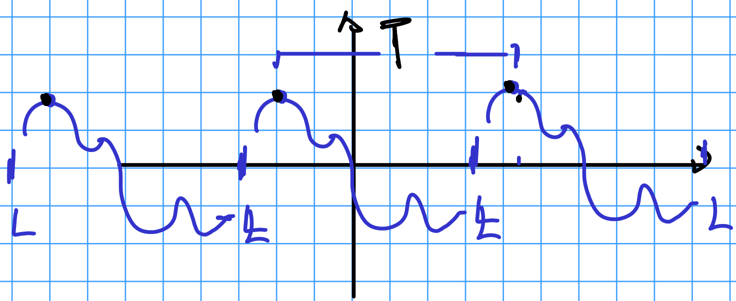
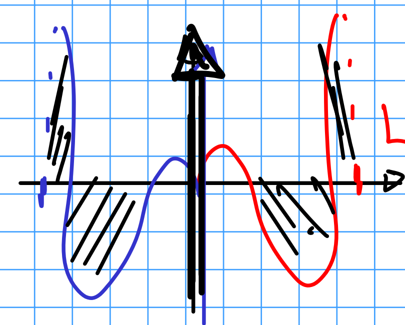
$f: \mathbb{R} \to \mathbb{R}$ es periodica con periodo T ssi

$$f(t+T) = f(t)$$

Solo necesito $f: [-T/2, T/2] \to \mathbb{R}$



$L = T/2$ (semi periodo)

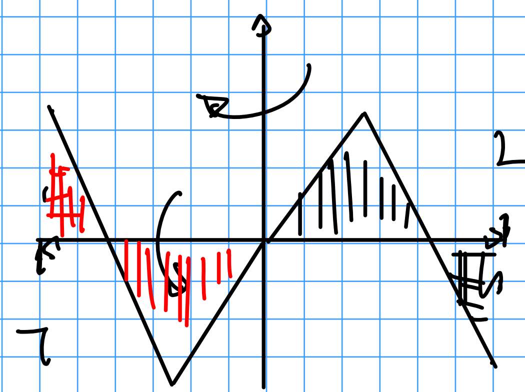


#1. f es par si $f(t) = f(-t)$

$$\int_{-L}^{L} f(x)\,dx = 2\int_{0}^{L} f(x)\,dx$$

#2. f es impar si $-f(t) = f(-t)$

$$\int_{-L}^{L} f(x)\,dx = 0$$



El producto interno en $\mathcal{F} = \{ f : [-L, L] \to \mathbb{C} \mid f$ es continua por partes $\}$

$$\langle f \mid g \rangle = \int_{-L}^{L} f^*(t) \cdot g(t)\,dt$$

Quiere decir que $\langle \alpha f + \beta g | h \rangle = \alpha^* \langle f | h \rangle + \beta^* \langle g | h \rangle$

$$\| f \| = \sqrt{\langle f | f \rangle} = \int_{-L}^{L} f^* f \, dt = \int_{-L}^{L} |f|^2 dt \geq 0$$

norma de f ó su energía

Recuerde $z = a + ib$ $\Rightarrow$ $|z|^2 = a^2 + b^2$

$z^* = a - ib$ $\quad = z^* \cdot z$

f es ortogonal a g si $\langle f | g \rangle = 0$.

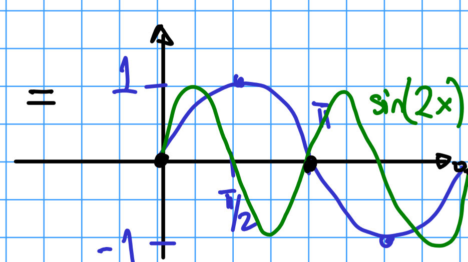
$\{e_1, e_2, \dots\}$ son ortogonales si $\langle e_i | e_j \rangle = 0$ cuando $i \neq j$.

Si $\phi_m = \cos\left(\frac{m\pi x}{L}\right)$, ϕ_m son ortogonales en $[-L, L]$

$\omega_m = \frac{m\pi}{L}$.

dem: Sup $m \neq n$. $\langle \phi_m | \phi_n \rangle = \int_{-L}^{L} \hat{c}(\omega_m x)^* \hat{c}(\omega_n x) \, dx$

$= \frac{1}{2} \int_{-L}^{L} \left[\hat{c}(\omega_m + \omega_n) x + \hat{c}(\omega_m - \omega_n) x \right] dx =$



$$\frac{1}{2}\left(\frac{\hat{S}(\omega_m+\omega_n)x}{\omega_m+\omega_n}+\frac{\hat{S}(\omega_m-\omega_n)x}{\omega_m-\omega_n}\right)\Bigg|_{-L}^{L}$$

$$\frac{1}{2}\left[\frac{\cancel{\hat{S}(m+n)\pi}^{\,0}}{\omega_m+\omega_n}+\frac{\cancel{\hat{S}(m-n)\pi}^{\,0}}{\omega_m-\omega_n}-\left(\frac{\hat{S}(m+n)(-\pi)}{\omega_m+\omega_n}+\frac{\hat{S}(m-n)(-\pi)}{\omega_m-\omega_n}\right)\right]=0$$

$$(\omega_m+\omega_n)L=$$

$$\left(\frac{\pi m}{L}+\frac{\pi n}{L}\right)L=$$

$$=\pi\underbrace{(m+n)}_{\in\mathbb{Z}}$$