Домашно 2

Александър Гуров

4 януари 2023 г.

Задача 3

От дефинициите на линейните изображения съставяме матрици:

$$\theta : U \longrightarrow V,$$

$$\theta(x_1e_1+x_2e_2+x_3e_3) = (2x_1+3x_2-x_3)f_1+(x_1-2x_2-x_3)f_2, \quad \forall x_1, x_2, x_3 \in F,$$

$$\text{Матрица на } \theta\text{: } A = \begin{pmatrix} 2 & 3 & -1 \\ 1 & -2 & -1 \end{pmatrix}$$

$$\varphi : U \longrightarrow U,$$

$$\varphi(x_1e_1+x_2e_2+x_3e_3) = (x_1-x_2+2x_3)e_1+(3x_1+x_2-x_3)e_2+(2x_1+x_2+x_3)e_3, \quad \forall x_1, x_2, x_3 \in F$$

$$\text{Матрица на } \varphi\text{: } B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 1 & -1 \\ 2 & 1 & 1 \end{pmatrix}$$

$$\psi : V \longrightarrow V,$$

$$\psi(y_1f_1+y_2f_2) = (5y_1-y_2)f_1+(2y_1-4y_2)f_2, \quad \forall y_1, y_2 \in F$$

$$\text{Матрица на } \psi\text{: } C = \begin{pmatrix} 5 & -1 \\ 2 & -4 \end{pmatrix}$$

Търсим векторите $u \in U$, за които е вярно$(\psi\theta - \theta\varphi)(u) = f_1$

$$u \in U \implies u = x.e = (x_1e_1, x_2e_2) = x_1e_1 + x_2e_2$$

$$\begin{gathered}(\psi\theta - \theta\varphi)(u) = \theta(\psi(u)) - \varphi(\theta(u)) = \theta(\psi(ex)) - \varphi(\theta(ex)) \\ = (f.A_{\psi\theta} - f.A_{\theta\varphi}).x = f.x.(A_\psi A_\theta - A_\theta A_\varphi) = \\ = f.x.(CA - AB)\end{gathered}$$

$$\begin{gathered}(\psi\theta - \theta\varphi)(u) = f_1 \\ f.x.(CA - AB) = f\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}(CA - AB)^{-1}\end{gathered}$$

$$CA = \begin{pmatrix} 5 & -1 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} 2 & 3 & -1 \\ 1 & -2 & -1 \end{pmatrix} \sim \begin{pmatrix} 9 & 17 & -4 \\ 0 & 14 & 2 \end{pmatrix}$$

$$AB = \begin{pmatrix} 2 & 3 & -1 \\ 1 & -2 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 & 2 \\ 3 & 1 & -1 \\ 2 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 9 & 0 & 0 \\ -7 & -4 & 3 \end{pmatrix}$$

$$CA - AB = \begin{pmatrix} 0 & 17 & -4 \\ 7 & 10 & -1 \end{pmatrix}$$

$$(CA - AB)^{-1} = \left(\begin{array}{ccc|ccc} 7 & 10 & -1 & 1 & 0 & 0 \\ 0 & 17 & -4 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 7 & 10 & -1 & 1 & 0 & 0 \\ 0 & 17 & 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 7 & 10 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{17} & \frac{4}{17} \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 7 & 0 & -1 & 1 & \frac{-10}{17} & \frac{-40}{17} \\ 0 & 1 & 0 & 0 & \frac{1}{17} & \frac{4}{17} \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 7 & 0 & 0 & 1 & \frac{-10}{17} & \frac{-23}{17} \\ 0 & 1 & 0 & 0 & \frac{1}{17} & \frac{4}{17} \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{7} & \frac{-10}{119} & \frac{-23}{119} \\ 0 & 1 & 0 & 0 & \frac{1}{17} & \frac{4}{17} \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} \frac{1}{7} & \frac{-10}{119} & \frac{-23}{119} \\ 0 & \frac{1}{17} & \frac{4}{17} \\ 0 & 0 & 1 \end{pmatrix}$$