Squaring Binary Polynomials

Module: Math Background

D-Wave Systems Training Course

Throughout our course we will use binary variables to represent our problems. We will need to write math expressions using these variables and simplify them to prepare our model for the quantum processing unit.

Binary Variables

Binary variables are variables that can take one of two possible values. For this document we will assume these values are 0 and 1. In other words, if our set of variables is $x_0, x_1, x_2, \ldots, x_n$ then each x_i equals either 0 or 1.

Working with these binary variables, there is a helpful trick that we can use to simplify expressions. Since $0^2 = 0$ and $1^2 = 1$, we can replace any squared binary variable with an "unsquared" version. In other words:

$$x_i^2 = x_i \tag{1}$$

Multiplying Polynomials

When we multiply two polynomials, remember that we must multiply each term in the first polynomial by every term in the second. You may recall this as the FOIL method for binomials (two-term polynomials). You can find a good refresher on how to multiply two polynomials on YouTube (https://youtu.be/jpD_BugTR6I)

Example:

$$\begin{aligned}(x_1 + 2x_2 + 3x_3)(x_1 + x_2 + x_4) &= x_1(x_1 + x_2 + x_4) + 2x_2(x_1 + x_2 + x_4) + 3x_3(x_1 + x_2 + x_4) \\ &= x_1^2 + x_1x_2 + x_1x_4 + 2x_1x_2 + 2x_2^2 + 2x_2x_4 + 3x_1x_3 + 3x_2x_3 + 3x_3x_4 \\ &= x_1^2 + 3x_1x_2 + 3x_1x_3 + x_1x_4 + 2x_2^2 + 3x_2x_3 + 2x_2x_4 + 3x_1x_3 + 3x_3x_4\end{aligned}$$

Helpful Formulas

We will need to frequently square polynomials that take the form $(\sum x_i) - C$, where C is some constant number. The general formula for this is the following.

$$\left(\left(\sum_{i=0}^{n} x_i\right) - C\right)^2 = \left(\sum_{i=0}^{n} x_i^2\right) + 2\left(\sum_{i=0}^{n}\sum_{j>i}^{n} x_i x_j\right) - 2C\left(\sum_{i=0}^{n} x_i\right) + C^2 \tag{2}$$