8a) $x(t) = \int_0^t \cos(x^2)\,dx \quad y(t) = \int_0^t \sin(x^2)\,dx$

$\int_0^1 f(z)\,dz$ in terms of $f(0), f(1), f(2)$

$$\int_0^1 f(z)\,dz \approx w_1 f(0) + w_2 f(1) + w_3 f(2)$$

integral = quadrature approx.

for $f(z) = z^0 = c$

$$\int_0^1 f(z)\,dz = [cz]_0^1 = c = w_1 f(0) + w_2 f(1) + w_3 f(2)$$

$$c = c(w_1 + w_2 + w_3)$$

$$1 = w_1 + w_2 + w_3$$

for $f(z) = z^1$

$$\int_0^1 f(z)\,dz = \left[\frac{z^2}{2}\right]_0^1 = \frac{1}{2} = w_2 + 2w_3$$

for $f(z) = z^2$

$$\int_0^1 f(z)\,dz = \left[\frac{z^3}{3}\right]_0^1 = \frac{1}{3} = w_2 + 4w_3 \qquad \frac{1}{2} + \frac{1}{3}$$

$$- \quad \frac{1}{2} = w_2 + 2w_3$$

$$\frac{1}{3} - \frac{1}{2} = 2w_3$$

$$-\frac{1}{12} = w_3 \qquad \frac{1}{2} - 2\left(-\frac{1}{12}\right) = w_2 = \frac{2}{3}$$

$$1 = w_1 + \frac{2 \cdot 4}{3 \cdot 4} - \frac{1}{12}$$

$$1 = w_1 + \frac{7}{12}$$

$$\frac{5}{12} = w_1$$

$$w_1 = \frac{5}{12}, \quad w_2 = \frac{2}{3}, \quad w_3 = -\frac{1}{12}$$