_{X|Z}(x|z) = \frac{P_{XZ}(x,z)}{P_Z(z)}$$

$$P_{XZ}(0,0) = P_{XYZ}(0,0,0) + P_{XYZ}(0,1,0) = 0.564 + 0.070 = 0.634$$

$$P_{XZ}(0,1) = P_{XYZ}(0,0,1) + P_{XYZ}(0,1,1) = 0.056 + 0.070 = 0.126$$

$$P_{XZ}(1,0) = P_{XYZ}(1,0,0) + P_{XYZ}(1,1,0) = 0.096 + 0.054 = 0.150$$

$$P_{XZ}(1,1) = P_{XYZ}(1,0,1) + P_{XYZ}(1,1,1) = 0.024 + 0.126 = 0.150$$

$$P_Z(0) = P_{XZ}(0,0) + P_{XZ}(1,0) = 0.634 + 0.150 = 0.784$$

$$P_Z(1) = P_{XZ}(0,1) + P_{XZ}(1,1) = 0.126 + 0.150 = 0.276$$

$$P_{X|Z}(0|0) = \frac{0.634}{0.784} = 0.809 \quad P_{X|Z}(0|1) = \frac{0.126}{0.276} = 0.457$$

$$P_{X|Z}(1|0) = \frac{0.150}{0.784} = 0.191 \quad P_{X|Z}(1|1) = \frac{0.150}{0.276} = 0.543$$

X	Z	$P_{X Z}(x z)$
0	0	0.809
0	1	0.457
1	0	0.191
1	1	0.543

3.2

$$(a) \quad P = \begin{pmatrix} P_{00} & P_{01} & P_{02} & P_{03} \\ P_{10} & P_{11} & P_{12} & P_{13} \\ P_{20} & P_{21} & P_{22} & P_{23} \\ P_{30} & P_{31} & P_{32} & P_{33} \end{pmatrix} = \begin{pmatrix} 0.9 & 0.1 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \\ 0.4 & 0.6 & 0 & 0 \\ 0 & 0 & 0.7 & 0.3 \end{pmatrix}$$

$$w = (w_0, w_1, w_2, w_3)$$

$$(w_0, w_1, w_2, w_3) = (w_0, w_1, w_2, w_3) \begin{pmatrix} 0.9 & 0.1 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \\ 0.4 & 0.6 & 0 & 0 \\ 0 & 0 & 0.7 & 0.3 \end{pmatrix}$$

$$\begin{cases} w_0 = 0.9w_0 + 0.4w_2 & \dots ① \\ w_1 = 0.1w_0 + 0.6w_2 & \dots ② \\ w_2 = 0.5w_1 + 0.7w_3 & \dots ③ \\ w_3 = 0.5w_1 + 0.3w_3 & \dots ④ \\ w_0 + w_1 + w_2 + w_3 = 1 & \dots ⑤ \end{cases}$$

$$① \& ② \Rightarrow w_0 = 4w_2 \dots ⑥$$

$$④ \& ⑤ \Rightarrow w_3 = \frac{5}{7}w_1 \dots ⑦$$

$$③ \& ⑦ \Rightarrow w_2 = 0.5w_1 + 0.7 \times \frac{5}{7}w_1 \\ = w_1 \dots ⑧$$

$$w_0 = 4w_1 \dots ⑨$$

$$⑥⑦⑧⑨ \& ⑤ \Rightarrow 1 = 1 \times 1$$

$$4w_1 + w_1 + w_1 + \frac{5}{7}w_1 = 1$$

$$\frac{47}{7}w_1 = 1$$

$$w_1 = \frac{1}{47}$$

$$w_2 = w_1 \& ⑧$$

$$w_2 = \frac{1}{47}$$

$$w_0 = \frac{28}{47} \quad w_1 = \frac{1}{47} \quad w_2 = \frac{1}{47} \quad w_3 = \frac{5}{47}$$

$$(b) \quad P_x(0) = 0.9w_0 + 0.5w_1 + 0.4w_2 + 0.3w_3$$

$$= 0.9 \times \frac{28}{47} + 0.5 \times \frac{1}{47} + 0.4 \times \frac{1}{47} + 0.3 \times \frac{5}{47}$$

$$= \frac{252}{470} + \frac{35}{470} + \frac{28}{470} + \frac{15}{470}$$

$$= \frac{330}{470} = \frac{33}{47}$$

$$P_x(0) = \frac{33}{47}$$