

# Non-linear and Non-stationary Noise Removal from GW Data



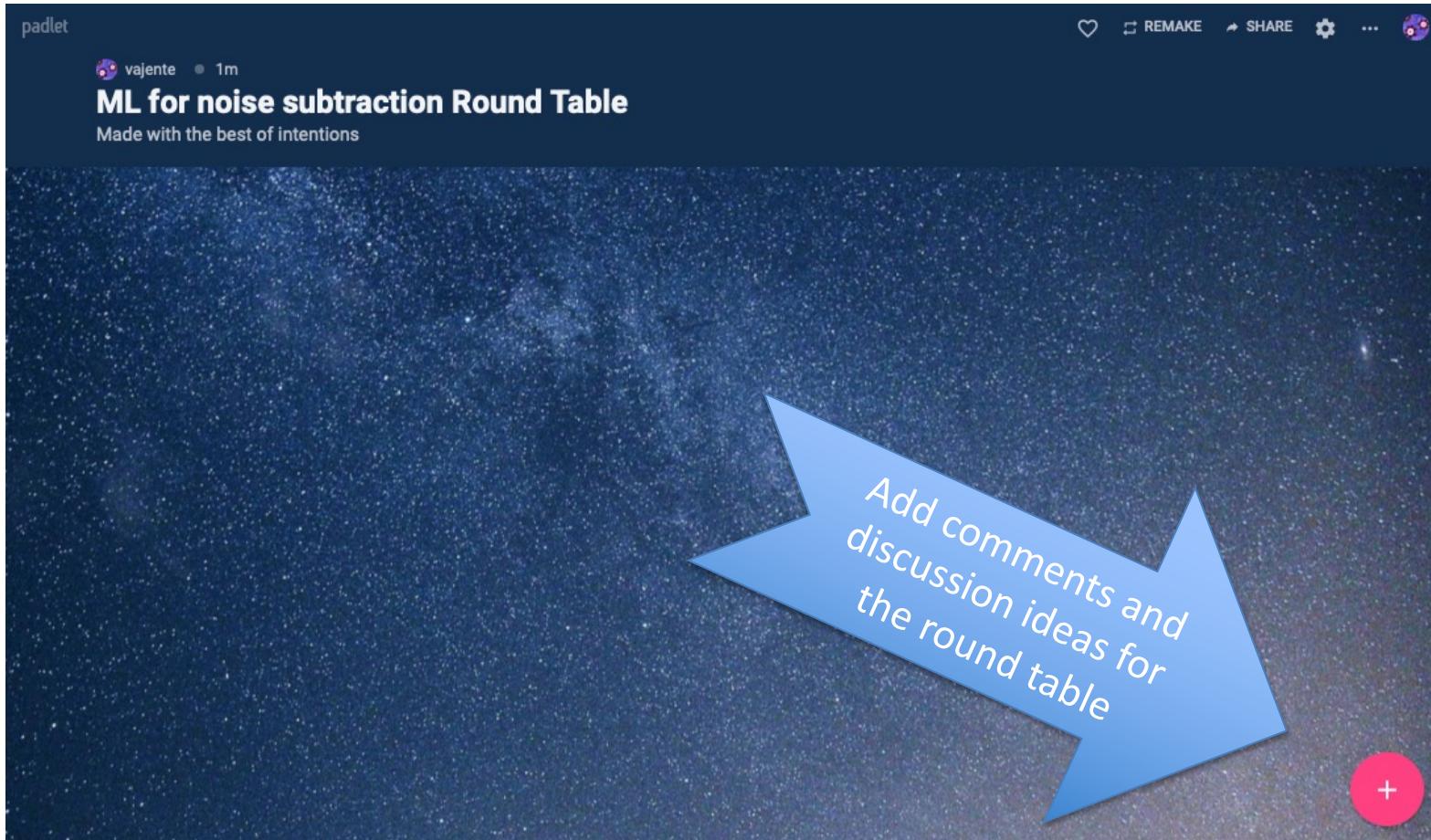
Gabriele Vajente ([vajente@caltech.edu](mailto:vajente@caltech.edu))

LIGO Laboratory – Caltech

g2net WG3 workshop on Machine Learning  
for Advanced Control Techniques

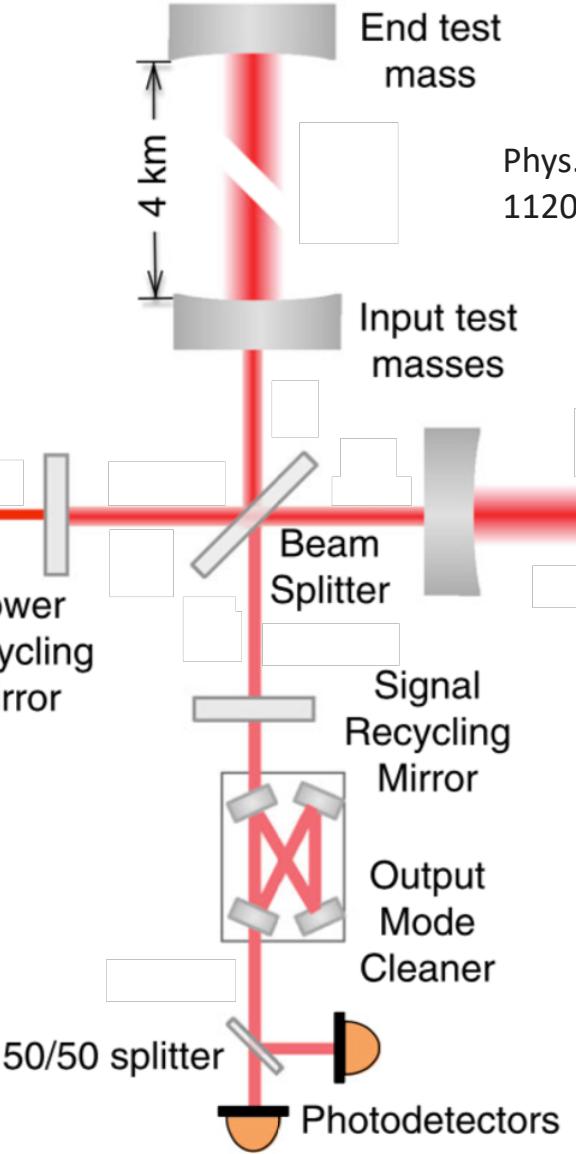
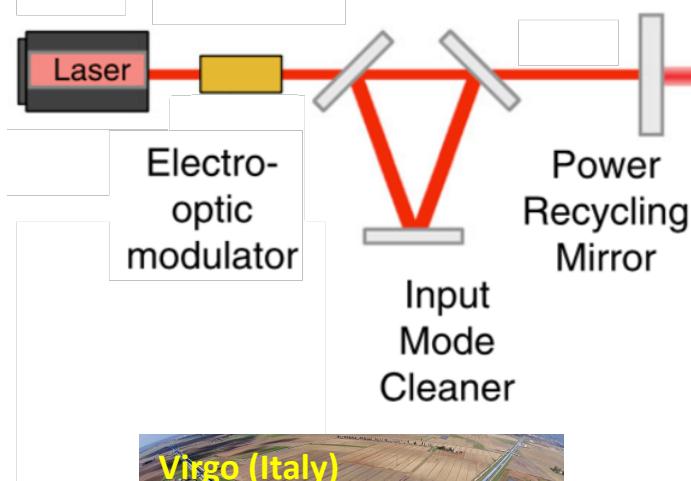
March 22<sup>nd</sup> 2021



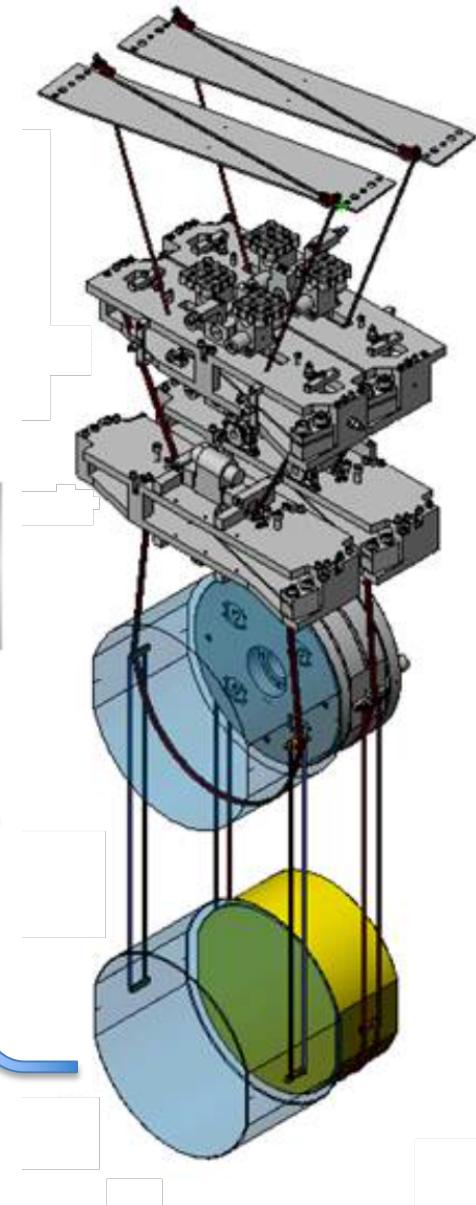


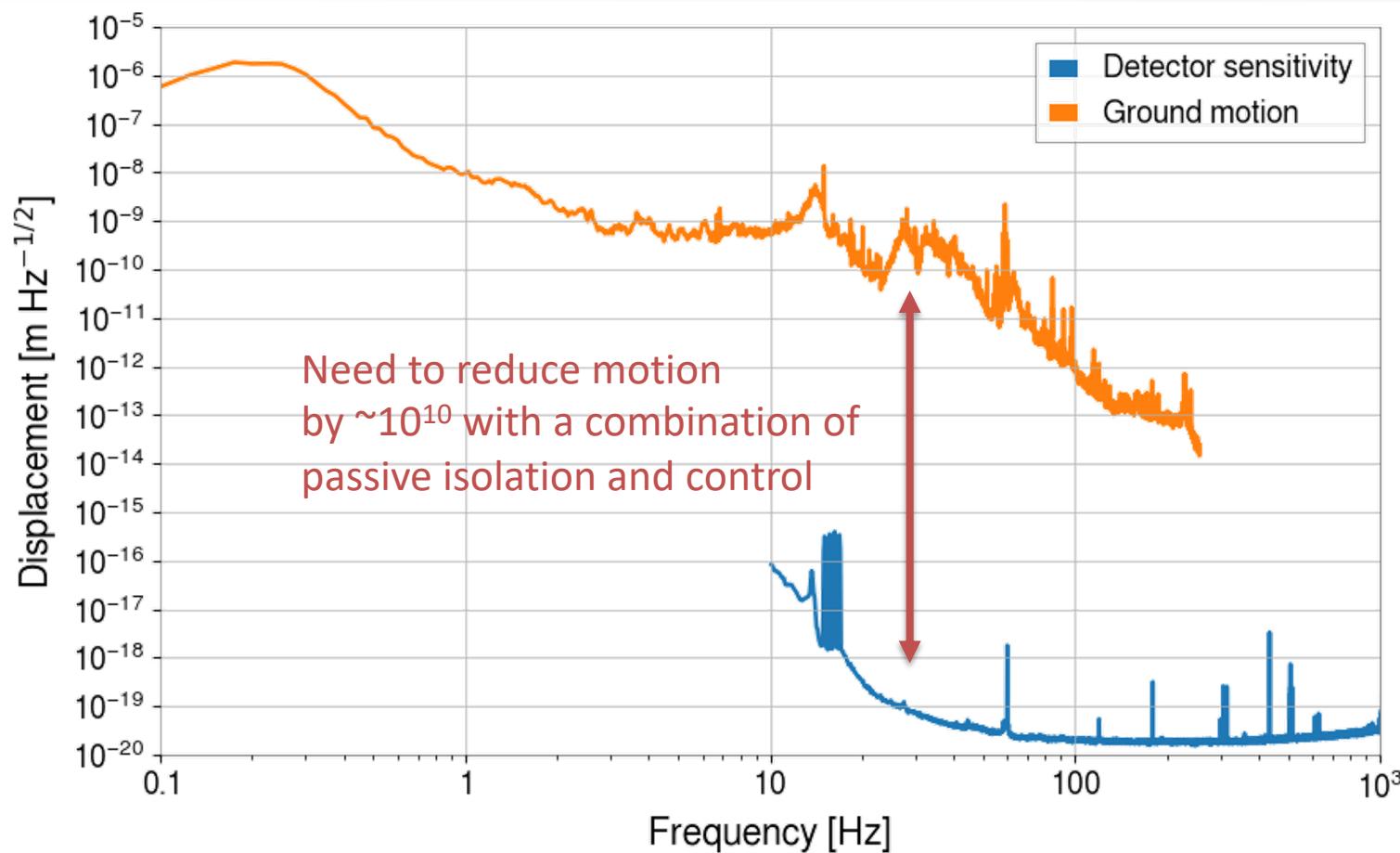
A screenshot of a Padlet board titled "ML for noise subtraction Round Table". The board was created by "vajente" 1m ago. It has a dark background with a starry field texture. A large blue arrow points diagonally across the board with the text: "Add comments and discussion ideas for the round table". In the bottom right corner, there is a red circular button with a white plus sign (+).

<https://padlet.com/vajente/whthqq0lxa7nfy52>



Phys. Rev. D 93,  
112004 (2016)



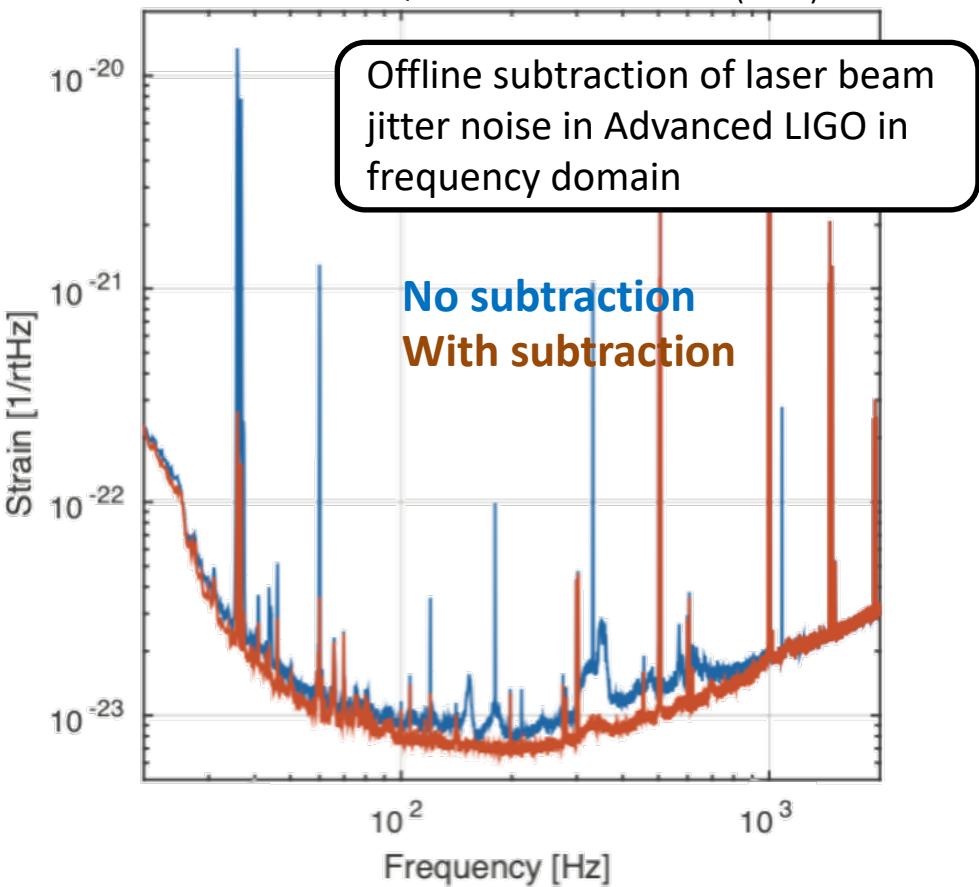


Compensate for **1-10  $\mu\text{m}$  of motion below  $\sim 1 \text{ Hz}$** , without introducing spurious displacement larger than  $\sim 10^{-19} \text{ m/VHz}$  above  $\sim 10 \text{ Hz}$

Feedback controls to **linearize** the system response and maintain the operating point close to optimal

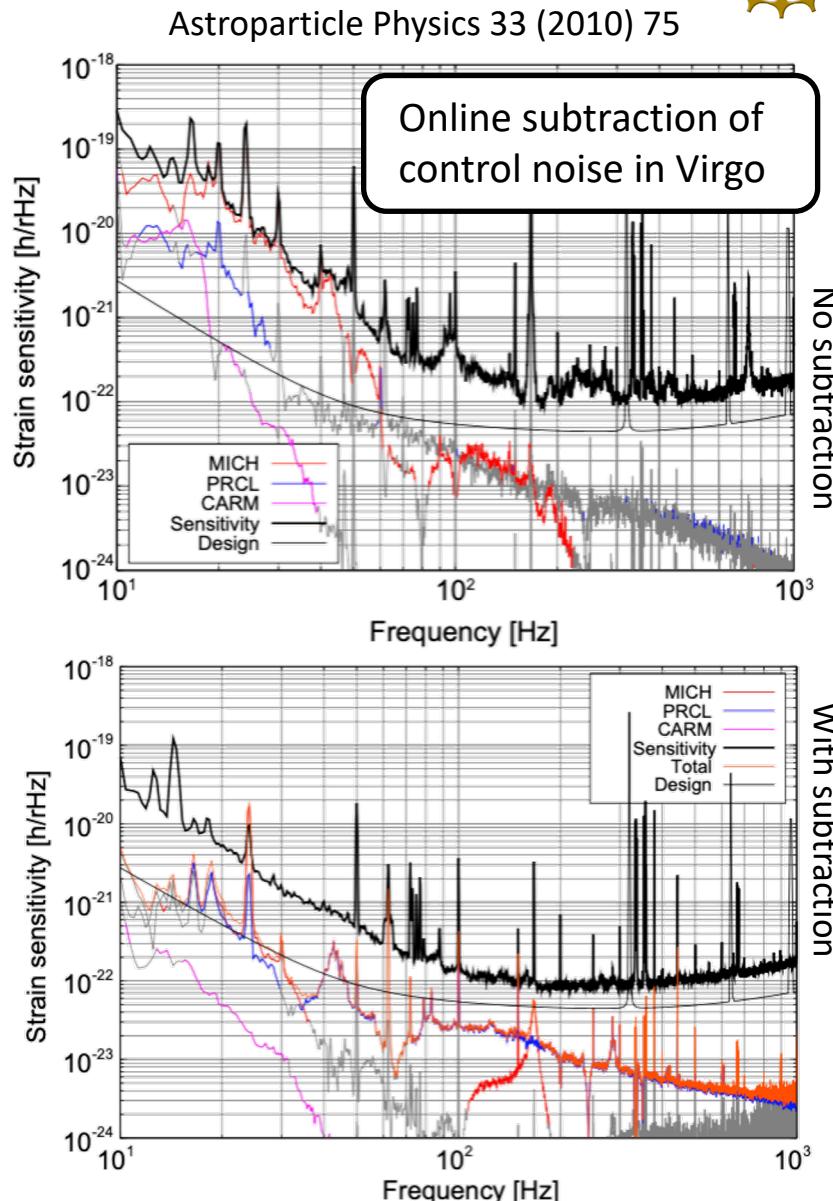
# Linear Noise Subtraction

Phys. Rev. D 99, 042001 (2019)  
Class. Quantum Grav. 36 055011 (2019)



If the coupling is **linear** and **stationary**:

- Coherence / Transfer Function analysis, Wiener filter, online or offline



## Linear

There's only one functional form

Simple parametrization

$$h(t) = \int_0^{+\infty} \alpha(\tau) x(t - \tau) d\tau$$

$$\tilde{h}(\omega) = \tilde{\alpha}(\omega) \tilde{x}(\omega)$$

$$\tilde{\alpha}(\omega) = k \frac{\prod_i (i\omega - z_i)}{\prod_i (i\omega - p_i)}$$

Coherence analysis is a powerful discovery tool

LIGO-G1500230

## Non-Linear

Only constraint from causality

$$h(t) = \mathcal{F}[x(\tau < t)]$$

Volterra Series:

$$h(t) = \sum_{n=0}^{+\infty} \int_0^{+\infty} \dots \int_0^{+\infty} \alpha_n(\tau_1, \dots, \tau_n) \prod_{i=1}^n x(t - \tau_i) d\tau_i$$

No easy parametrization

No discovery tool (higher order coherences are ill-defined and numerically unstable)

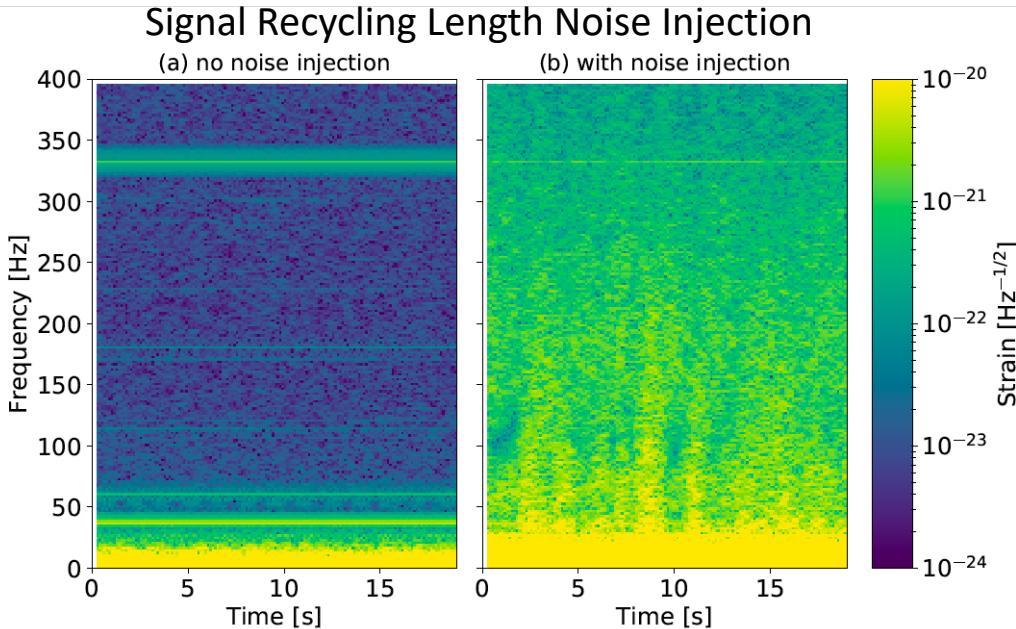
- Noise in auxiliary channels can couple in a **non-linear** way

$$h(t) = \varepsilon_B(t) + \mathcal{F} [w_1(\tau < t), \dots, w_N(\tau < t)]$$

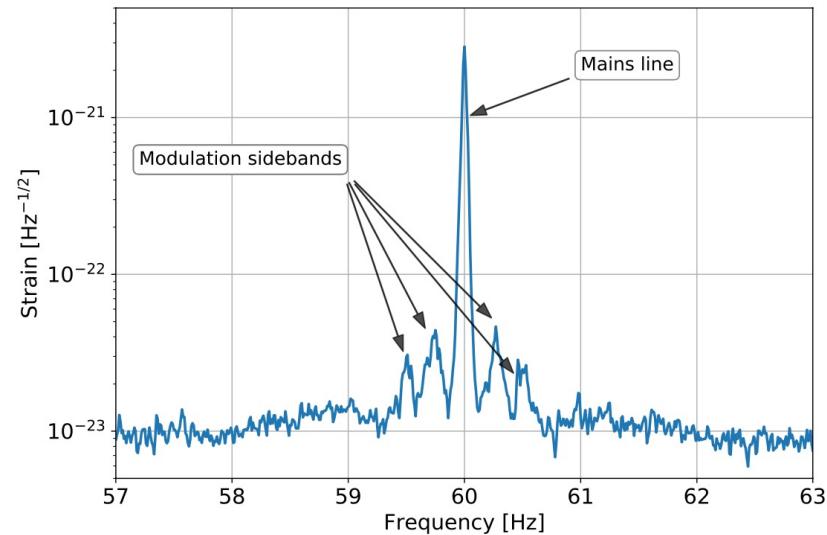
Some other background noise



- However, (in most cases\*) we expect small deviations from linearity: **quadratic coupling** or **non-stationary coupling**



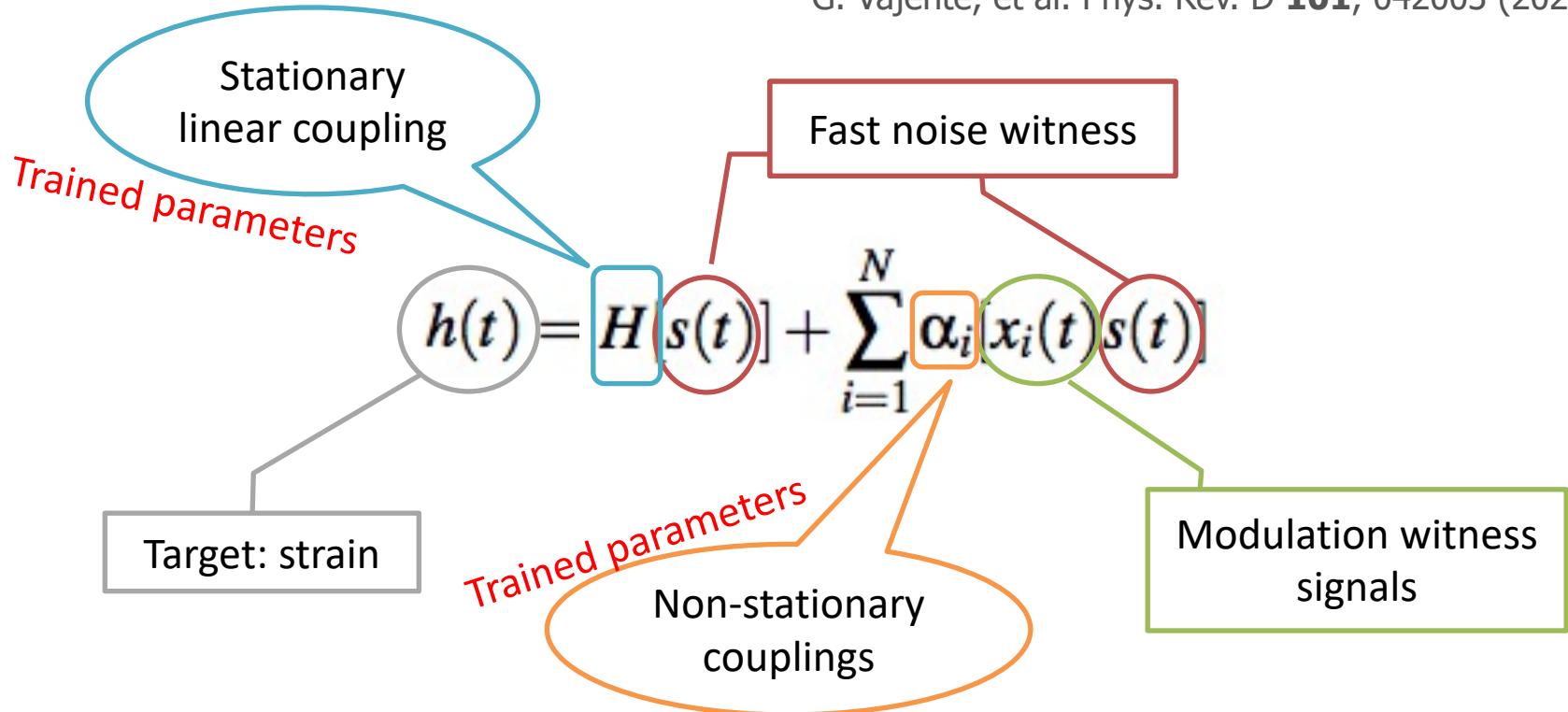
Phys. Rev. D 101, 042003 (2020)



\*scattered light is an important exception: arXiv:2007.14876 (2020), Opt. Expr. 21 10546 (2013)

- One source of noise with power in the frequency band of interest (e.g. 10-200 Hz)
- Coupling is instantaneously linear but modulated at low frequencies (e.g. <1 Hz)

G. Vajente, et al. Phys. Rev. D **101**, 042003 (2020)



- Parametrize the couplings and use ML-inspired cost optimization algorithms to minimize the noise in the strain signal

G. Vajente, et al. Phys. Rev. D **101**, 042003 (2020)

- Describe each transfer function using a **parametrized form**

- Laplace domain:  $\alpha_h(s) = \frac{\sum_{i=0}^{N_N} b_i s^i}{\sum_{j=0}^{N_D} a_j s^j}$  Or a more efficient parametrization!

$$\tilde{r}(\omega) = \tilde{h}(\omega) - \tilde{y}(\omega) = \tilde{h}(\omega) - \sum_{i=0}^N \tilde{\alpha}_i(\omega) \tilde{s}_i(\omega)$$

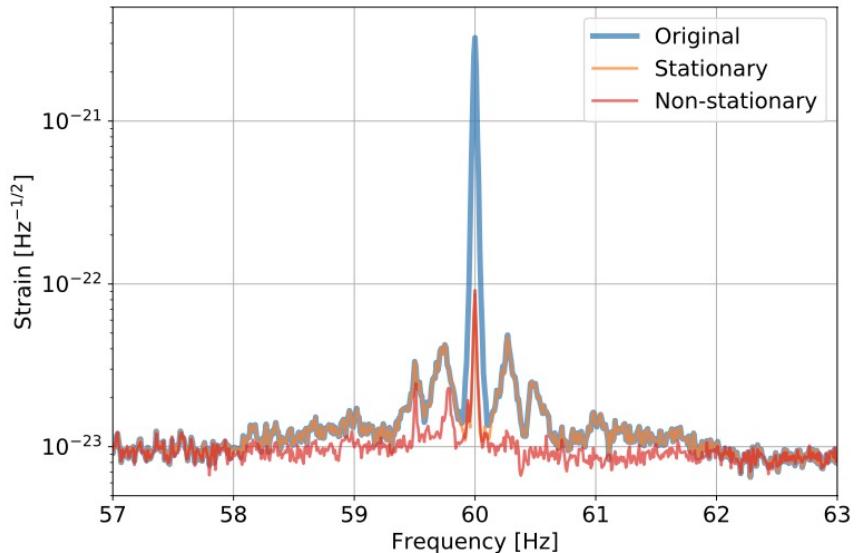
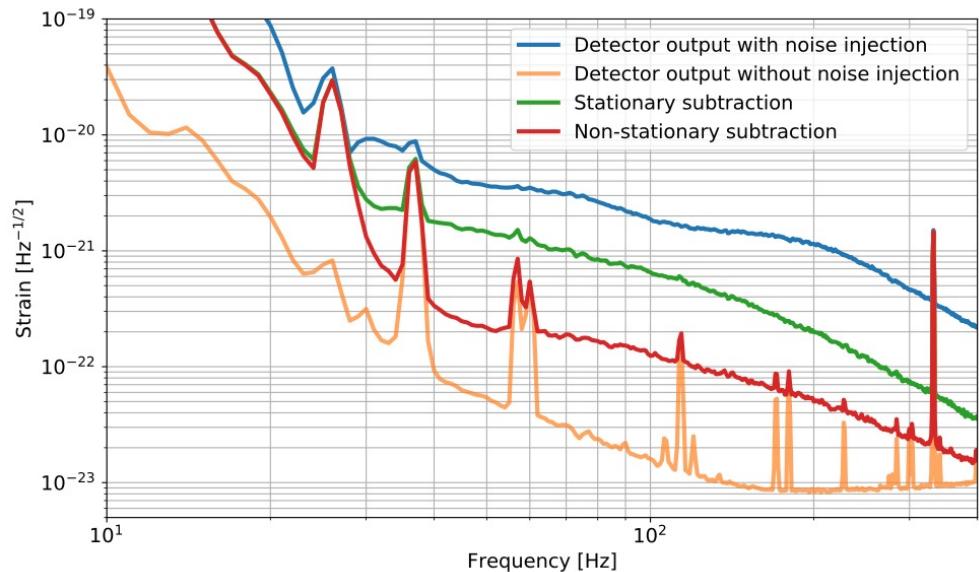
- Define a weighted frequency-integrated cost function

$$\mathcal{C}(\theta_{h,i}) = \frac{1}{\omega_2 - \omega_1} \int_{\omega_1}^{\omega_2} W(\omega) S_{rr}(\omega) d\omega \quad W(\omega) = \frac{1}{S_{hh}(\omega)}$$

- Use gradient-based optimization algorithms to minimize residual (ADAM)

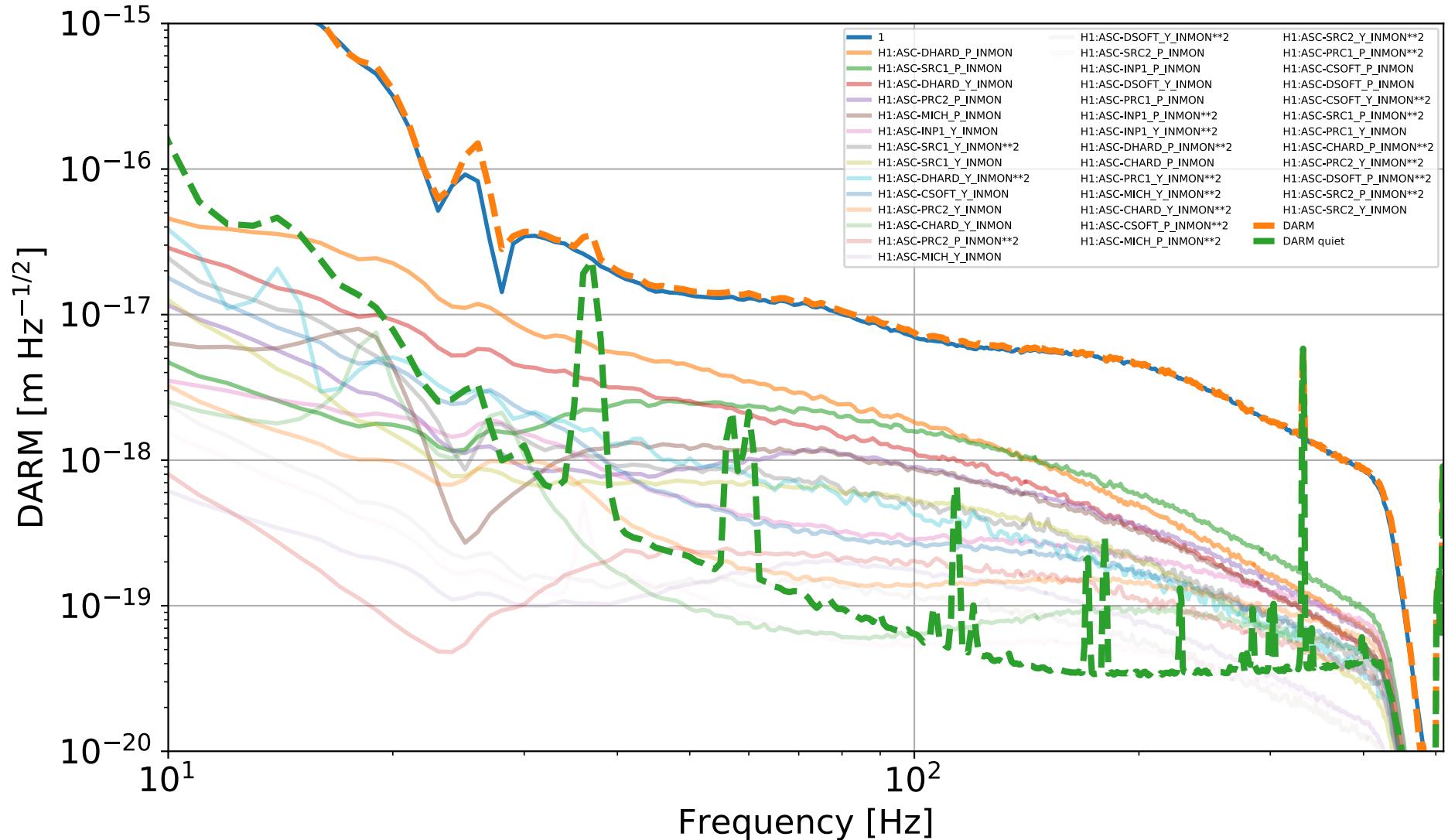
$$\frac{\partial \mathcal{C}}{\partial \theta_{h,n}} = \frac{1}{\omega_2 - \omega_1} \int_{\omega_1}^{\omega_2} W(\omega) \left[ \frac{\partial S_{rr}}{\partial \alpha_h} \cdot \frac{\partial \alpha_h}{\partial \theta_{h,n}} + \frac{\partial S_{rr}}{\partial \alpha_h^*} \cdot \frac{\partial \alpha_h^*}{\partial \theta_{h,n}} \right] d\omega$$

G. Vajente, et al. Phys. Rev. D **101**, 042003 (2020)

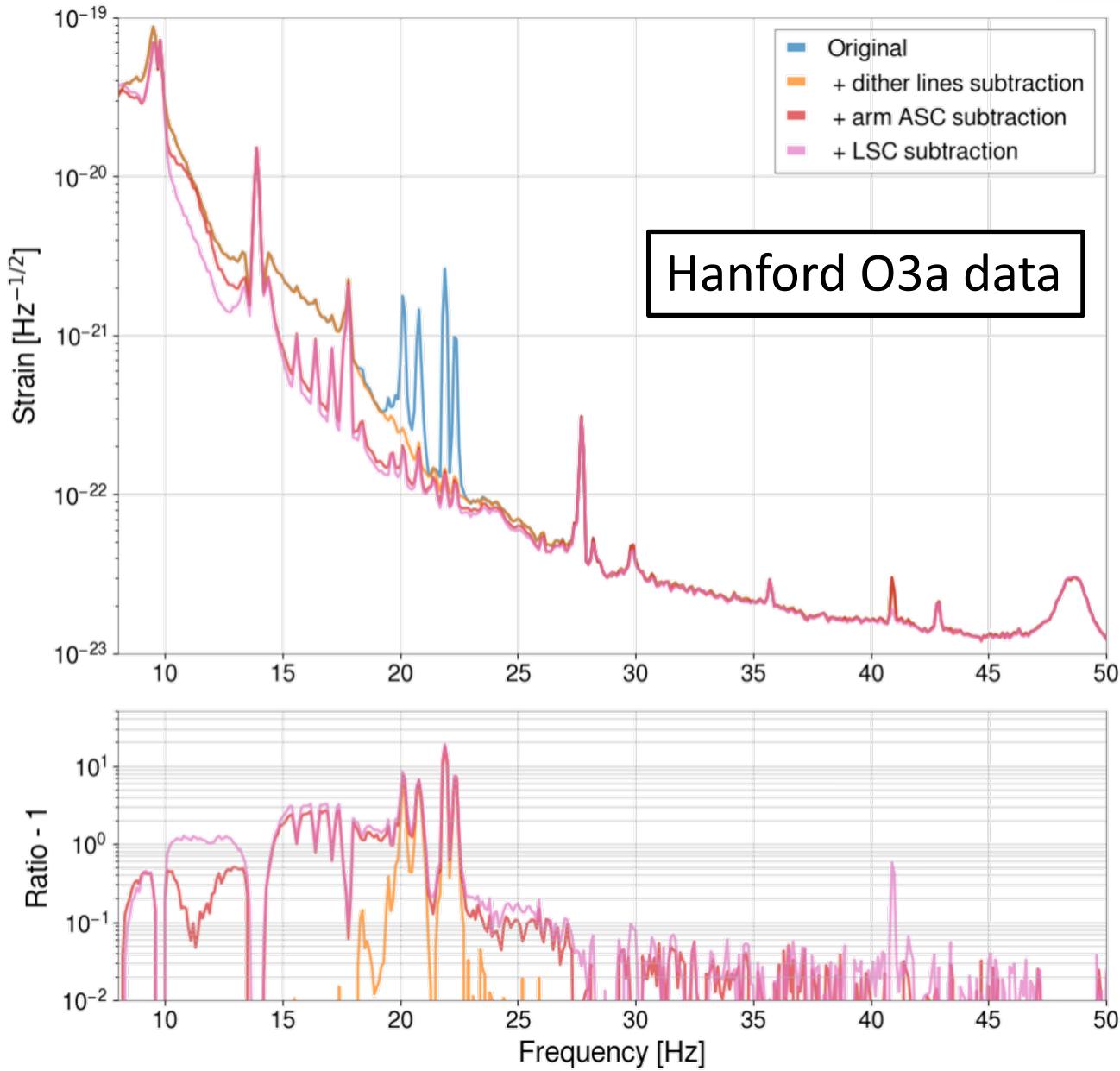


- Non-stationary noise modelling and subtraction **outperforms** the linear subtraction
- Not the most general non-linear model, but “**good enough**” to improve Advanced LIGO sensitivity

- **Interpretable results:** we can learn what degree of freedom modulates the coupling
  - **Time domain implementation**

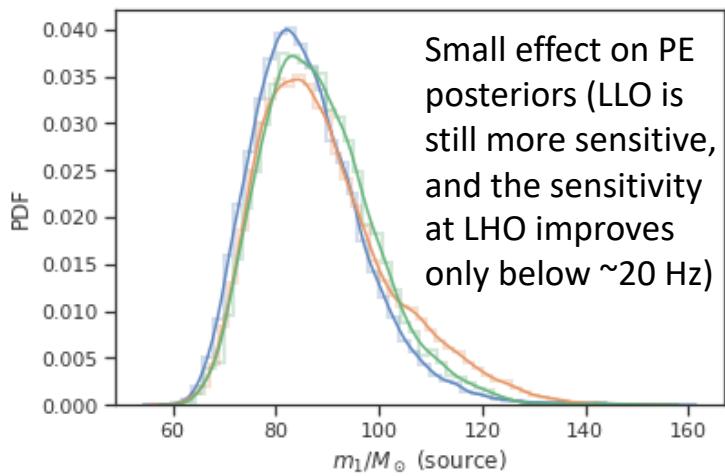
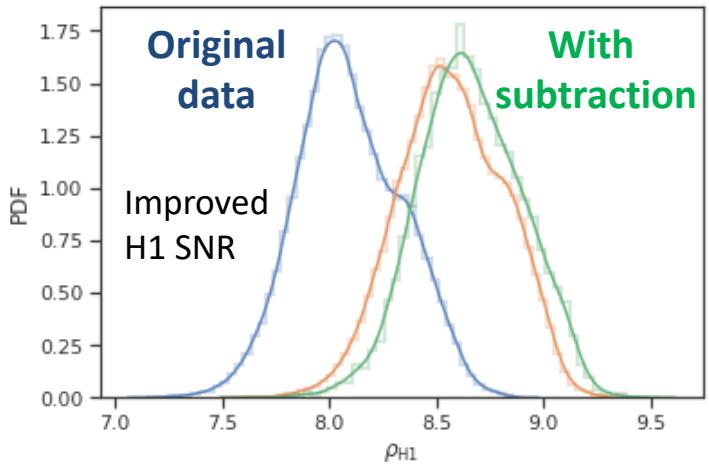


<https://alog.ligo-wa.caltech.edu/aLOG/index.php?callRep=57503>  
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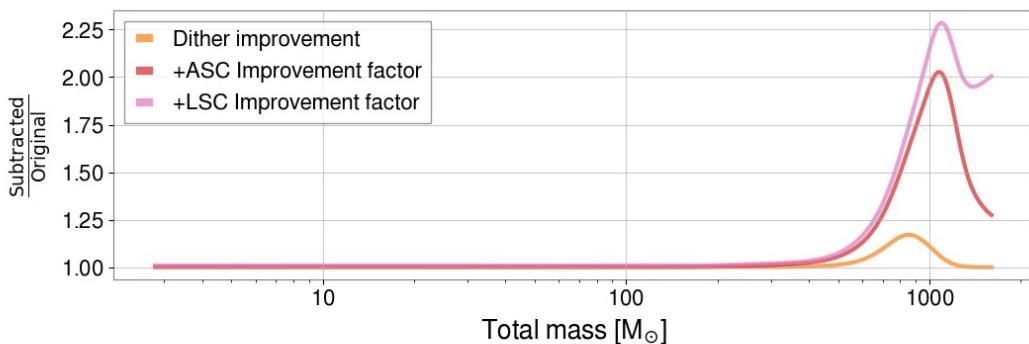
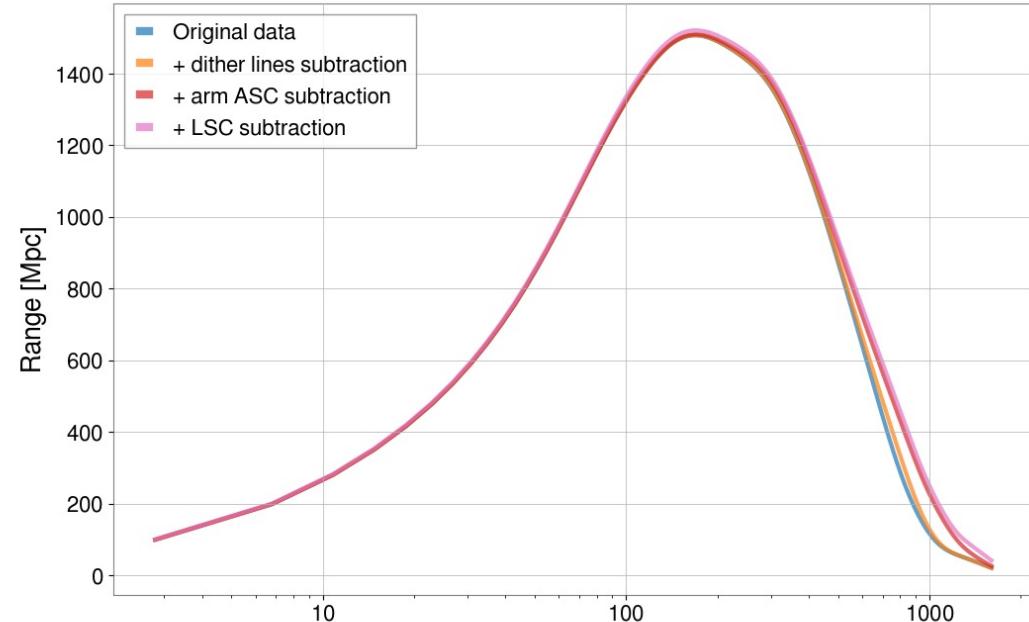


High mass event GW190521

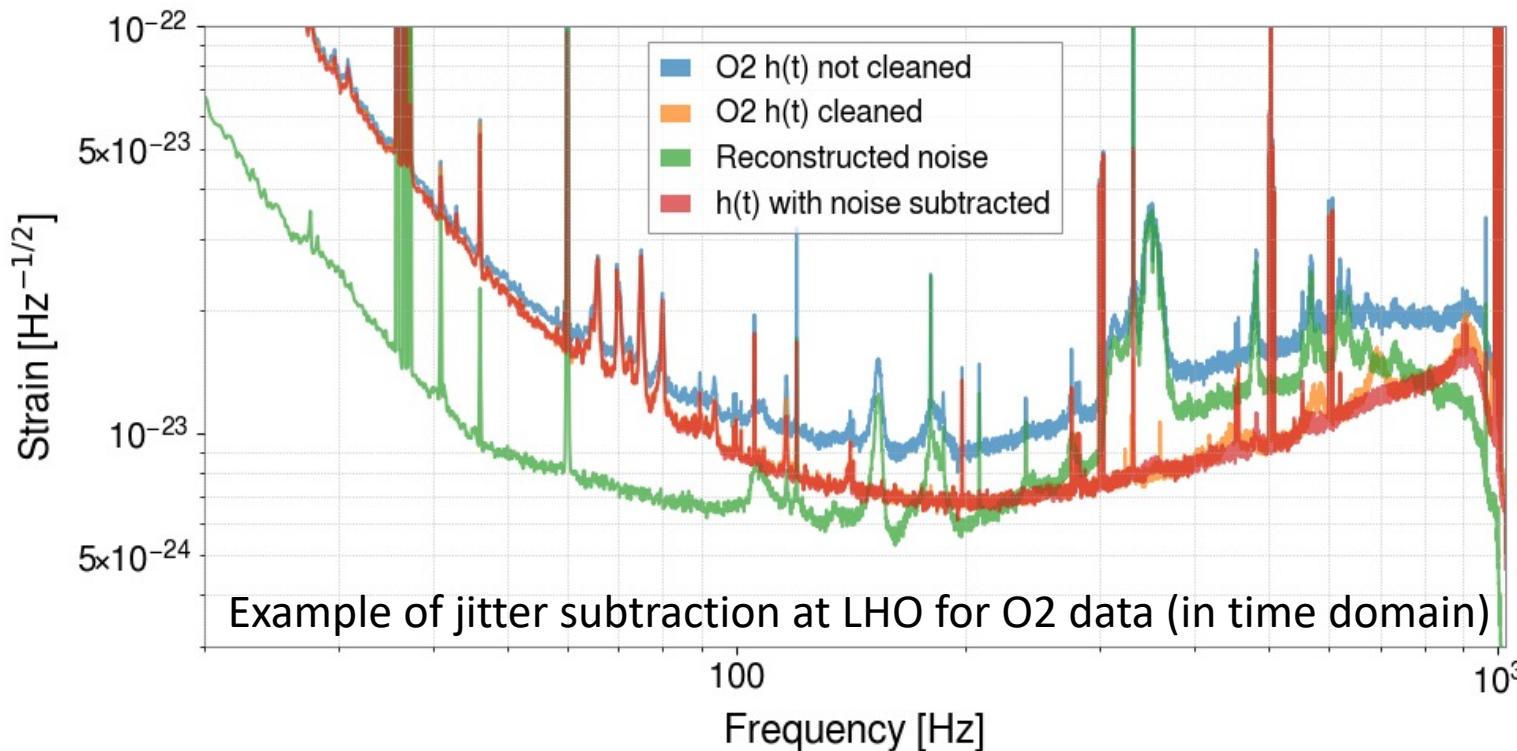
Parameter Estimation by M. Lisi



## Effect on range for high masses



- This is now a developed tool that can be used for **offline** noise subtraction (**stationary** and **non-stationary**)
- It could be adapted to run **online** by implementing IIR filters in the **real time frontends** or in **low latency pipelines**
- It could be made **adaptive** (with some more work and tests)

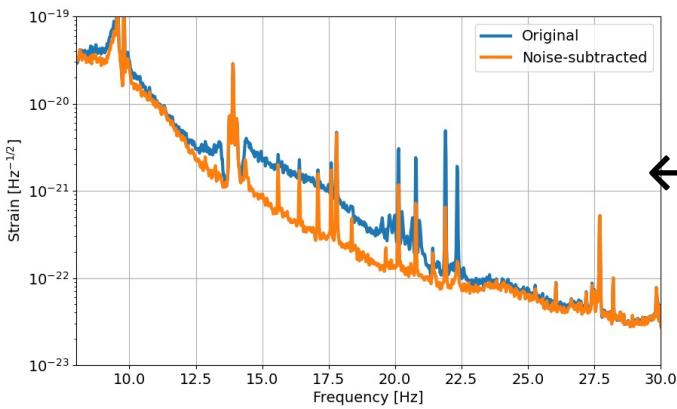


20x Modulation  
witnesses

ASC-CHARD\_P\_OUT\_DQ  
ASC-CHARD\_Y\_OUT\_DQ  
ASC-DHARD\_P\_OUT\_DQ  
ASC-DHARD\_Y\_OUT\_DQ  
ASC-CSOFT\_P\_OUT\_DQ  
ASC-DSOFT\_P\_OUT\_DQ

6x Noise witnesses

ASC-INP1\_P\_INMON  
ASC-INP1\_Y\_INMON  
ASC-MICH\_P\_INMON  
ASC-MICH\_Y\_INMON  
ASC-PRC1\_P\_INMON  
ASC-PRC1\_Y\_INMON  
ASC-PRC2\_P\_INMON  
ASC-PRC2\_Y\_INMON  
ASC-SRC1\_P\_INMON  
ASC-SRC1\_Y\_INMON  
ASC-SRC2\_P\_INMON  
ASC-SRC2\_Y\_INMON  
ASC-DHARD\_P\_INMON  
ASC-DHARD\_Y\_INMON  
ASC-CHARD\_P\_INMON  
ASC-CHARD\_Y\_INMON  
ASC-DSOFT\_P\_INMON  
ASC-DSOFT\_Y\_INMON  
ASC-CSOFT\_P\_INMON  
ASC-CSOFT\_Y\_INMON



**Sum and  
subtract ←  
from h(t)**

6x Stationary signals + 120x Modulated signals

ASC-CHARD\_P\_OUT\_DQ  
ASC-CHARD\_Y\_OUT\_DQ  
ASC-DHARD\_P\_OUT\_DQ  
...  
ASC-CHARD\_P\_OUT\_DQ \* ASC-INP1\_P\_INMON  
ASC-CHARD\_P\_OUT\_DQ \* ASC-INP1\_Y\_INMON  
ASC-CHARD\_P\_OUT\_DQ \* ASC-MICH\_P\_INMON  
...  
ASC-CHARD\_Y\_OUT\_DQ \* ASC-INP1\_P\_INMON  
ASC-CHARD\_Y\_OUT\_DQ \* ASC-INP1\_Y\_INMON  
ASC-CHARD\_Y\_OUT\_DQ \* ASC-MICH\_P\_INMON  
...

$\alpha_1$  [ASC-CHARD\_P\_OUT\_DQ ]  
 $\alpha_2$  [ASC-CHARD\_Y\_OUT\_DQ ]  
 $\alpha_3$  [ASC-DHARD\_P\_OUT\_DQ ]  
...  
 $\alpha_7$  [ASC-CHARD\_P\_OUT\_DQ \* ASC-INP1\_P\_INMON ]  
 $\alpha_8$  [ASC-CHARD\_P\_OUT\_DQ \* ASC-INP1\_Y\_INMON ]  
 $\alpha_9$  [ASC-CHARD\_P\_OUT\_DQ \* ASC-MICH\_P\_INMON ]  
...  
 $\alpha_{27}$  [ASC-CHARD\_Y\_OUT\_DQ \* ASC-INP1\_P\_INMON ]  
 $\alpha_{28}$  [ASC-CHARD\_Y\_OUT\_DQ \* ASC-INP1\_Y\_INMON ]  
 $\alpha_{29}$  [ASC-CHARD\_Y\_OUT\_DQ \* ASC-MICH\_P\_INMON ]  
...

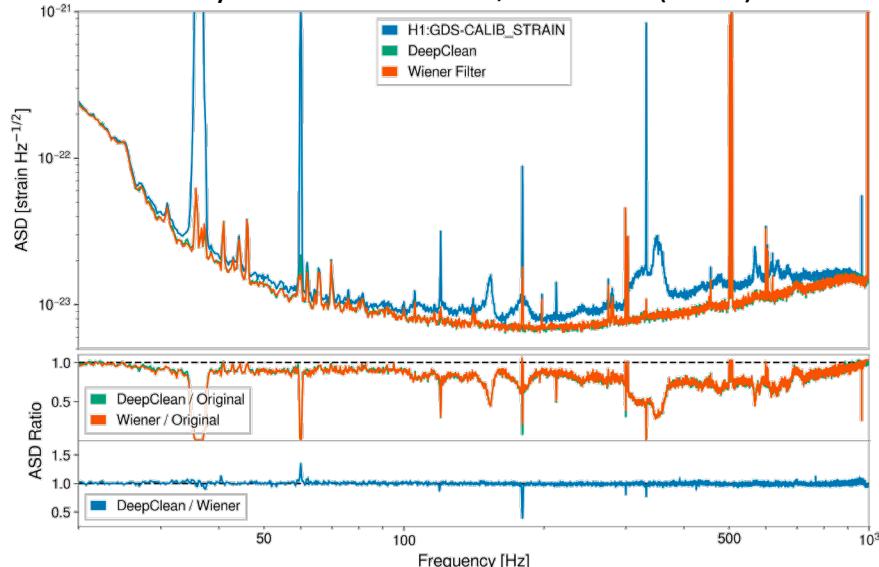
126x IIR filters (each is 12<sup>th</sup> order)

- Implemented as a python code, with **mature user interface** [**NonSENS: Non-Stationary Estimation of Noise Subtraction**]  
<https://git.ligo.org/gabriele-vajente/nonsens>
- See in the examples folder for instructions and examples on how to use the package
- The algorithm compute coefficients of filters in Laplace or z-domain: direct implementation of **time-domain subtraction**
  - It can be done online and even adaptively
- The source of coupling modulation is explicitly exposed: allows understanding of the IFO dynamics

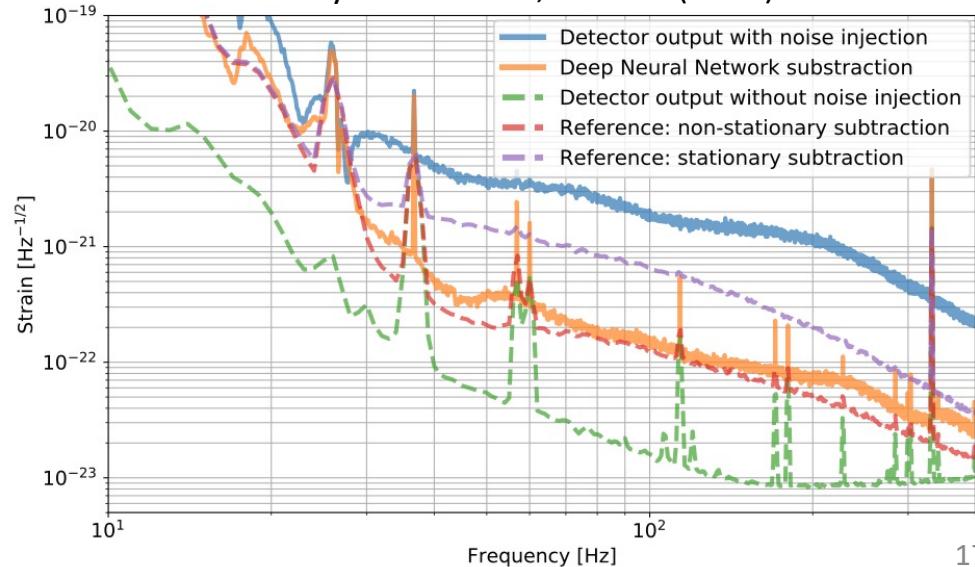
- In theory, a **Deep Neural Network** can express arbitrarily complicated non-linear functions
- Need to include time-dependency and history, therefore Convolutional NN or Recurrent NN
- DNN shown to reproduce linear subtraction performance
- Although a RNN is in principle capable of expressing a slowly modulated coupling, it's hard to train, at the end of training it underperforms, and it is not easily interpretable

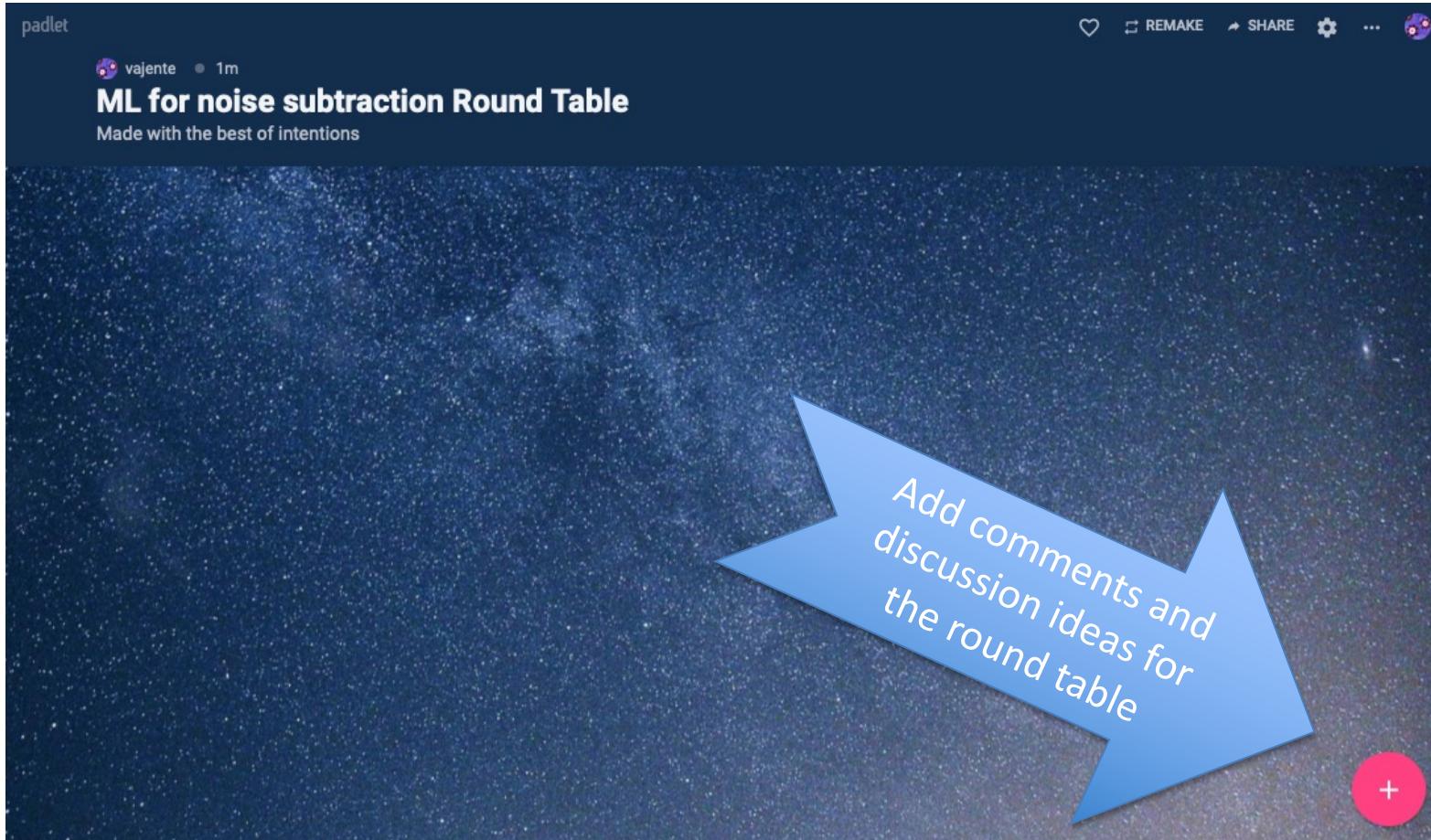
**Sampling problem:** need fast sampling to capture noise and long times to capture variations.  
**Training problem:** CNN and RNN do not scale well with very long time series

Phys. Rev. Research 2, 0330666 (2020)



Phys. Rev. D 101, 042003 (2020)





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