

Introduction to Solving QUBO Problems



Visualization in the Leap IDE: The Two-Qubit Problem

<https://ide.dwavesys.io/#https://github.com/dwave-training/two-qubit-interactions>

Problem One: Favor qubits with **same** values

1. On left, click *two_same.py*
2. On top right, click the **green** arrow
3. You should see the four solutions and their energies

Do these solutions match what you expect from the previous course?

```
LEAP File Edit Selection View Go Debug Terminal Workspace Help  
EXPLORER: TWO-Q... 🔍 ⌂ ... Preview README.md two_same.py  
> tests  
  LICENSE  
  README.md  
  requirements.txt  
  two_different.py  
  two_implies.py  
  two_same.py  
10  # This software is provided under the terms of the D-Wave Software License Agreement.  
11  # distributed under the License is distributed on an "AS IS" BASIS,  
12  # WITHOUT WARRANTIES OR CONDITIONS OF ANY KIND, either express or implied.  
13  # See the License for the specific language governing permissions and  
14  # limitations under the License.  
15  import dimod  
16  exactsolver = dimod.ExactSolver()  
17  
18  Q = {(0, 0): 1, (1, 1): 1, (0, 1): -2}  
19  
20  results = exactsolver.sample_qubo(Q)  
21  
22  # print the results  
23  for sample, energy in results.data(['sample', 'energy']):  
24      print(sample, energy)  
25  
26  
Problems > /workspace/two-qubit-interactions > Python × Open Ports  
Leap IDE /workspace/two-qubit-interactions $ /usr/local/bin/python /workspace/two-qubit-interactions/two_same.py  
{0: 0, 1: 0} 0.0  
{0: 1, 1: 1} 0.0  
{0: 1, 1: 0} 1.0  
{0: 0, 1: 1} 1.0  
Leap IDE /workspace/two-qubit-interactions $
```

Problem Two: Favor qubits with **different** values

1. On left, click *two_different.py*
2. On top right, click the **green** arrow
3. You should see the four solutions and their energies

Do these solutions match what you expect from the previous course?

The screenshot shows the Leap IDE interface. The top navigation bar includes File, Edit, Selection, View, Go, Debug, Terminal, Workspace, and Help. The left sidebar has icons for files, search, and workspace. The Explorer panel shows a folder structure with tests, LICENSE, README.md, requirements.txt, two_different.py (selected), two_implies.py, and two_same.py. The main editor area displays the contents of two_different.py:

```
11  # WITHOUT WARRANTIES OR CONDITIONS OF ANY KIND, either express or implied.
12  # See the License for the specific language governing permissions and
13  # limitations under the License.
14
15 import dimod
16
17 exactsolver = dimod.ExactSolver()
18
19 Q = {(0, 0): -1, (1, 1): -1, (0, 1): 2}
20
21 results = exactsolver.sample_qubo(Q)
22
23 # print the results
24 for sample, energy in results.data(['sample', 'energy']):
25     print(sample, energy)
```

The bottom terminal window shows the output of running the script:

```
Leap IDE /workspace/two-qubit-interactions $ /usr/local/bin/python /workspace/two-qubit-interactions/two_different.py
{0: 1, 1: 0} -1.0
{0: 0, 1: 1} -1.0
{0: 0, 1: 0} 0.0
{0: 1, 1: 1} 0.0
Leap IDE /workspace/two-qubit-interactions $
```

Did you notice that the two problems have different minimum energy solutions?

- When we solved the problem in the previous course, we used

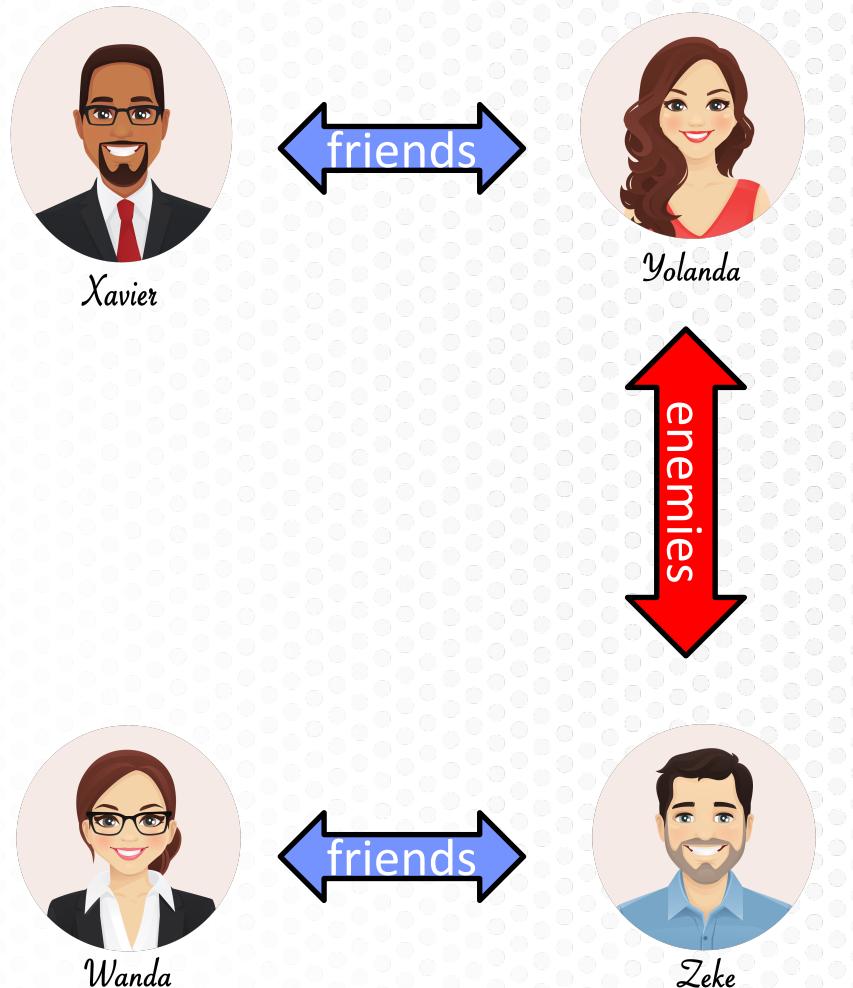
$$\text{objective} = 2xy - x - y + 1$$

- Note that in Problem Two (previous slide), we used

$$\text{objective} = 2xy - x - y$$

- The constant 1 is the **energy offset**
- It is the constant term in the QUBO model or the Ising model
- We can use it to adjust the values of the energy results
 - Example: Traveling Salesman, so that result is in kilometers

Problem Three: Friends & Enemies

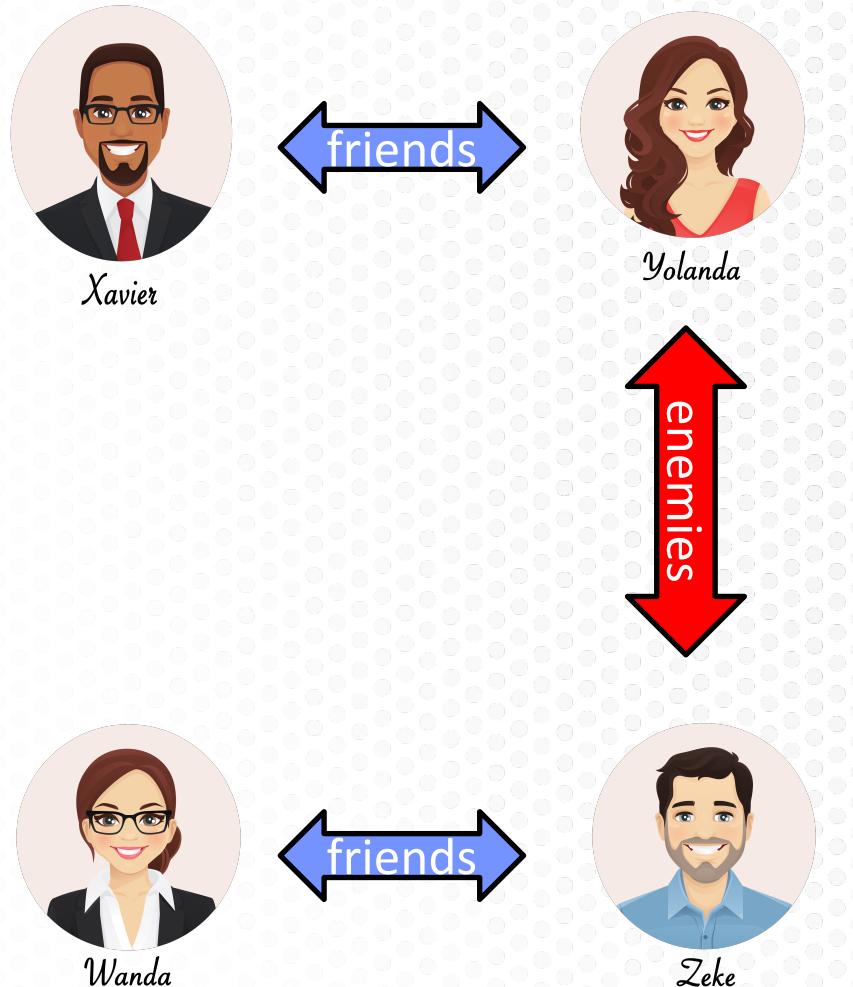


Problem:

Given a set of people and information about some pairs of people that are friends and some pairs that are enemies.....

How do we write a program that finds a solution in which all the friend and enemy relationships are true?

Problem Three: Friends & Enemies

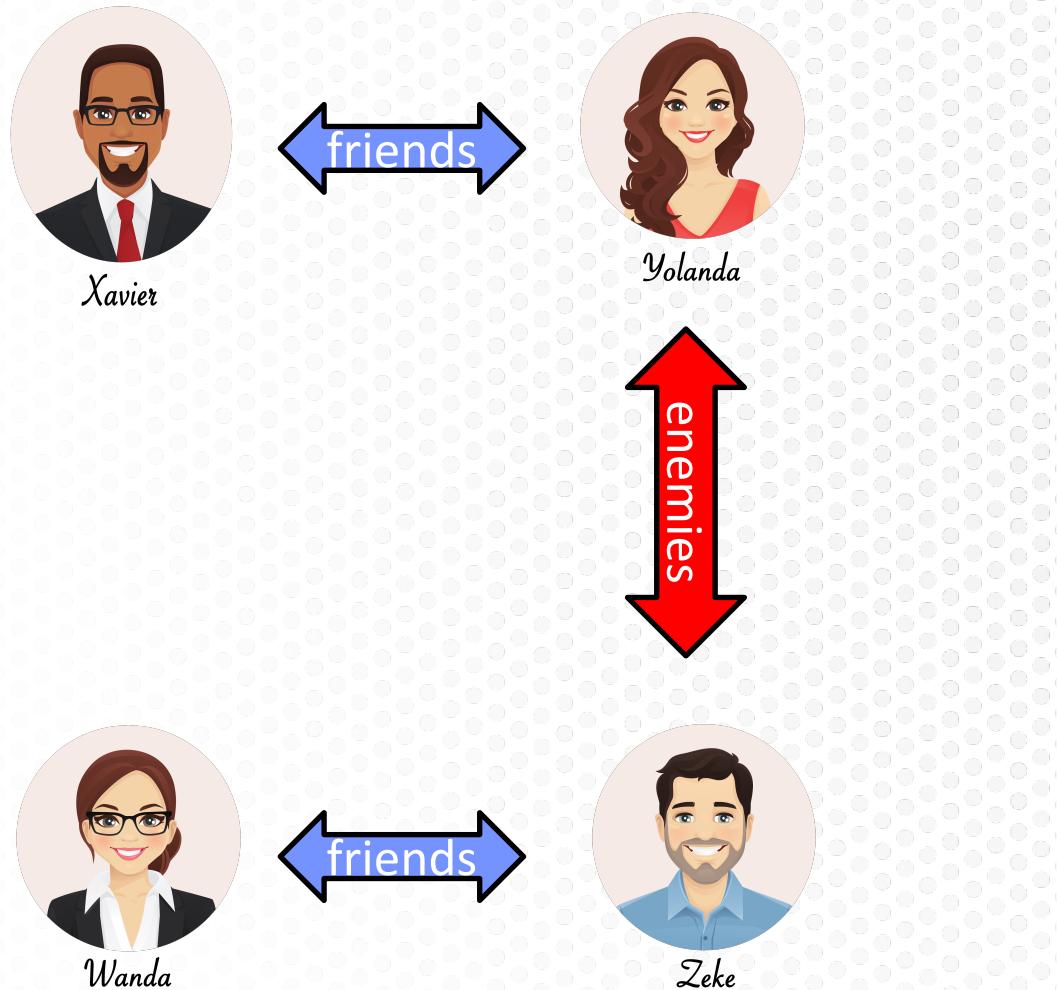


Problem Statement:

Find qubit values (0 or 1) such that:

- X (Xavier) and Y (Yolanda) have **same** values
- Y (Yolanda) and Z (Zeke) have **different** values
- Z (Zeke) and W (Wanda) have **same** values

Problem Three: Friends & Enemies



An Example Solution:

X (Xavier): 0

Y (Yolanda): 0

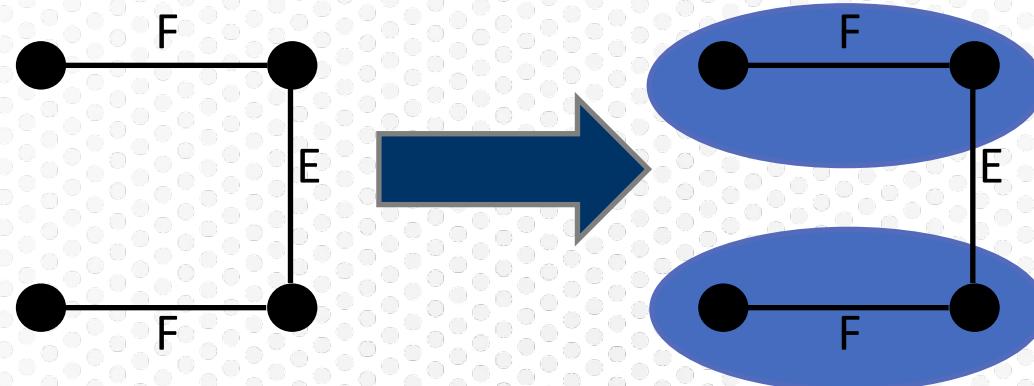
Z (Zeke): 1

W (Wanda): 1

X & Y and W & Z : Friends (same qubit value)
 Y & Z : Enemies (different qubit value)

This is a **valid** solution
Can the D-Wave system find
this solution for us?

Convert to graphical representation



We want the lowest possible energy:

- 1 (top pair of friends in same group)
- 1 (bottom pair of friends in same group)
- 1 (right pair of enemies in different groups)

-1 is an arbitrary choice

-
- 3 (will the QPU find a solution?)

Convert to QUBO: friends

The diagram illustrates the conversion of a binary matrix representing friend relationships between two individuals, Xavier and Yolanda, into a quadratic unconstrained binary optimization (QUBO) equation.

Two circular icons represent the individuals: Xavier (a man with glasses and a suit) and Yolanda (a woman with brown hair).

A 4x3 grid table shows the binary values of their interactions:

Xavier	Yolanda	$ax + by + cxy + d$
0	0	-1
0	1	0
1	0	0
1	1	-1

An arrow points from the table to the general QUBO equation $ax + by + cxy + d$, indicating the mapping of the matrix values to the coefficients a, b, c, d .

The QUBO equation is given as:

$$+1 \quad -2 \quad +1 \quad \text{const} = -1$$

with variables:

$$\begin{aligned} a &= +1 \\ b &= +1 \\ c &= -2 \\ d &= -1 \end{aligned}$$

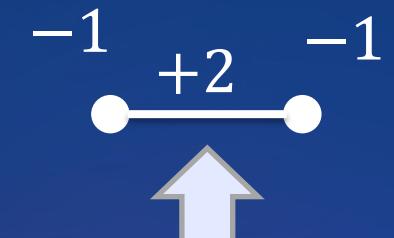
and constraints:

$$\begin{aligned} -1 &= d \\ 0 &= b + d \\ 0 &= a + d \\ -1 &= a + b + c + d \end{aligned}$$

Convert to QUBO: enemies

Yolanda	Zeke	$ey + fz + gyz + h$
0	0	0
0	1	-1
1	0	-1
1	1	0

Most general
QUBO



$$e = -1$$

$$f = -1$$

$$g = +2$$

$$h = 0$$

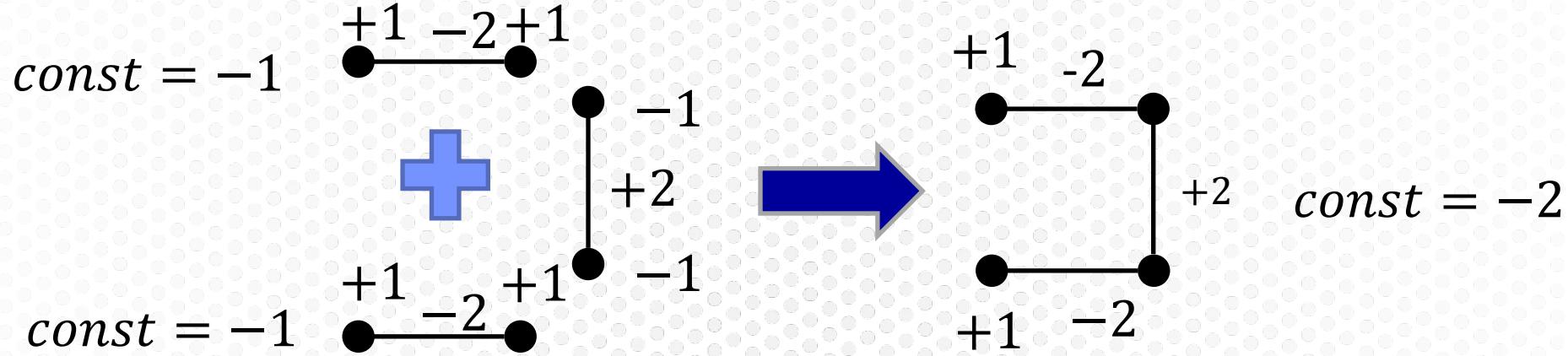
$$0 = h$$

$$-1 = f + h$$

$$-1 = e + h$$

$$0 = e + f + g + h$$

Combine sub-QUBOs



Note that the linear terms -1 and +1 on the upper right node add to 0 – they vanish! Likewise for the linear terms -1 and +1 on the lower right node.

Evaluate the QUBO: some of the 16 possible answers

x	y	z	w	objective
0	0	0	0	-2
0	0	0	1	-1
0	0	1	0	-2
0	0	1	1	-3
0	1	0	0	-2
0	1	0	1	-1
0	1	1	0	0
0	1	1	1	-1
1	1	0	0	-3
1	1	0	1	-2

$$\text{QUBO} = -2 + x + w - 2xy + 2yz - 2zw$$

Interpret the answers

- $(x = 0, y = 0, z = 1, w = 1)$:
 - Xavier and Yolanda are the same (good)
 - Yolanda and Zeke are different (good)
 - Zeke and Wanda are the same (good)
- $(x = 1, y = 1, z = 0, w = 0)$:
 - Xavier and Yolanda are the same (good)
 - Yolanda and Zeke are different (good)
 - Zeke and Wanda are the same (good)
- Do these solutions make sense?
- Should there be more than two?
- Can you work out additional solutions that satisfy all the constraints?

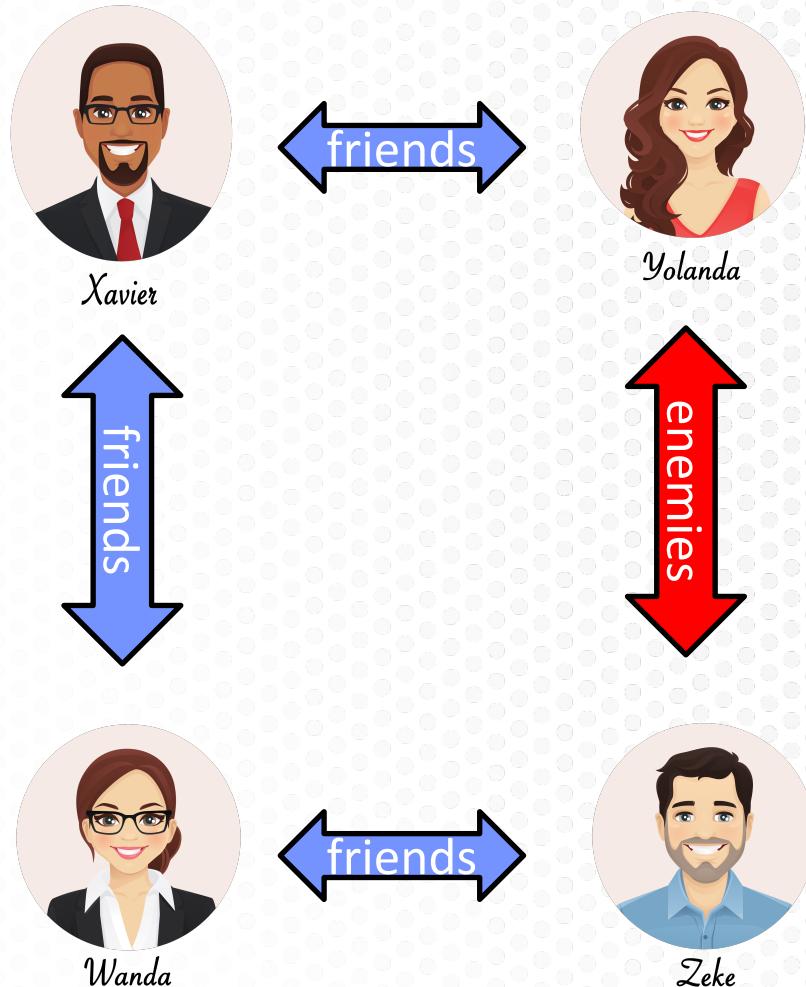
Did we follow a process in this problem?

- We expressed a problem in domain terms
(X is a friend of Y, Y is an enemy of Z...)
- We developed a graphical representation
- We expressed it mathematically with binary variables
- We solved the QUBO
- We interpreted the results

Now – what about Xavier and Wanda?

- ($x = 0, y = 0, z = 1, w = 1$):
- ($x = 1, y = 1, z = 0, w = 0$):
- In both solutions, Xavier and Wanda are enemies
- But we assumed that there was no relationship between Xavier and Wanda
- If we add an “enemies sub-QUBO” between Xavier and Wanda, we get these two lowest energy solutions
- What happens if Xavier and Wanda are *friends*?

Problem Four: More Friends & Enemies



A Possible Solution:

X (Xavier): 0
 Y (Yolanda): 0
 Z (Zeke): 1
 W (Wanda): 1

X & Y and W & Z : Friends (same qubit value)
 Y & Z : Enemies (different qubit value)
But X and W do not have the same value!

This is **not** a valid solution
Can the D-Wave system find a solution for us?

Looking at The Math

X and Y: $-2xy + x + y - 1$

Y and Z: $-y - z + 2yz$

Z and W: $-2zw + z + w - 1$

W and X: $-2wx + w + x - 1$

QUBO:

$-2xy - 2xw + 2yz - 2zw + 2x + 2w - 3$

Spectrum & gap

x	y	z	w	objective
0	0	0	0	-3
0	0	0	1	-1
0	0	1	0	-3
0	0	1	1	-3
0	1	0	0	-3
0	1	0	1	-1
0	1	1	0	-1
0	1	1	1	-1
1	0	0	0	-1
1	0	0	1	-1
1	0	1	0	-1
1	0	1	1	-3
1	1	0	0	-3
1	1	0	1	-3
1	1	1	0	-1
1	1	1	1	-3

Interpret the answers with objective = -3

- $(x = 0, y = 0, z = 0, w = 0)$:
 - Xavier and Yolanda are the same (good)
 - Yolanda and Zeke are the same (**not good**)
 - Zeke and Wanda are the same (good)
 - Wanda and Xavier are the same (good))
- $(x = 0, y = 0, z = 1, w = 1)$:
 - Xavier and Yolanda are the same (good)
 - Yolanda and Zeke are different (good)
 - Zeke and Wanda are the same (good)
 - Wanda and Xavier are different (**not good**)

Do these solutions make sense? Why are there eight?

What do all the objective = -3 solutions have in common?

Is there any way to solve all the constraints? (*frustration*)

Conclusions

- We will write the QUBO model or Ising model in our Python code
- The offset, or energy constant, may be very useful
- It's important to interpret the solutions, versus the original problem