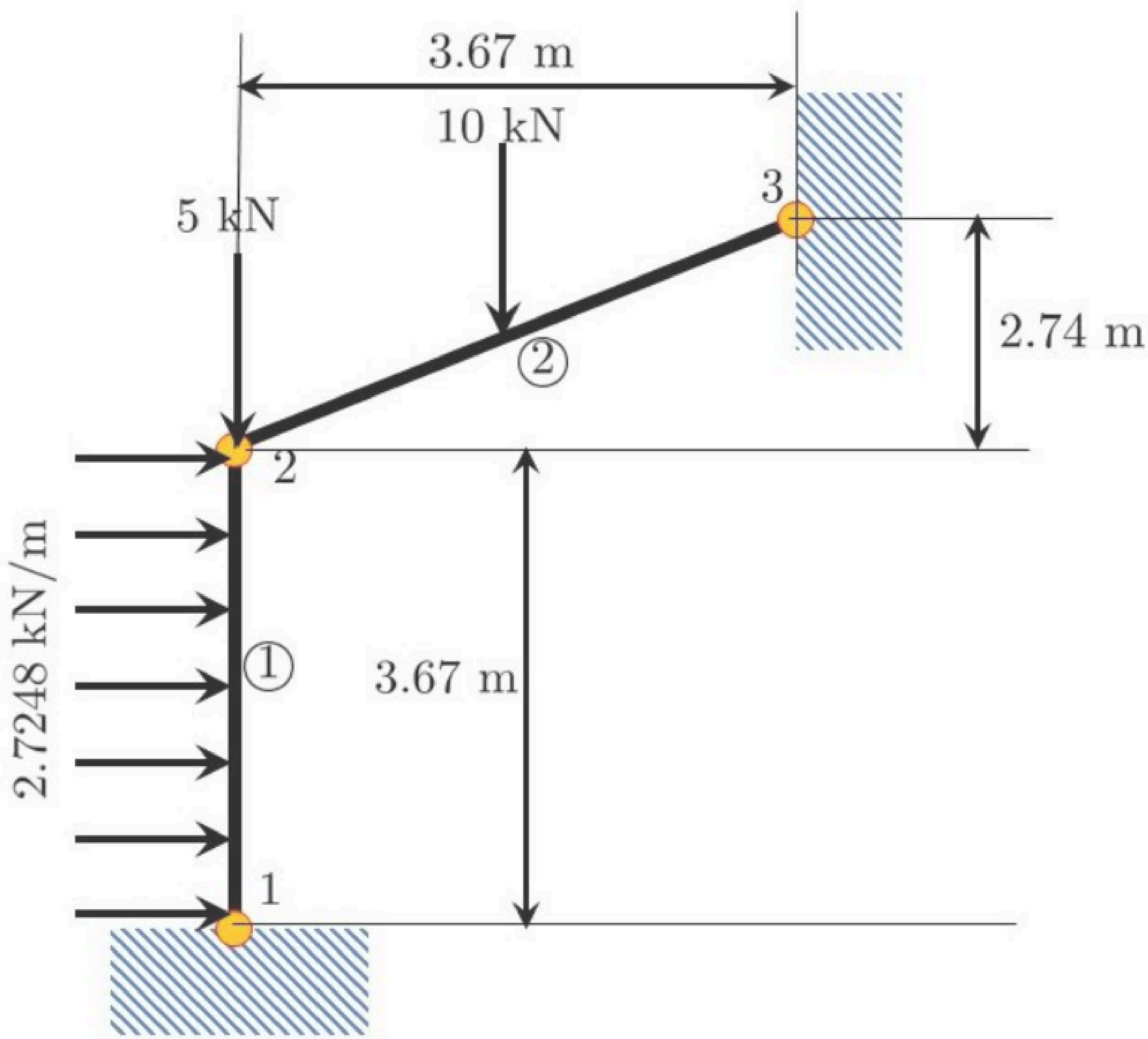
Consider the plane frame shown below. It is discretised into two elements as shown. Each element is a combintion of a Bernoulli beam and a truss, i.e. each node has 3 degrees of freedom (u, v, θ). The 10 kN load is applied at the midpoint of element 2. The origin is at node 1, with the x-axis pointing to the right and y-axis towards node 2. Use conventions that are used in the class. Both the elements have the same material properties and cross sectional properties. Specifically, Young's modulus $E = 200 \times 10^9$ Pa, area $A = 6.5 \times 10^{-3}$ m^2 and moment of intertia $I = 3.5 \times 10^{-6}$ m^4.

Find equivalent nodal loads for the loading shown.

1. Formulate the destination array
2. Formulate the stiffness matrices of each element ($\mathbf{K}_1, \mathbf{K}_2$), the global stiffness $\mathbf{K}_g$.
3. Formulate the global load vector $\mathbf{F}_g$ equivalent to the loading shown, apply boundary conditions and determine the solution vector $\mathbf{U}$. While formulating $\mathbf{F}_g$ use the ad-hoc method to determine the nodal equivalent loads. In other words, neglect the statically equivalent moments, if they arise.
4. Find the magnitude of the reaction moment M_{3z} at node 3.



```
%mdsem 2023
L=3.67; %in m
L1=2.74; %in m
E=200e09; %in Pa
A=6.5e-03 % in m^2
I=3.5e-6; %in m^4
nelem=2;
ndof=2;
x=[0 0 L];
y=[0 L L+L1];
nodeperelem=2;
conn=[1 2;2 3];
dest(1:2,1:6)=0;
dest = [1 2 3 4 5 6; 4 5 6 7 8 9];
Kg(1:9,1:9)=0;
% loop over elements
for ii=1:nelem

   theta=atan2( y(conn(ii,2))-y(conn(ii,1)),
x(conn(ii,2))-x(conn(ii,1))); % angle the element makes with the x axis
   len= sqrt( (x(conn(ii,2))-x(conn(ii,1)))^2 +
(y(conn(ii,2))-y(conn(ii,1)))^2 ); % length of the element

   stiff_beam= (E*I/len^3)*[12 6*len -12 6*len;
                        6*len 4*len^2 -6*len 2*len^2;
                        -12 -6*len 12 -6*len;
                        6*len 2*len^2 -6*len 4*len^2]; % stiffness matrix
of the beam 4X4
   stiff_bar= (E*A/len)*[1 -1; -1 1]; % stiffness matrix of the bar 2X2
   stiffe(1:6,1:6)=0;
   stiffe([1 4],[1 4] ) = stiff_bar;
   stiffe([2 3 5 6],[2 3 5 6] ) = stiff_beam;
   T1=[cos(theta) sin(theta) 0;
  -sin(theta) cos(theta) 0;
   0 0 1 ]; % transformation matrix 3X3
   T(1:6,1:6)=0;
   T(1:3,1:3 )=T1;
   T(4:6,4:6 )=T1;
   Ke= T'*stiffe*T; % local stiffness of element ii in the global
coordinate system
   if ii == 1
       K1=Ke;
   end
```

```
   if ii== 2
       K2=Ke;
   end

   % assemble local stiffness for element ii
    for a=1:6
       for b=1:6
   Kg(dest(ii,a),dest(ii,b) )= Kg(dest(ii,a),dest(ii,b) )+ Ke(a,b);
       end
    end

end


Fg=[5 0 0 5 -10 0 0 -5 0]'*1e3; % global force vector written as 1X9

% Convert to column for solving

% find reaction forces for element 2
uspec=[1 2 3 7 8 9];

Kgprime = Kg;
Fgprime = Fg;
% Remove constrained rows first
Kgprime(uspec,:) = [];
Fgprime(uspec) = [];

% Then remove corresponding columns
Kgprime(:,uspec) = [];

U=Kgprime\Fgprime;

%U = U';

%Ufull=zeros(9,1);
%Ufull(setdiff(1:9,uspec))=U;


Ufree =U;
U = zeros(9,1);                          % must be 9x1
U(setdiff(1:9,uspec)) = Ufree;

% reactions at node 3
```

```
R2=Kg*U - Fg;
M3z=abs(R2(9));     % magnitude of reaction moment at node 3.

Fg=Fg'
```