The Hybrid Solver Service

D-Wave
The Quantum Computing Company™

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Agenda

- Introduction to The Hybrid Solver Service
- Problem – Knapsack
- Conclusions

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Introduction to the Hybrid Solver Service

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Leap^{TM} Hybrid Solver Service: Expands Problem Size & Ease Of Use

Submit complex problems and get good answers without having to know anything about the D-Wave quantum processing unit (QPU):

- Run problems with up to 20,000 variables fully connected
- Problems run on classical and quantum computing resources using an algorithm portfolio
- Easily accessed in Leap user interface as a hybrid solver
- Shorter learning curve
  - No embedding or processor parameters to worry about

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Hybrid Quantum-Classical Approach

The portfolio approach:

- Hybrid solvers target different categories of inputs and use cases
- During its operation, each heuristic solver formulates some number of quantum queries, which are sent to the QPU
- The classical and quantum components in each solver communicate asynchronously so that one part of the system does not block another

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Hybrid Quantum-Classical Approach: Quantum Acceleration

At the end of the time limit:

- All solvers stop
- Best solution is returned to the user
- The user did not have to decide which solution approach is best for any given input

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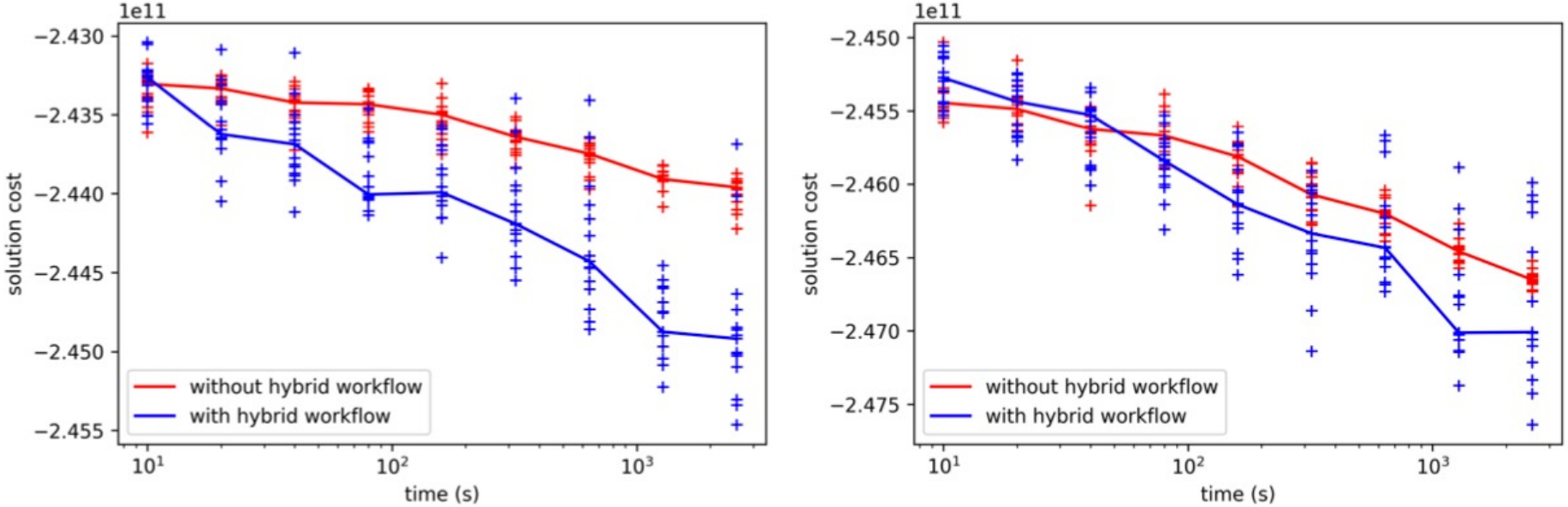
Hybrid Approach Wins Over Heuristic-Only (White Paper)

- Reduced solution cost (measures solution quality)
- Both dense (left graph) and sparse (right graph)
- Quantum queries lead to more rapid gains in solution quality



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The Time Limit Parameter *T*

There is only one user parameter: time limit *T*, in units of seconds

- The system calculates a minimum time limit that scales with input problem size (and it may be used by default)
- The minimum time limit ensures that each hybrid solver has enough time to both perform a first step and to query and have a good chance to receive at least one response from the D-Wave QPU
- Currently the minimum time limit for all problems is 3 seconds
- Currently the maximum time is 24 hours

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How To Solve A QUBO Problem?

Formulate a QUBO (quadratic unconstrained binary optimization) for the problem

Write a Python program using the Ocean API, which implements the QUBO

Run the Python program on a sampler/solver

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Using BQM Solver Is Easy

```
from dwave.system import LeapHybridSampler
sampler = LeapHybridSampler()
response = sampler.sample(bqm, time_limit=5)
```

Things to notice:

- No embedding to worry about
- No *chain_strength* to think about
- Starting *time_limit* at the minimum; increase if needed

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Knapsack problem

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Problem Specifications

- We want to pack a shipping container or a knapsack or a backpack…
- We have a collection of objects
- Each object has a weight w_i and a value v_i
- Fill up the container so that we respect a maximum weight W
- Maximize the total value in the container

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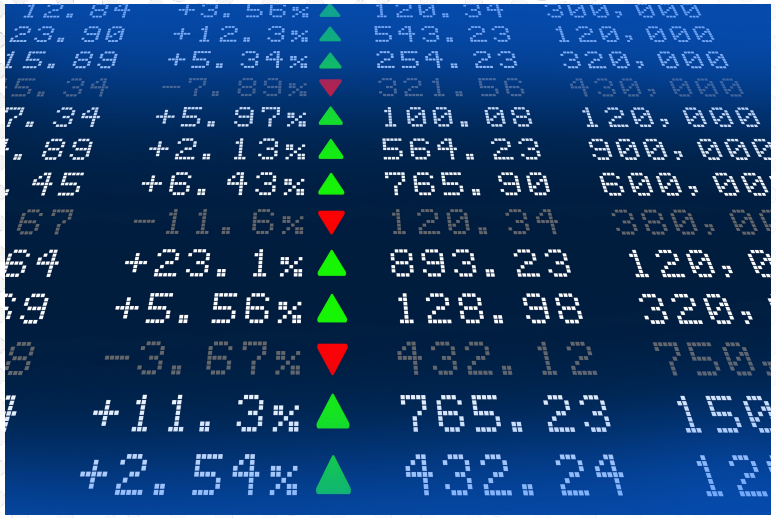
Example Knapsack Problems

Choosing which ads to bid on in Internet ad auctions, on fixed budget

Forestry service wants to bid on land packages with fixed budget

Financial portfolios: maximize your profits on a fixed budget



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QUBO Step 1

Objective:

– Maximize the total value of the selected objects

Constraints:

– The total weight must be less than or equal to *W*

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QUBO Step 2: Binary Variables

Binary Variables:

For each object we ask: “Is this object in the knapsack or not?”

We can use the binary variables to “label” the objects.

$$x_i = \begin{cases} 1, \text{if object } i \text{ is in the knapsack} \\ 0, \text{if object } i \text{ is not in the knapsack} \end{cases}$$

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The QUBO – Write The Objective First

$$H = -\sum_{\alpha} v_{\alpha}\, x_{\alpha}$$

Minimizes the sum of values – makes it as negative as possible – hence the most value possible

Remember that the x_{α} are binary variables, and will be returned by the solver (output)

The v_{α} are the values of the individual objects (input)

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The QUBO – Try To Write The Constraint

$$H = A\,(W - \sum_\alpha w_\alpha\, x_\alpha)^2 - \sum_\alpha v_\alpha\, x_\alpha$$

What's wrong with this?

This assumes *W* is *equal* to the sum of the weights, but we want it to be less than or equal to

We need to add *auxiliary variables* – so that it may be less than by one, or less than by two …

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The QUBO Formulation*

$$H = -\sum_i v_i x_i \tag{1}$$

$$\text{s.t.} \tag{2}$$

$$\sum_i w_i x_i \leq W \tag{3}$$

$$H = -\sum_i v_i x_i + A \left(\sum_i c_i y_i - \sum_i w_i x_i \right)^2 \tag{4}$$

$$c_i = 2^i \quad \text{for} \quad i = 1, ..., M-1 \tag{5}$$

$$c_M = W + 1 - 2^M \tag{6}$$

$$M = \lceil \log_2 W \rceil \tag{7}$$

- Andrew Lucas, Ising formulations of many NP problems, Front. Phys., 12 Feb. 2014.
- Andrew Lucas, "NP-hard combinatorial problems as Ising spin glasses", Workshop on Classical and Quantum Optimization; ETH Zuerich - August 20, 2014

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Python Code

```
df = pd.read_csv(data_file_name, header=None)
df.columns = ['cost', 'weight']
bqm = knapsack_bqm(df['cost'], df['weight'], weight_capacity)
sampler = LeapHybridSampler()
response = sampler.sample(bqm)
process_results(response)
```

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Run Using HSS On Small Data Set With Max Weight 50

Value	Weight(kg)
35	12
85	27
30	11
50	17
70	20
80	10
55	15

Solution:

Choose [20, 10, 15]

Energy = -205

HSS finds the solution promptly with time_limit = 3

The energy looks good – matches the total value

The weights add up to less than or equal to 50 kg

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Run Using HSS On Larger Data Set With Max Weight 70

Value	Weight(kg)
84	4
78	20
91	1
96	19
83	14
92	10
77	95
44	70
…	

68 objects in data set
Problem cannot be embedded on DW-2000Q

The hybrid solver finds solution with run_time and charge_time close to 3 seconds

Solution is weights [4, 20, 1, 19, 14, 10]
Solution satisfies constraints (less than 70 kg)
Energy equals the summed objective – values add to -524

The hybrid solver finds the known optimal solution (found elsewhere by classical methods)

This example is in the github.com/dwave-examples repository

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Analysis

- This problem is too big for the QPU – does not embed
- The hybrid solver makes use of the QPU
- Research question: will the hybrid solver find the optimal solution every time if we give it longer run times

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Conclusions

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Conclusions

- HSS is very easy to use
- HSS runs on very large problems
- We saw it do very well on Knapsack

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