Ejemplo: $\| e^{imx} \|^2 = \langle e^{imx} | e^{imx} \rangle$ en $[-\pi, \pi]$

$$e^{imx} = \hat{c}mx + i\hat{s}mx$$
$$(e^{imx}) = \hat{c}mx - i\hat{s}mx$$
$$e^{-imx}$$

$$= \int_{-\pi}^{\pi} (e^{imx})^* e^{imx}\, dx =$$

$$= \int_{-\pi}^{\pi} e^{-imx} e^{imx}\, dx = \int_{-\pi}^{\pi} dx = 2\pi.$$

Ejemplo: $\phi_m = \cos(\omega_m x)$ en $[-L, L]$ son ortogonales

$$\omega_m = \frac{\pi m}{L}.$$

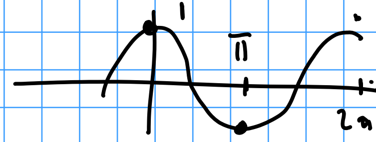
$$\cos(\omega_m x) = \frac{1}{2}\left(e^{i\omega_m x} + e^{-i\omega_m x}\right)$$

$$\int_{-L}^{L} \cos(\omega_m x) \cos(\omega_n x)\, dx \qquad \text{para } m \neq n$$

$$= \frac{1}{4} \int_{-L}^{L} (e^{i\omega_m x} + e^{-i\omega_m x})(e^{i\omega_n x} + e^{-i\omega_n x})\, dx$$

$$= \frac{1}{4} \int_{-L}^{L} \left[e^{i(\omega_m + \omega_n)x} + e^{i(\omega_n - \omega_m)x} + e^{i(\omega_m - \omega_n)x} + \right.$$

$$e^{-i(\omega_m+\omega_n)x}\Big]\,dx =$$



$$\Bigg[\int_{-L}^{L} e^{\pm i(\omega_m \pm \omega_n)x}\,dx = \left.\frac{e^{\pm i(\omega_m \pm \omega_n)x}}{\pm i(\omega_m \pm \omega_n)}\right|_{-L}^{L}$$

$$(\omega_m \pm \omega_n)L = \pi(m \pm n) \Rightarrow e^{\pm\pi(m\pm n)i} =$$

$$\cos\big(\pi\underbrace{(m\pm n)}_{\in\mathbb{Z}}\big) \pm i\sin\big(\pi\underbrace{(m\pm n)}_{\in\mathbb{Z}}\big) = (-1)^{m\pm n} \quad \text{sust.}$$

$$\underbrace{\cancel{\frac{(-1)^{m+n}-(-1)^{m+n}}{i(\omega_m+\omega_n)}}}_{0} + \underbrace{\cancel{\frac{(-1)^{n-m}-(-1)^{n-m}}{i(\omega_n-\omega_m)}}}_{0} + \underbrace{\cancel{\frac{(-1)^{m-n}-(-1)^{m-n}}{i(\omega_m-\omega_n)}}}_{0} + \underbrace{\cancel{\frac{(-1)^{m+n}-(-1)^{m+n}}{-i(\omega_m+\omega_n)}}}_{0} \;\square$$

$$= 0.\ \square$$

Integración por partes:

$$\frac{d}{dx}(f\cdot g) = \frac{df}{dx}\cdot g + f\cdot\frac{dg}{dx} \quad // \int \ldots$$

$$f\cdot g = \int g\,df + \int f\,dg$$

$$\Rightarrow \int f\,dg = f\cdot g - \int g\,df$$

$$dv = g \qquad u = \frac{df}{dx} \qquad\qquad \int g = G_1$$

$$fg - G_1\frac{df}{dx} + \int G_1 \frac{d^2f}{dx^2}\,dx$$

$$dv = G_1 \qquad u = \frac{d^2f}{dx^2} \qquad\qquad G_2 = \int G_1\,dx$$

$$fg - G_1\frac{df}{dx} + G_2\frac{d^2f}{dx^2} - \int G_2 \cdot \frac{d^3f}{dx^3}\,dx = \ldots$$

$$\int f\cdot dg \qquad \begin{array}{ccc} + & f & dg \\ & \downarrow & \downarrow \int \\ - & f' & g \\ & \downarrow & \downarrow \int \\ + & f'' & G_1 \\ & \vdots & \vdots \end{array}$$

Ejemplo: Calcule $\int_{-\pi}^{\pi} (x^3-1)\, e^{imx}\, dx$

Truco de Legendre:

$+$	$u = x^3 - 1$	$dv = e^{imx}$
$-$	$3x^2$	$\frac{1}{im} e^{imx}$
$+$	$6x$	$\frac{-1}{m^2} e^{imx}$
$-$	6	$\frac{-1}{im^3} e^{imx}$
$+$	0	$\frac{1}{m^4} e^{imx}$

$$e^{imx}\Big|_{-\pi}^{\pi} = \cos m\pi + i\underbrace{\sin(m\pi)}_{0} - \big(\cos(m\pi) - i\underbrace{\sin(m\pi)}_{0}\big)$$

$$= 0$$

$$\left(\frac{x^3-1}{im} e^{imx} + \frac{3x^2}{m^2} e^{imx} - \frac{6x}{im^3} e^{imx} - \frac{6}{m^4} e^{imx}\right)\Bigg|_{-\pi}^{\pi} = 0.$$

$$\int_0^{\pi} (1-x^2)\cos x \, dx = \quad \text{por Legendre}$$

	u	dv
$+$	$1-x^2$	$\cos x$
$-$	$-2x$	$\sin x$
$+$	-2	$-\cos x$
$-$	0	$-\sin x$

$$= \left((1-x^2)\sin x - 2x\cos x + 2\sin x\right)\Big|_0^{\pi}$$

$$(0 + 2\pi + 0) - (0 - 0 + 0) = 2\pi$$

$$\int_0^{\pi} (1-x^2)\cos x \, dx = 2\pi \quad \square$$

$$\phi_m = \cos\frac{m\pi x}{L} \qquad \psi_m = \sin\frac{m\pi x}{L}. \quad \text{en } [-L, L]$$

#1. $\langle \phi_m | \psi_n \rangle$

#2. ψ_m son ortogonales.

#3. Calcular $\|\psi_m\|^2$

#4. $\displaystyle\int_0^L \sin\left(\frac{16\pi x}{L}\right)\cos\left(\frac{7\pi x}{L}\right) dx$