

$$\min_{\beta \in \mathbb{R}^d} \sum_{i=1}^n w_i (\underbrace{z_i^T \beta}_{\text{scalar}} - y_i)^2 + \underbrace{\lambda \sum_{j=1}^d \beta_j^2}_{\lambda \beta^T \beta} \quad \left. \vphantom{\sum_{i=1}^n} \right\} \text{convert to matrix form}$$

matrix form:  $(Z^T \beta - Y)^T W (Z^T \beta - Y) + \lambda \beta^T \beta =$

$$(\beta^T Z - Y^T) W (Z^T \beta - Y) + \lambda \beta^T \beta =$$

$$(\beta^T Z W - Y^T W) (Z^T \beta - Y) + \lambda \beta^T \beta =$$

$$\beta^T Z W Z^T \beta - \beta^T Z W Y - Y^T W Z^T \beta + \cancel{Y^T W Y} + \lambda \beta^T \beta =$$

→ take derivative

$$(Z W Z^T + (Z W Z^T)^T) \beta - Z W Y - Y^T W Z^T + 2\lambda \beta = 0$$

$$\begin{aligned} & (W Z^T)^T Z^T \\ & \downarrow Z W^T Z^T \end{aligned}$$

$$Z (Z W)^T$$

$$Z W^T Z^T \rightarrow$$

since  $W$  is symmetrical,  
 $W = W^T$

$$(Z W Z^T + (Z W Z^T)^T + 2\lambda) \beta = Z W Y + Y^T W Z^T$$

$$(Z W Z^T + Z W Z^T + 2\lambda) \beta = Z W Y + Y^T W Z^T$$

$$\boxed{\beta = (2Z W Z^T + 2\lambda)^{-1} (Z W Y + Y^T W Z^T)}$$