Introduction to Solving QUBO Problems

The Quantum Computing Company™

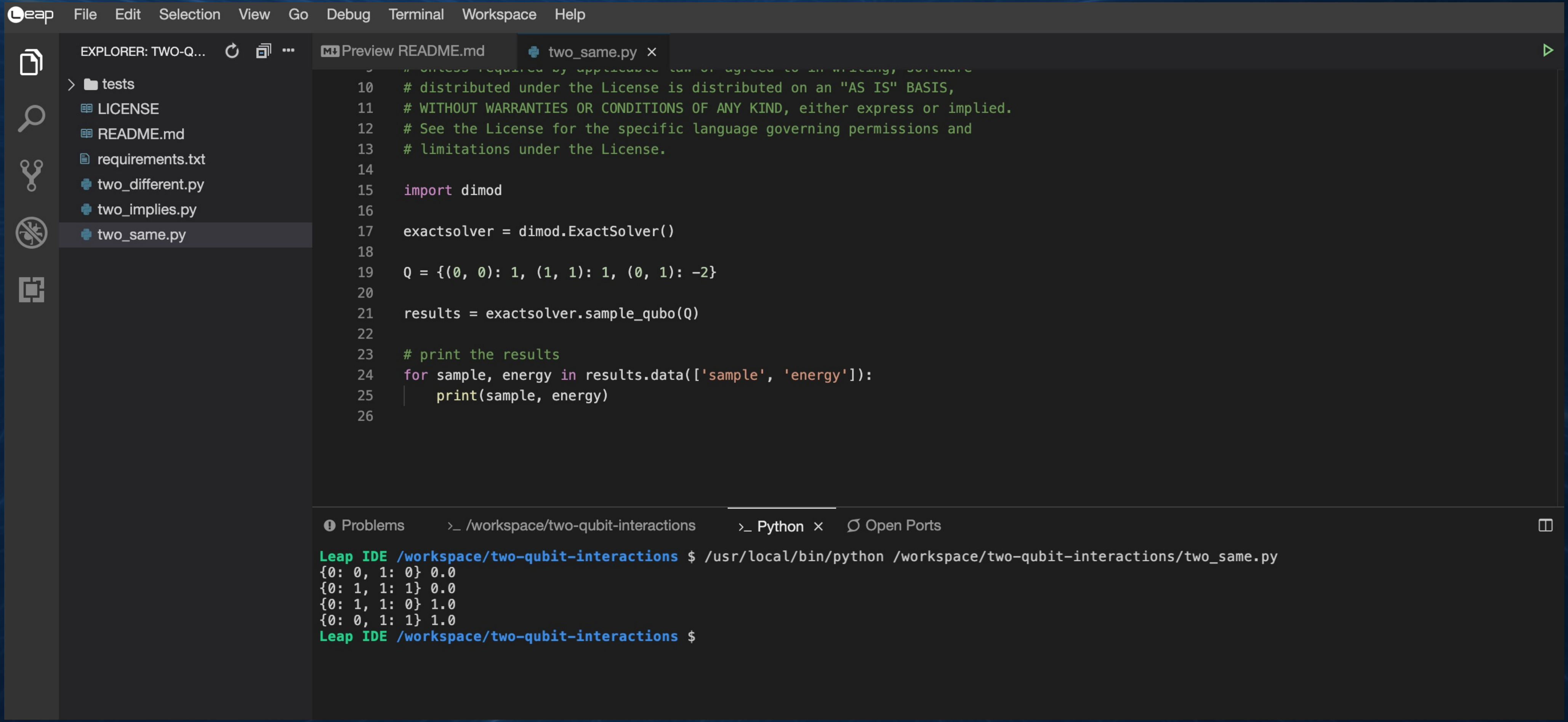
Visualization in the Leap IDE: The Two-Qubit Problem

https://ide.dwavesys.io/#https://github.com/dwave-training/two-qubit-interactions

Problem One: Favor qubits with same values

1. On left, click *two_same.py*
2. On top right, click the green arrow
3. You should see the four solutions and their energies

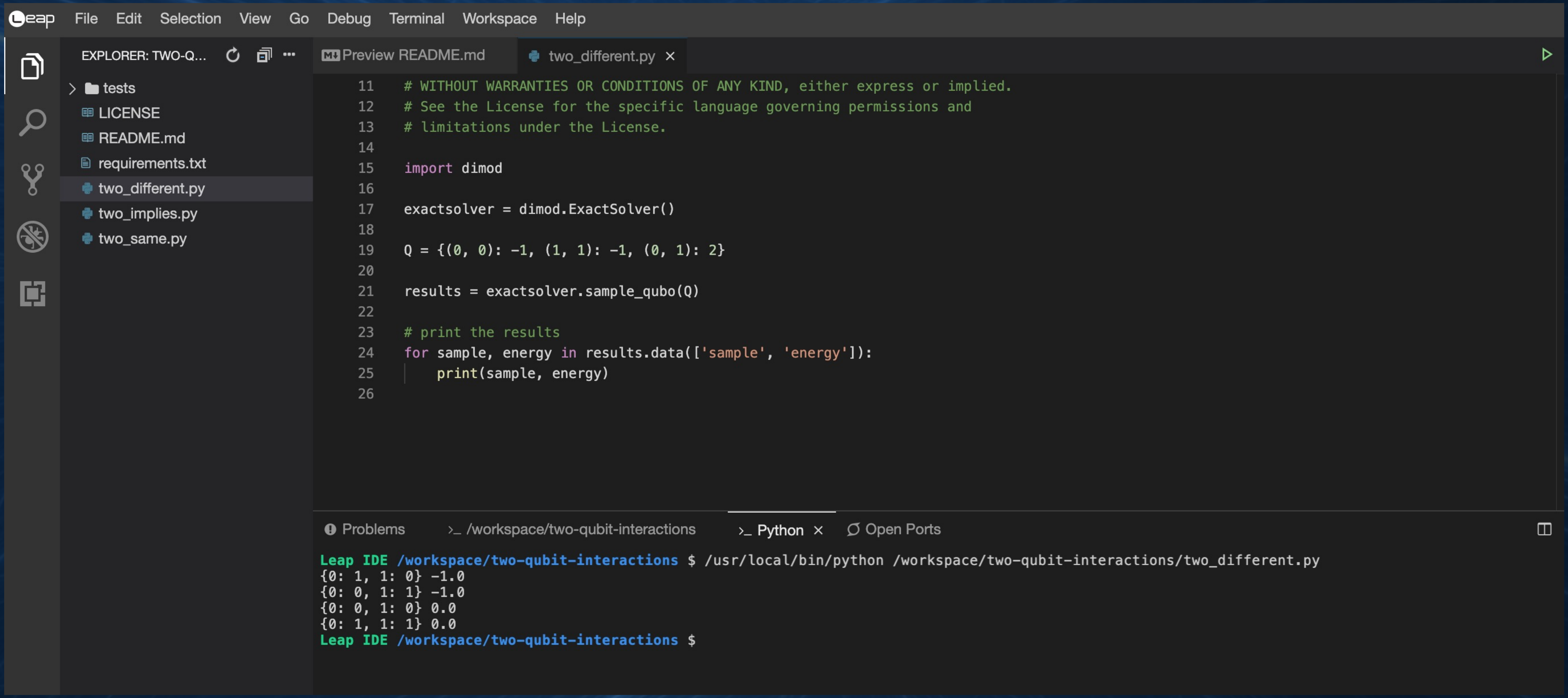
Do these solutions match what you expect from the previous course?



Problem Two: Favor qubits with different values

1. On left, click *two_different.py*
2. On top right, click the green arrow
3. You should see the four solutions and their energies

Do these solutions match what you expect from the previous course?



Did you notice that the two problems have different minimum energy solutions?

- When we solved the problem in the previous course, we used

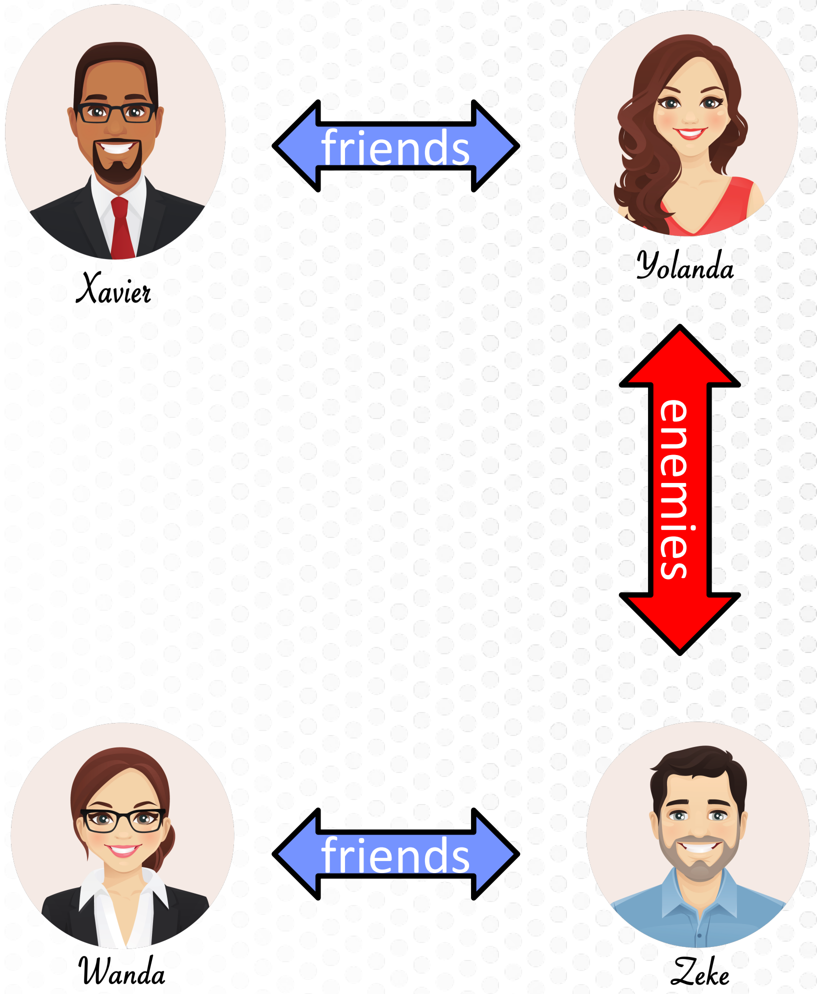
$$\textbf{objective} = 2xy - x - y + 1$$

- Note that in Problem Two (previous slide), we used

$$\textbf{objective} = 2xy - x - y$$

- The constant 1 is the **energy offset**
- It is the constant term in the QUBO model or the Ising model
- We can use it to adjust the values of the energy results
  - Example: Traveling Salesman, so that result is in kilometers

Problem Three:
Friends & Enemies

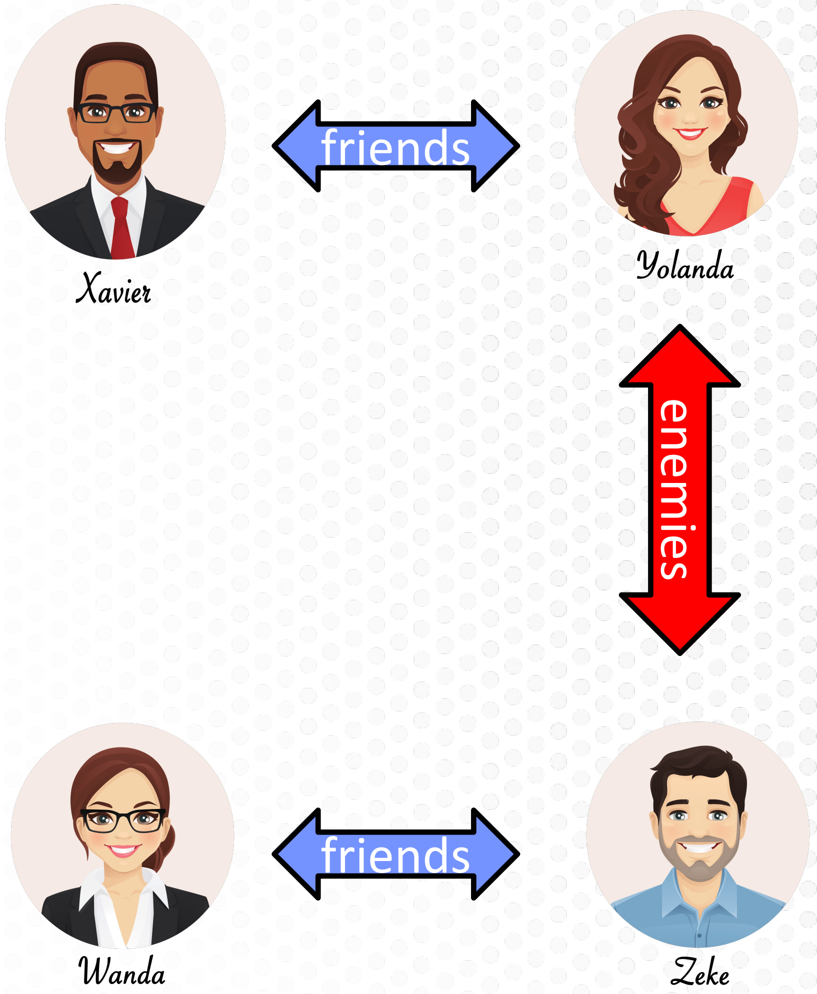


Problem:

Given a set of people and information about some pairs of people that are friends and some pairs that are enemies…..

How do we write a program that finds a solution in which all the friend and enemy relationships are true?

Problem Three: Friends & Enemies

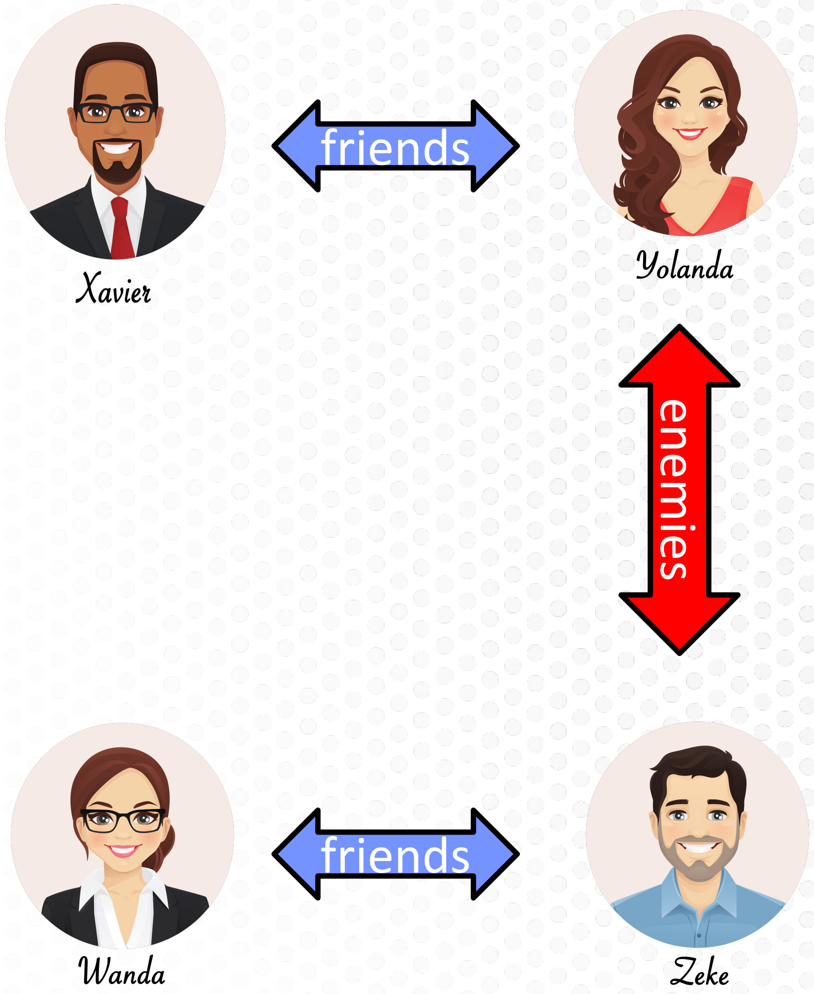


Problem Statement:

Find qubit values (0 or 1) such that:

- *X* (Xavier) and *Y* (Yolanda) have same values
- *Y* (Yolanda) and *Z* (Zeke) have different values
- *Z* (Zeke) and *W* (Wanda) have same values

Problem Three:
Friends & Enemies



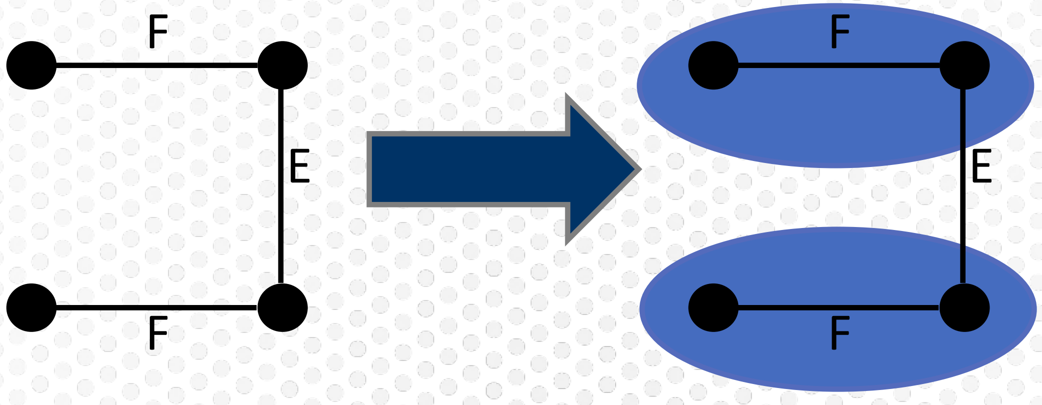
An Example Solution:

X (Xavier): 0
Y (Yolanda): 0
Z (Zeke): 1
W (Wanda): 1

X & *Y* and *W* & *Z*: Friends (same qubit value)
Y & *Z*: Enemies (different qubit value)

This is a valid solution
Can the D-Wave system find this solution for us?

Convert to graphical representation



We want the lowest possible energy:

-1 is an arbitrary choice

-1 (top pair of friends in same group)
-1 (bottom pair of friends in same group)
-1 (right pair of enemies in different groups)

-3 (will the QPU find a solution?)

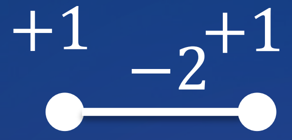
Convert to QUBO: friends

Most general QUBO

Xavier	Yolanda	$ax + by + cxy + d$
0	0	-1
0	1	0
1	0	0
1	1	-1

$$-1 = d$$
$$0 = b + d$$
$$0 = a + d$$
$$-1 = a + b + c + d$$

$$a = +1$$
$$b = +1$$
$$c = -2$$
$$d = -1$$



$$const = -1$$

Convert to QUBO: enemies

Yolanda

Zeke

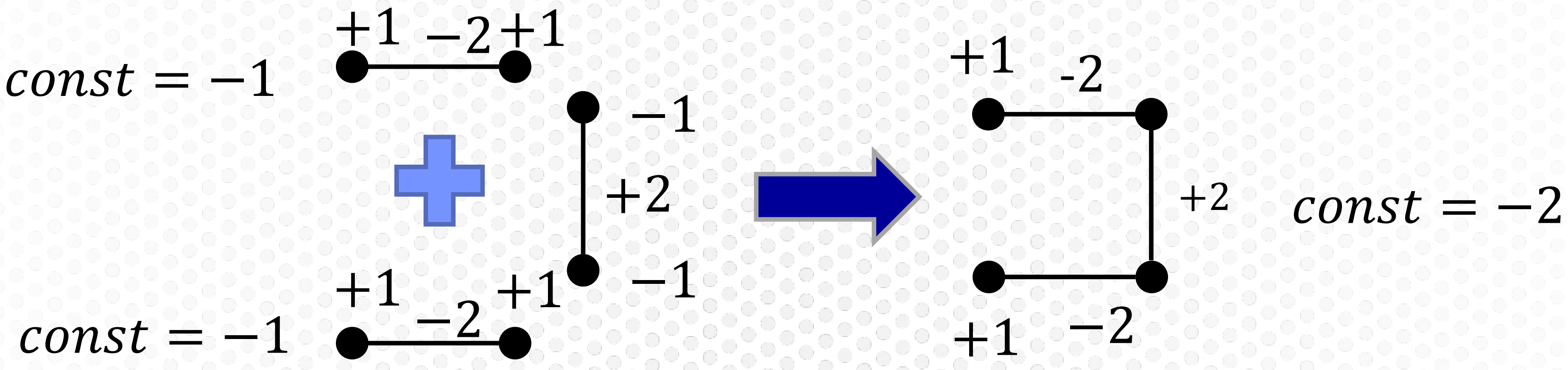
Most general QUBO

Yolanda	Zeke	$ey + fz + gyz + h$
0	0	0
0	1	-1
1	0	-1
1	1	0

$$0 = h$$
$$-1 = f + h$$
$$-1 = e + h$$
$$0 = e + f + g + h$$

$$e = -1$$
$$f = -1$$
$$g = +2$$
$$h = 0$$

Combine sub-QUBOs



Note that the linear terms -1 and +1 on the upper right node add to 0 – they vanish! Likewise for the linear terms -1 and +1 on the lower right node.

Evaluate the QUBO: some of the 16 possible answers

x	y	z	w	objective
0	0	0	0	-2
0	0	0	1	-1
0	0	1	0	-2
0	0	1	1	-3
0	1	0	0	-2
0	1	0	1	-1
0	1	1	0	0
0	1	1	1	-1
1	1	0	0	-3
1	1	0	1	-2

QUBO = $-2 + x + w - 2xy + 2yz - 2zw$

Interpret the answers

- *(x = 0, y = 0, z = 1, w = 1):*
  - Xavier and Yolanda are the same (good)
  - Yolanda and Zeke are different (good)
  - Zeke and Wanda are the same (good)

- *(x = 1, y = 1, z = 0, w = 0):*
  - Xavier and Yolanda are the same (good)
  - Yolanda and Zeke are different (good)
  - Zeke and Wanda are the same (good)

- Do these solutions make sense?
- Should there be more than two?
- Can you work out additional solutions that satisfy all the constraints?

Did we follow a process in this problem?

- We expressed a problem in domain terms

  (*X* is a friend of *Y*, *Y* is an enemy of *Z*…)
- We developed a graphical representation
- We expressed it mathematically with binary variables
- We solved the QUBO
- We interpreted the results

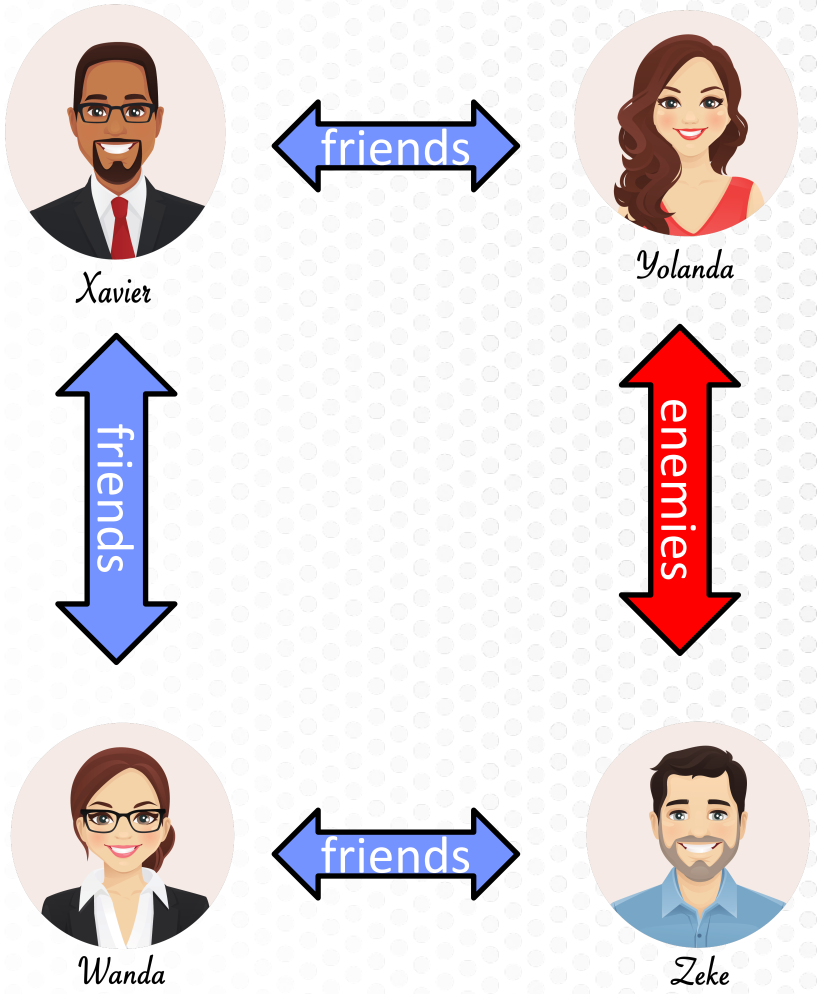
Now – what about Xavier and Wanda?

- *$(x = 0, y = 0, z = 1, w = 1)$:*
- *$(x = 1, y = 1, z = 0, w = 0)$:*

- In both solutions, Xavier and Wanda are enemies
- But we assumed that there was no relationship between Xavier and Wanda
- If we add an “enemies sub-QUBO” between Xavier and Wanda, we get these two lowest energy solutions

- What happens if Xavier and Wanda are *friends*?

Problem Four:
More Friends & Enemies



A Possible Solution:

X (Xavier): 0
Y (Yolanda): 0
Z (Zeke): 1
W (Wanda): 1

X & *Y* and *W* & *Z*: Friends (same qubit value)
Y & *Z*: Enemies (different qubit value)
But *X* and *W* do not have the same value!

This is not a valid solution
Can the D-Wave system find a solution for us?

Looking at The Math

***X* and *Y*:** $-2xy + x + y - 1$

***Y* and *Z*:** $-y - z + 2yz$

***Z* and *W*:** $-2zw + z + w - 1$

***W* and *X*:** $-2wx + w + x - 1$

QUBO:

$$-2xy - 2xw + 2yz - 2zw + 2x + 2w - 3$$

Spectrum & gap

x	y	z	w	objective
0	0	0	0	-3
0	0	0	1	-1
0	0	1	0	-3
0	0	1	1	-3
0	1	0	0	-3
0	1	0	1	-1
0	1	1	0	-1
0	1	1	1	-1
1	0	0	0	-1
1	0	0	1	-1
1	0	1	0	-1
1	0	1	1	-3
1	1	0	0	-3
1	1	0	1	-3
1	1	1	0	-1
1	1	1	1	-3

Interpret the answers with objective = -3

- *(x = 0, y = 0, z = 0, w = 0):*
  - Xavier and Yolanda are the same (good)
  - Yolanda and Zeke are the same (**not good**)
  - Zeke and Wanda are the same (good)
  - Wanda and Xavier are the same (good))
- *(x = 0, y = 0, z = 1, w = 1):*
  - Xavier and Yolanda are the same (good)
  - Yolanda and Zeke are different (good)
  - Zeke and Wanda are the same (good)
  - Wanda and Xavier are different (**not good**)

Do these solutions make sense? Why are there eight?
What do all the objective = -3 solutions have in common?
Is there any way to solve all the constraints? (*frustration*)

Conclusions

- We will write the QUBO model or Ising model in our Python code
- The offset, or energy constant, may be very useful
- It’s important to interpret the solutions, versus the original problem