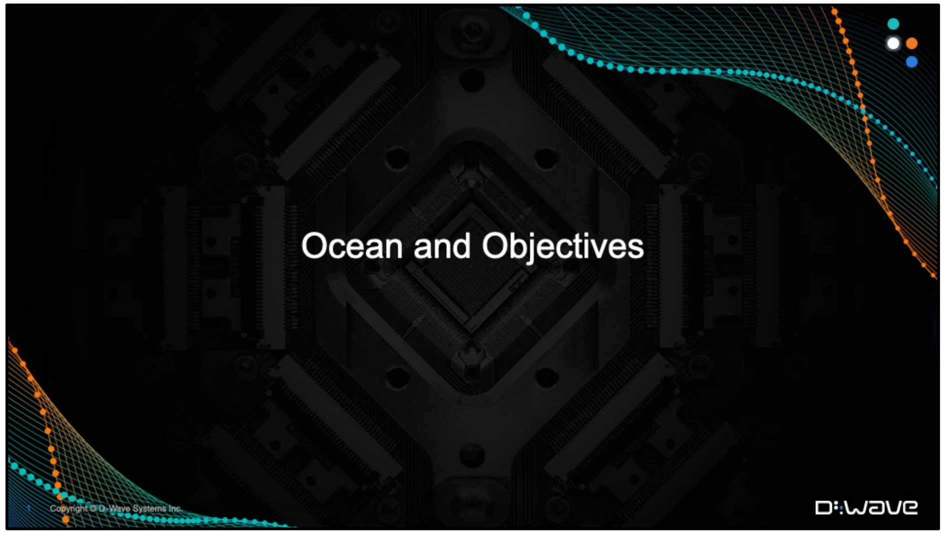
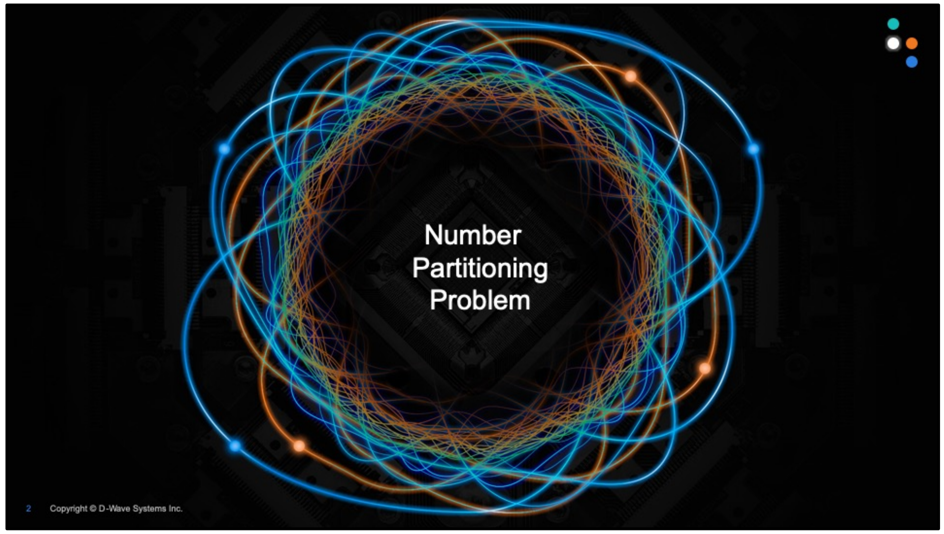
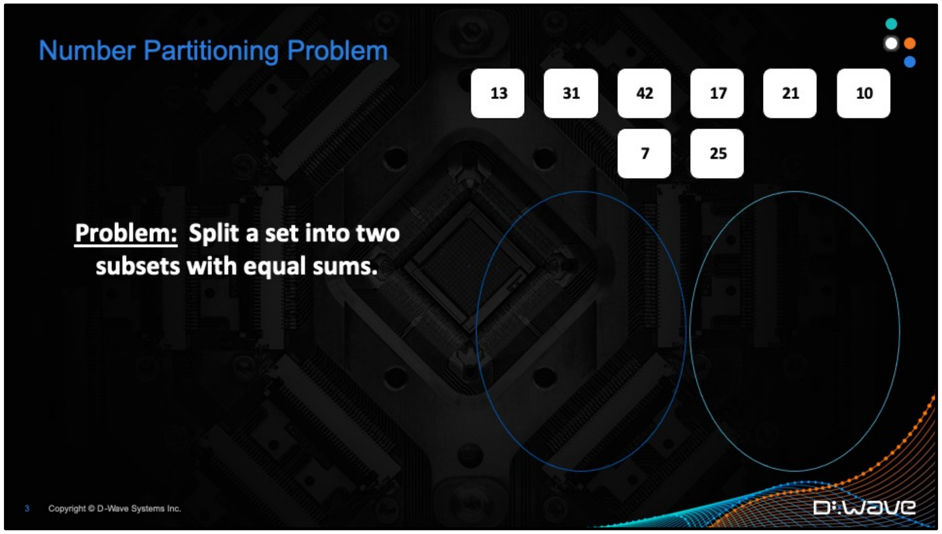


In this module, we'll look at how to write Ocean programs for QUBOs that have an objective function and no constraints.

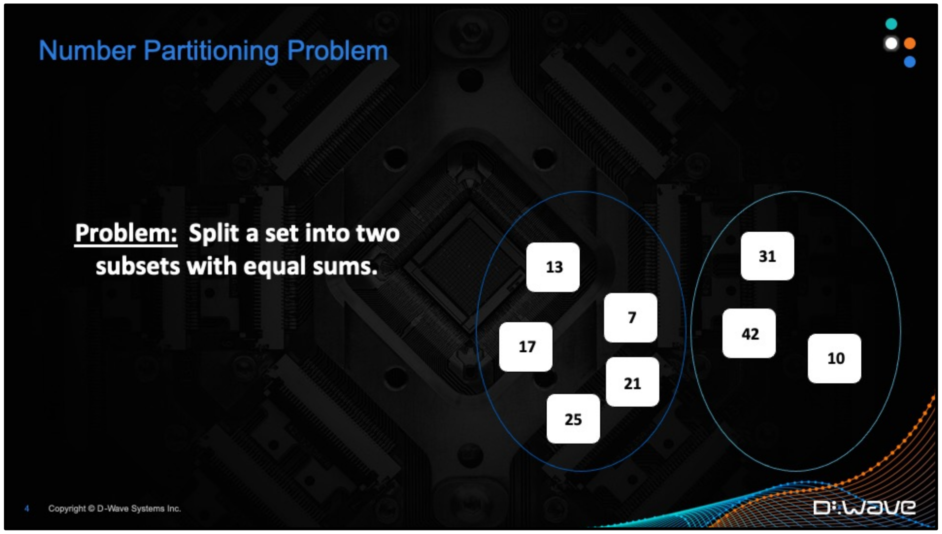


We’ll go through this module by exploring a problem called the number partitioning problem.

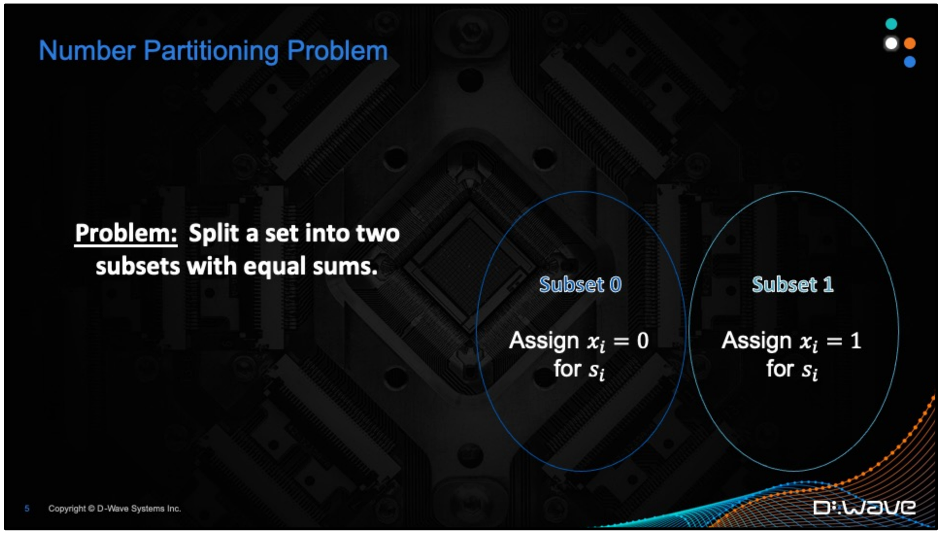
During this lecture we’ll go over the QUBO formulation for this problem and your homework assignment will be to write the Ocean program to run it on the QPU.



The number partitioning problem asks you to take a set of numbers and split it into two subsets with equal sums.



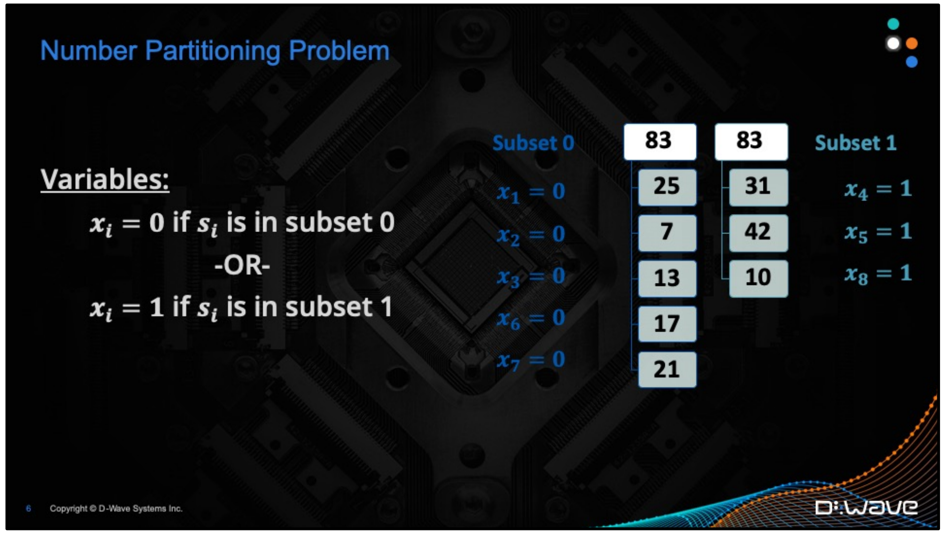
For example, if we have this set of 8 numbers we might split it as shown here. With this split each subset has a sum of 83.



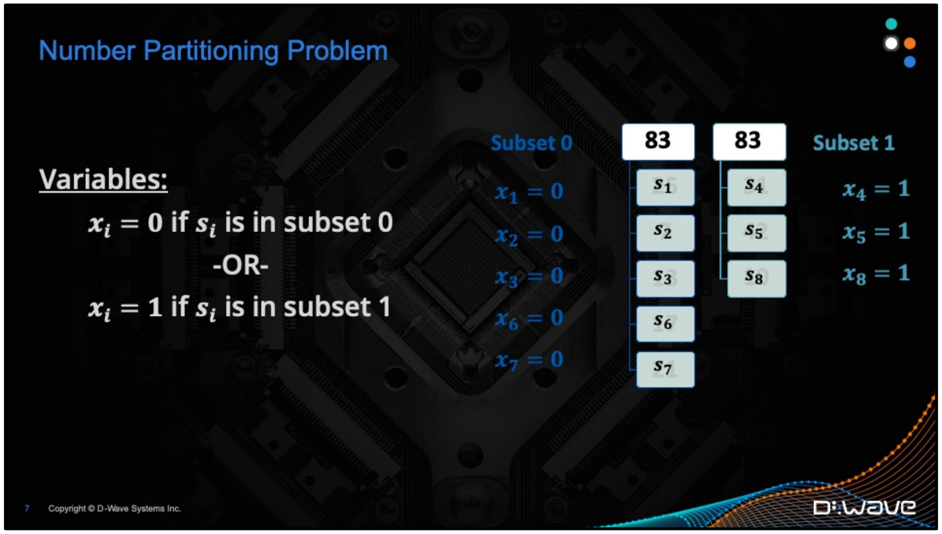
To write our QUBO for this problem we'll choose our binary variables to help us split the complete set into two subsets.

We label the subsets "0" and "1" and make a binary variable for each number in the complete set.

We put all of the numbers with binary variable equal to 0 in Subset 0, and all of the numbers with binary variable equal to 1 in Subset 1.



For our complete set of 8 numbers that we saw earlier, this is how our partition into Subset 0 and Subset 1 looks.

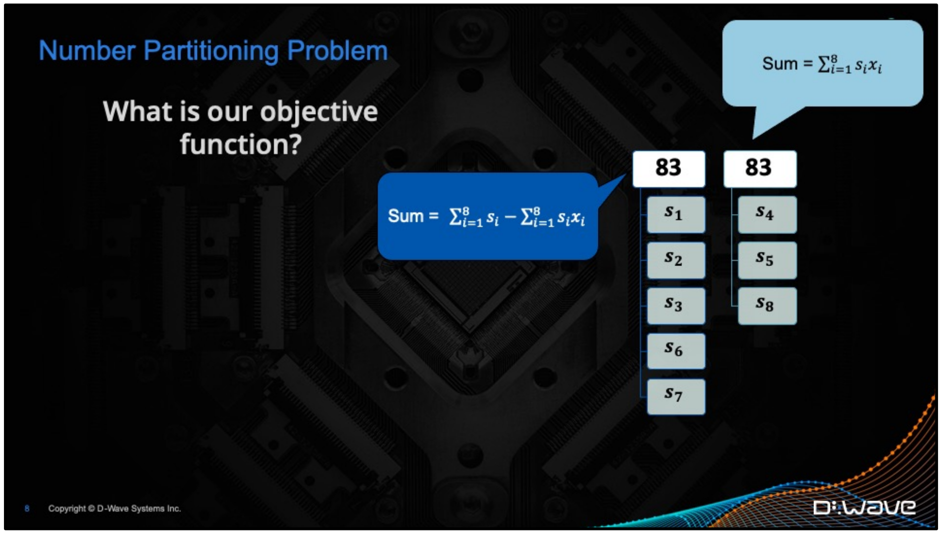


Since we're writing a program we might want to make it more general so that it will work for any set of numbers instead of this specific list.

To do that we can use the variable s_i to represent a number in our set.

Instead of a list of numbers like 13, 31, 42, 17 and so on, we have the set s1, s2, s3, s4, and so on.

Keep in mind that these s variables are fixed numbers that are handed to us, while the x variables are our binary variables that the system will return.

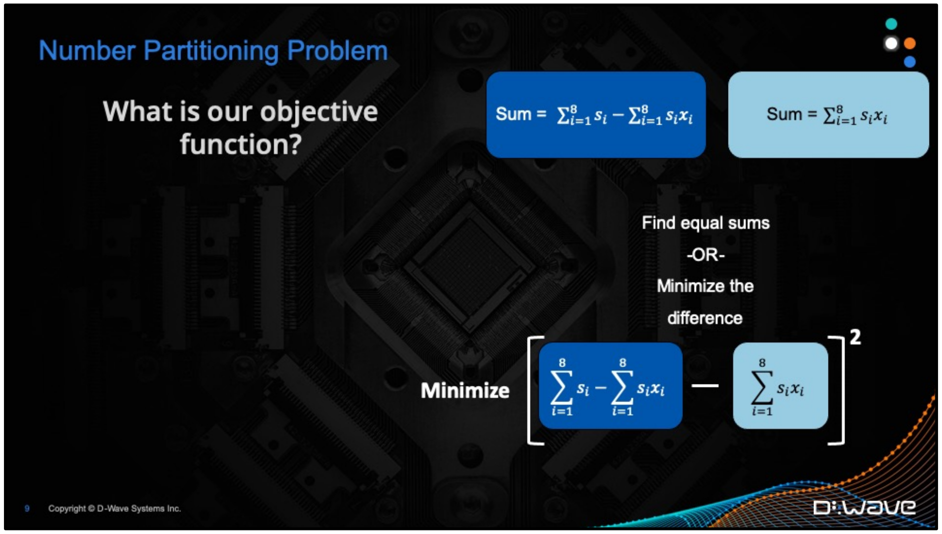


How do we write our objective function using these variables that we've set up? We need to think of a way get the sum of each subset so that we can check if they are equal.

Starting with Subset 1 is a bit easier, since we can do a weighted sum as shown. This summation of si times xi will add up only the numbers in subset 1.

For subset 0 we have to be a little bit more creative. There are a few ways to think about this, but the easiest way is to remember that subset 0 is everything that is NOT in subset 1.

If we add up all of the numbers and subtract the ones in subset 1 we are left with sum of subset 0. So here we see the summation of all of the numbers si minus what we figured out was the sum of subset 1.



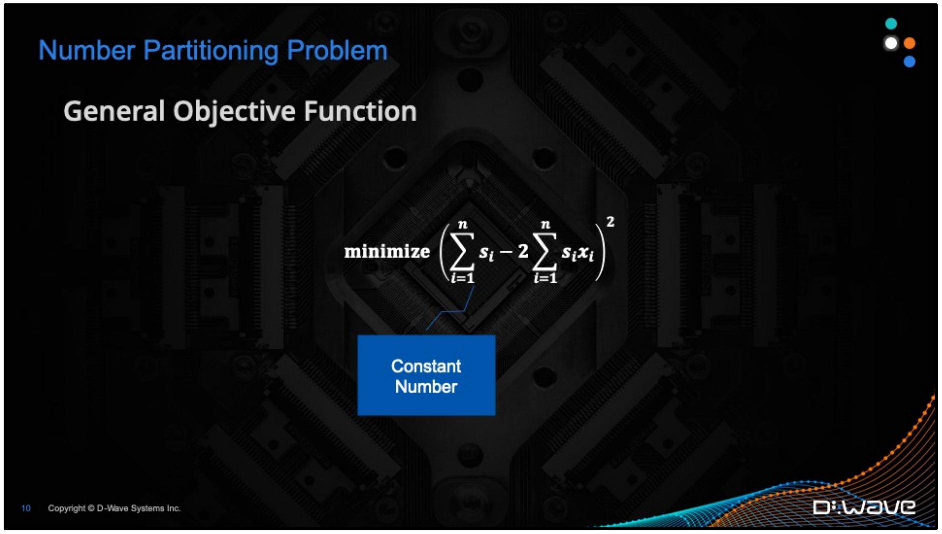
Now to figure out our objective function we think back to the problem at hand.

When are these two sums equal?

Finding equal sums is the same as minimizing the difference, so we can use subtraction.

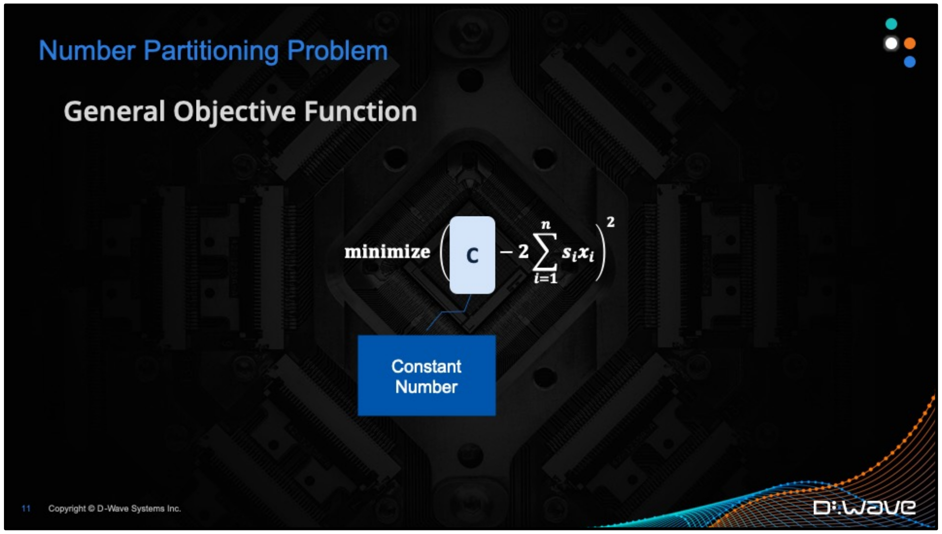
Our goal is to figure out when this difference is 0, so we need to square this expression to make sure that the smallest possible value is 0.

You can think about this like taking the absolute value of the difference – the smallest possible difference here is 0 which is just what we're looking for.



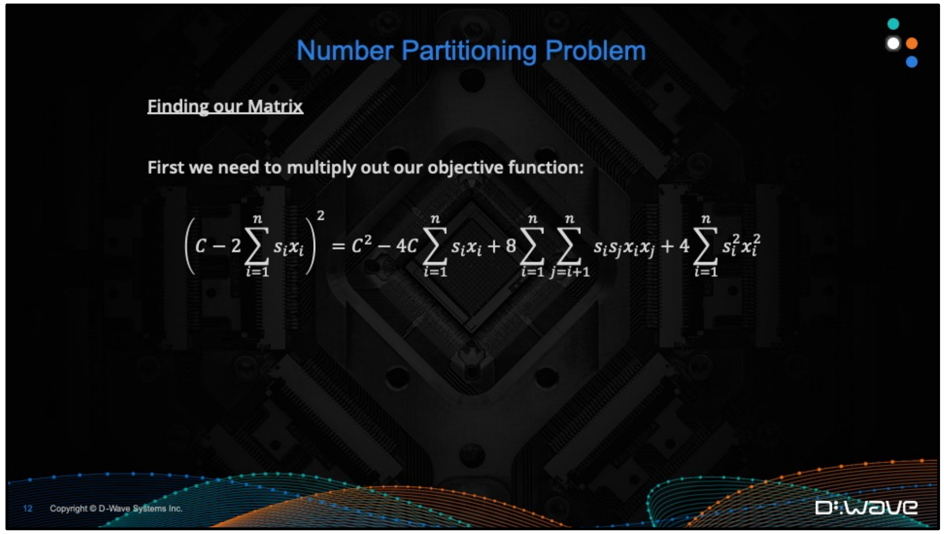
Now that we have our objective function we need to get it ready for an Ocean program.

Our requirement is that we have the linear and quadratic coefficients in a matrix, so we need to multiply this all out and simplify to figure out what those coefficients are.



To make things easier on the eyes, I'll replace this summation of the si's with a single constant C.

Since there are no binary variables in this term it will just be a constant number that is found by adding up all of the numbers in our complete set.

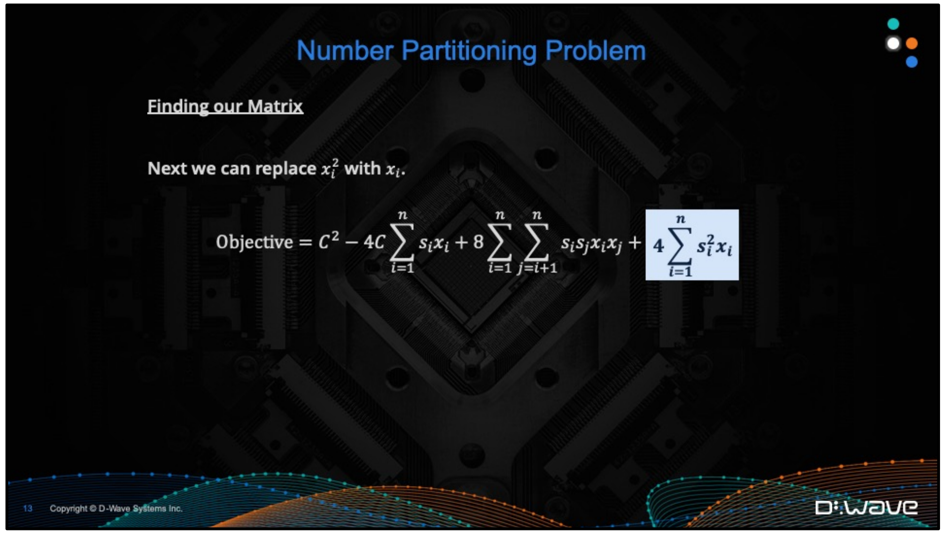


Since the math gets a bit hairy I'll step through the multiplication and simplifying here for you.

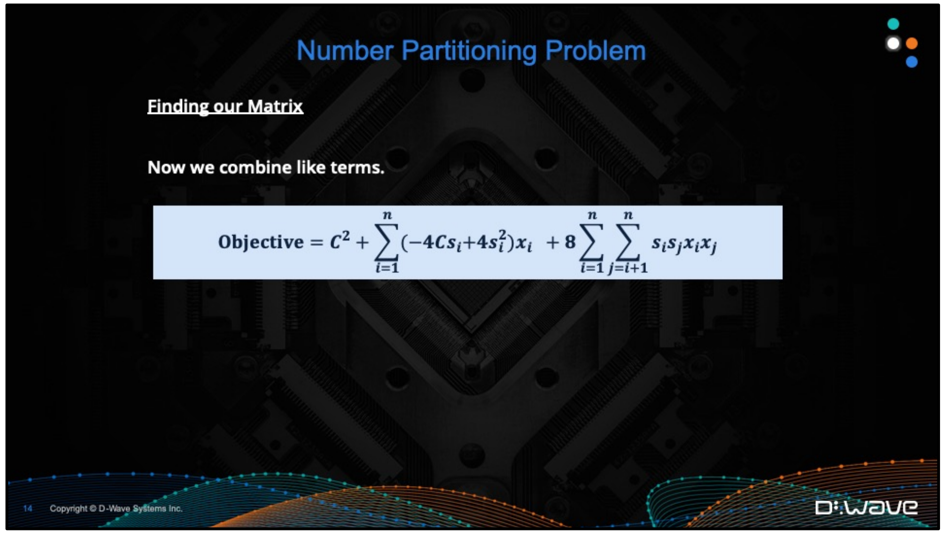
Remember that Wolfram Alpha can come in handy for this!

The first step is to multiply everything out.

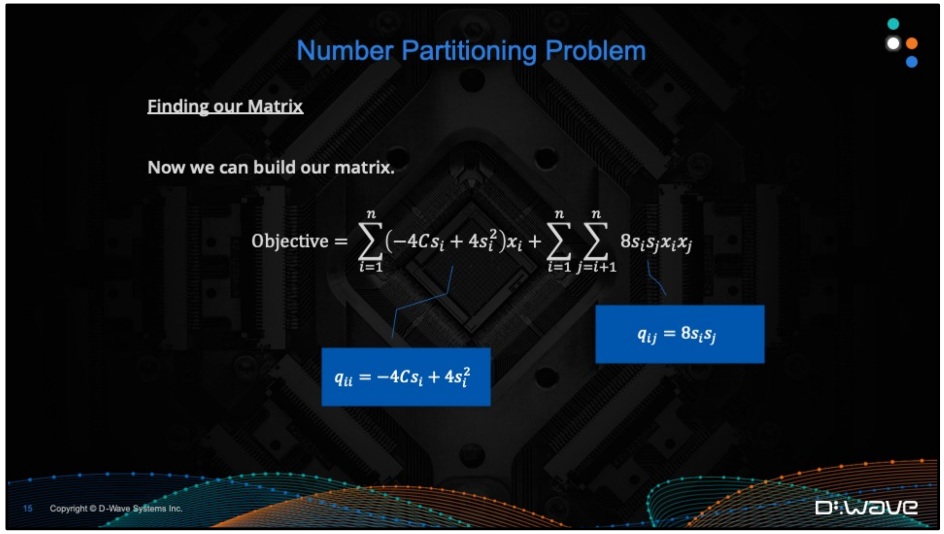
When we square this polynomial we get this big expression on the right, and remember that formula is on the written guide in the math refresh module.



Next we can replace any squared binary variables with the linear version since 0 squared equals 0 and 1 squared equals 1.

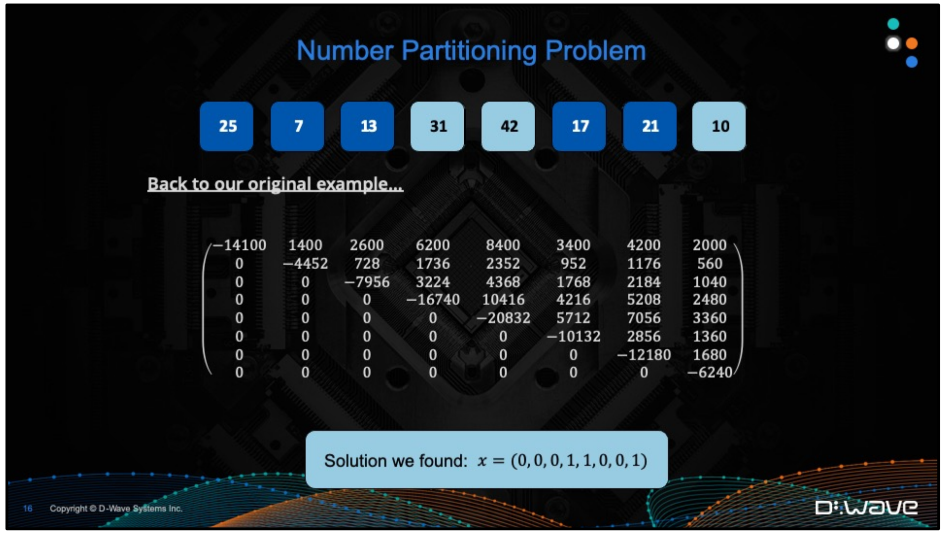


Now we have linear terms that we can combine so that they are all together.



Finally we can ignore the constant that is added at the beginning, this C squared, since it's not attached to any binary variables.

Now we can clearly see our linear terms that will go along the diagonal and the quadratic terms.



Back to our original example our matrix would look like this shown here.

Notice that I've written this matrix as an upper-triangular matrix: the bottom left is all 0's.

This means that if I have the term 5x1x2 in my QUBO I put the 5 in row 1, column 2, instead of row 2, column 1.

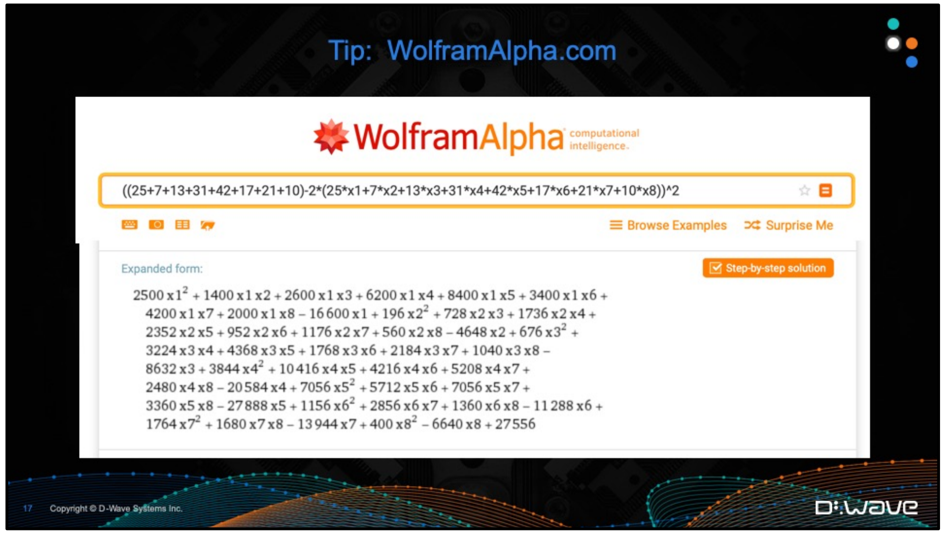
Does this matter?

The answer is yes and no. It doesn't matter if you put the 5 in position (1,2) in the matrix or position (2,1), but you cannot put it in both places.

The Ocean tools will take your QUBO matrix and add everything from the bottom half (the 0's here) to the top half.

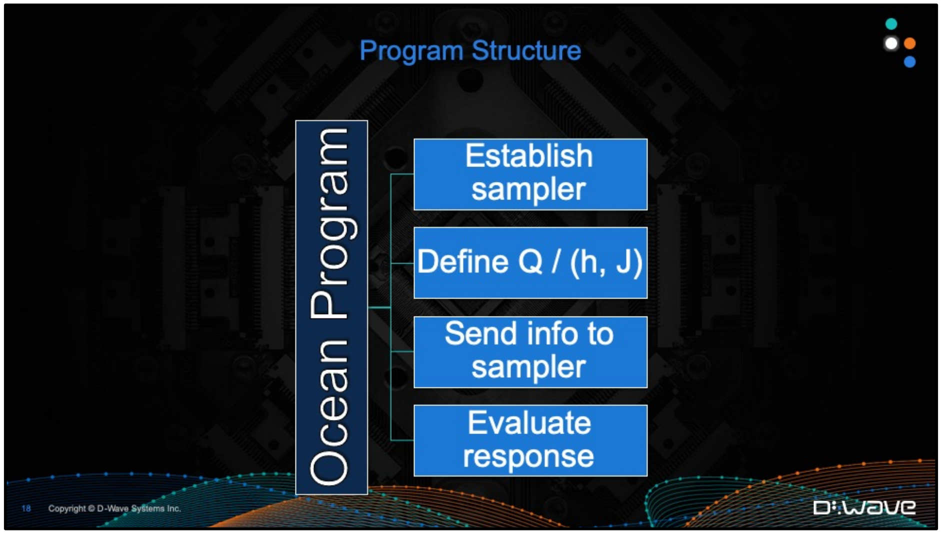
If you put your coefficient in both places then you're actually doubling all of the quadratic coefficients in your QUBO!

The solution we showed earlier with equal subset sums would be returned from the QPU as a binary string like this one shown.



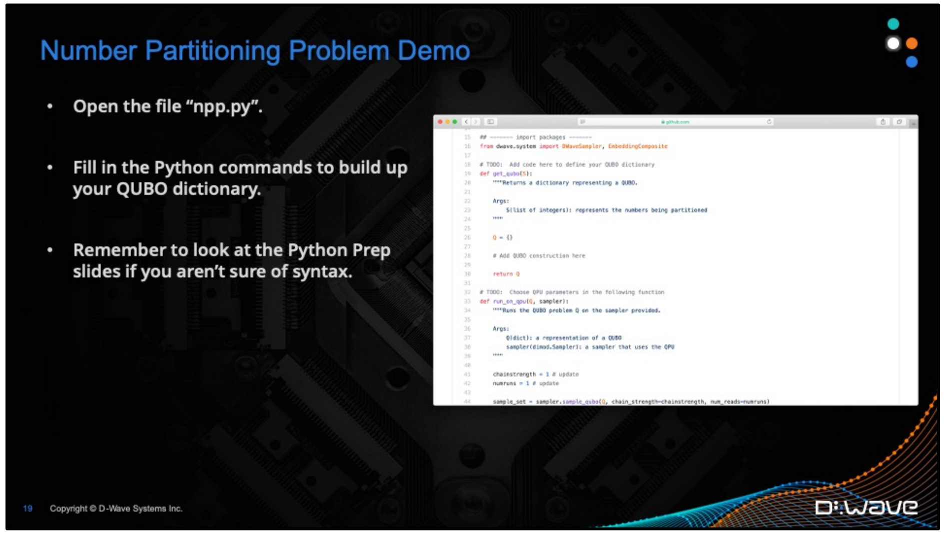
To build up this matrix without any programming so that you can check your work, Wolfram Alpha is an amazing resource.

It's a good idea to have an example matrix handy so that you can check that you have written up your python program correctly.



Now that we have our matrix we're ready to write our Ocean program.

Remember the four basic components of the program that we'll need to have: first we set up our sampler, we build our QUBO matrix, then we send the QUBO matrix to the sampler and look at the results.

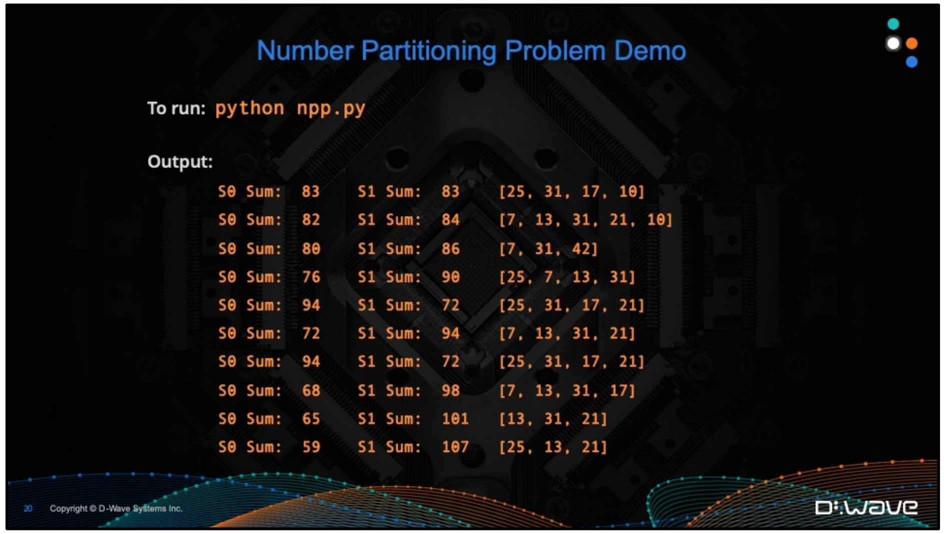


For your assignment head over to the github repository shown on Canvas.

You’ll be editing the file “npp.py”.

I’ve provided a program outline for you so that you only need to fill in a few sections.

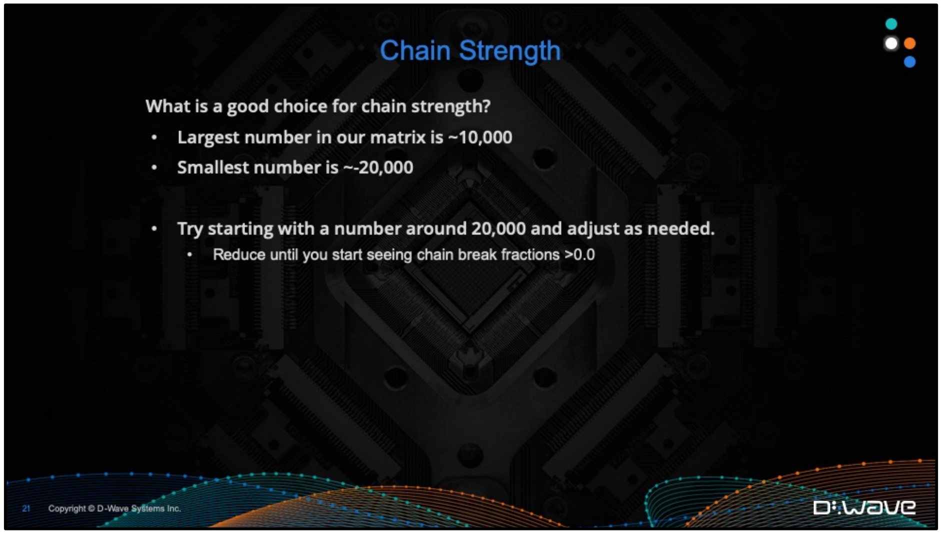
The directions for what you need to complete and how it will be graded are on Canvas.



If you run the file npp.py either by clicking the green “play” button in the IDE or by typing “python npp.py” on the command line you’ll see output like this.

Each line gives you one answer that was returned: it tells you the sum of each subset and the elements that are in one of the subsets.

The lowest energy solutions are listed first, so if you don’t see your best answers at the top you’ll want to double-check your QUBO matrix in your program.



Last but not least, you might want to try some different values for the chain strength parameter to get good results on this assignment. The default chain strength tool is a great start, but try tuning the parameter manually to see how it goes.

Remember from our embedding module that chain strength is what controls how strongly qubits are tied together to represent one of our binary variables.