MAP3121- MÉTODOS NUMÉRICOS E APLICAÇÕES

Um problema inverso para obtenção de distribuição de Temperatura

Parte 1 - EP

Guilherme Akira Alves dos Santos 11027484

Letícia Mendonça Carraro 10374480

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SUMÁRIO

1. Introdução

Este relatório trata da documentação do projeto proposto pela disciplina de Métodos Numéricos e Aplicações. Tal projeto consiste da resolução de um problema direto.

Problemas diretos são aqueles nos quais busca-se descobrir o efeito que dada ação trará, ao passo que problemas inversos são aqueles que, dados efeitos, tenta-se buscar sua causa. Nesta primeira parte do exercício é proposto que seja solucionado o problema direto da equação de calor.

2. A equação do calor

As seguintes equações são utilizadas no enunciado para descrever a distribuição do calor numa barra metálica

$$u_t(t,x) = u_{xx}(t,x) + f(t,x) \ em \ [0,T] \times [0,1]$$

$$u(0,x) = u_0(x) \ em \ [0,1]$$

$$u(t,0) = g_1(t) \ em \ [0,T]$$

$$u(t,1) = g_2(t) \ em \ [0,T]$$

Após essas equações terem sido apresentadas, foi aplicado o método das diferenças finitas para obter uma equação capaz de aproximar o valor numérico das variáveis de interesse.

Contudo, o método não é capaz de fornecer um resultado com perfeita precisão, havendo assim uma equação para calculá-lo. Deve ser lembrado que o objetivo é fazer com que tal erro seja o menor possível.

3. Métodos

Foi decidido pela dupla que a linguagem a ser utilizada no projeto seria C, devido a familiaridade que os membros possuem com esta. Do ponto de vista computacional, tal linguagem também apresenta vantagens de tempo de execução e desempenho.

O projeto foi inicialmente programado na plataforma Linux (distribuição Manjaro e Xubuntu) e quando foi testado em Windows (Windows 10), foram encontrados alguns problemas na execução. Estes problemas foram corrigidos, contudo o programa apresenta irregularidades quando a parte de plotar gráficos é executada.

4. Tarefas

4.1. Primeira Tarefa

A primeira tarefa consiste na implementação do método

com M e N sendo variáveis.

$$u_i^{k+1} = u_i^k + \Delta t\left(\frac{u_{i-1}^k - 2u_i^k + u_{i+1}^k}{\Delta x^2} + f(x_i, t_k)\right) \ , \ i = 1 \ , \ ... \ , N - 1 \ e \ k = 0 \ , \ ... \ , M - 1$$

Um dos problemas computacionais enfrentados ao implementar essa tarefa foi o valor de N. Percebeu-se que até N=160 o programa era capaz de plotar os gráficos. Entretanto, quando N chega ao valor de 320, a dupla não possuía poder computacional suficiente para plotar o gráfico.

O número de passos temporais M dá-se pela relação:

$$M = \frac{TN^2}{\lambda}$$

Assim, para T=1 e N=640, obtemos:

$$\lambda = 0.5 \Rightarrow M = \frac{640^2}{0.5} = 819200 \quad | \quad \lambda = 0.25 \Rightarrow M = \frac{640^2}{0.25} = 1638400$$

Primeiramente, foi sugerido que o programa fosse testado com algumas funções conhecidas. Os resultados obtidos estão mostrados a seguir:

T = 1

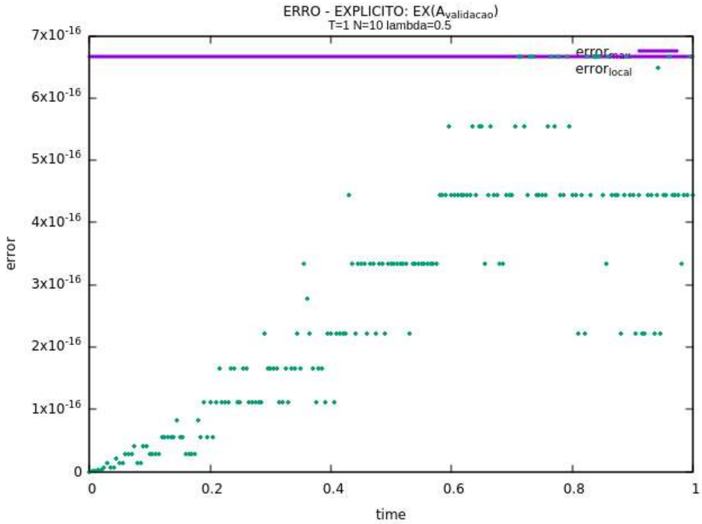
Fonte: $f(t, x) = 10x^2(x - 1) - 60xt + 20t$

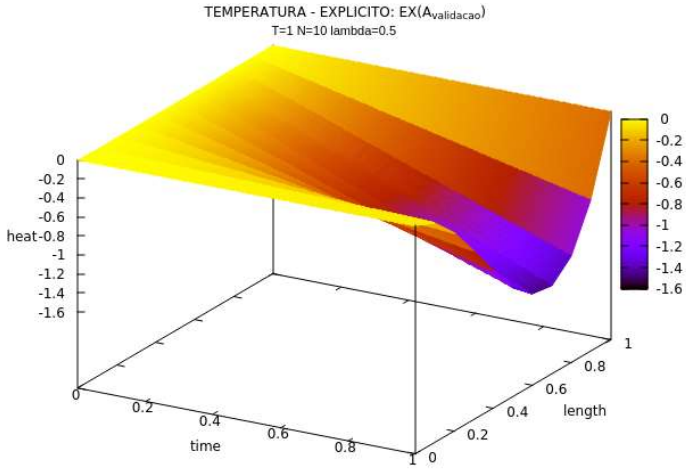
Condição inicial: $u_0(x) = 0$

Condições nulas na fronteira.

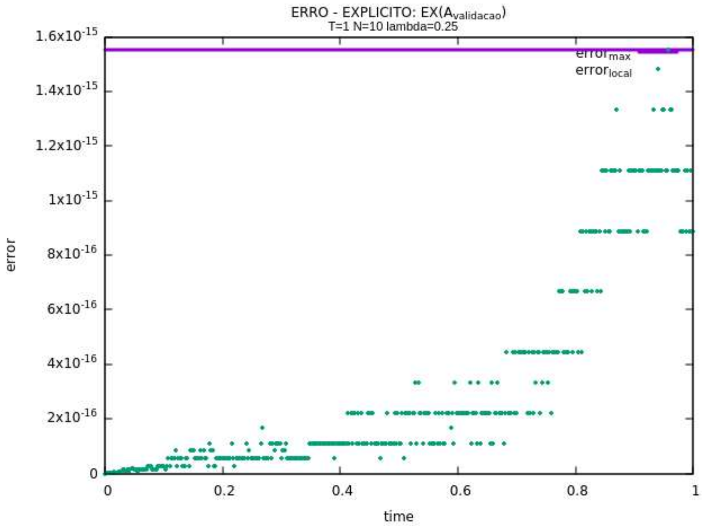
Solução exata: $u(t, x) = 10tx^2(x - 1)$

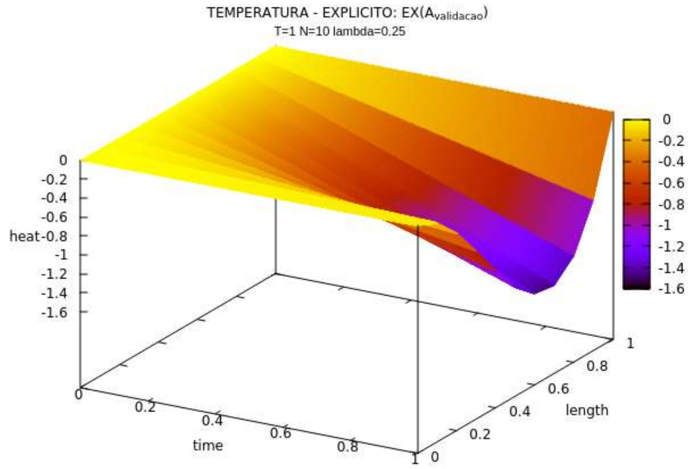
N= 10 e λ = 0.5



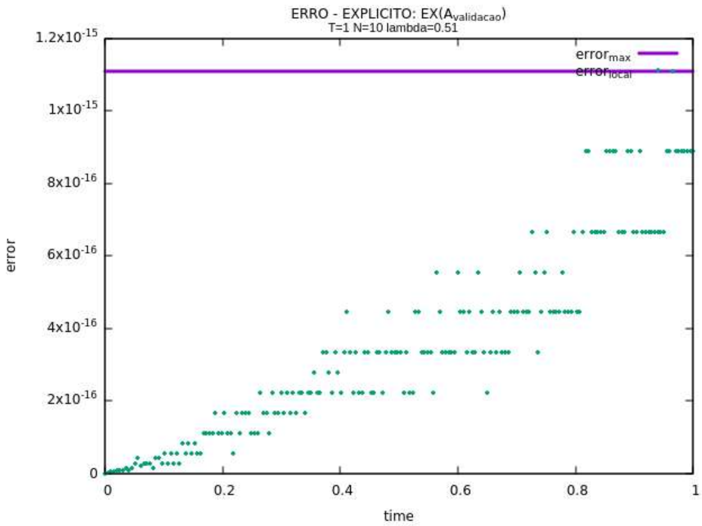


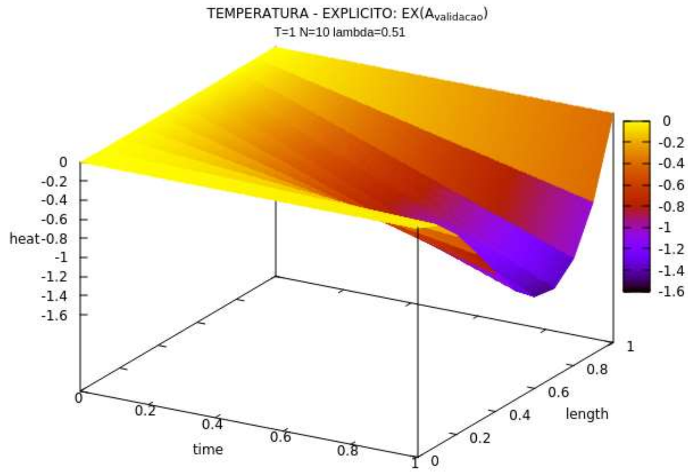
N= 10 e λ = 0.25



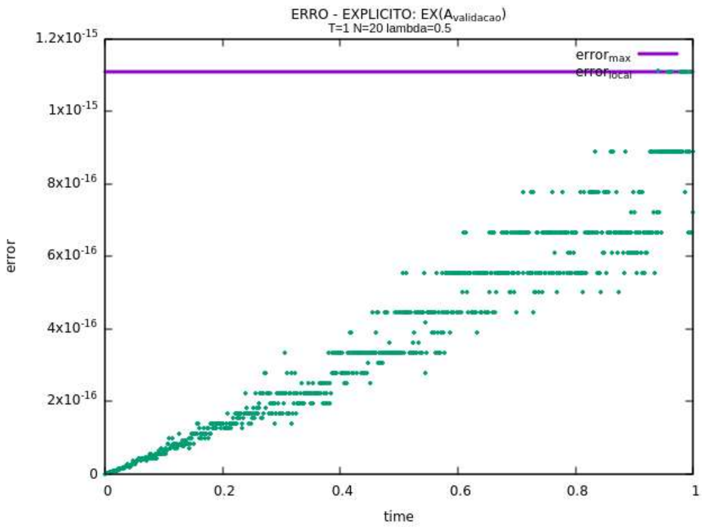


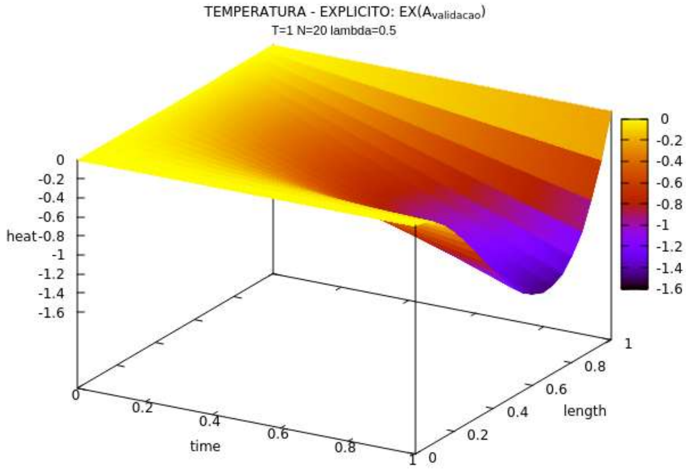
N= 10 e λ = 0.51





N= 20 e λ = 0.5





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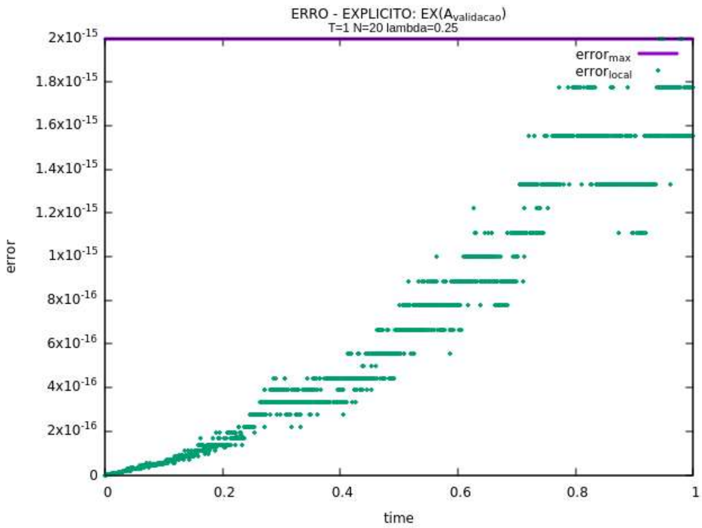

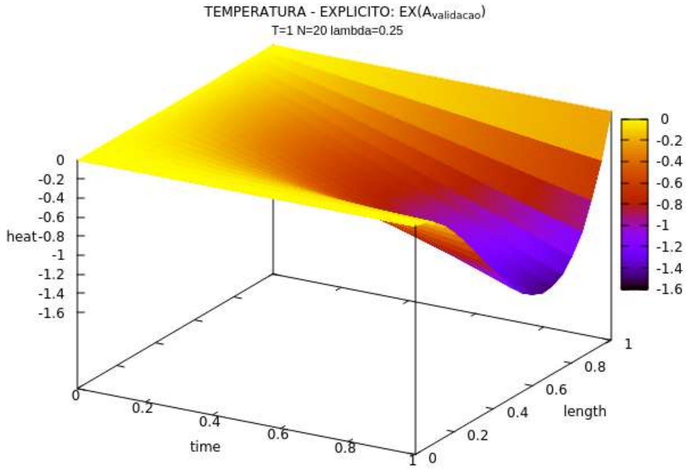

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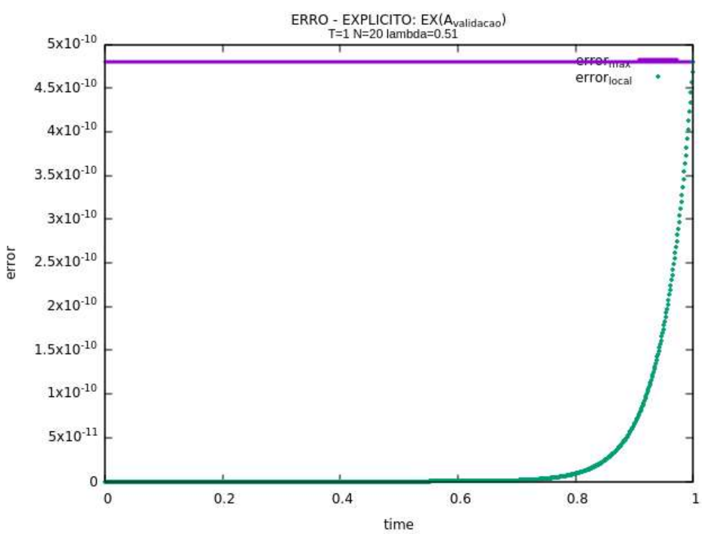
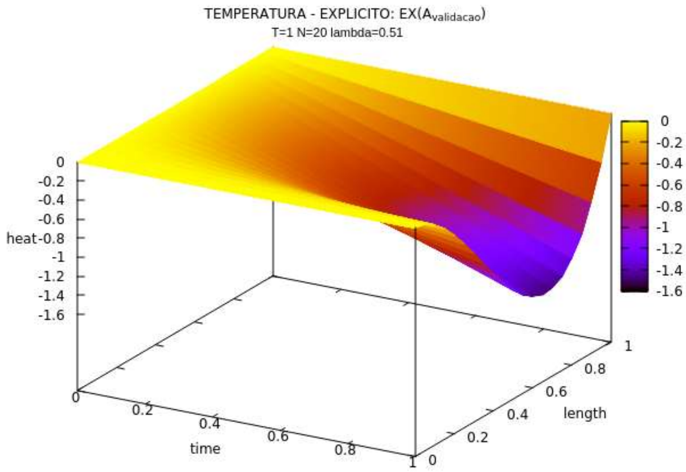
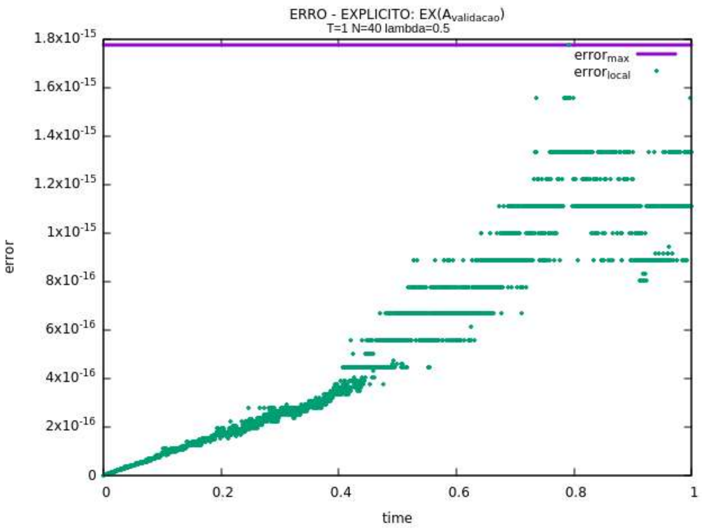
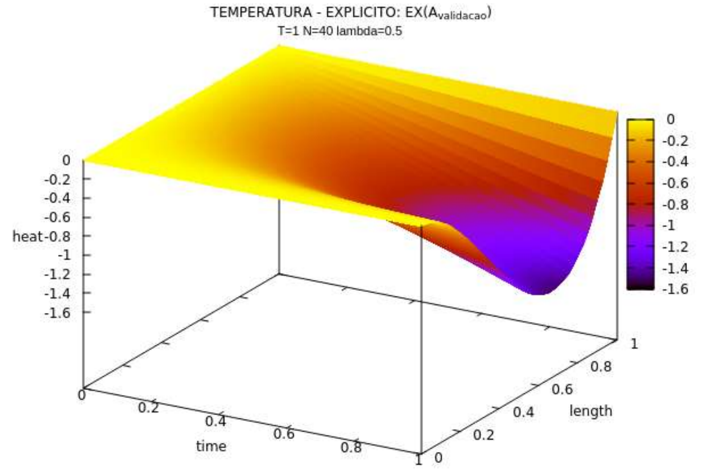




N= 40 e λ = 0.5





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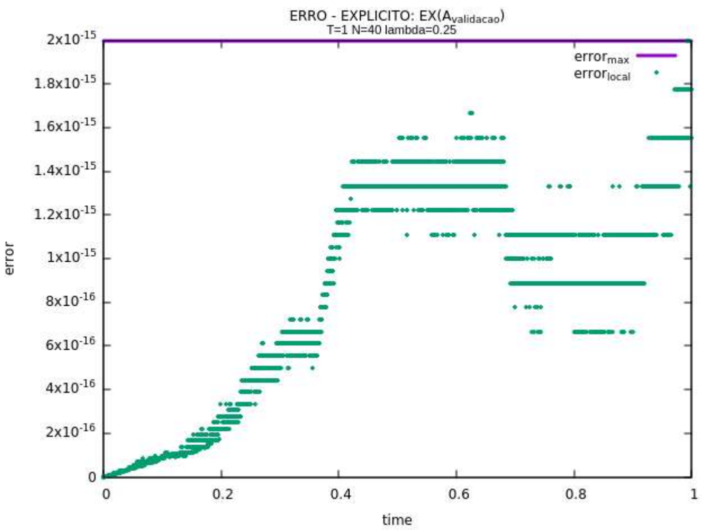

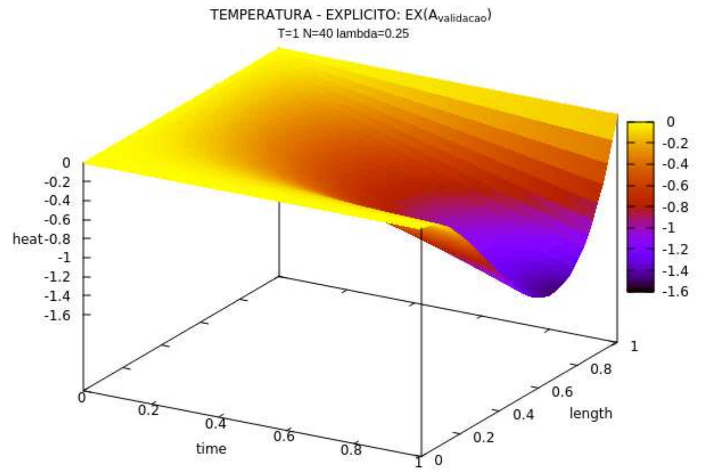

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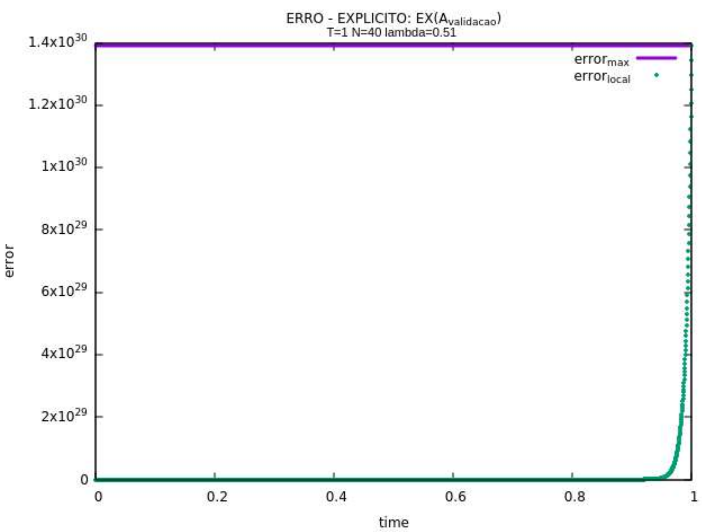
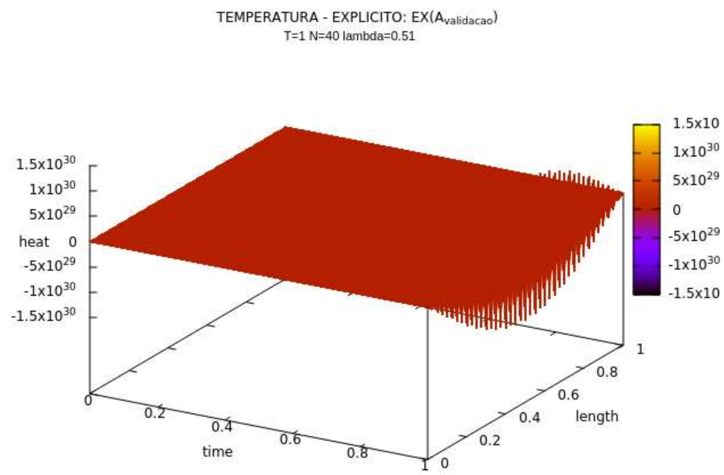
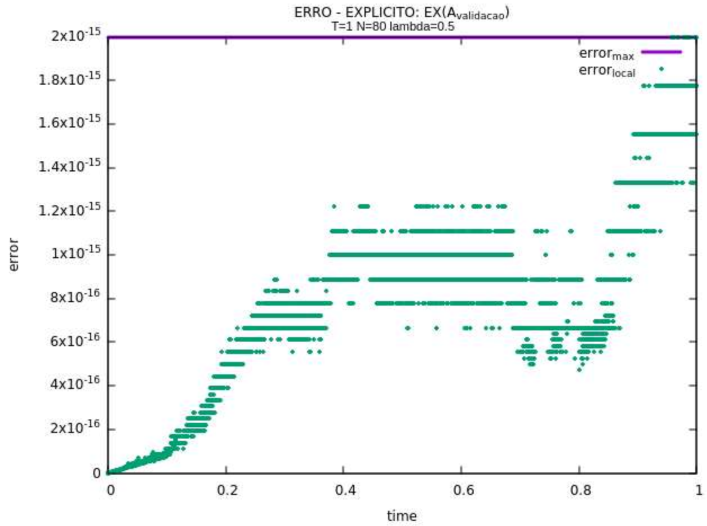
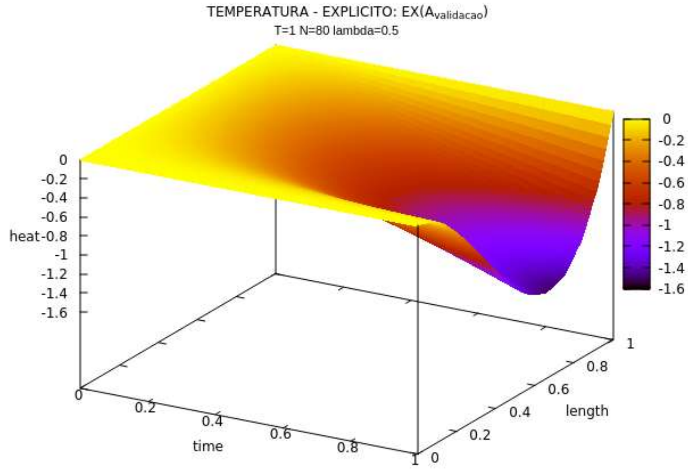


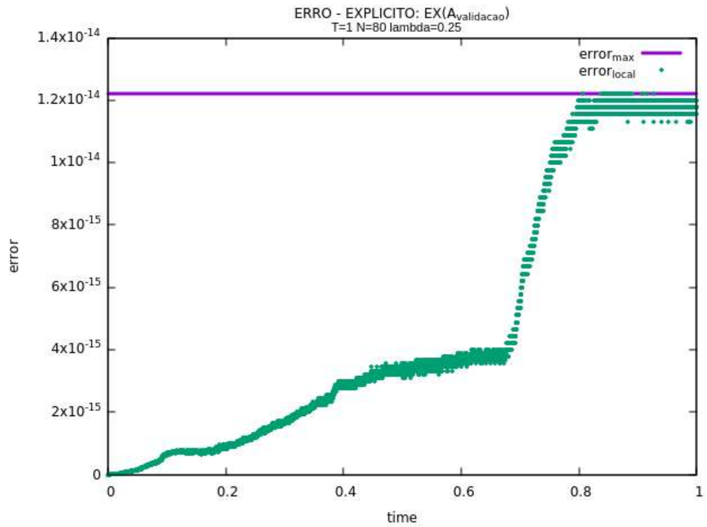


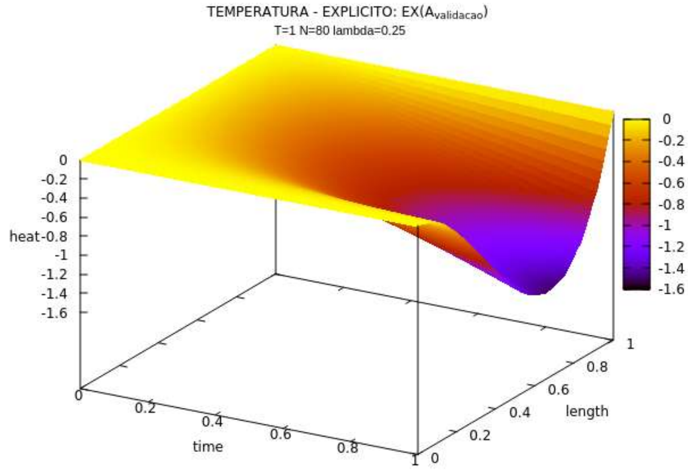
N= 80 e λ = 0.5





N= 80 e λ = 0.25





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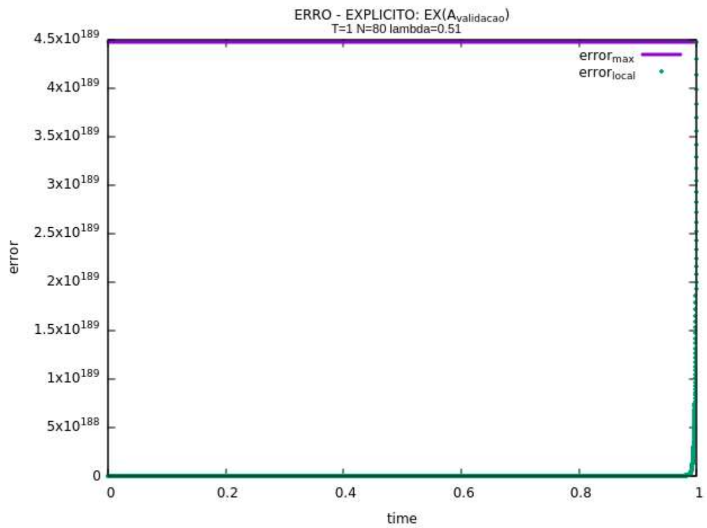
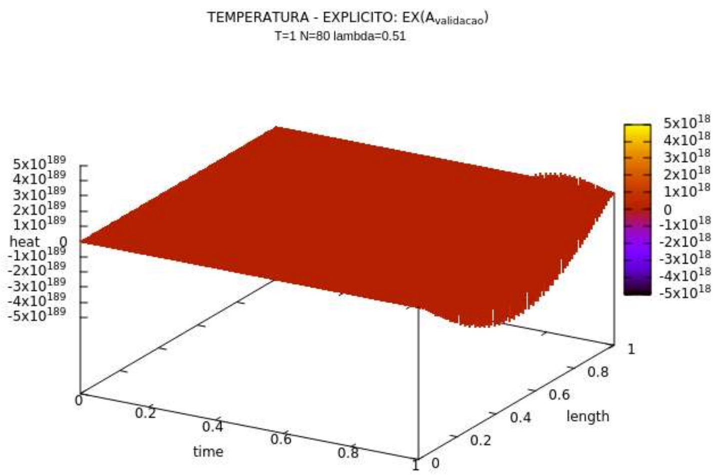
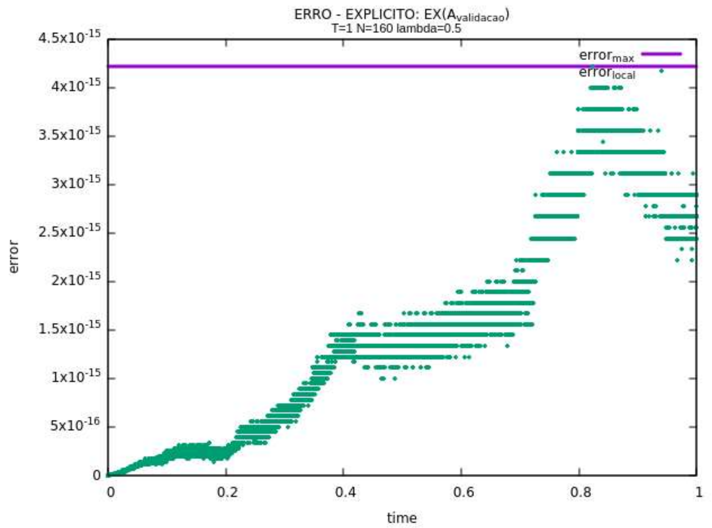
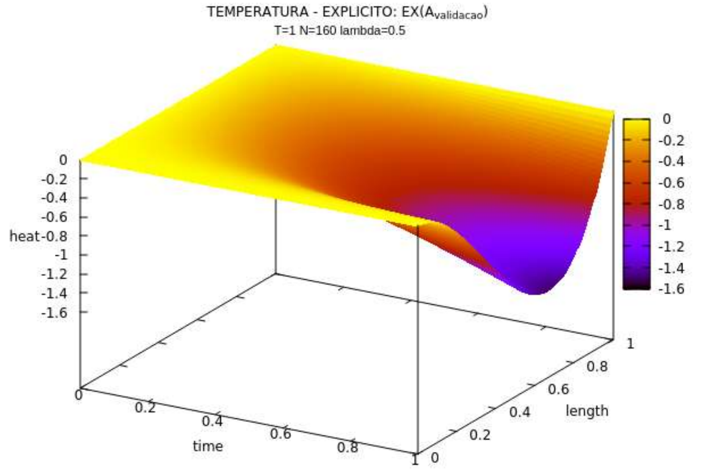




N= 160 e λ = 0.5





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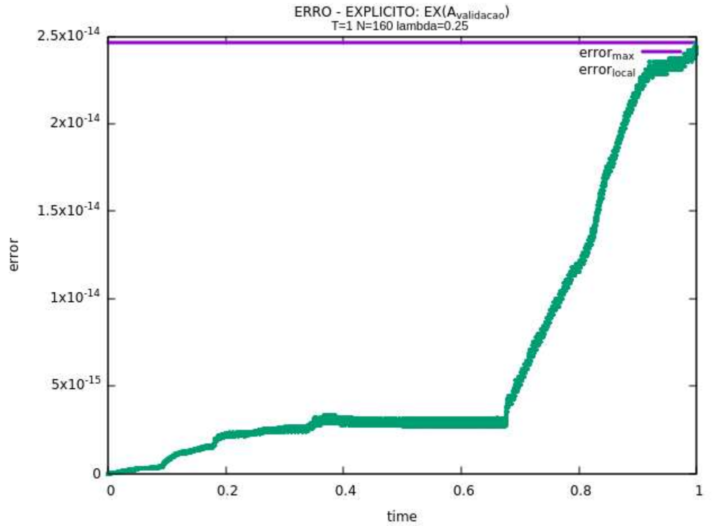

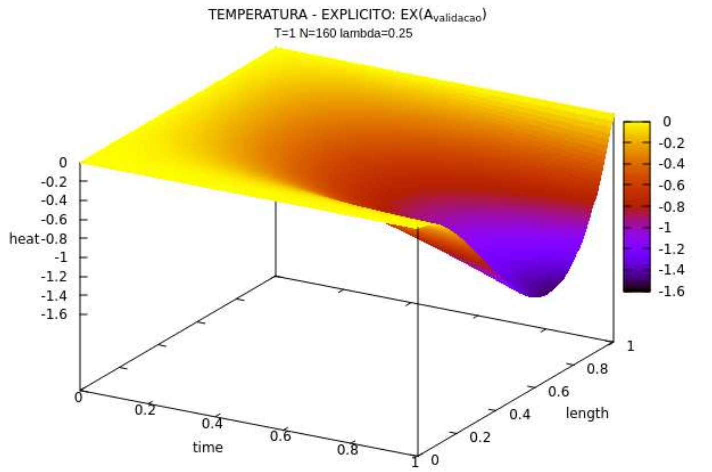

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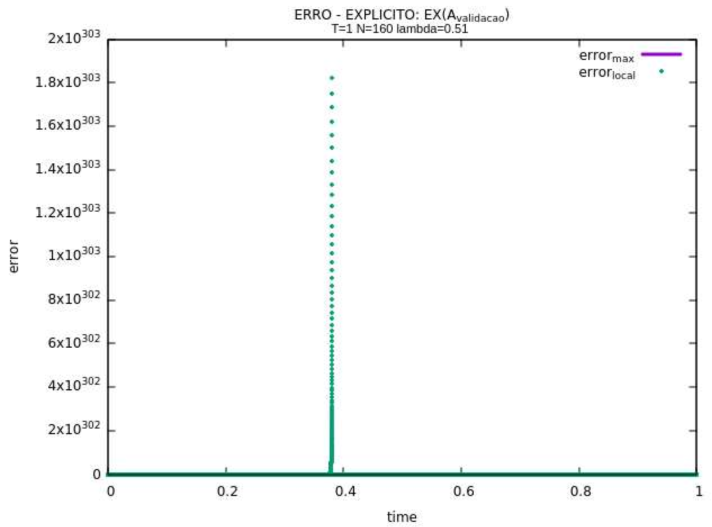
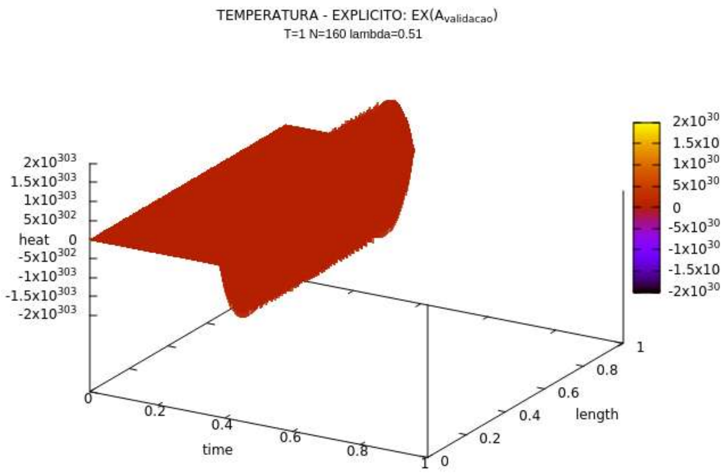




Os gráficos a seguir não foram obtidos devido a falta de poder computacional da dupla (no entanto, estes se mantêm praticamente iguais aos gráficos obtidos anteriormente):

- $N= 320$ e $\lambda = 0.5$
- $N= 320$ e $\lambda = 0.25$
- $N= 320$ e $\lambda = 0.51$

4.1.1. Item a

T = 1

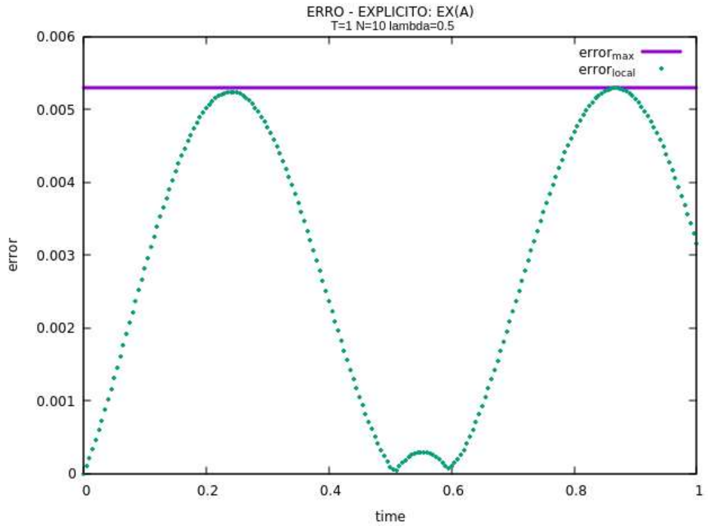
Fonte: $f(t,x) = 10cos(10t)x^2(1-x)^2 - (1+sin(10t))(12x^2 - 12x + 2)$

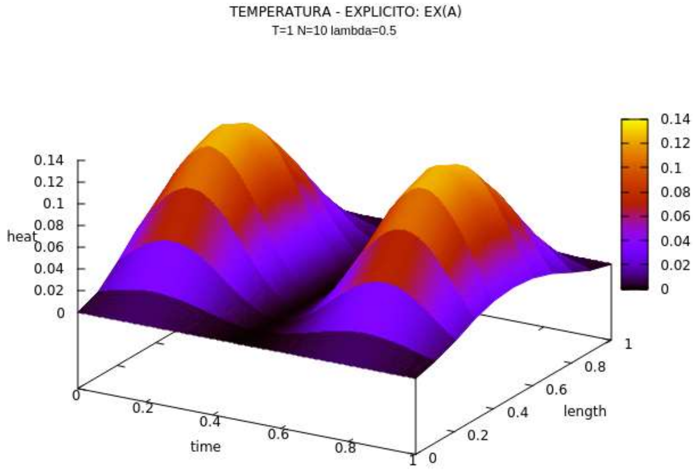
Condição inicial: $u_0(x) = x^2(1-x)^2$

Condições nulas na fronteira.

Solução exata: $u(t,x) = (1+sin(10t))x^2(1-x)^2$

N= 10 e λ = 0.5





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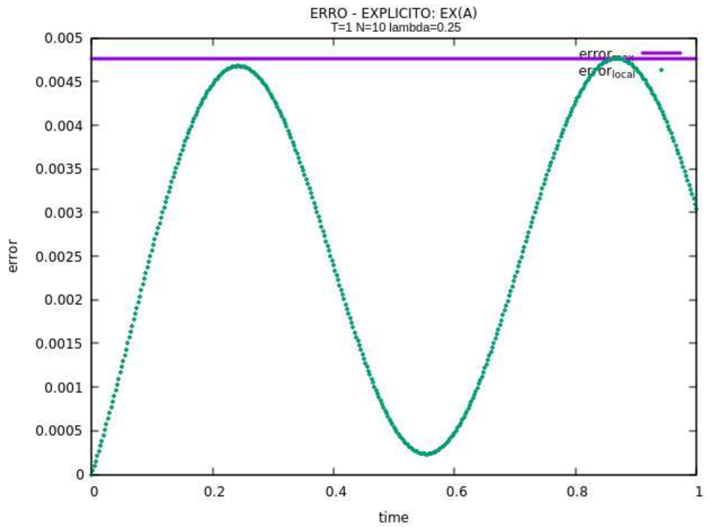

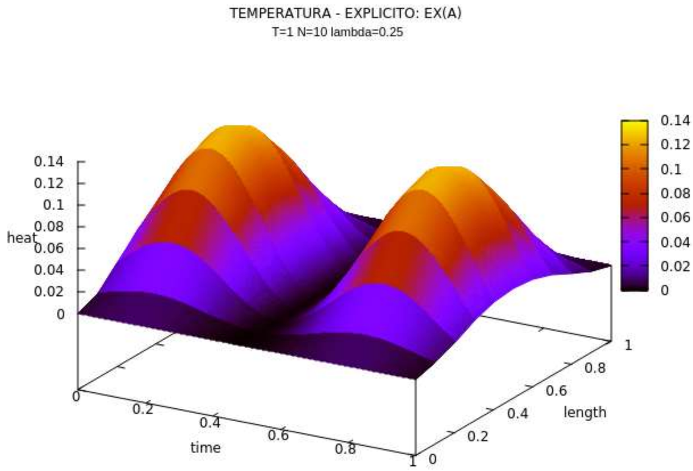

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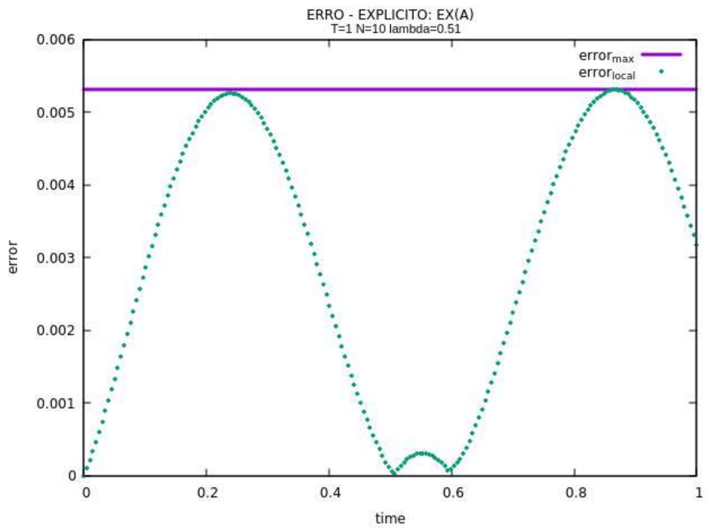
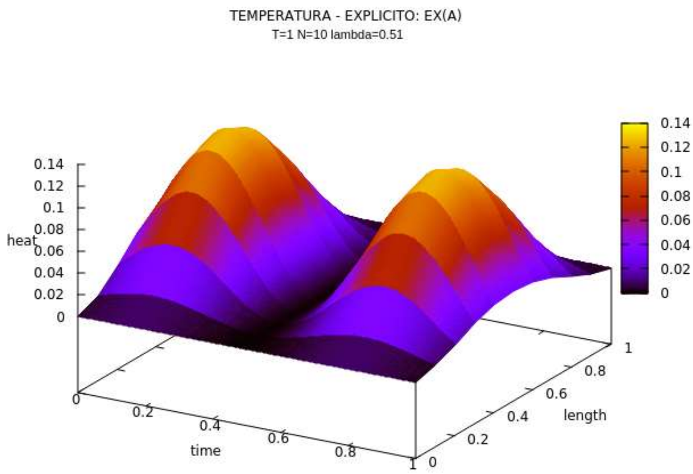
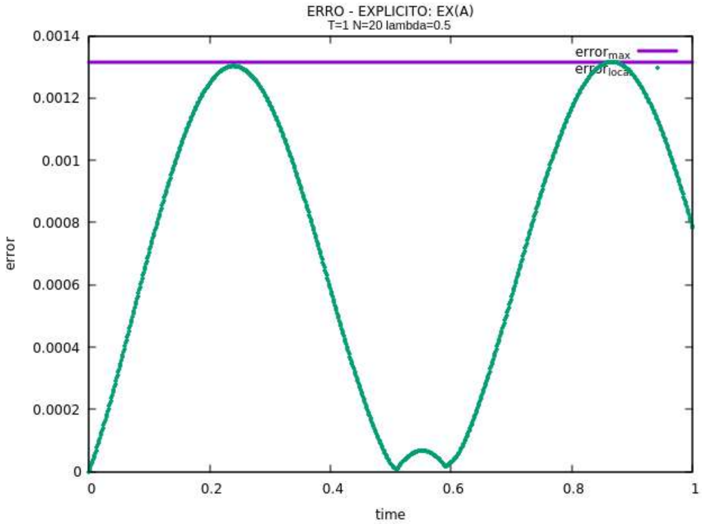
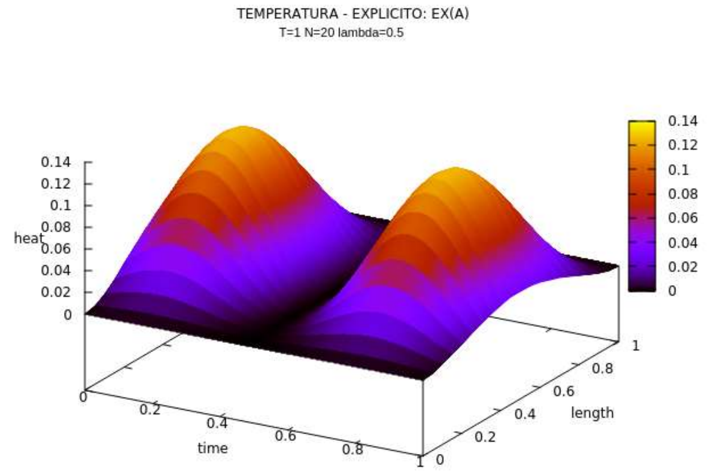




N= 20 e λ = 0.5





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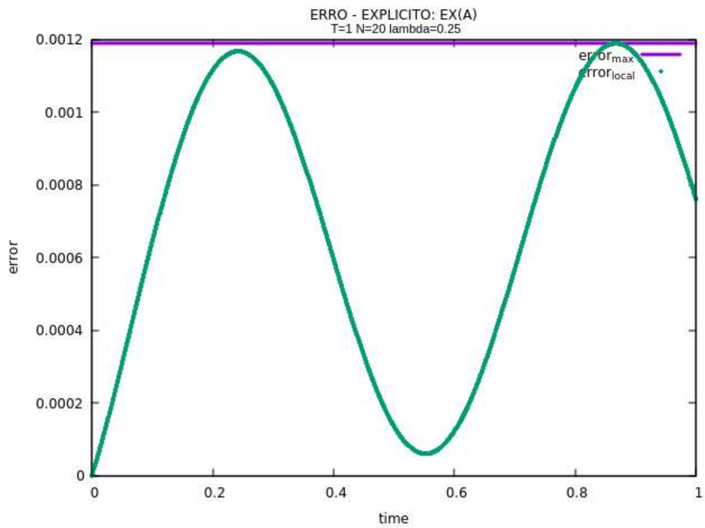

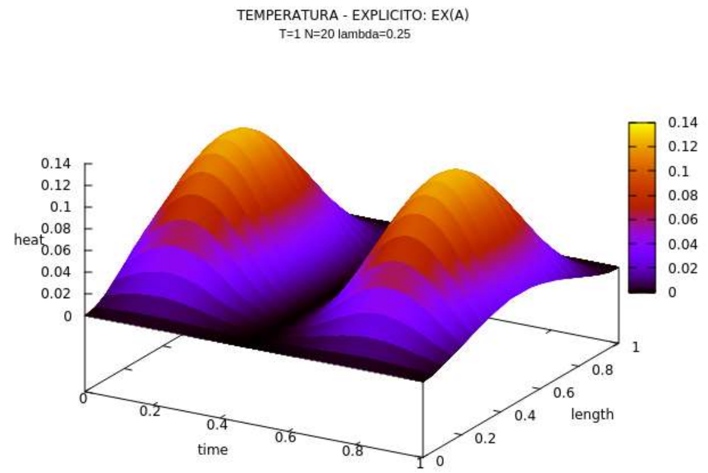

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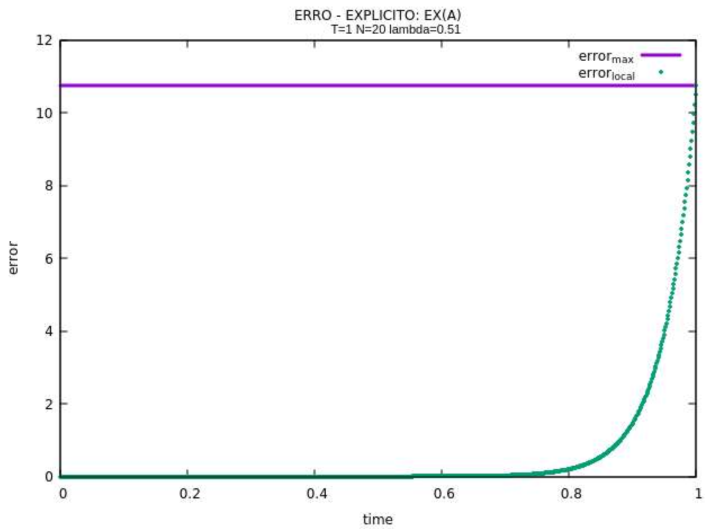
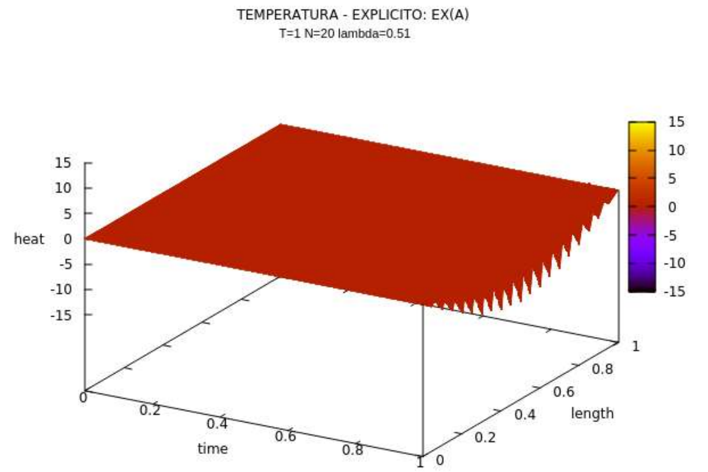
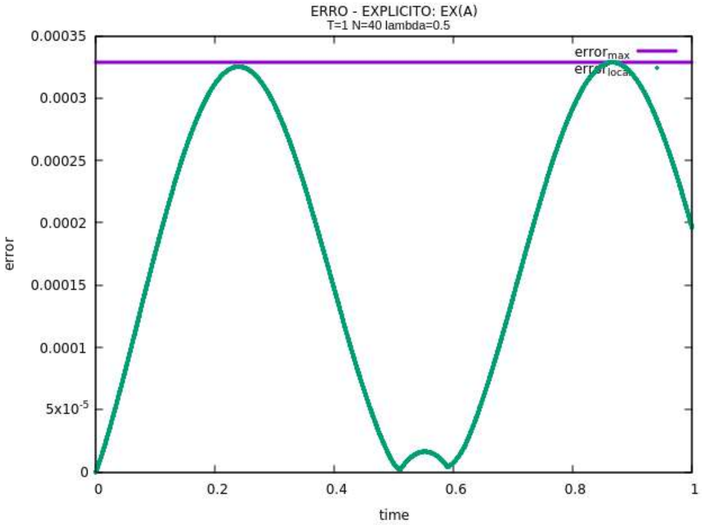
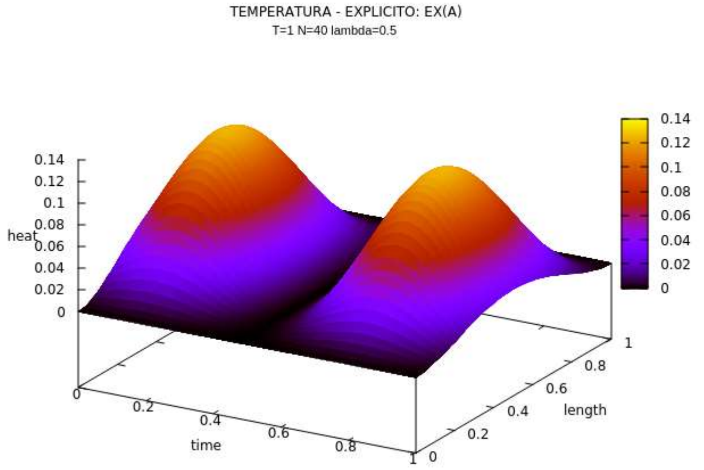




N= 40 e λ = 0.5





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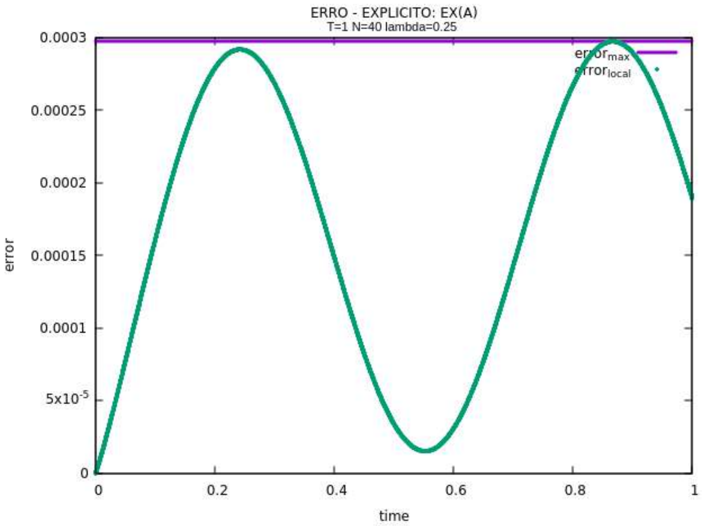

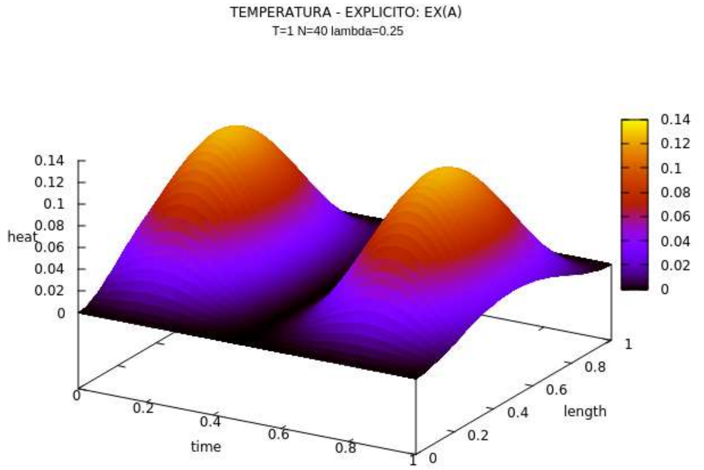

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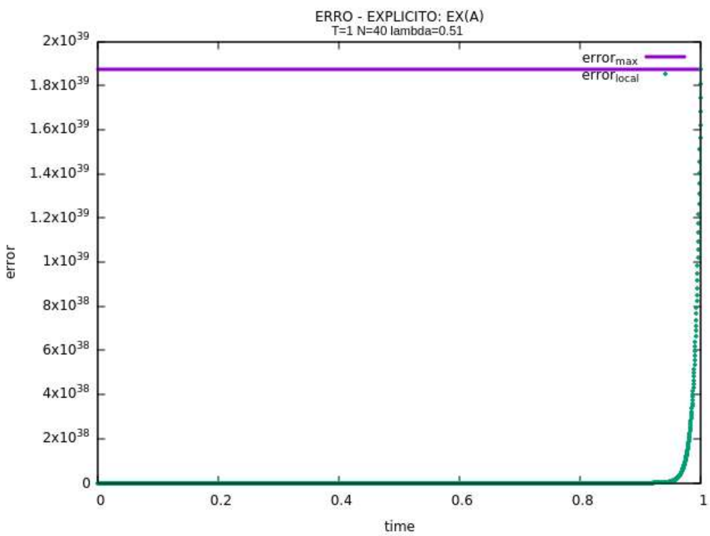

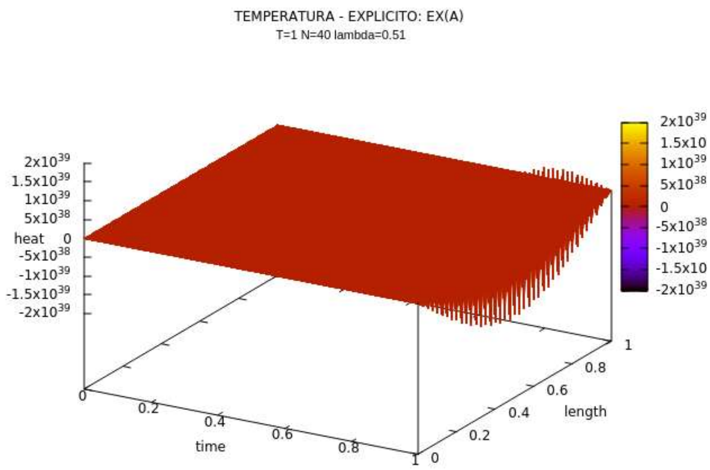

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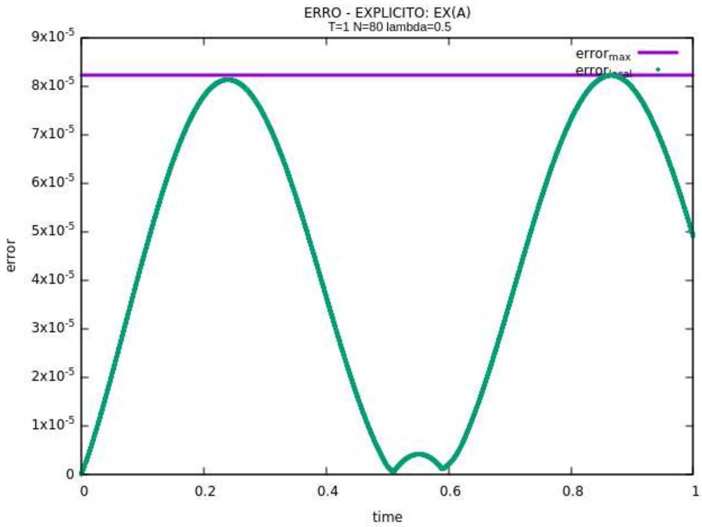

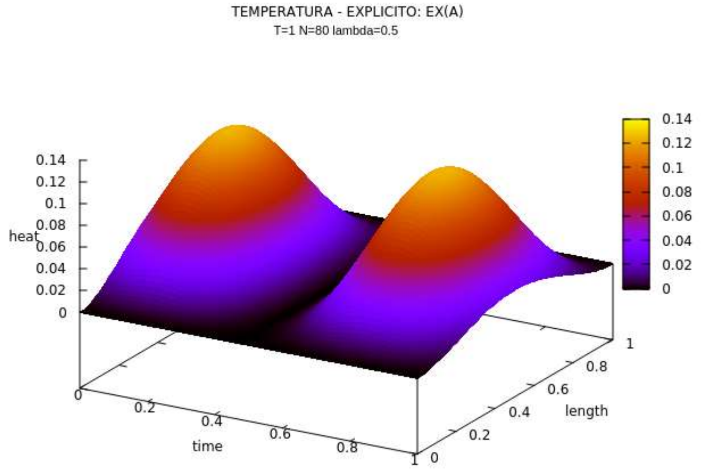

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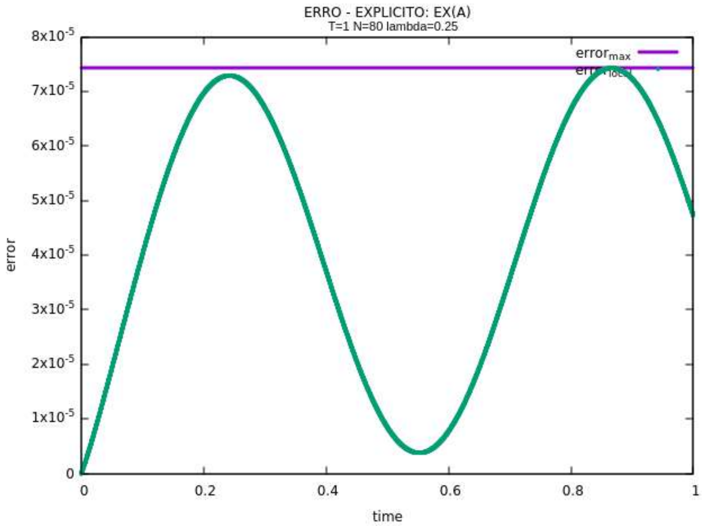

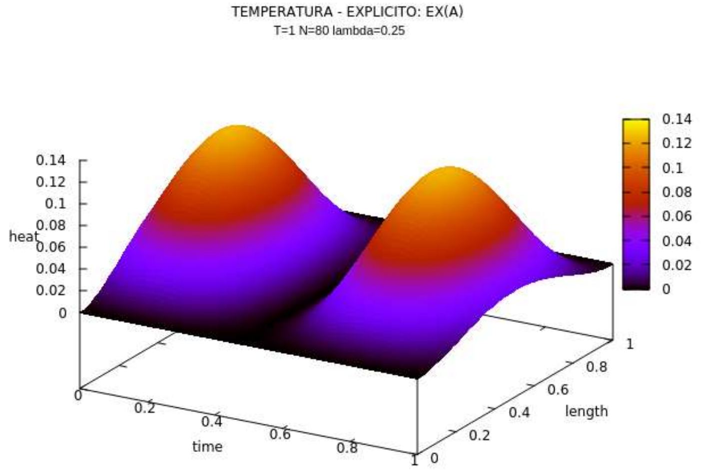

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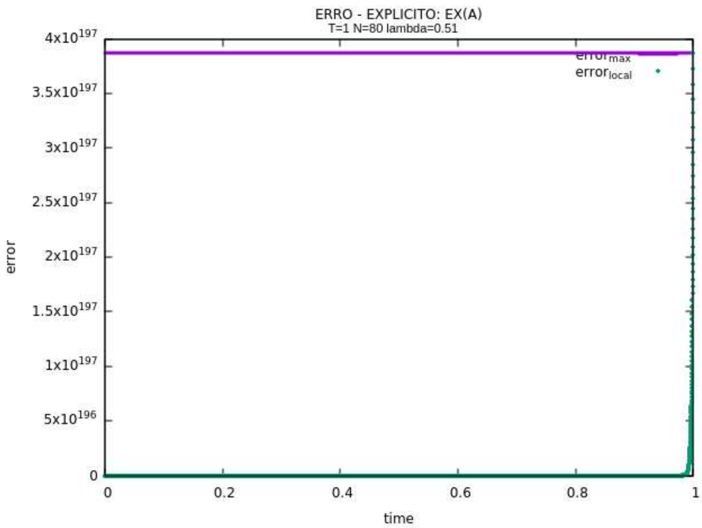

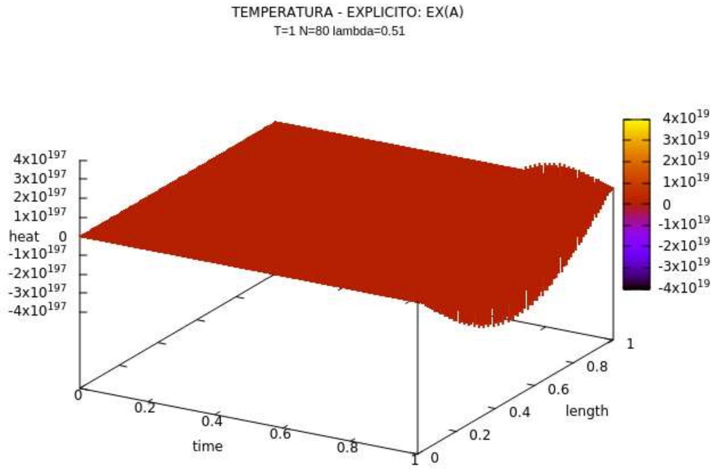

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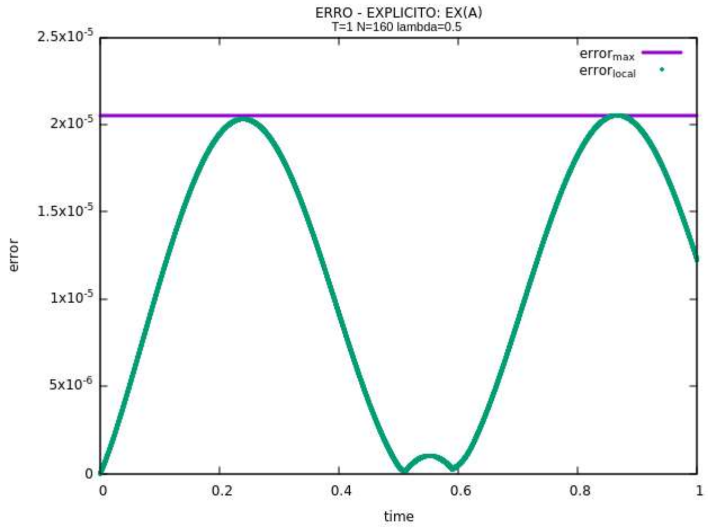

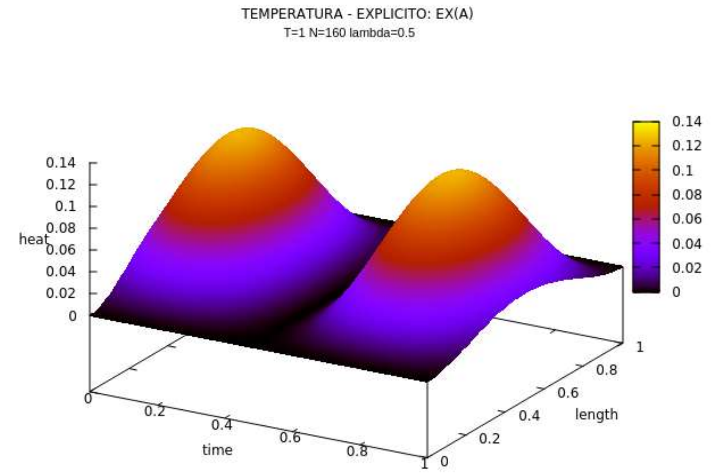

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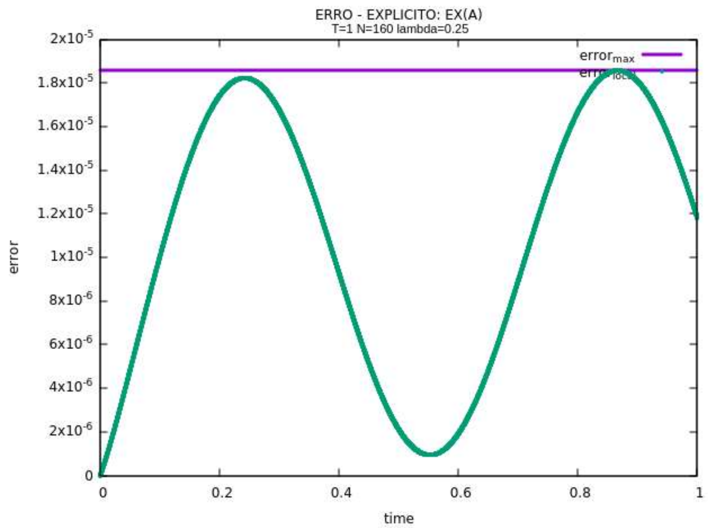

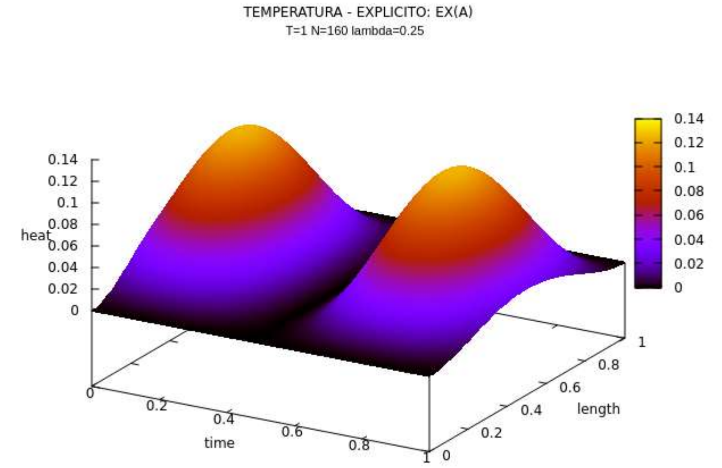

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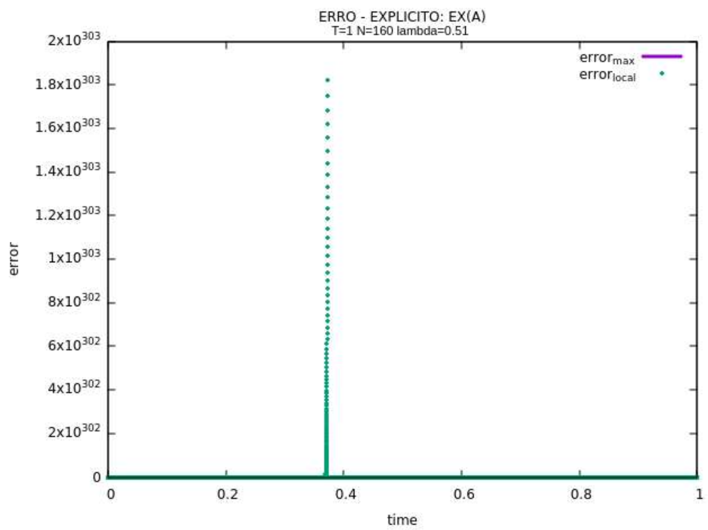
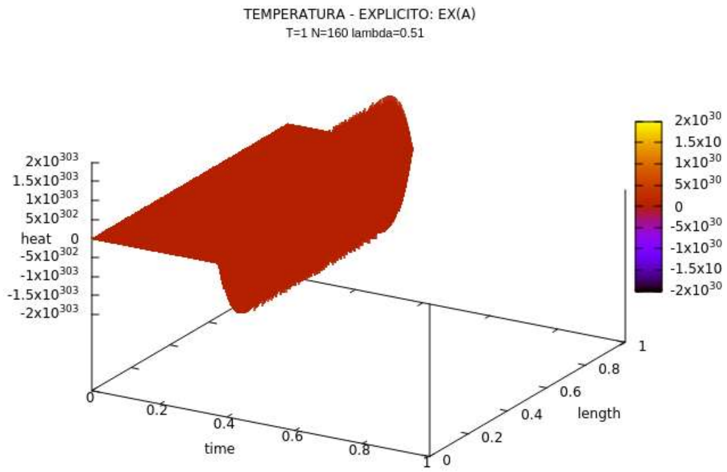




Os gráficos a seguir não foram obtidos devido a falta de poder computacional da dupla (no entanto, estes se mantêm praticamente iguais aos gráficos obtidos anteriormente):

- N= 320 e $\lambda = 0.5$
- N= 320 e $\lambda = 0.25$
- N= 320 e $\lambda = 0.51$

4.1.2. Item b

Determinar $u_0(x),\ g_1(t),\ g_2(t)\ e\, f(t,x)$ onde:

Solução exata: $u(t,x) =\ e^{t-x}cos(5tx)$

Condição inicial:

$u_0(x) = u(0,x)$

$u_0(x) = e^{-x}$

Fronteira inferior:

$g_1(t) = u(t,0)$

$g_1(t) = e^t$

Fronteira superior:

$g_2(t) = u(t,1)$

$g_2(t)\ = e^{t-1}cos(5t)$

Fonte:

$f(t,x) = \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2}$

Sendo $u(t,x) =\ e^{t-x}cos(5tx)$, *ao realizar suas derivadas parciais* :

$\frac{\partial u}{\partial t} =\ e^{t-x}[cos(5tx) - 5xsin(5tx)]$

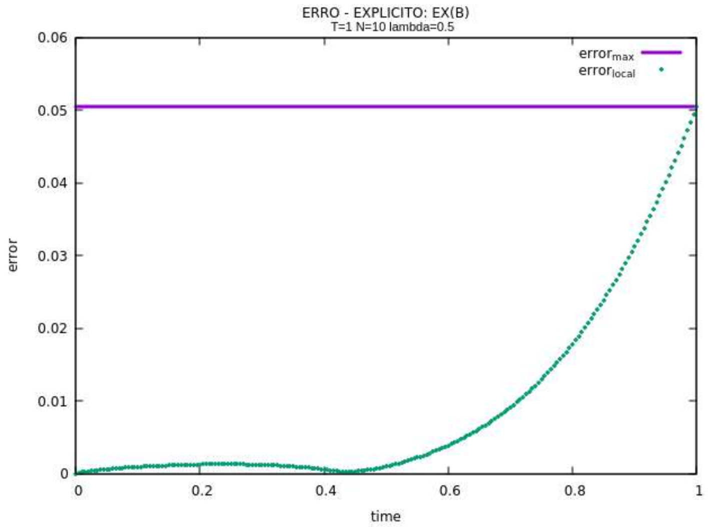
$\frac{\partial^2 u}{\partial x^2} =\ e^{t-x}[(1-25t^2)cos(5tx) + 10tsin(5tx)]$

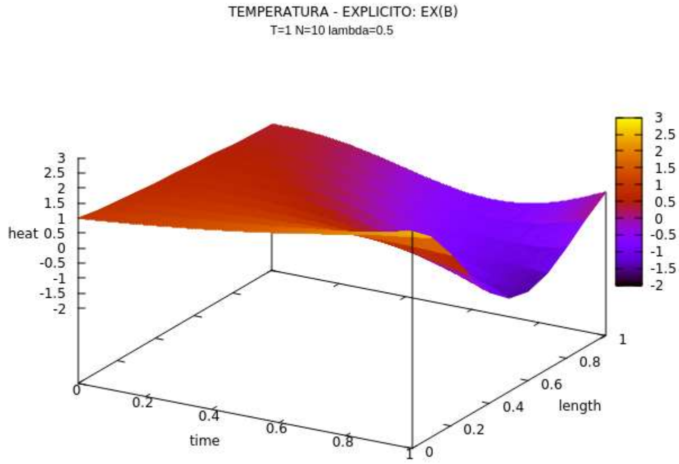
$\therefore$

$f(t,x) =\ e^{t-x}[cos(5tx) - 5xsin(5tx)] -\ e^{t-x}[(1-25t^2)cos(5tx) + 10tsin(5tx)]$

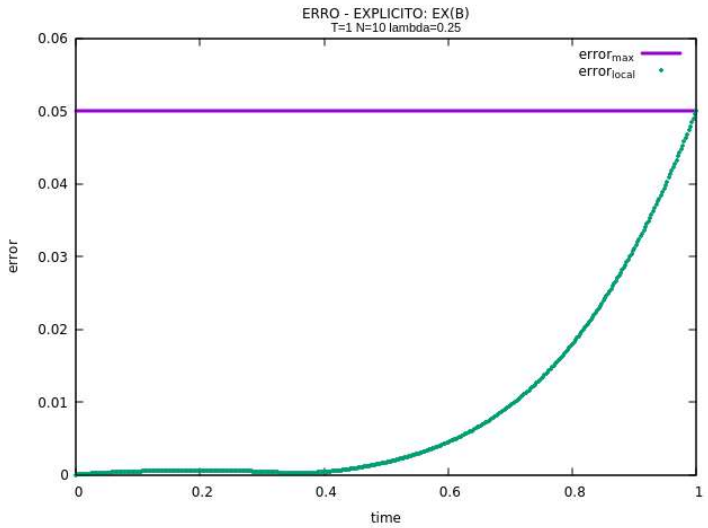
$f(t,x) =\ 5e^{t-x}[5t^2cos(5tx) - (2t+x)sin(5tx)]$

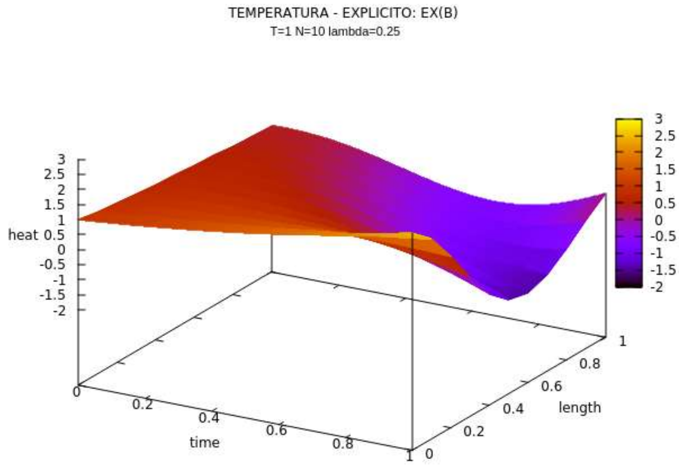
N= 10 e λ = 0.5



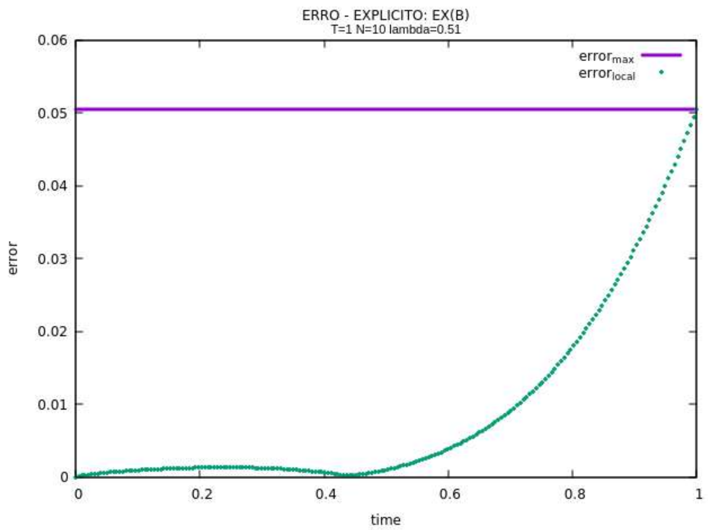


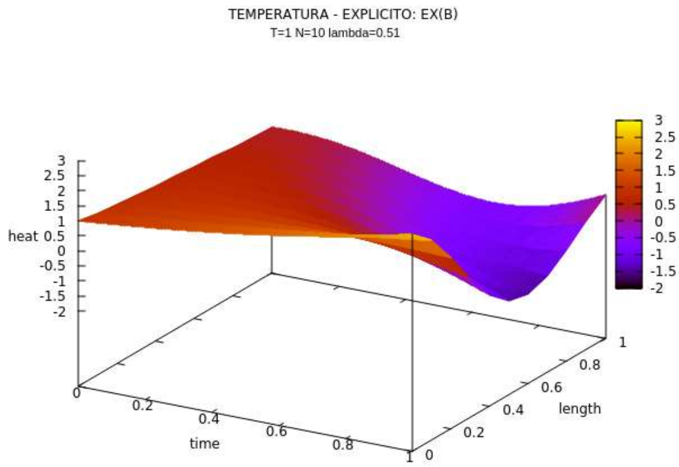
N= 10 e λ = 0.25



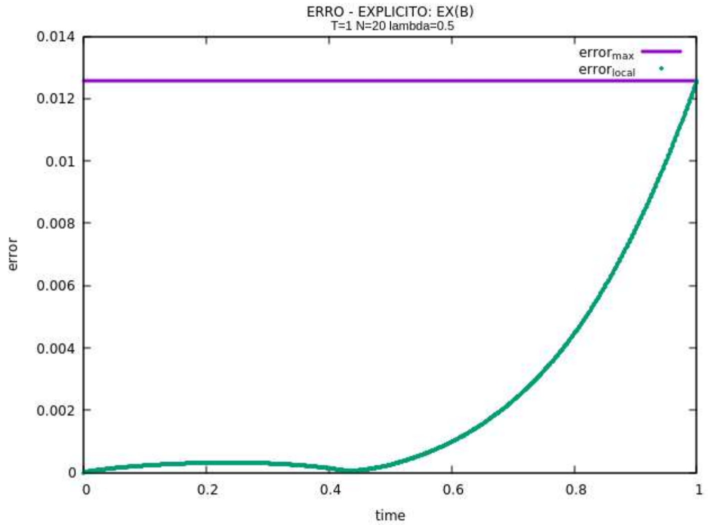


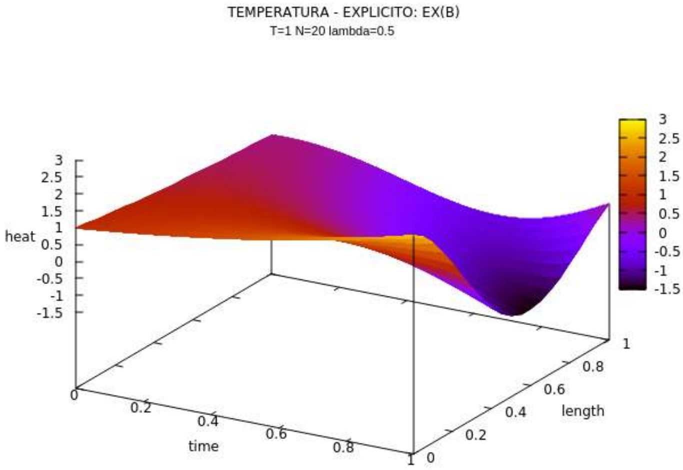
N= 10 e λ = 0.51



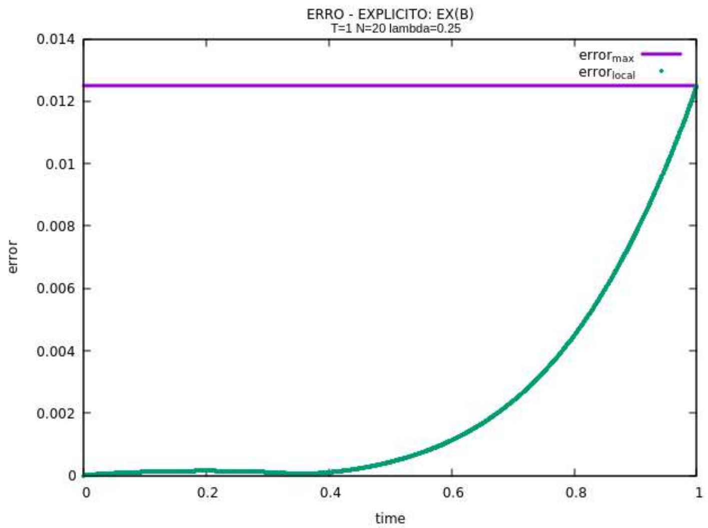


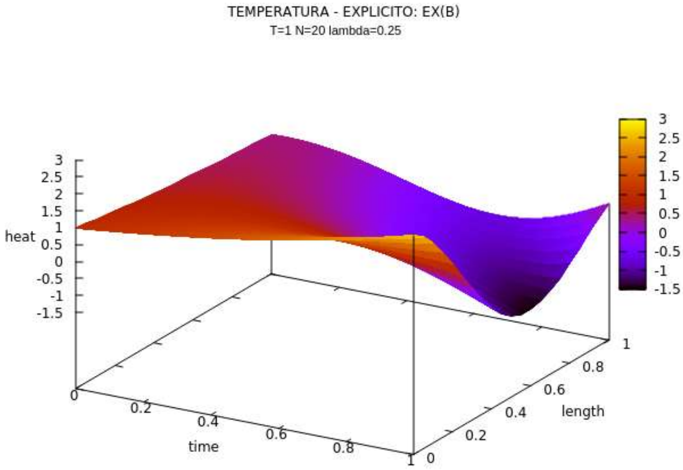
N= 20 e λ = 0.5



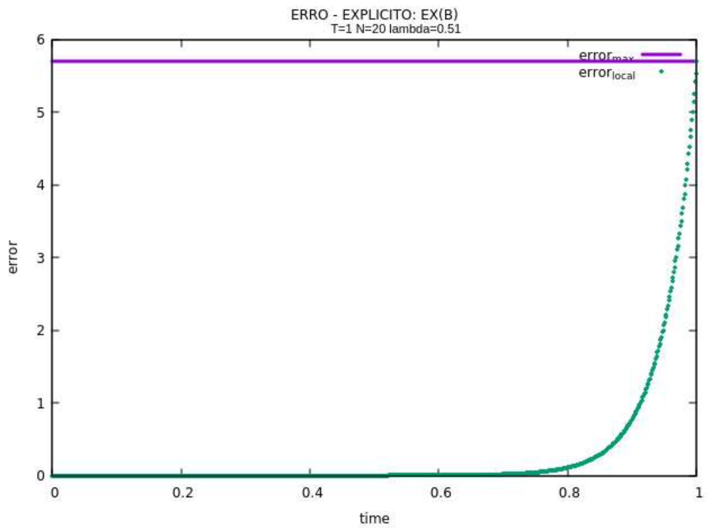


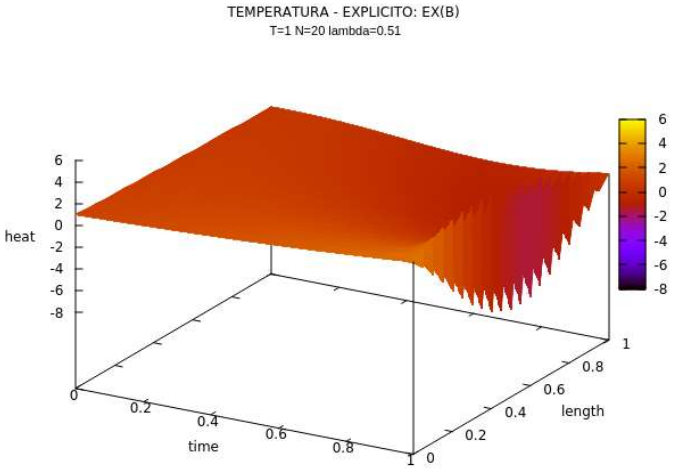
N= 20 e λ = 0.25



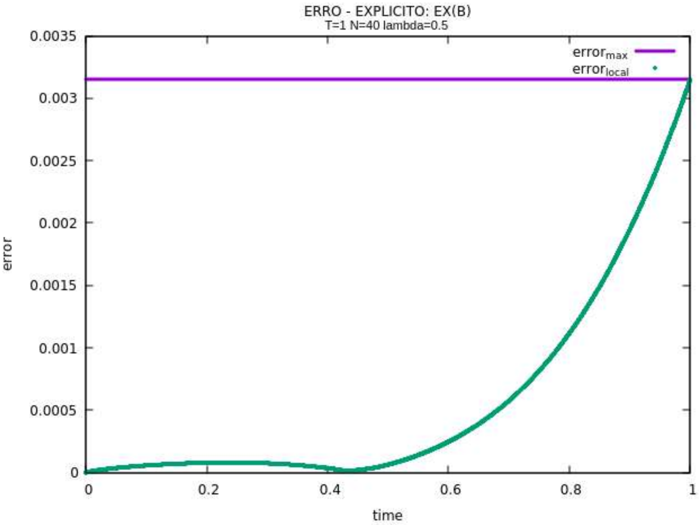


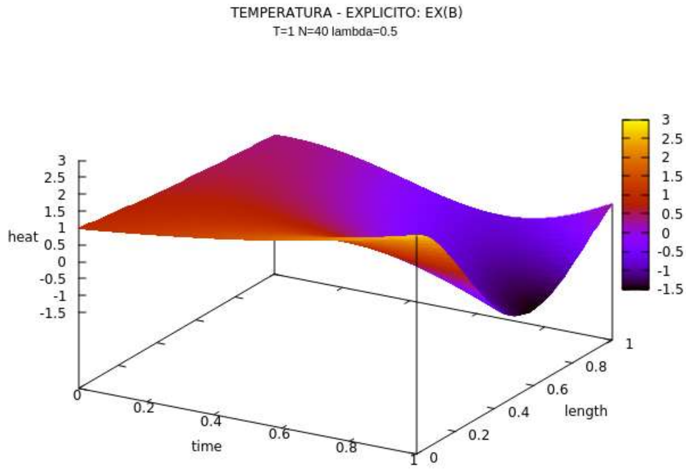
N= 20 e λ = 0.51



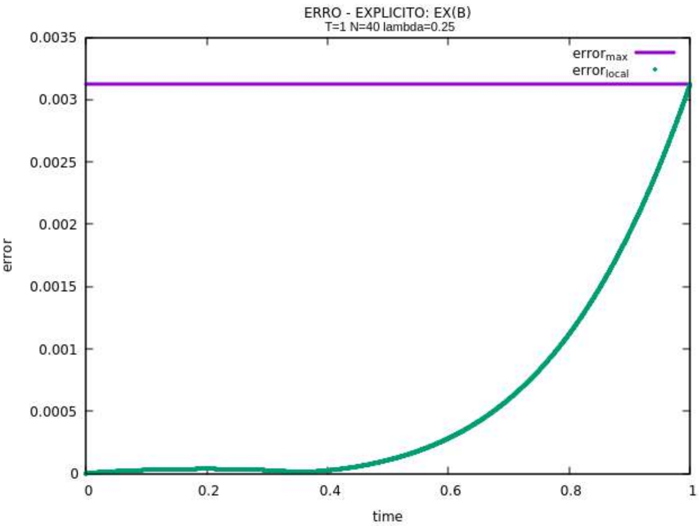


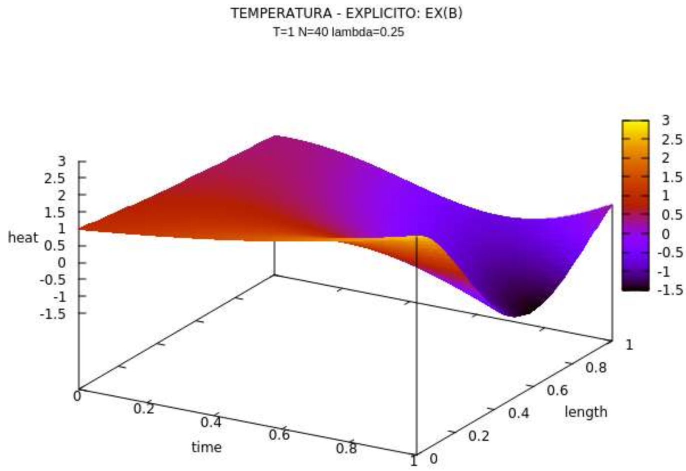
N= 40 e λ = 0.5



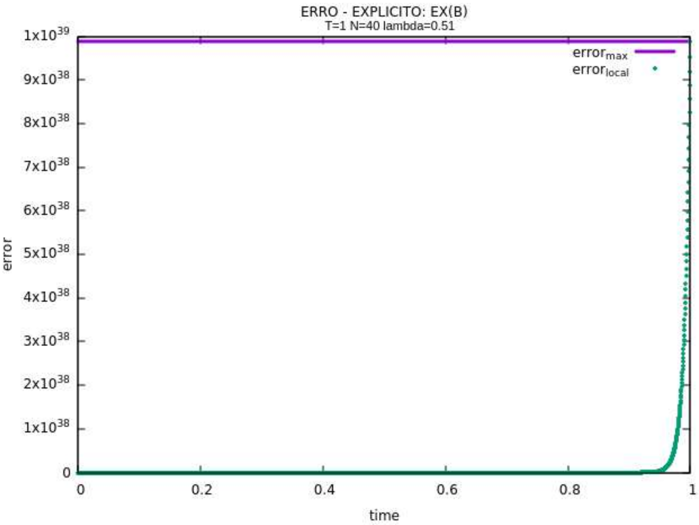


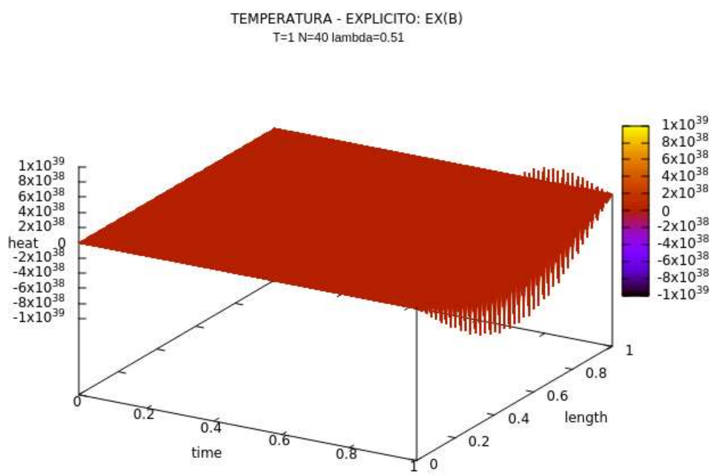
N= 40 e λ = 0.25



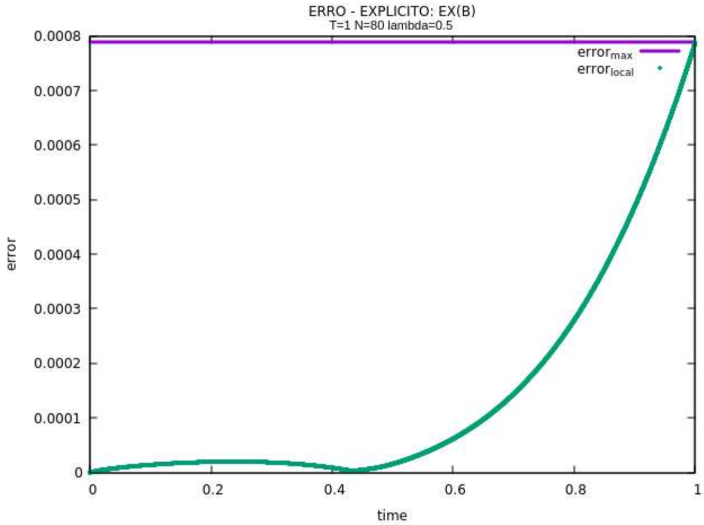


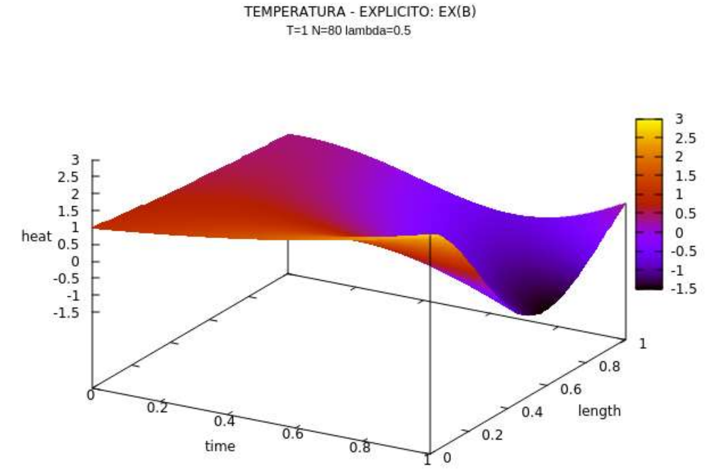
N= 40 e λ = 0.51



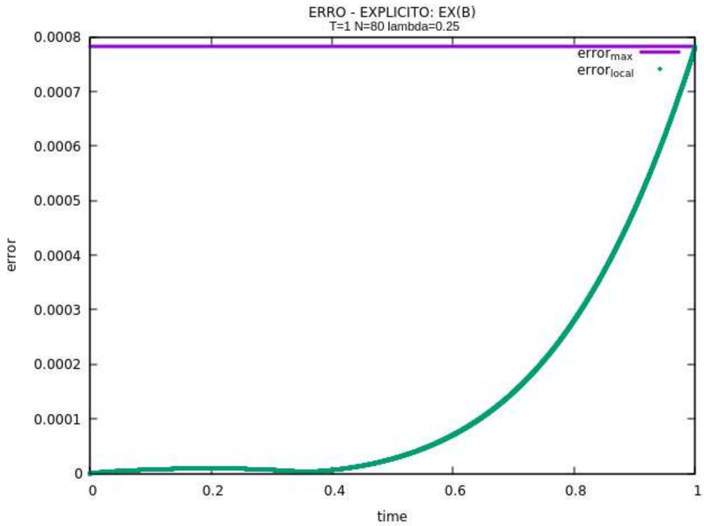


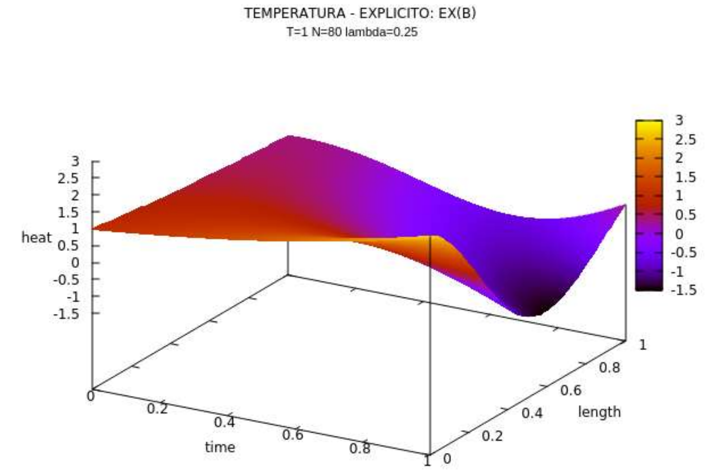
N= 80 e λ = 0.5



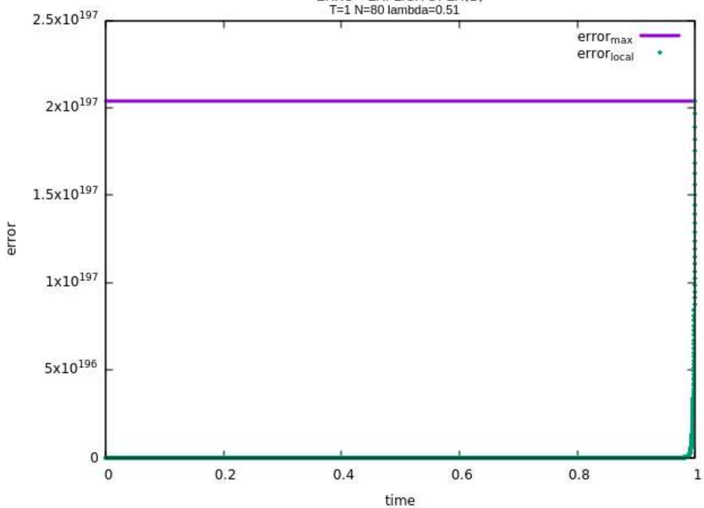


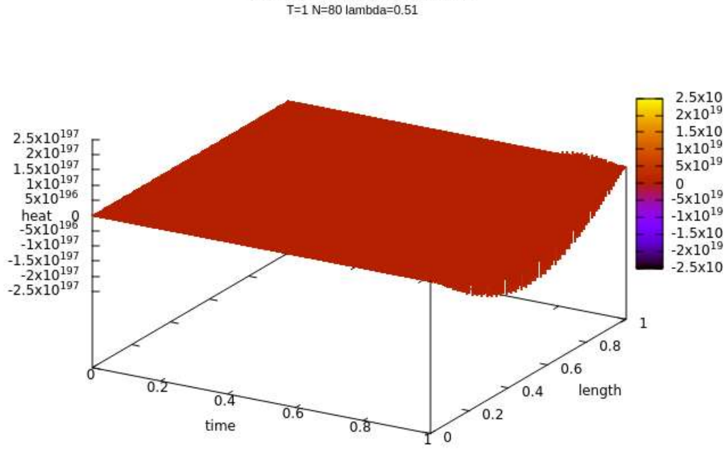
N= 80 e λ = 0.25



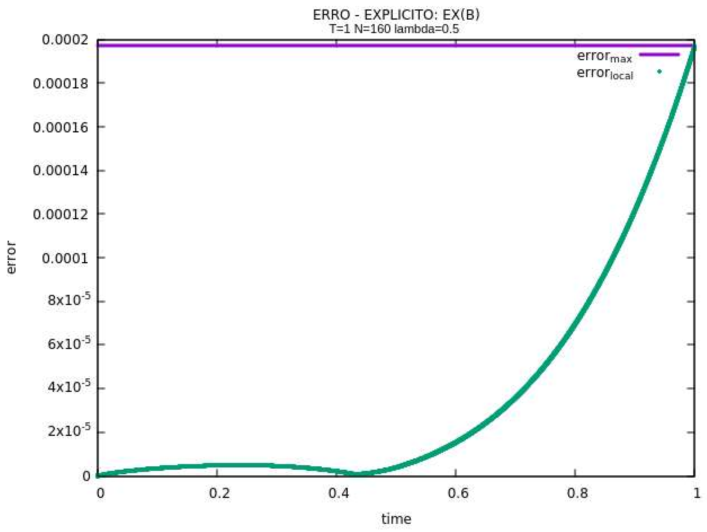


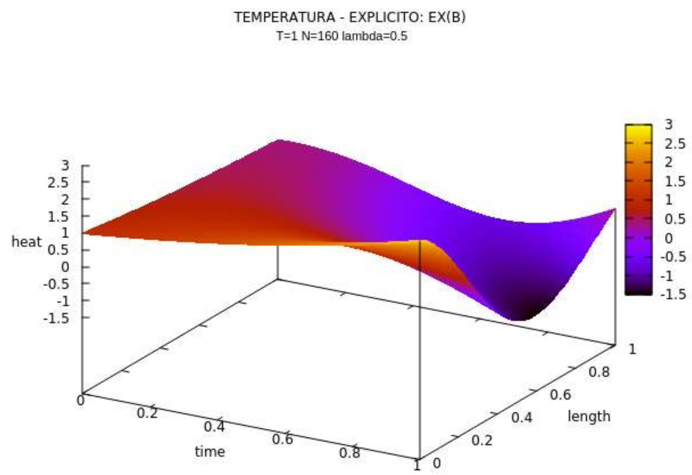
N= 80 e λ = 0.51



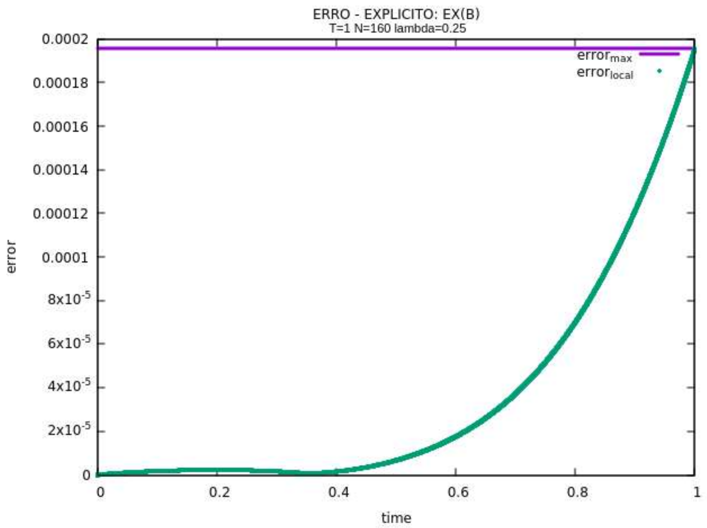


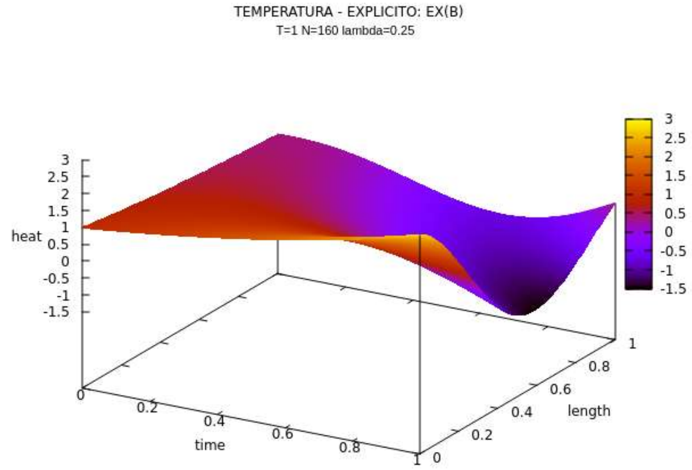
N= 160 e λ = 0.5



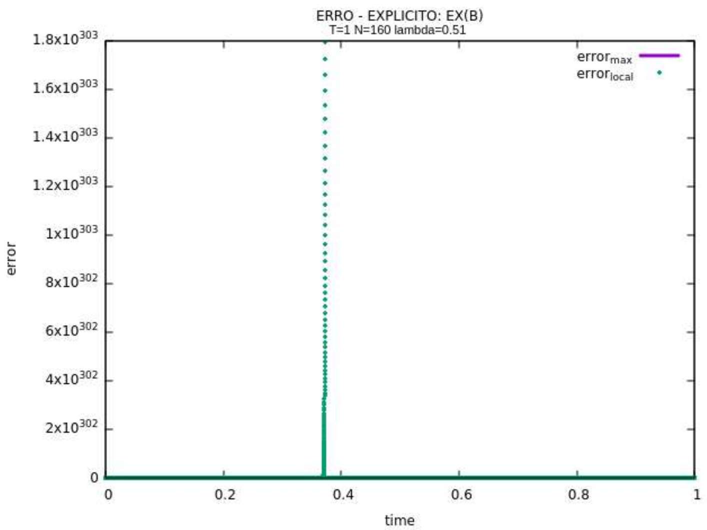


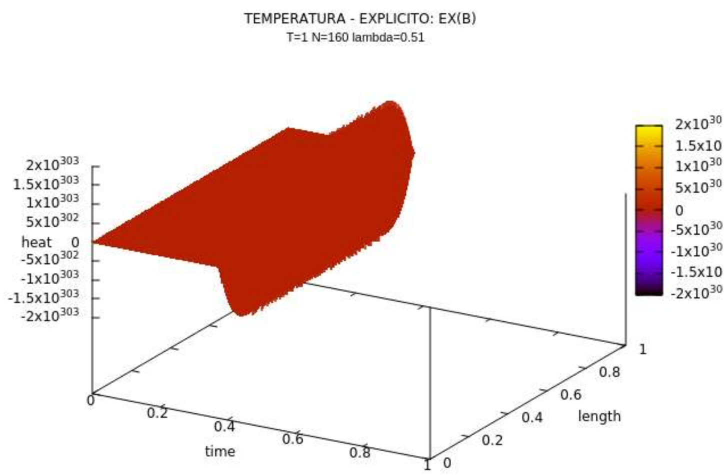
N= 160 e λ = 0.25





N= 160 e λ = 0.51





Os gráficos a seguir não foram obtidos devido a falta de poder computacional da dupla (no entanto, estes se mantêm praticamente iguais aos gráficos obtidos anteriormente):

- N= 320 e $\lambda = 0.5$
- N= 320 e $\lambda = 0.25$
- N= 320 e $\lambda = 0.51$

4.1.3. Item c

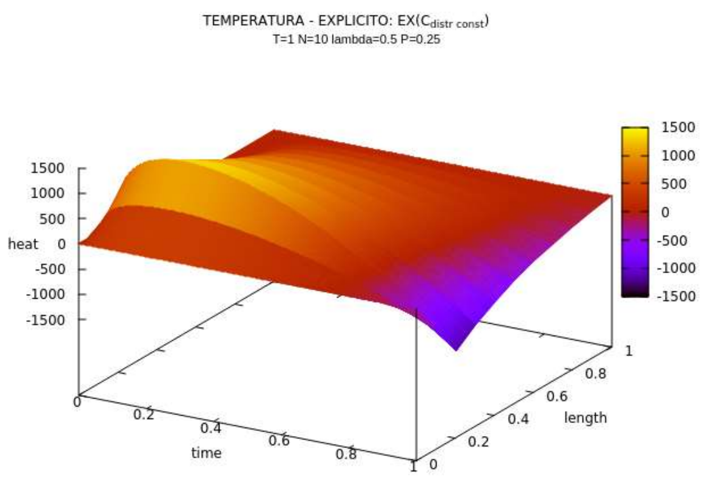
Neste item são fornecidas duas possibilidades de distribuição de calor. Sendo T=1 e condição inicial nula, a partir de uma fonte pontual de intensidade $r(t) = 10000 * (1 - 2t^2)$, pode-se ver a força de duas formas. A fonte pontual e descrita por $f(t,x) = r(t)g_h(x)$, com $h = \Delta x$ e $p = 0.25$. Na fronteira $g_1(t) = g_2(t) = 0$.

4.1.3.1 Distribuição constante

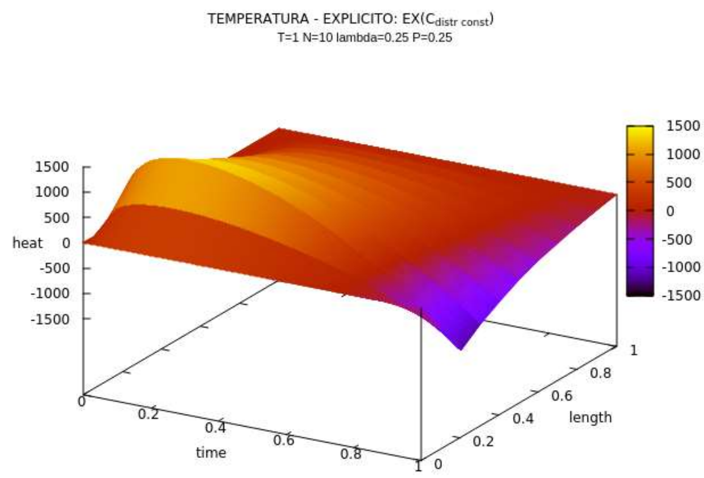
$g_h(x) = \frac{1}{h},\ se\ p - \frac{h}{2} \leq x \leq p + \frac{h}{2}$

$g_h(x) = 0,\ caso\ contrário$

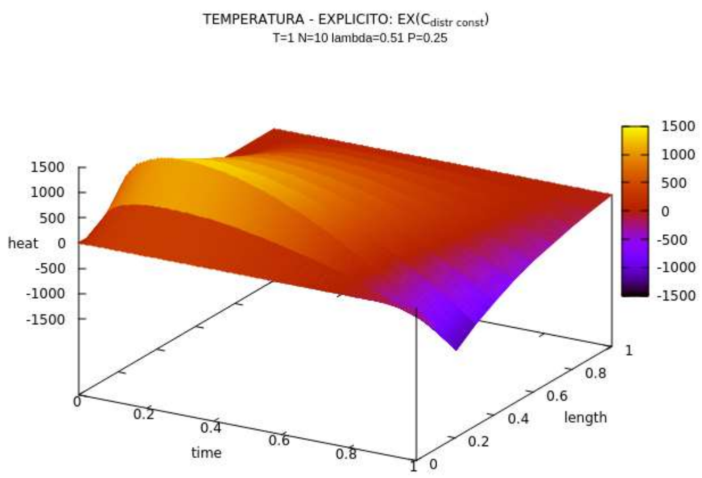
N=10 e λ = 0.5



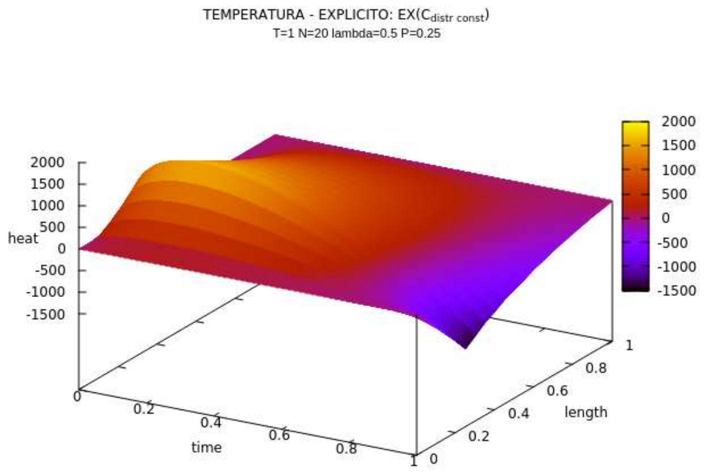
N=10 e λ = 0.25



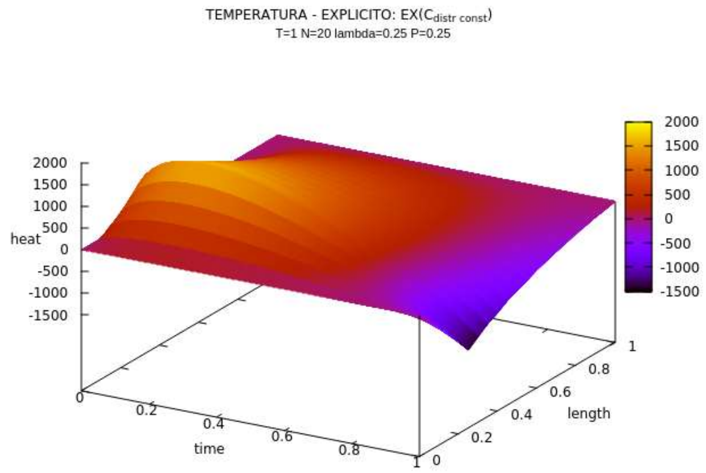
N=10 e λ = 0.51



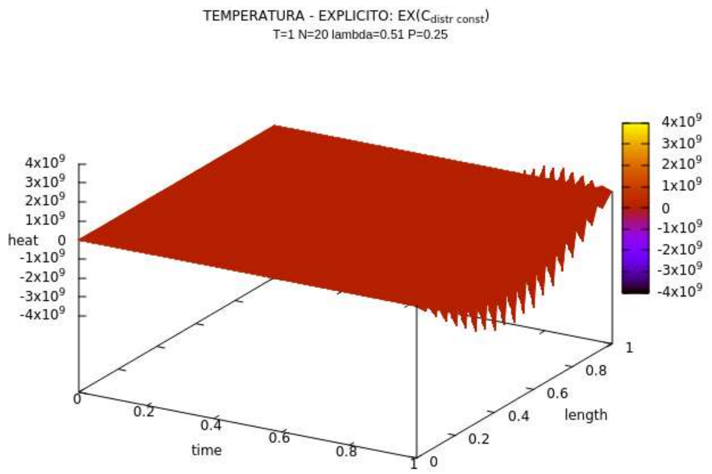
N=20 e λ = 0.5



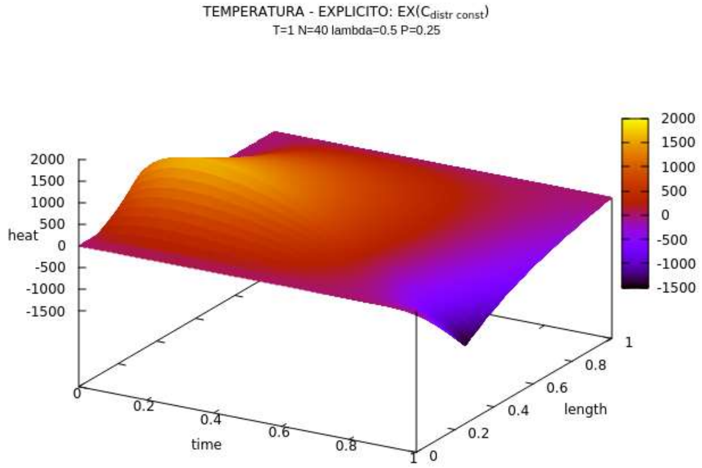
N=20 e λ = 0.25



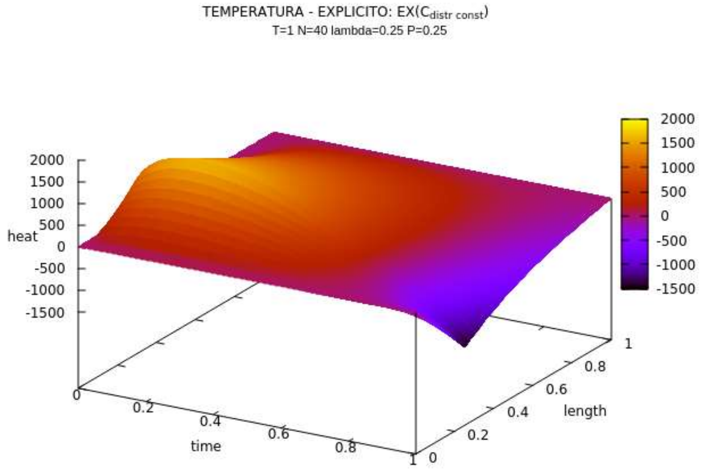
N=20 e λ = 0.51



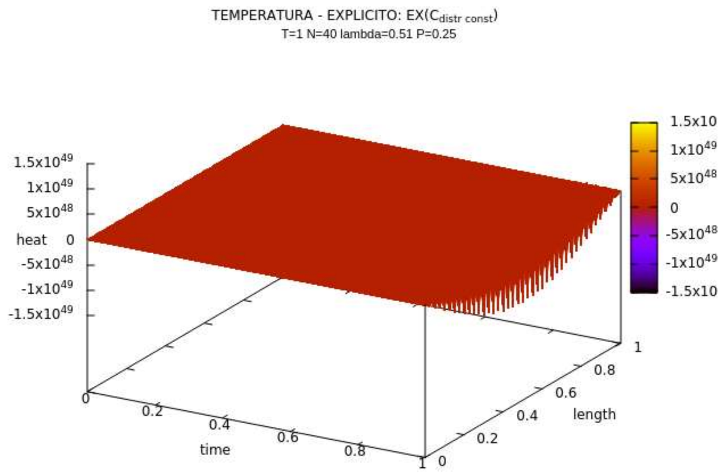
N=40 e λ = 0.5



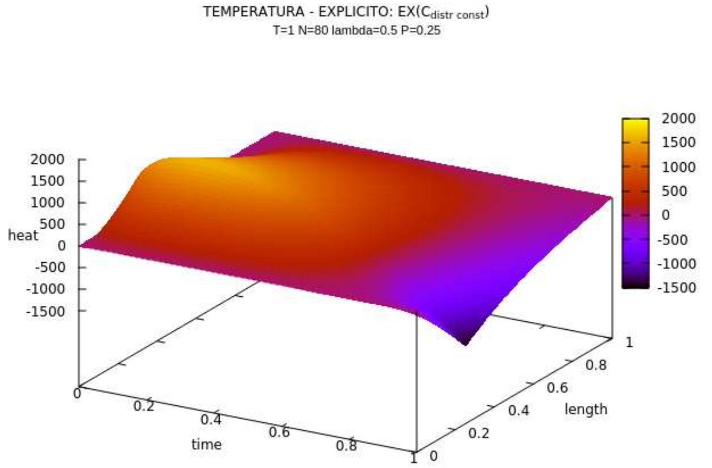
N=40 e λ = 0.25



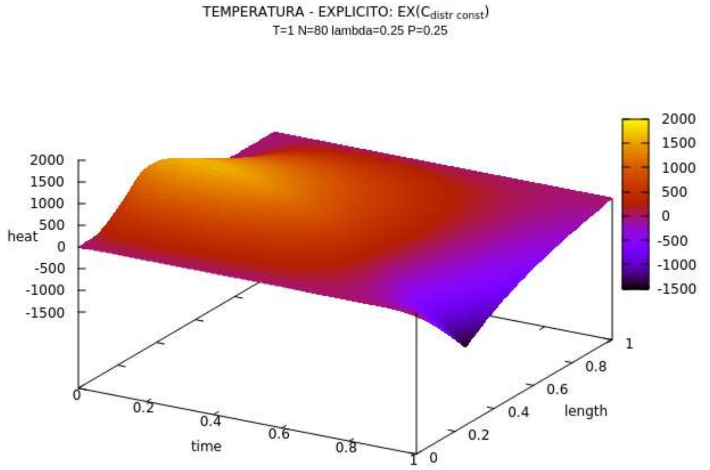
N=40 e λ = 0.51



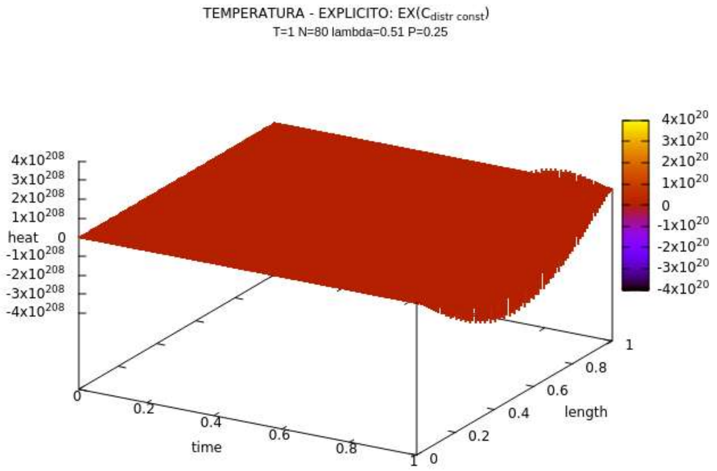
N=80 e λ = 0.5



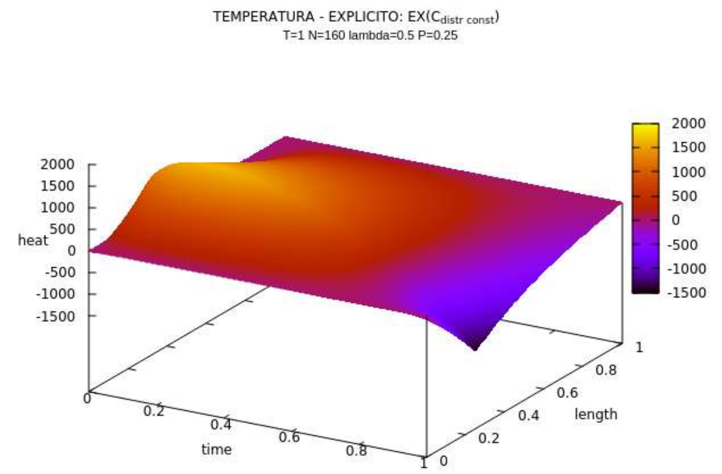
N=80 e λ = 0.25



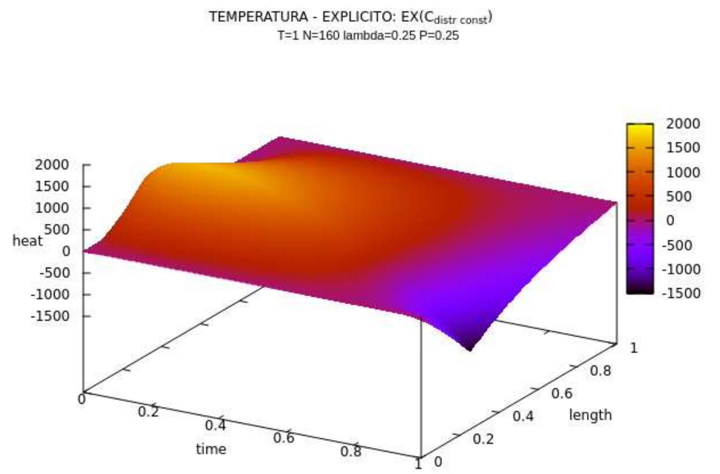
N=80 e λ = 0.51



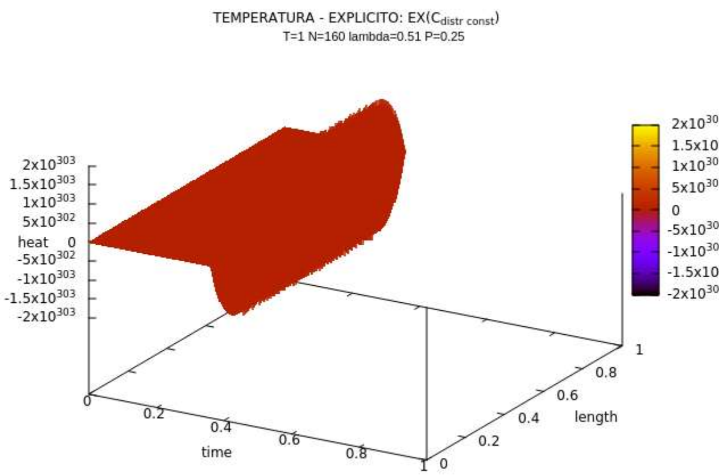
N=160 e λ = 0.5



N=160 e λ = 0.25



N=160 e λ = 0.51



Os gráficos a seguir não foram obtidos devido a falta de poder computacional da dupla (no entanto, estes se mantêm praticamente iguais aos gráficos obtidos anteriormente):

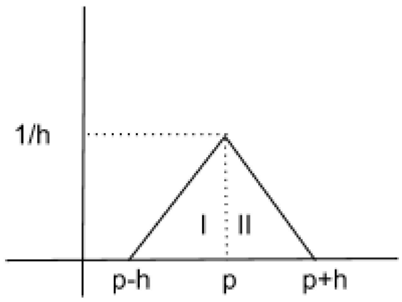
- N= 320 e λ = 0.5
- N= 320 e λ = 0.25
- N= 320 e λ = 0.51

4.1.3.2. Distribuição linear

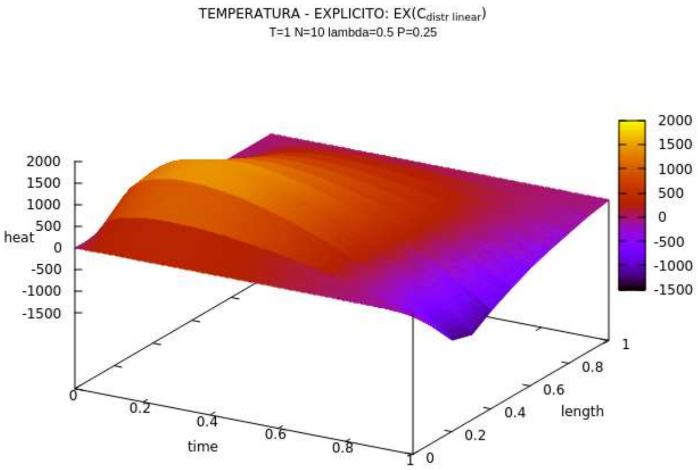
$$g_h(x) = \tfrac{1}{h^2}[x - (p - h)]\ ,\ se\ p - h \le x \le p \qquad \text{(I)}$$

$$g_h(x) = -\tfrac{1}{h^2}[x - (p + h)]\ ,\ se\ p < x \le p + h \qquad \text{(II)}$$

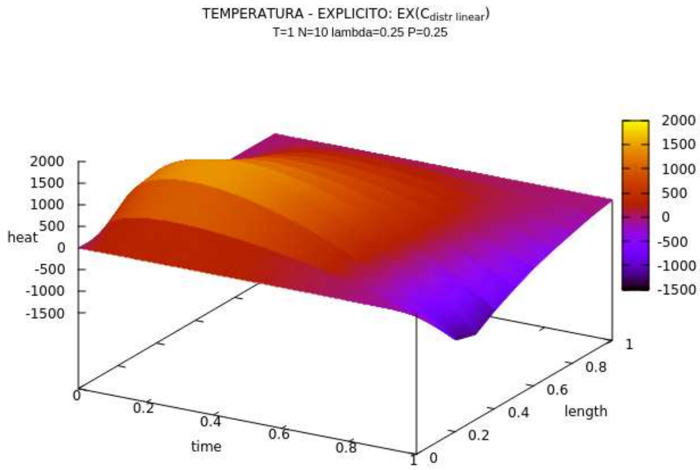
$$g_h(x) = 0,\ caso\ contrário$$



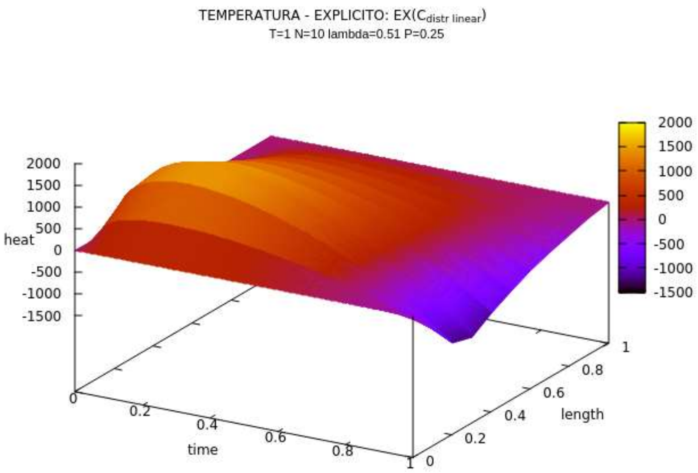
N=10 e λ = 0.5



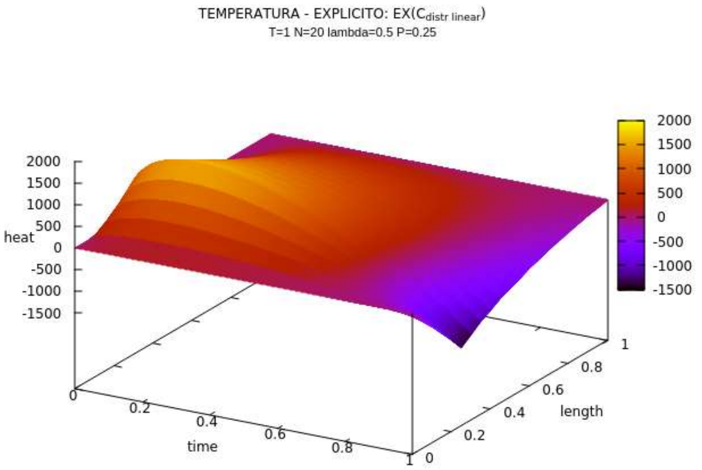
N=10 e λ = 0.25



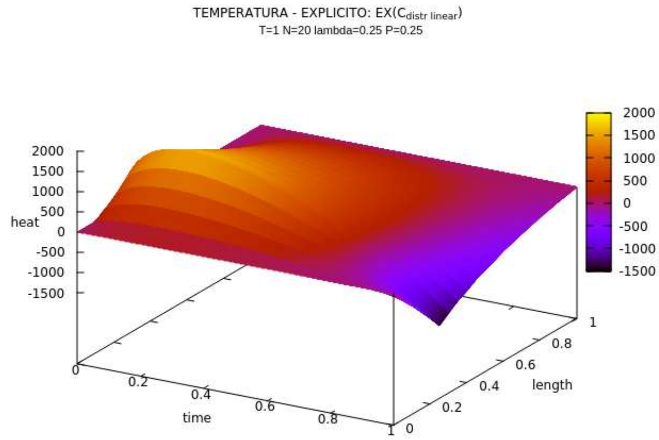
N=10 e λ = 0.51



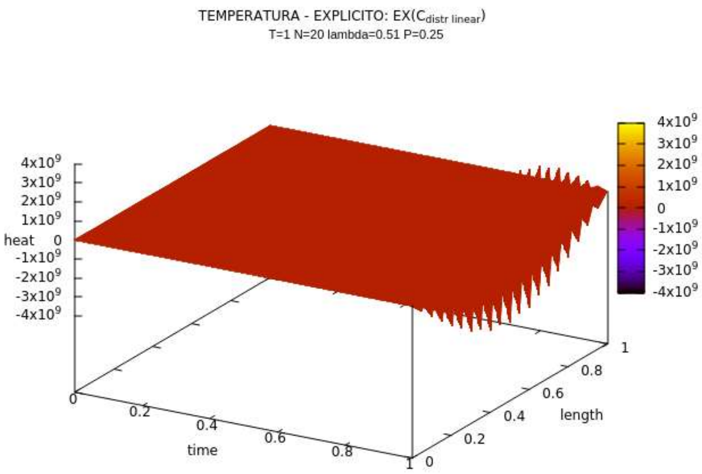
N=20 e λ = 0.5



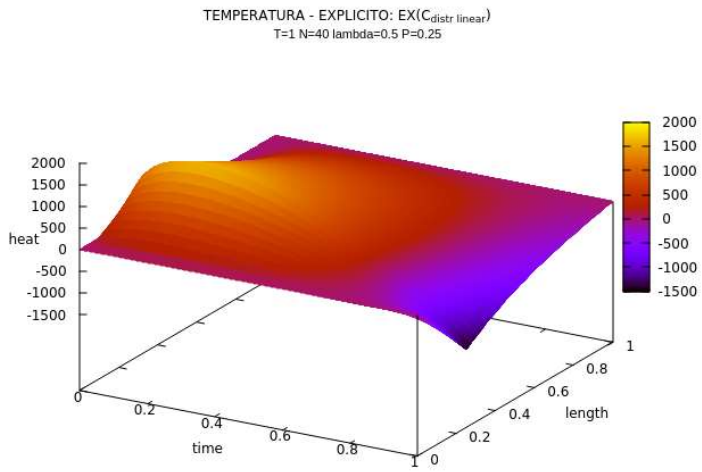
N=20 e λ = 0.25



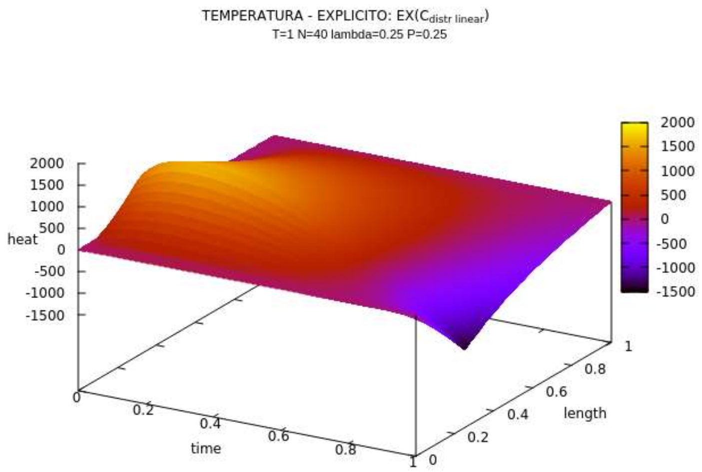
N=20 e λ = 0.51



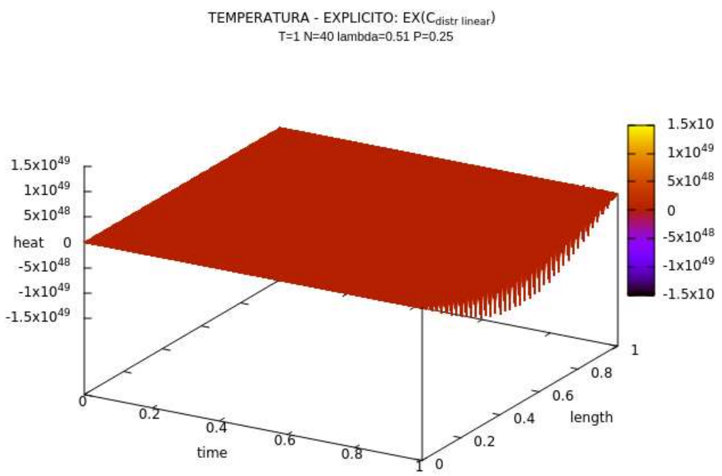
N=40 e λ = 0.5



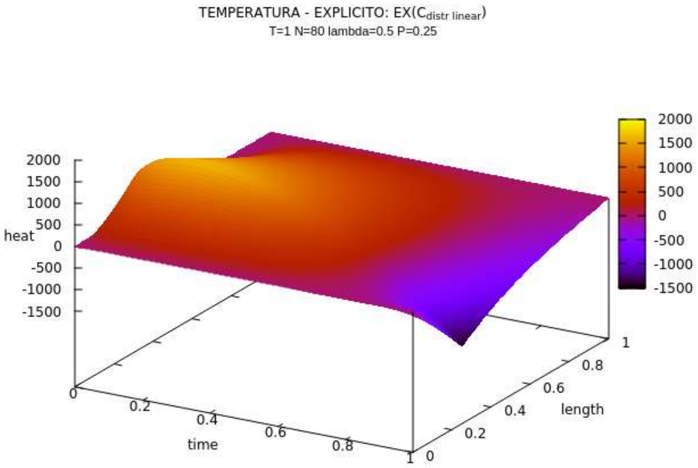
N=40 e λ = 0.25



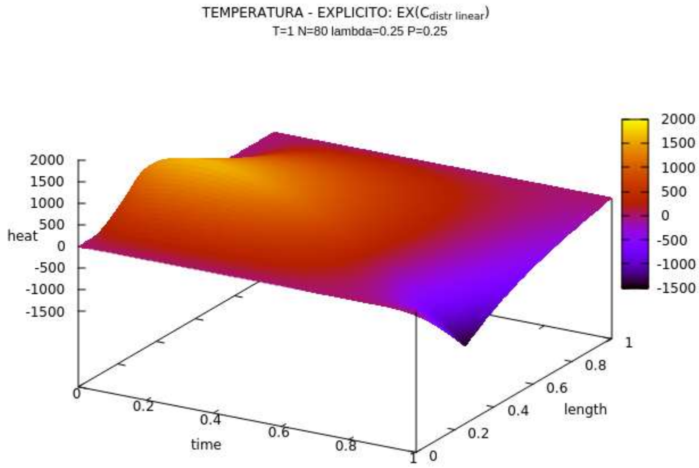
N=40 e λ = 0.51



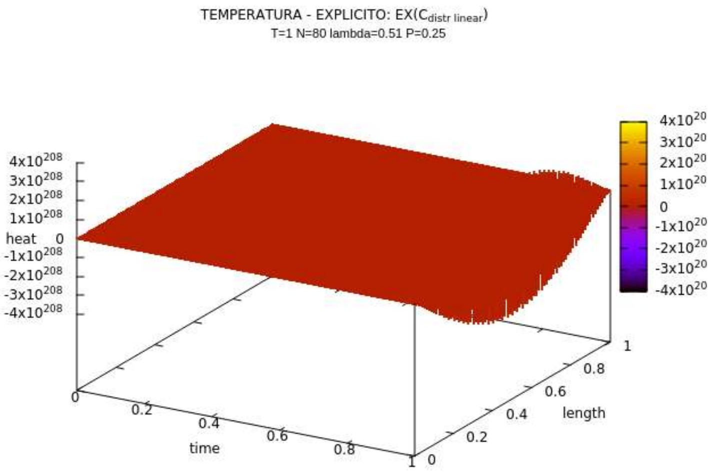
N=80 e λ = 0.5



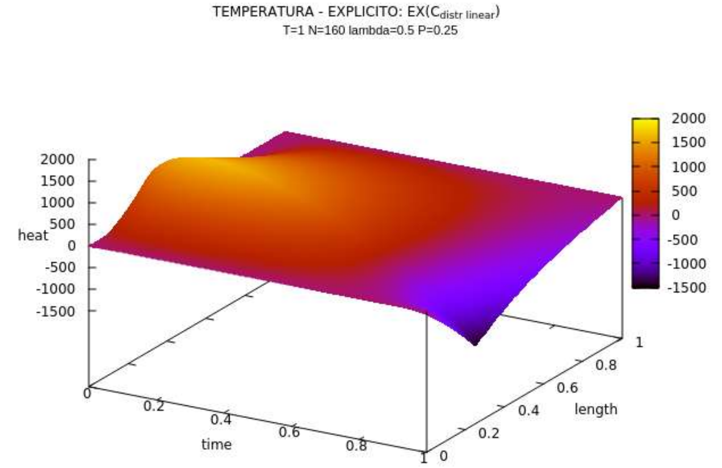
N=80 e λ = 0.25



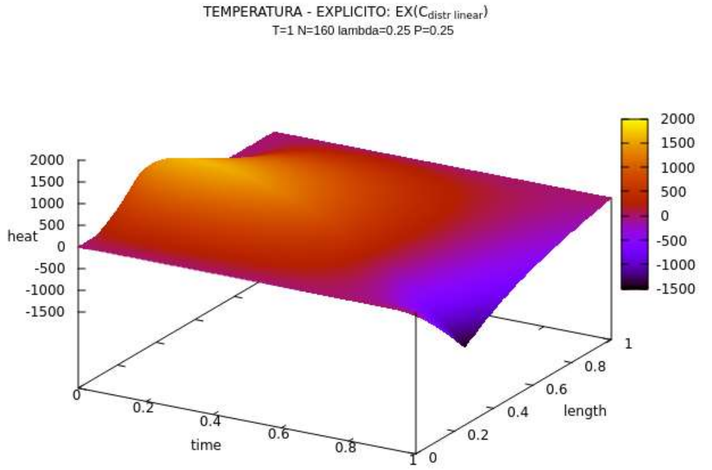
N=80 e λ = 0.51



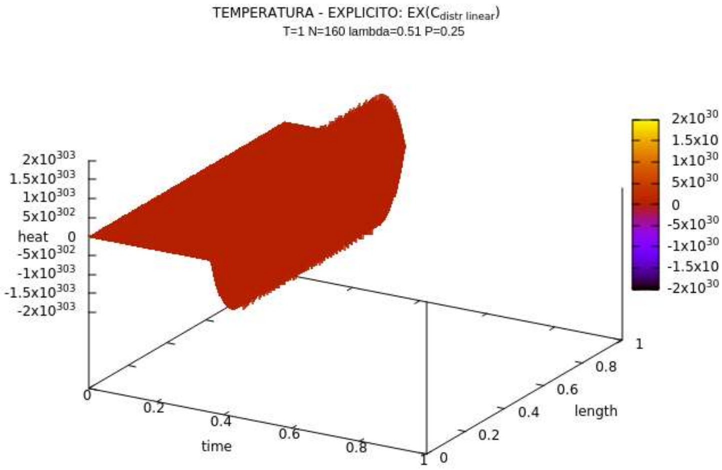
N=160 e λ = 0.5



N=160 e λ = 0.25



N=160 e λ = 0.51



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- N= 320 e λ = 0.5
- N= 320 e λ = 0.25
- N= 320 e λ = 0.51

4.2. Segunda tarefa

Apesar dos resultados obtidos no item anterior serem satisfatórios, o tempo de execução pode aumentar indefinidamente. Por este motivo, um método convergente de ordem 2 torna-se mais adequado. Entretanto, para que isso seja feito, é necessária a aplicação de um método implícito. Nesta segunda tarefa é proposta a implementação dos seguintes métodos implícitos: Euler e Crank-Nicolson.

4.2.1. Item a- Decomposição LDL^T

Dada uma matriz A, tridiagonal simétrica de tamanho $N-1 \times N-1$, onde:

$$\begin{bmatrix} a_{11} & a_{12} & 0 & \dots & 0 \\ a_{21} & a_{22} & a_{23} & \dots & \dots \\ 0 & \dots & \dots & \dots & 0 \\ \dots & \dots & a_{(N-2)(N-3)} & a_{(N-2)(N-2)} & a_{(N-2)(N-1)} \\ 0 & \dots & 0 & a_{(N-1)(N-2)} & a_{(N-1)(N-1)} \end{bmatrix}$$

Onde $a_{ij} = a_{ji}$, $i \neq j$. Assim, pode-se decompô-la na forma LDL^T:

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ l_2 & 1 & \dots & \dots \\ 0 & \dots & \dots & 0 \\ 0 & 0 & l_{(N-1)} & 1 \end{bmatrix} \begin{bmatrix} d_1 & 0 & \dots & 0 \\ 0 & d_2 & \dots & \dots \\ 0 & \dots & \dots & 0 \\ 0 & \dots & 0 & d_{N-1} \end{bmatrix} \begin{bmatrix} 1 & l_2 & 0 & 0 \\ 0 & \dots & \dots & 0 \\ \dots & \dots & 1 & l_{N-1} \\ 0 & \dots & 0 & 1 \end{bmatrix}$$

Podemos encontrar os valores para l_i e d_j:

$$d_1 = a_{11}$$
$$l_i = a_{i(i-1)} \div d_{i-1}\ ,\ i = 2\ ,\ \dots\ ,\ N-1$$
$$d_j = a_{jj} - (l_j^2 * d_{j-1})\ ,\ j = 2\ ,\ \dots\ ,\ N-1$$

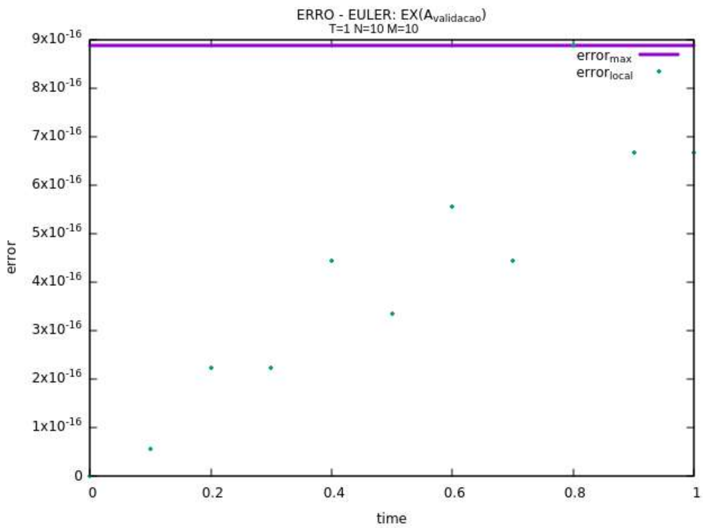
Com esta decomposição, a solução para o sistema $Ax = b$ se divide em três sistemas lineares simples de serem resolvidos computacionalmente:

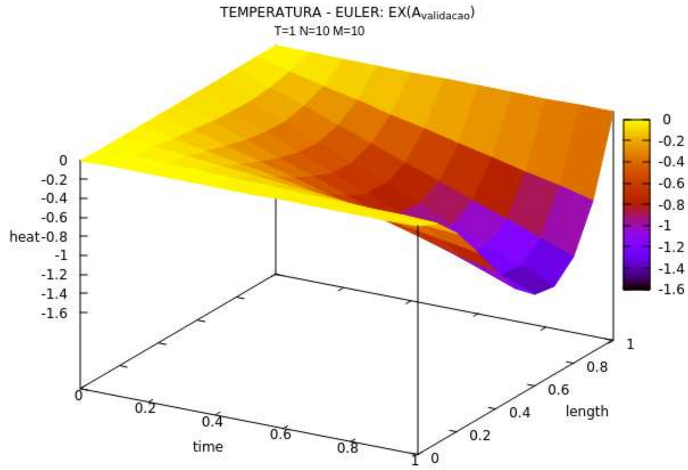
$$Lz = b$$
$$Dy = z$$
$$L^T x = y$$

4.2.2. Item b- Método de Euler

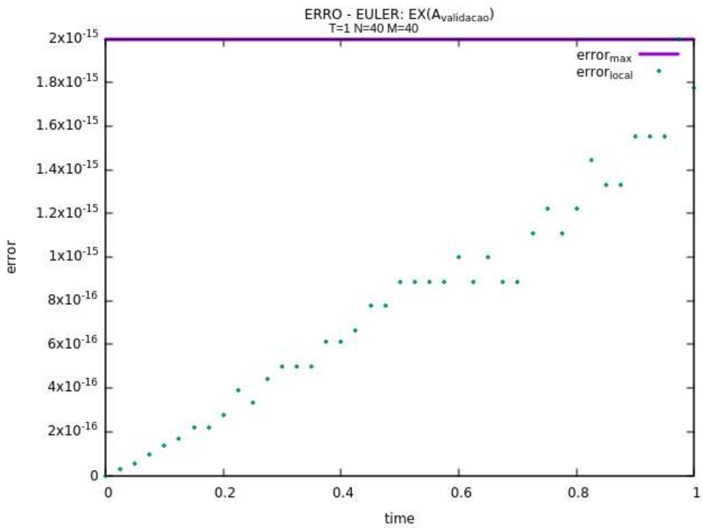
Teste) T = 1 com a fonte $f(t,x) = 10x^2(x-1) - 60xt + 20t$ a partir de $u_0(x) = 0$ e condições de fronteira nulas.

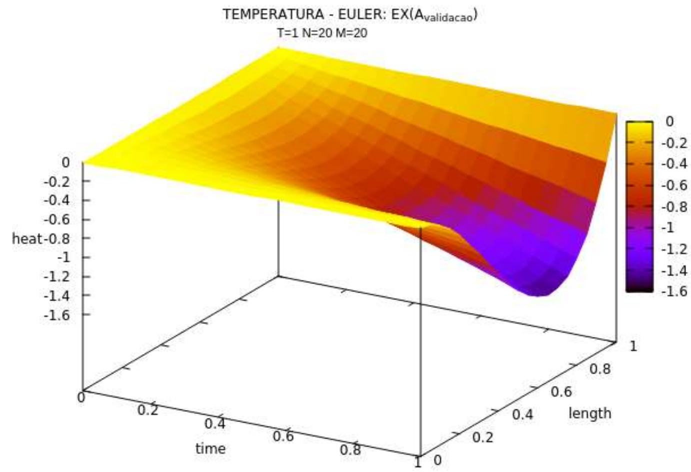
N=10



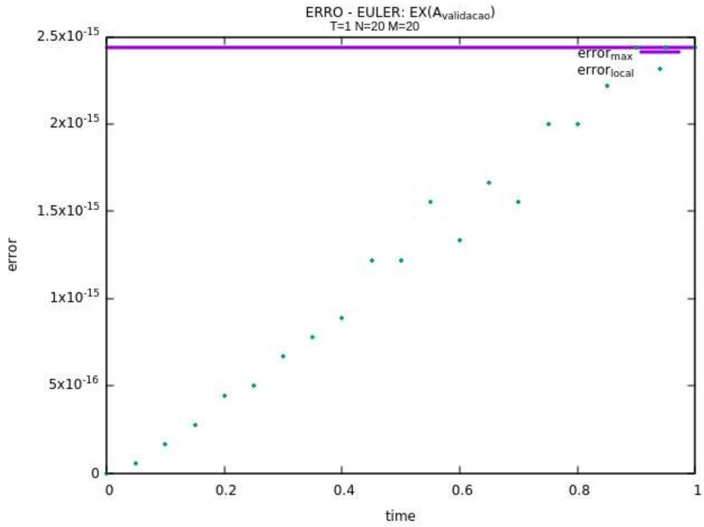


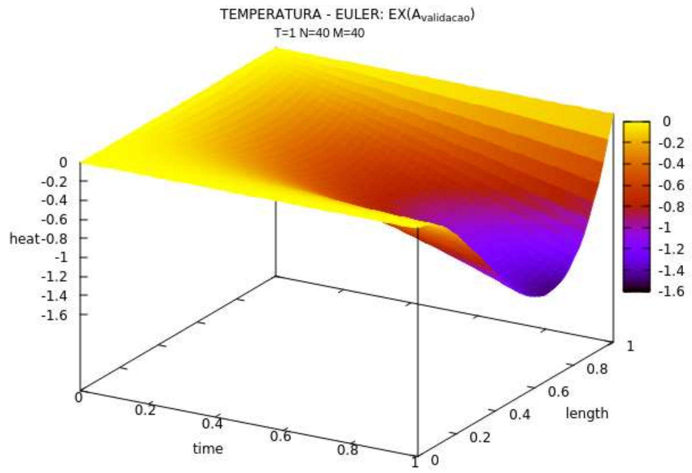
N=20



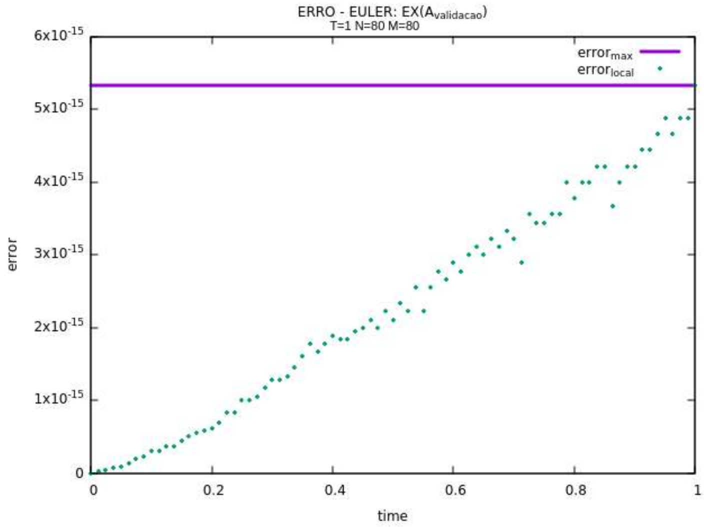


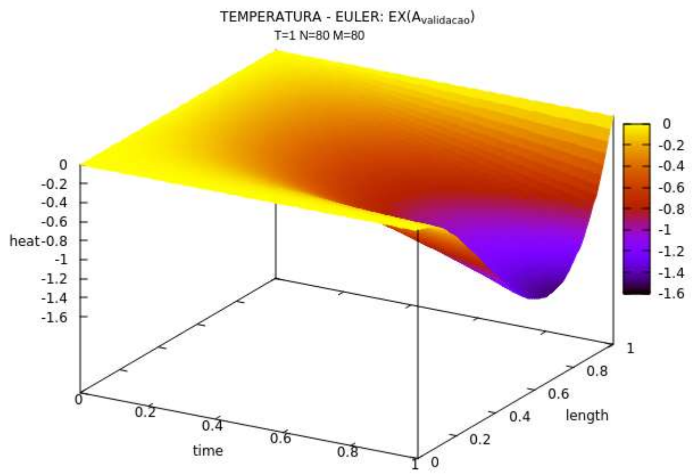
N=40


ERRO - EULER: EX($A_{validacao}$)
T=1 N=20 M=20
$error_{max}$
$error_{local}$
error
time

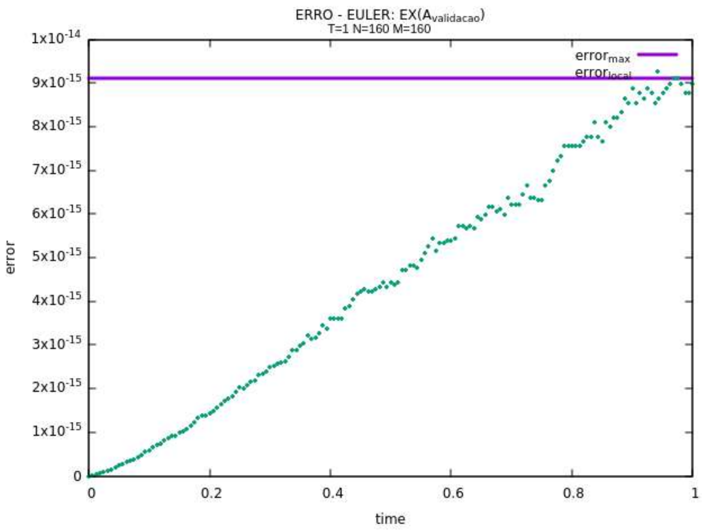

TEMPERATURA - EULER: EX($A_{validacao}$)
T=1 N=40 M=40
heat
time
length

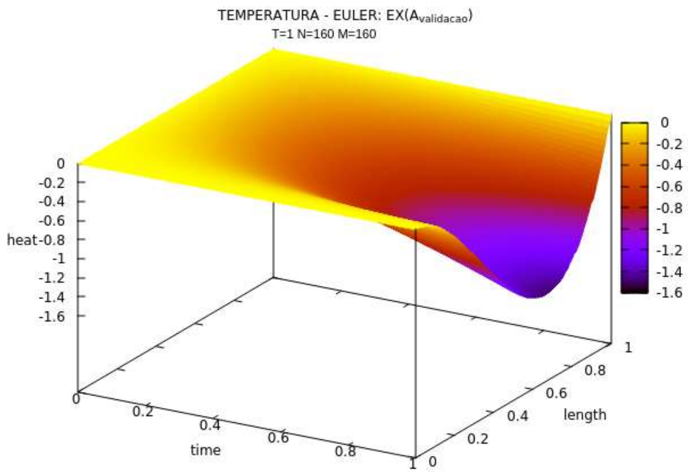
N=80


ERRO - EULER: EX($A_{validacao}$)
T=1 N=80 M=80
$error_{max}$
$error_{local}$
error
time

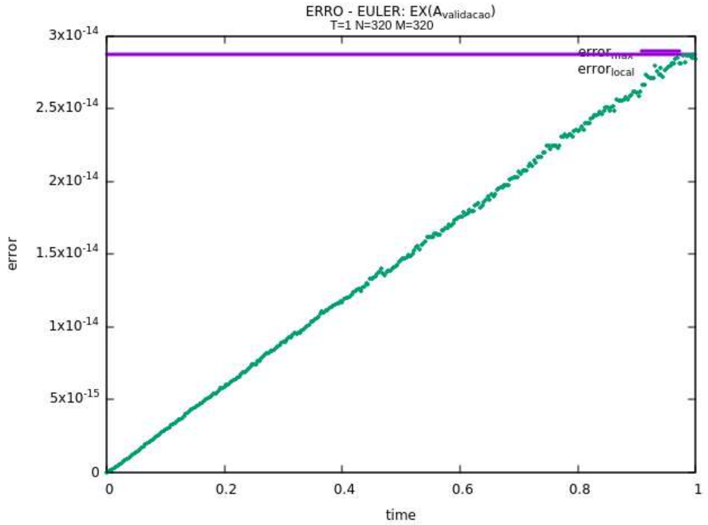

TEMPERATURA - EULER: EX($A_{validacao}$)
T=1 N=80 M=80
heat
time
length

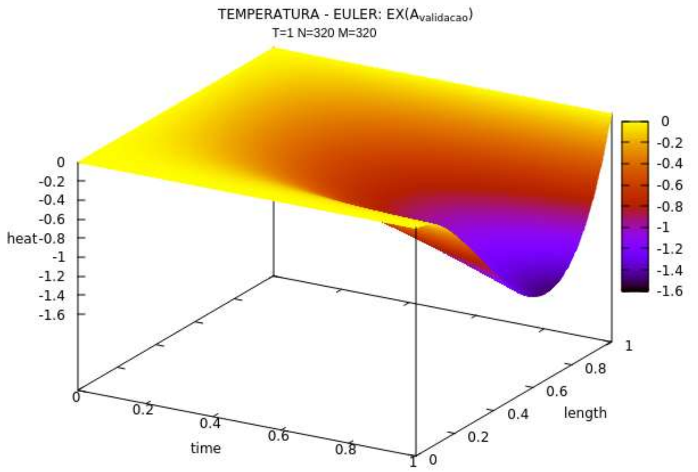
N=160


ERRO - EULER: EX($A_{validacao}$)
T=1 N=160 M=160
$error_{max}$
$error_{local}$
error
time


TEMPERATURA - EULER: EX($A_{validacao}$)
T=1 N=160 M=160
heat
time
length

N=320





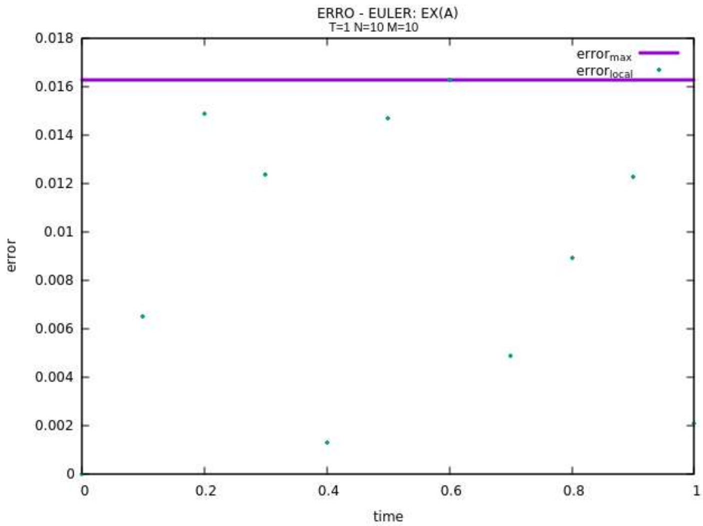
Exercício a) Fonte $f(t,x) = 10cos(10t)x^2(1-x)^2 - (1+sin(10t))(12x^2 - 12x + 2)$

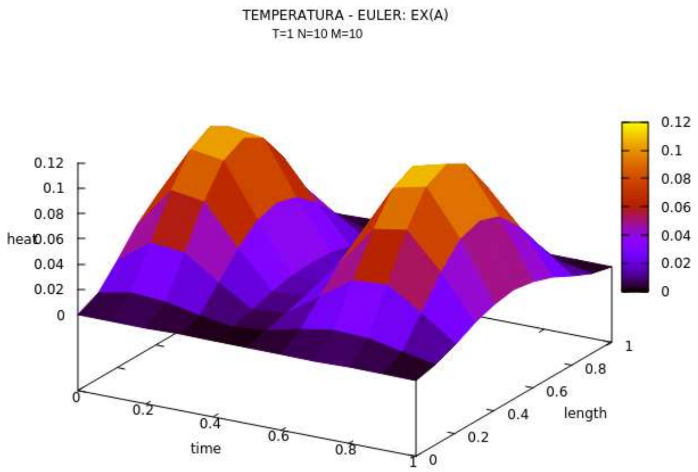
Solução exata $u(t,x) = (1+sin(10t))x^2(1-x)^2$

Valor inicial $u_0(x) = x^2(1-x)^2$

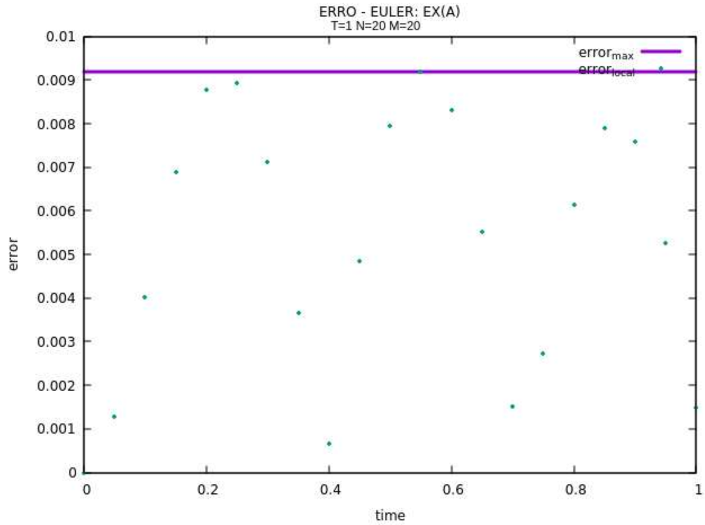
Condições nulas na fronteira.

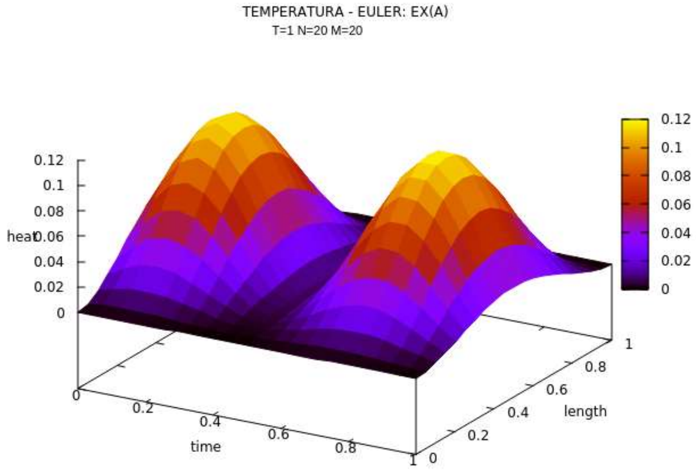
N=10



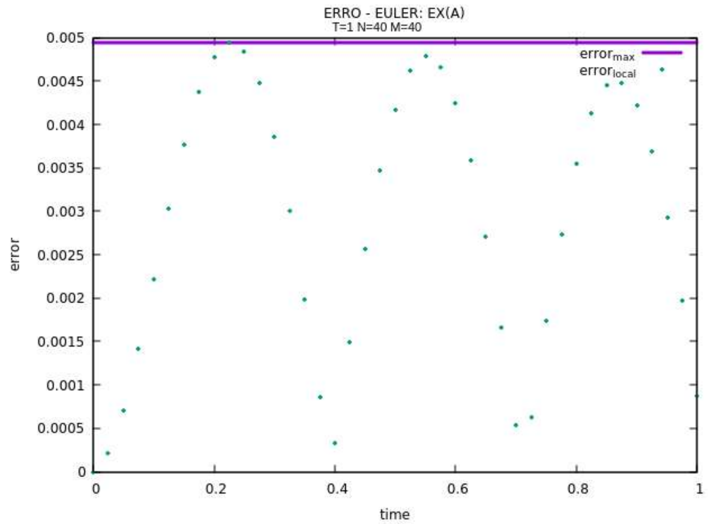


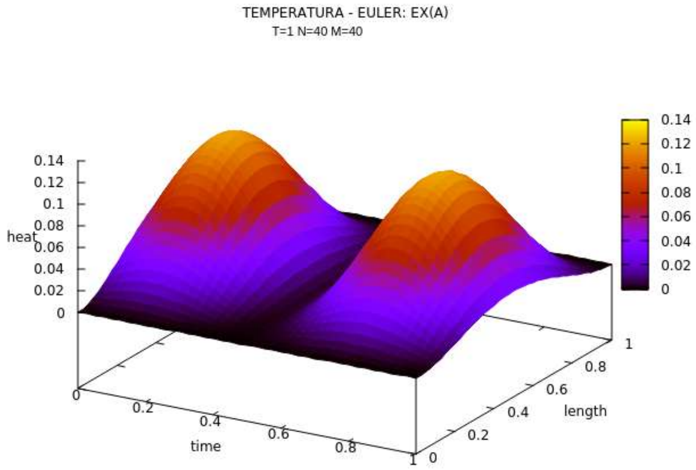
N=20


ERRO - EULER: EX(A)
T=1 N=20 M=20
$error_{max}$
$error_{local}$
error
time

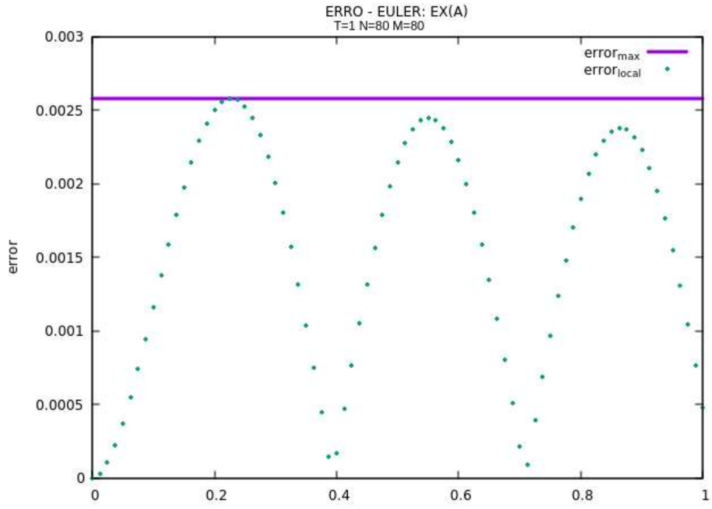

TEMPERATURA - EULER: EX(A)
T=1 N=20 M=20
heat
time
length

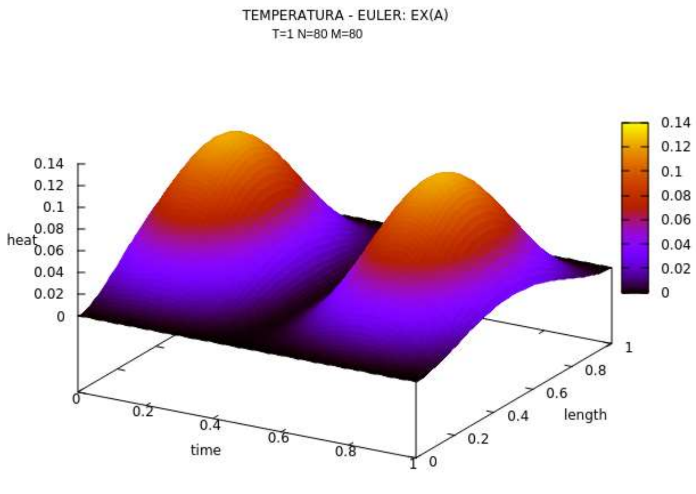
N=40


ERRO - EULER: EX(A)
T=1 N=40 M=40
$error_{max}$
$error_{local}$
error
time

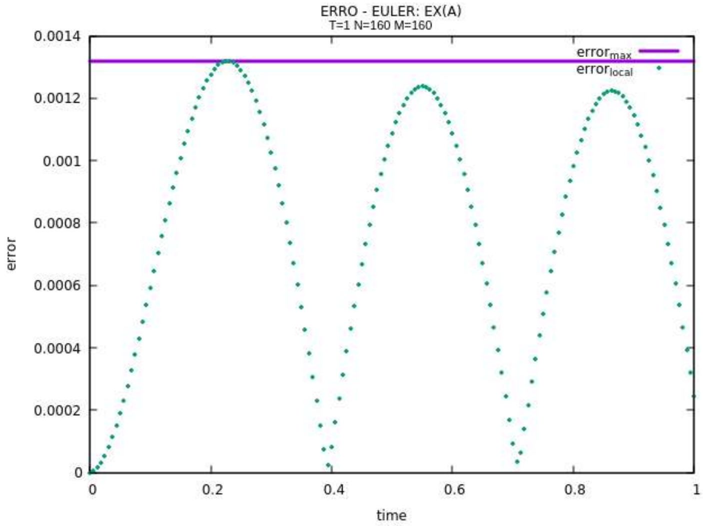

TEMPERATURA - EULER: EX(A)
T=1 N=40 M=40
heat
time
length

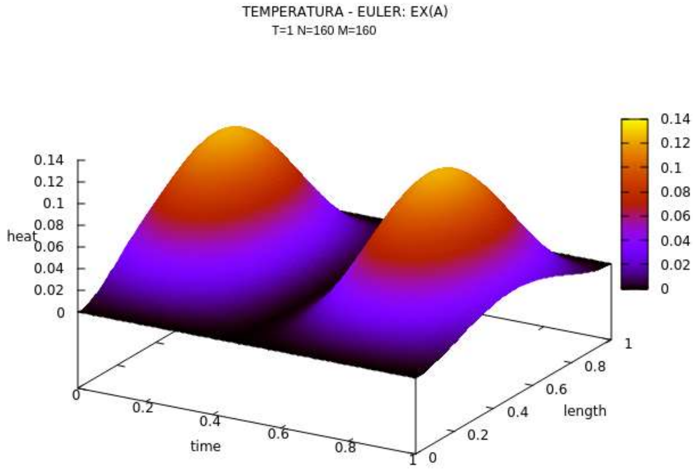
N=80


ERRO - EULER: EX(A)
T=1 N=80 M=80
$error_{max}$
$error_{local}$
error

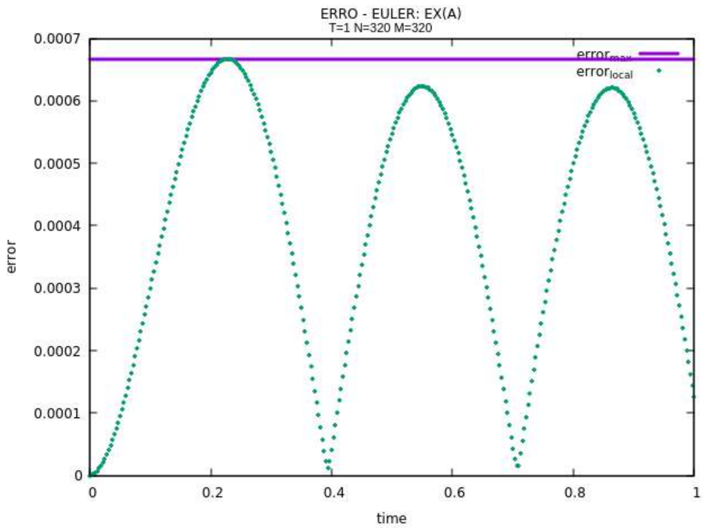

TEMPERATURA - EULER: EX(A)
T=1 N=80 M=80
heat
time
length

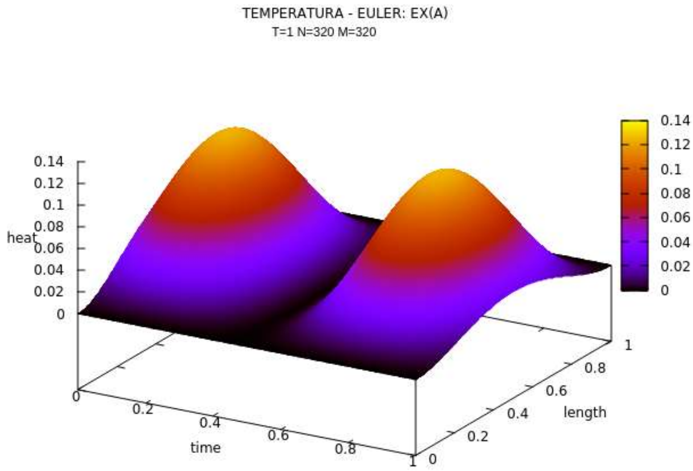
N=160


ERRO - EULER: EX(A)
T=1 N=160 M=160
$error_{max}$
$error_{local}$
error
time
0.0014
0.0012
0.001
0.0008
0.0006
0.0004
0.0002
0
0
0.2
0.4
0.6
0.8
1


TEMPERATURA - EULER: EX(A)
T=1 N=160 M=160
heat
time
length
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0

N=320


ERRO - EULER: EX(A)
T=1 N=320 M=320
$error_{max}$
$error_{local}$
error
time
0.0007
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
0
0
0.2
0.4
0.6
0.8
1


TEMPERATURA - EULER: EX(A)
T=1 N=320 M=320
heat
time
length
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0

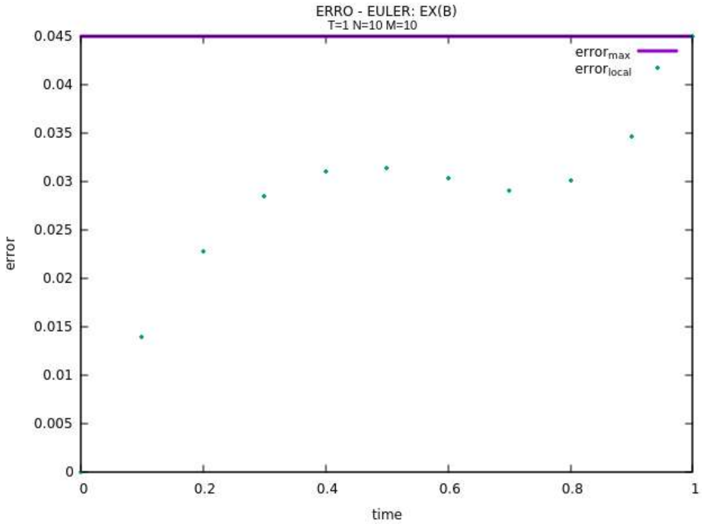
Exercício b) Fonte $f(t,x) = 10cos(10t)x^2(1-x)^2 - (1+sin(10t))(12x^2 - 12x + 2)$

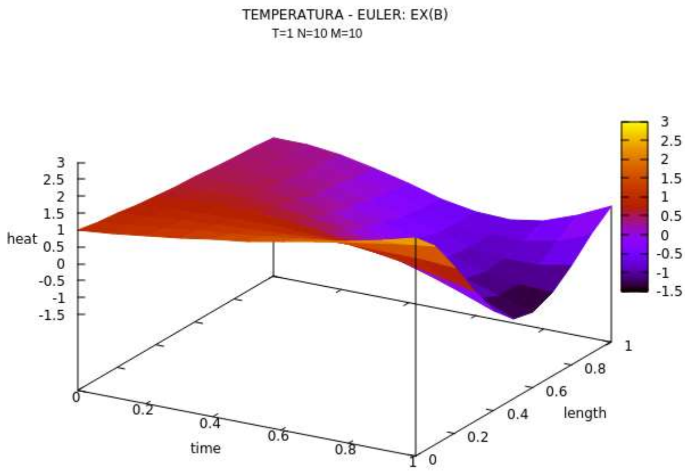
Solução exata $u(t,x) = (1+sin(10t))x^2(1-x)^2$

Valor inicial $u_0(x) = x^2(1-x)^2$

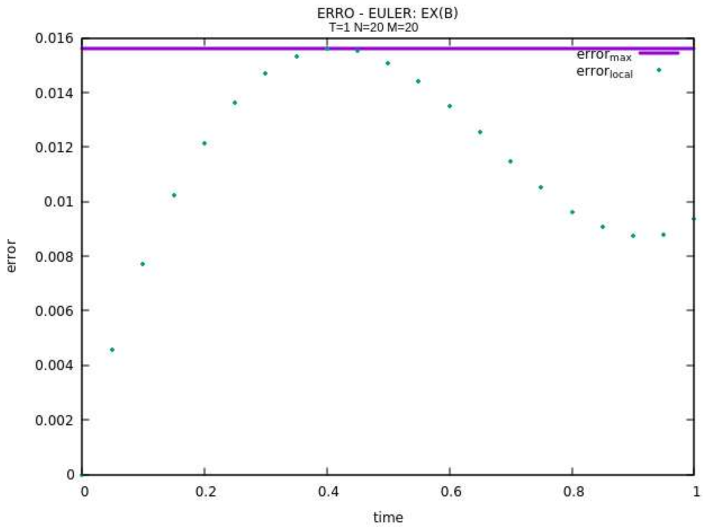
Condições nulas na fronteira.

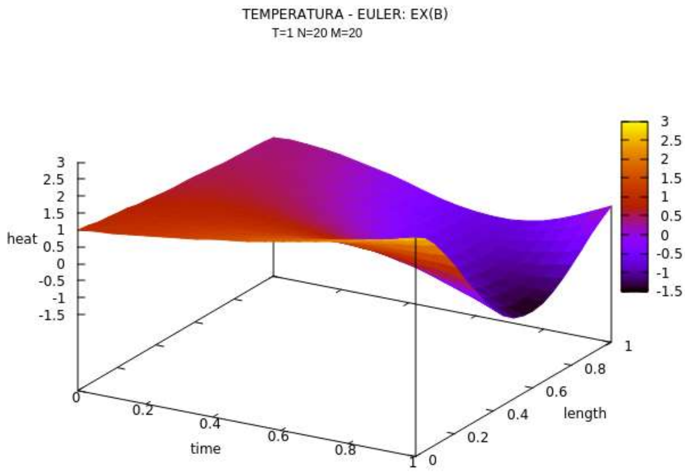
N=10



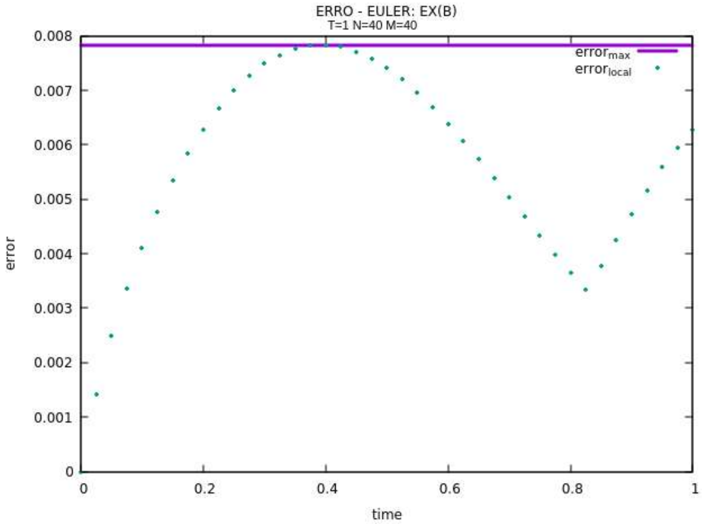


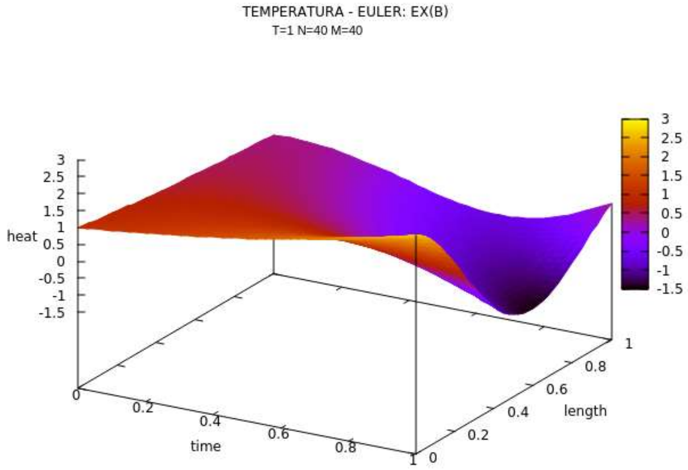
N=20



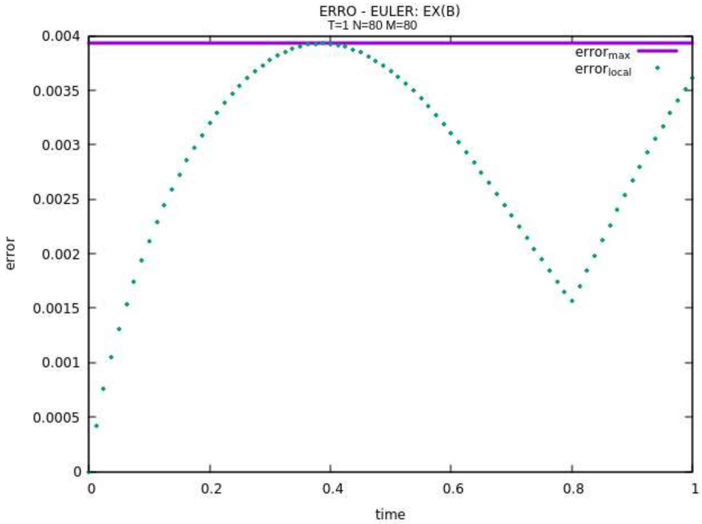


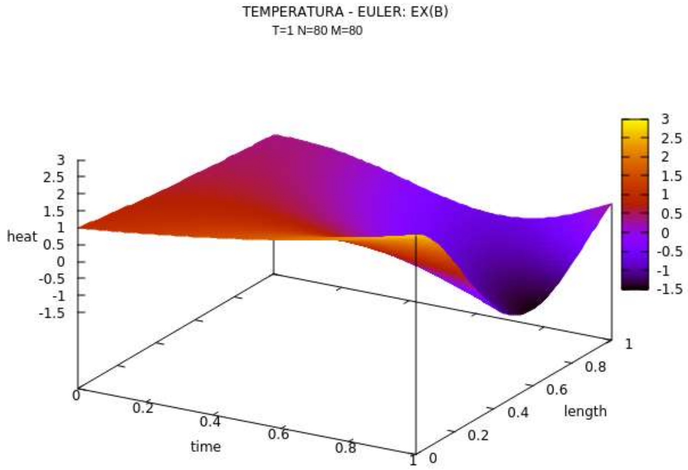
N=40


ERRO - EULER: EX(B)
T=1 N=40 M=40
error
time
error_max
error_local

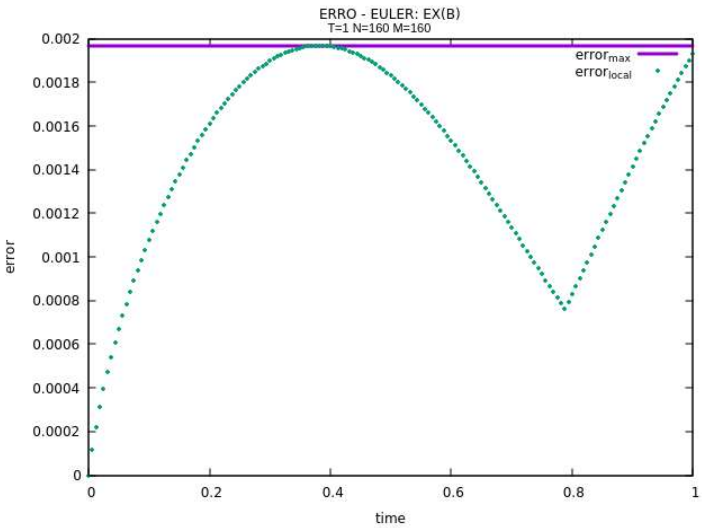

TEMPERATURA - EULER: EX(B)
T=1 N=40 M=40
heat
time
length

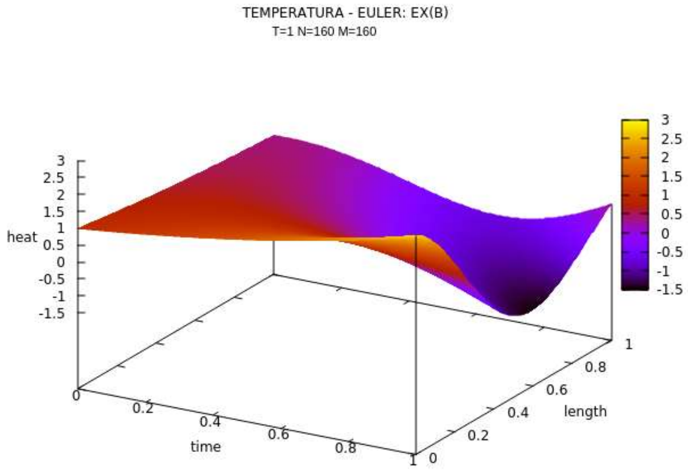
N=80


ERRO - EULER: EX(B)
T=1 N=80 M=80
error
time
error_max
error_local

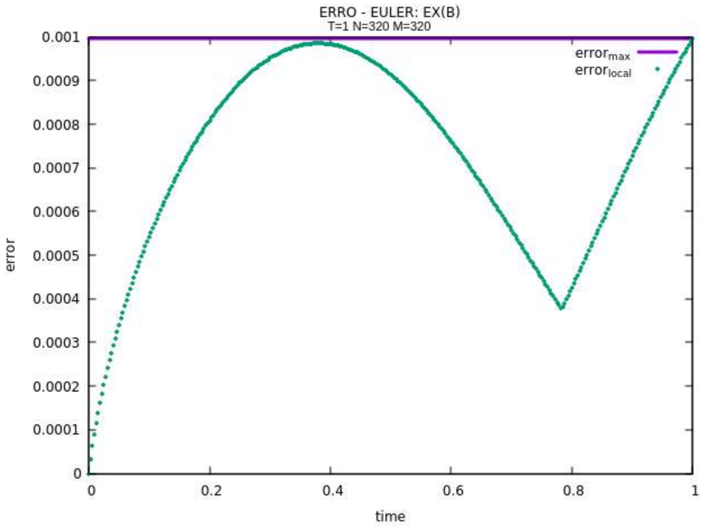

TEMPERATURA - EULER: EX(B)
T=1 N=80 M=80
heat
time
length

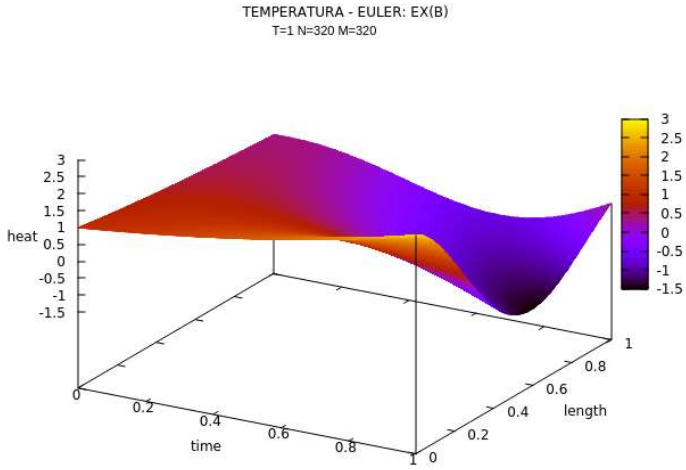
N=160


ERRO - EULER: EX(B)
T=1 N=160 M=160
error
time
error_max
error_local


TEMPERATURA - EULER: EX(B)
T=1 N=160 M=160
heat
time
length

N=320





Exercício c) T=1

Condição inicial nula

Fonte pontual de intensidade $r(t) = 10000 * (1 - 2t^2)$

$f(t, x) = r(t)g_h(x)$, com $h = \Delta x$ e $p = 0.25$

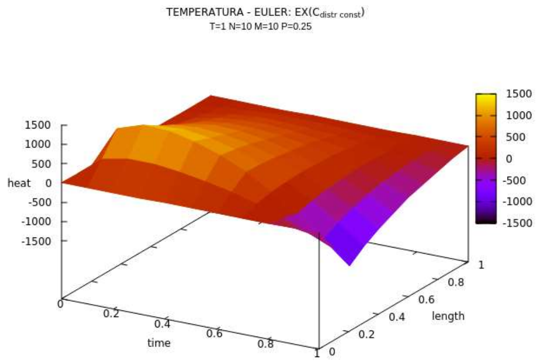
Na fronteira $g_1(t) = g_2(t) = 0$.

Exercício c.1) Distribuição constante

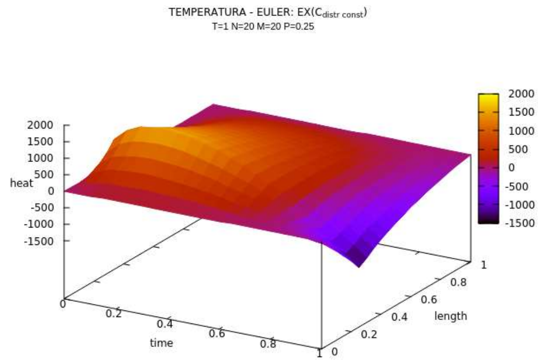
$$g_h(x) = \frac{1}{h},\ se\ p - \frac{h}{2} \leq x \leq p + \frac{h}{2}$$

$$g_h(x) = 0,\ caso\ contrário$$

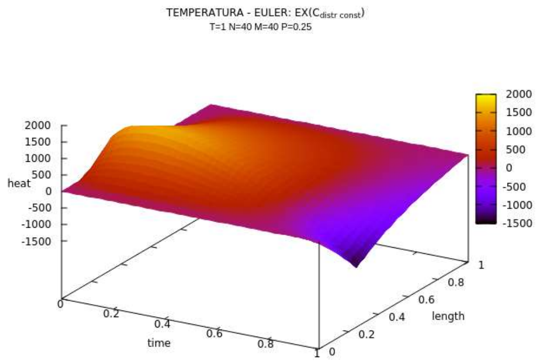
N=10



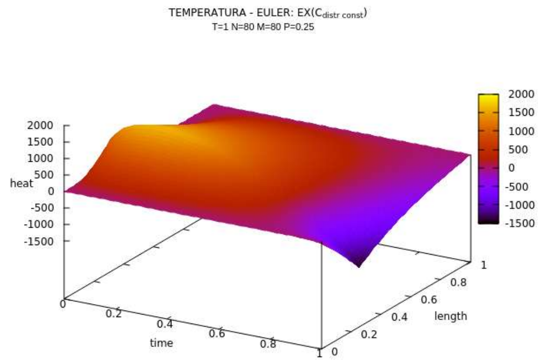
N=20



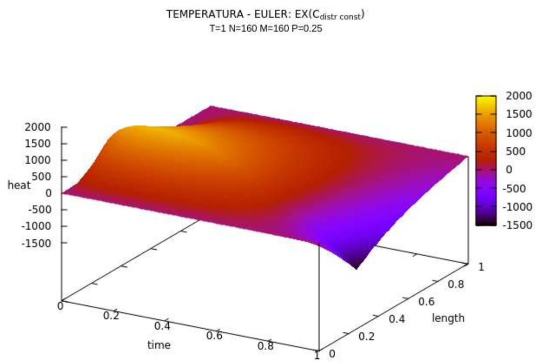
N=40



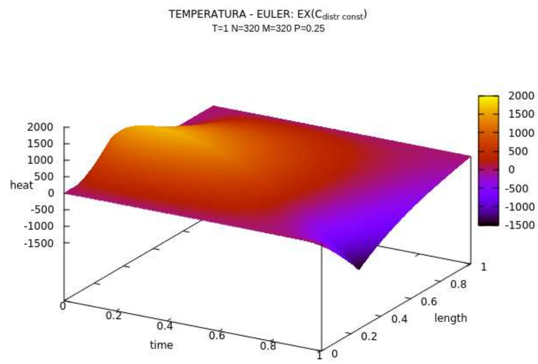
N=80



N=160



N=320



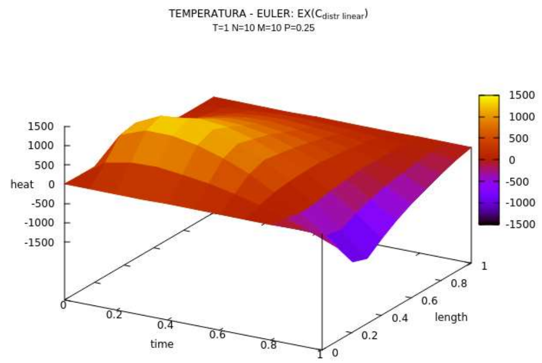
Exercício c.2) Distribuição linear

$$g_h(x) = \frac{1}{h^2}[x - (p - h)]\ ,\ se\ p - h \le x \le p$$

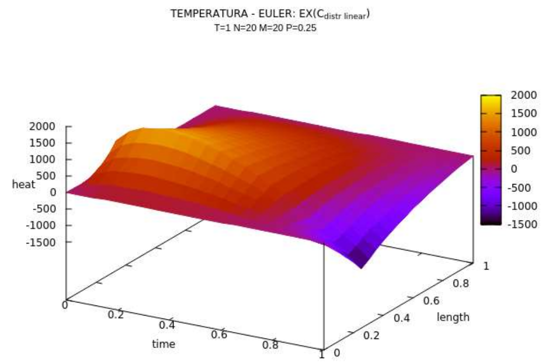
$$g_h(x) = \ -\frac{1}{h^2}[x - (p + h)]\ ,\ se\ p < x \le p + h$$

$$g_h(x) = 0,\ caso\ contrário$$

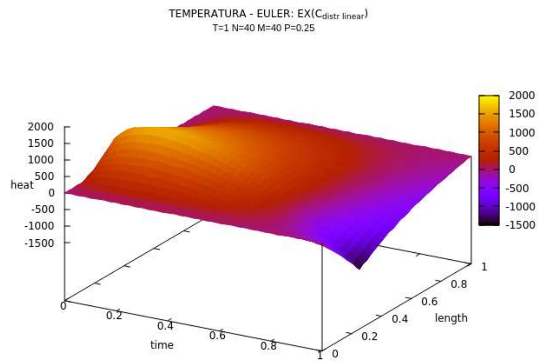
N=10



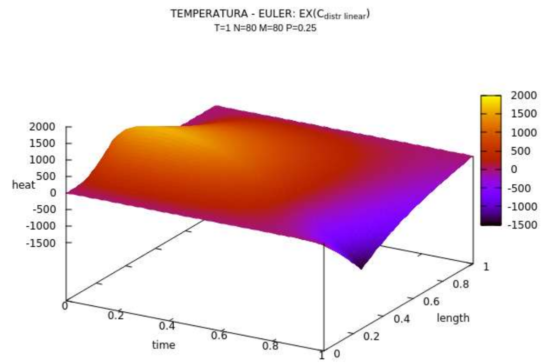
N=20



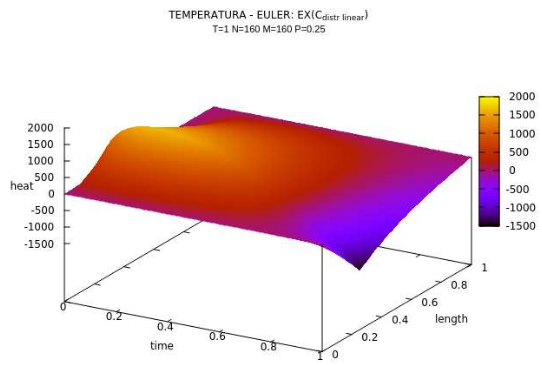
N=40



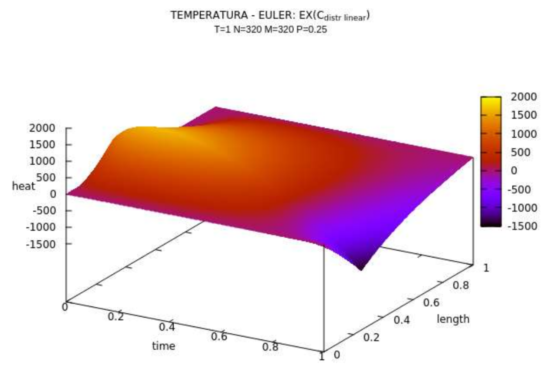
N=80



N=160



N=320



4.2.3. Item c - Método de Crank-Nicolson

$$u_i^{k+1} = u_i^k + \tfrac{\lambda}{2}((u_{i-1}^{k+1} - 2u_i^{k+1} + u_{i+1}^{k+1}) + (u_{i-1}^k - 2u_i^k + u_{i+1}^k)) + \tfrac{\Delta t}{2}(f(x_i, t_k) + f(x_i, t_{k+1}))$$
$$-\tfrac{\lambda}{2}u_{i-1}^{k+1} + (1+\lambda)u_i^{k+1} - \tfrac{\lambda}{2}u_{i+1}^{k+1} = \tfrac{\lambda}{2}u_{i-1}^k + (1-\lambda)u_i^k + \tfrac{\lambda}{2}u_{i+1}^k + \tfrac{\Delta t}{2}(f(x_i, t_k) + f(x_i, t_{k+1}))$$

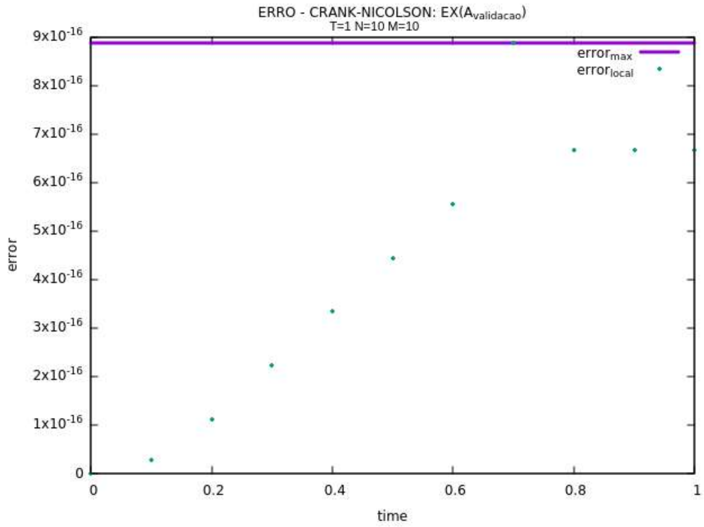
$$\begin{bmatrix} (1+\lambda) & -\frac{\lambda}{2} & 0 & \ldots & 0 \\ -\frac{\lambda}{2} & (1+\lambda) & -\frac{\lambda}{2} & \ldots & \ldots \\ 0 & \ldots & \ldots & \ldots & 0 \\ \ldots & \ldots & -\frac{\lambda}{2} & (1+\lambda) & -\frac{\lambda}{2} \\ 0 & \ldots & 0 & -\frac{\lambda}{2} & (1+\lambda) \end{bmatrix} \text{x} \begin{bmatrix} u_1^{k+1} \\ u_2^{k+1} \\ \ldots \\ u_{N-2}^{k+1} \\ u_{N-1}^{k+1} \end{bmatrix}$$

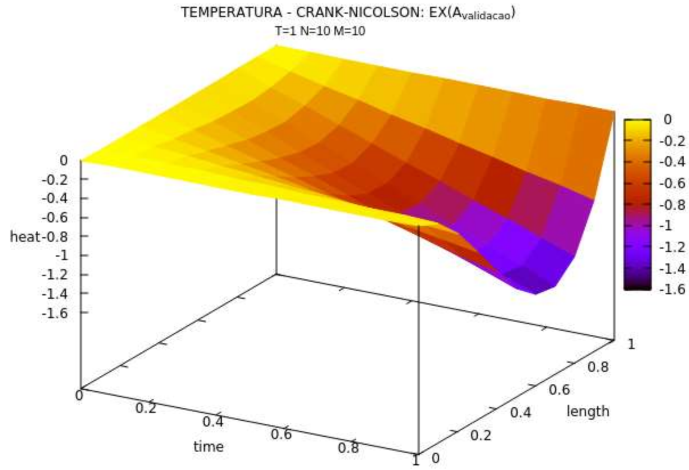
=

$$\begin{bmatrix} \frac{\lambda}{2}\ u_0^{k+1} + (1-\lambda)\ u_1^k + \frac{\lambda}{2}\ u_2^k + \frac{\Delta t}{2}(f_1^k + f_1^{k+1}) + \frac{\lambda}{2}\ g_1^{k+1} \\ \frac{\lambda}{2}\ u_1^{k+1} + (1-\lambda)\ u_2^k + \frac{\lambda}{2}\ u_3^k + \frac{\Delta t}{2}(f_2^k + f_2^{k+1}) \\ \ldots \\ \frac{\lambda}{2}\ u_{N-3}^{k+1} + (1-\lambda)\ u_{N-2}^k + \frac{\lambda}{2}\ u_{N-1}^k + \frac{\Delta t}{2}(f_{N-2}^k + f_{N-2}^{k+1}) \\ \frac{\lambda}{2}\ u_{N-2}^{k+1} + (1-\lambda)\ u_{N-1}^k + \frac{\lambda}{2}\ u_N^k + \frac{\Delta t}{2}(f_{N-1}^k + f_{N-1}^{k+1}) + \frac{\lambda}{2} g_2^{k+1} \end{bmatrix}$$

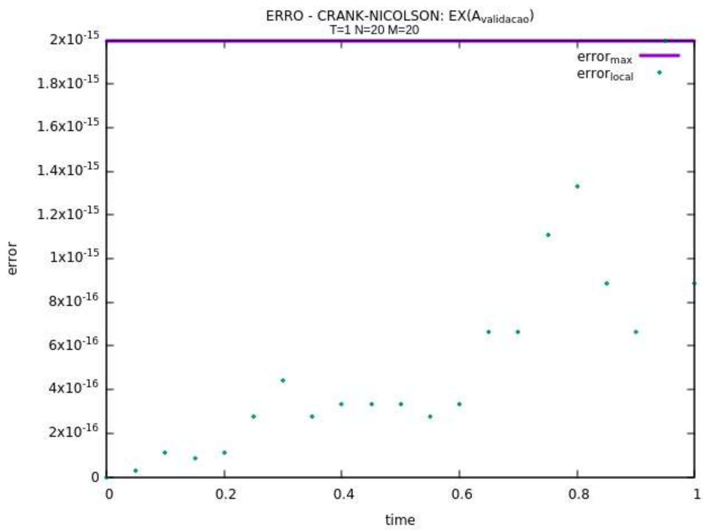
Teste) T = 1 com a fonte $f(t, x) = 10x^2(x-1) - 60xt + 20t$ a partir de $u_0(x) = 0$ e condições de fronteira nulas.

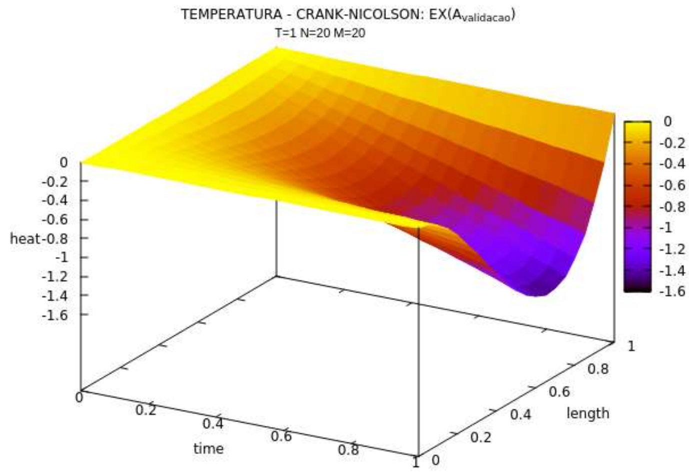
N=10


ERRO - CRANK-NICOLSON: EX($A_{validacao}$)
T=1 N=10 M=10
$error_{max}$
$error_{local}$
error
time

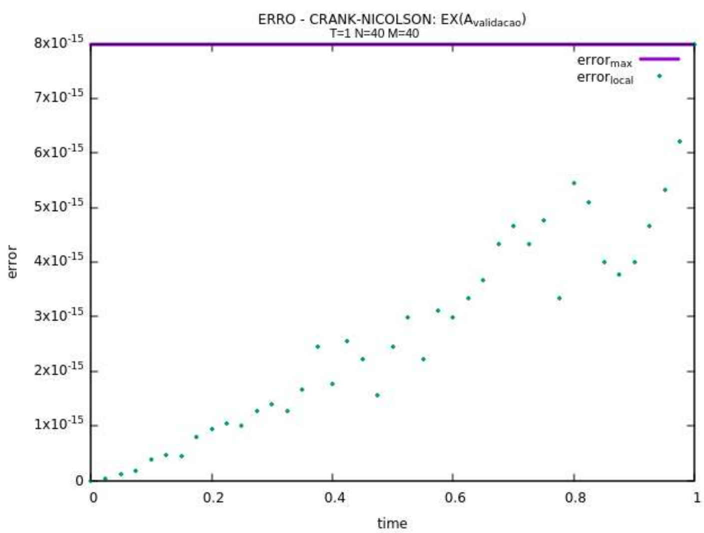

TEMPERATURA - CRANK-NICOLSON: EX($A_{validacao}$)
T=1 N=10 M=10
heat
time
length

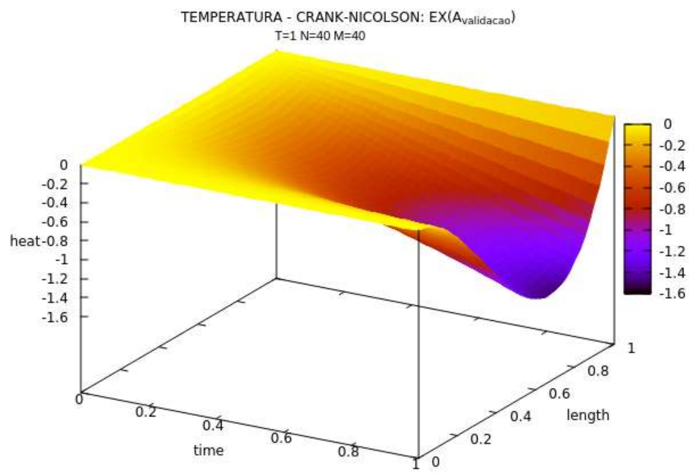
N=20


ERRO - CRANK-NICOLSON: EX($A_{validacao}$)
T=1 N=20 M=20
$error_{max}$
$error_{local}$
error
time

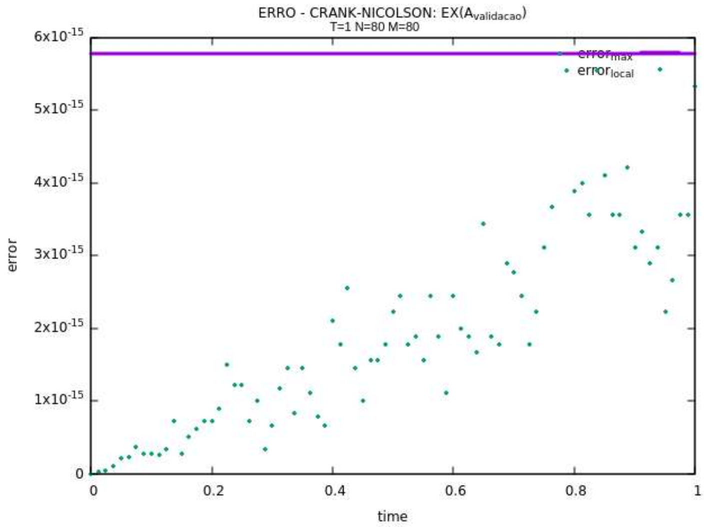

TEMPERATURA - CRANK-NICOLSON: EX($A_{validacao}$)
T=1 N=20 M=20
heat
time
length

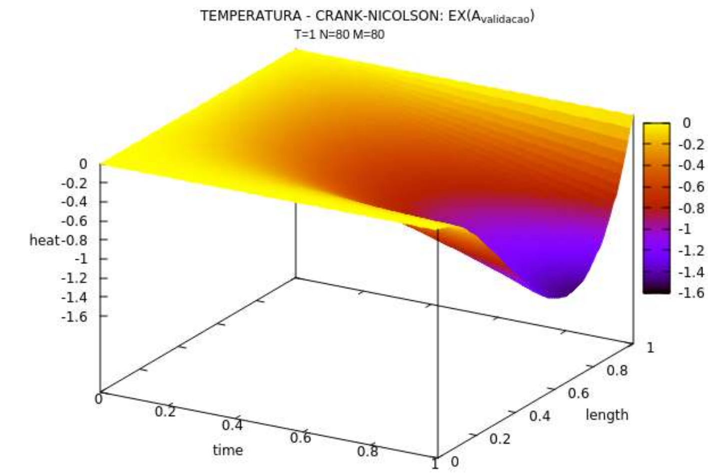
N=40


ERRO - CRANK-NICOLSON: EX($A_{validacao}$)
T=1 N=40 M=40
$error_{max}$
$error_{local}$
error
time

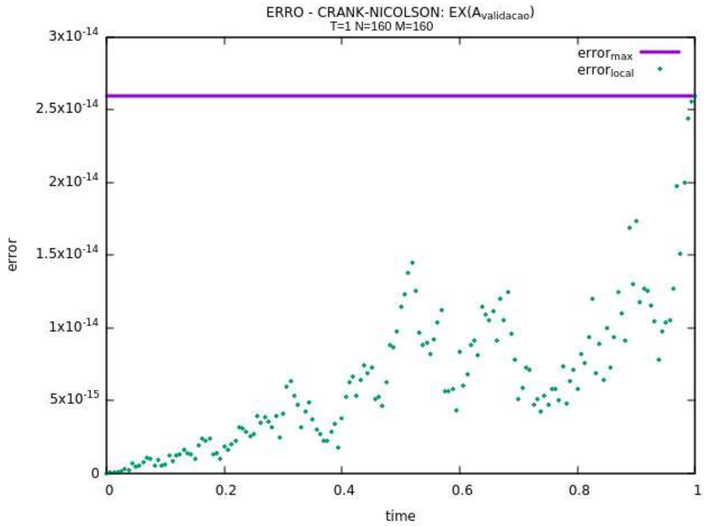

TEMPERATURA - CRANK-NICOLSON: EX($A_{validacao}$)
T=1 N=40 M=40
heat
time
length

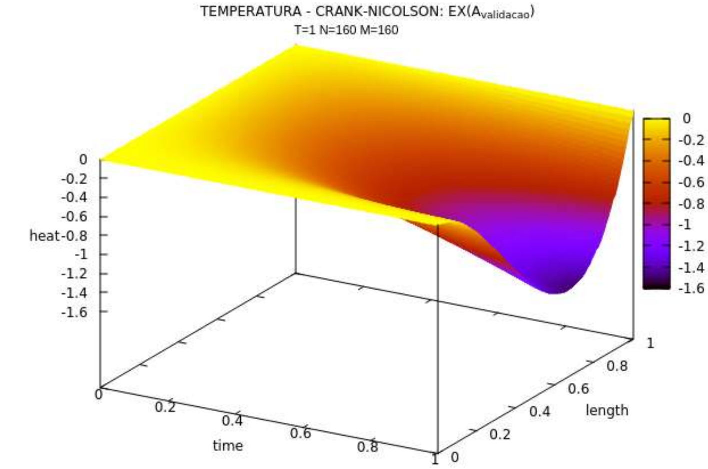
N=80



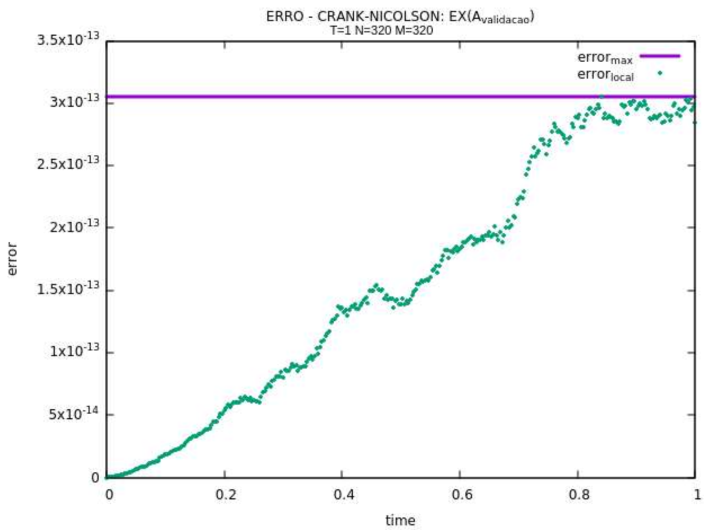


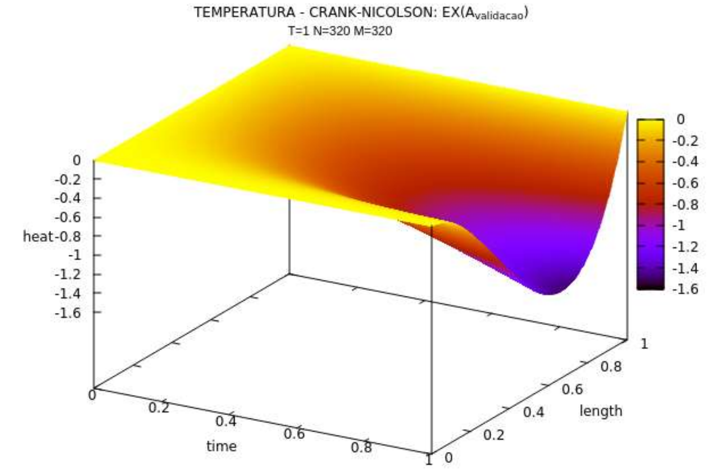
N=160





N=320





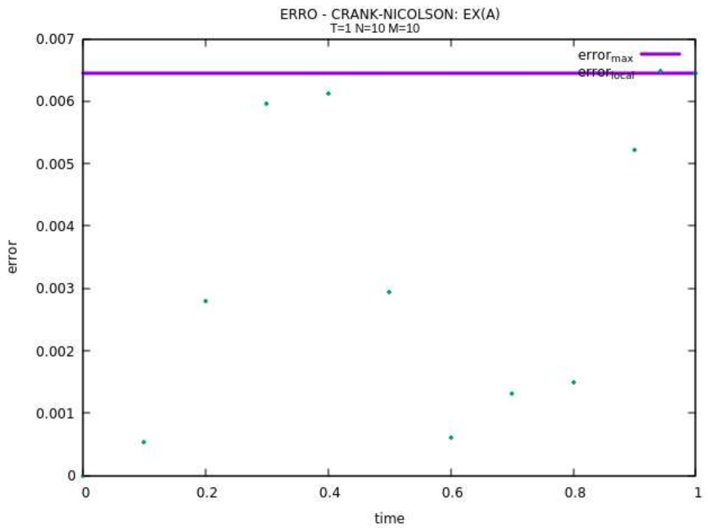
Exercício a) Função $f(t,x) = 10cos(10t)x^2(1-x)^2 - (1+sin(10t))(12x^2 - 12x + 2)$

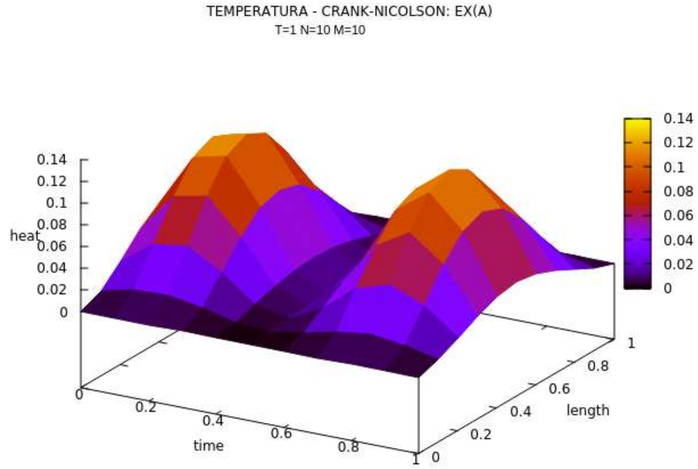
Solução exata $u(t,x) = (1+sin(10t))x^2(1-x)^2$

Valor inicial $u_0(x) = x^2(1-x)^2$

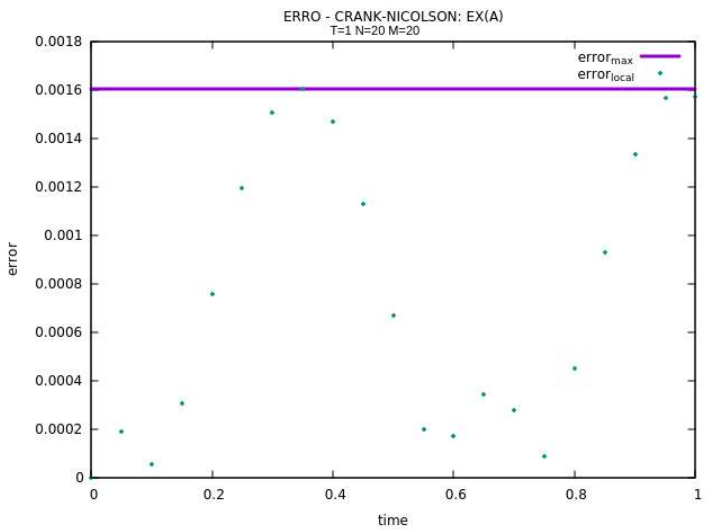
Condições nulas na fronteira.

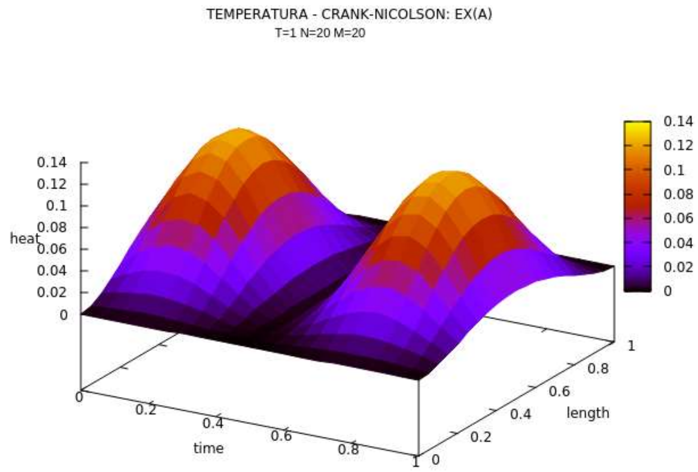
N=10



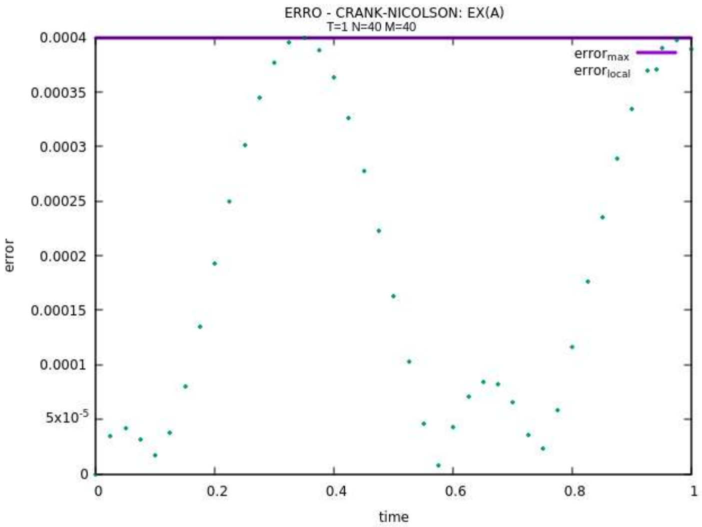


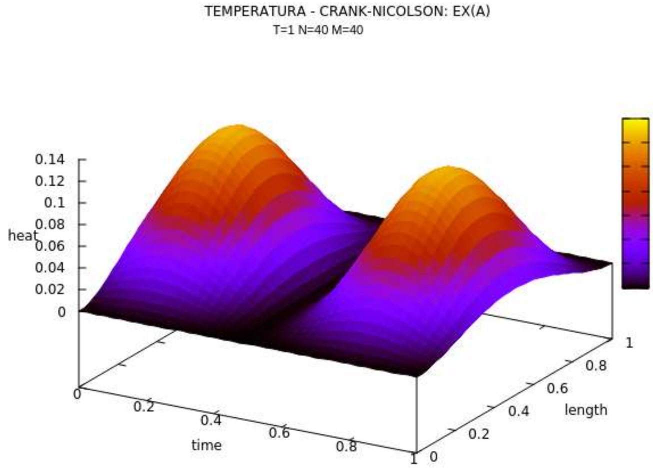
N=20



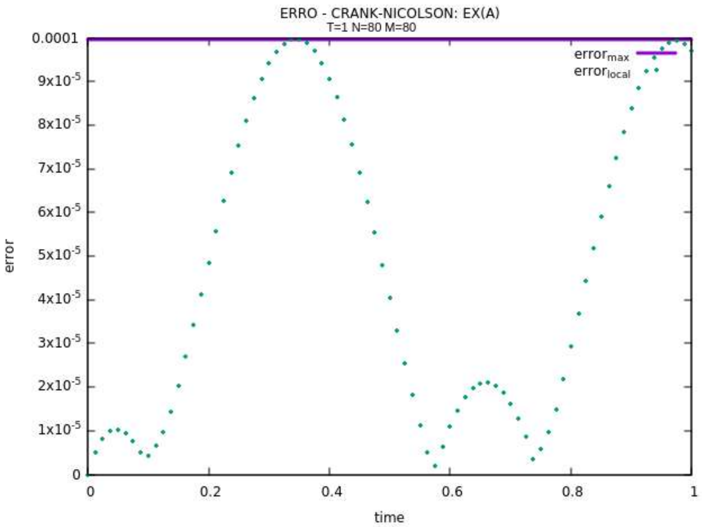


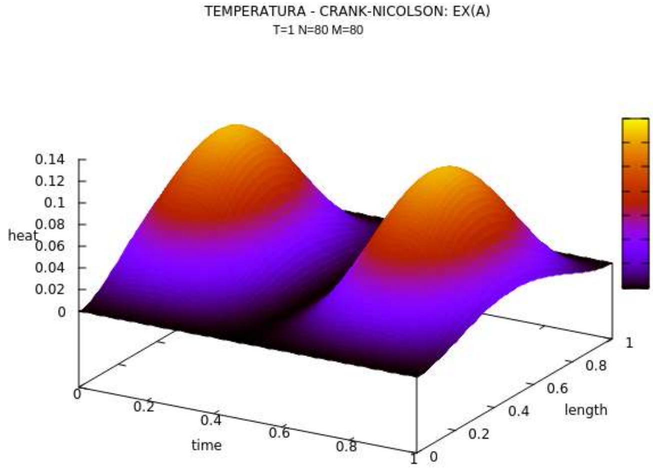
N=40


ERRO - CRANK-NICOLSON: EX(A)
T=1 N=40 M=40
error_max
error_local
error
time

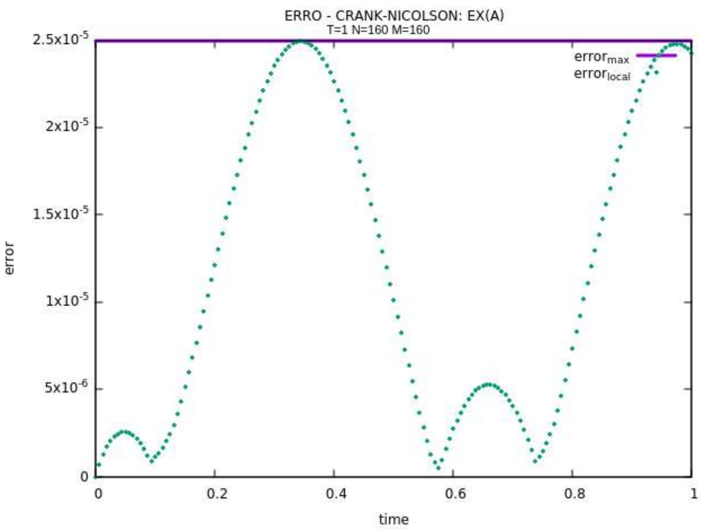

TEMPERATURA - CRANK-NICOLSON: EX(A)
T=1 N=40 M=40
heat
time
length

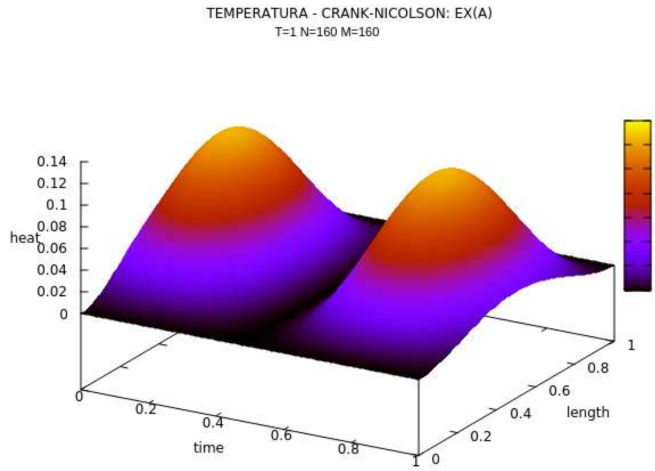
N=80


ERRO - CRANK-NICOLSON: EX(A)
T=1 N=80 M=80
error_max
error_local
error
time

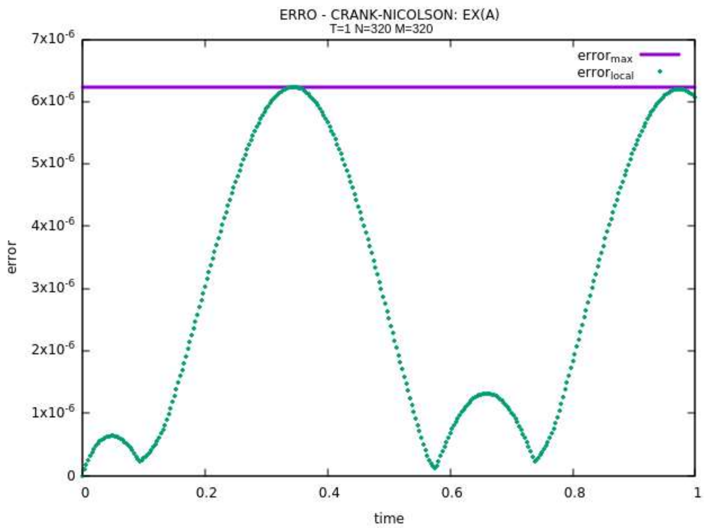

TEMPERATURA - CRANK-NICOLSON: EX(A)
T=1 N=80 M=80
heat
time
length

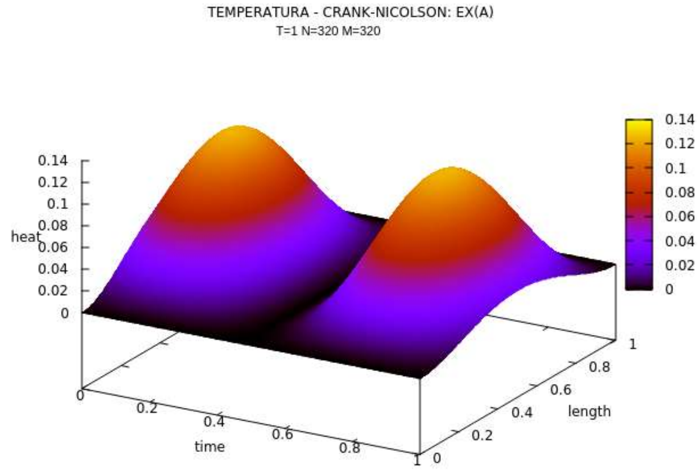
N=160


ERRO - CRANK-NICOLSON: EX(A)
T=1 N=160 M=160
error_max
error_local
error
time


TEMPERATURA - CRANK-NICOLSON: EX(A)
T=1 N=160 M=160
heat
time
length

N=320





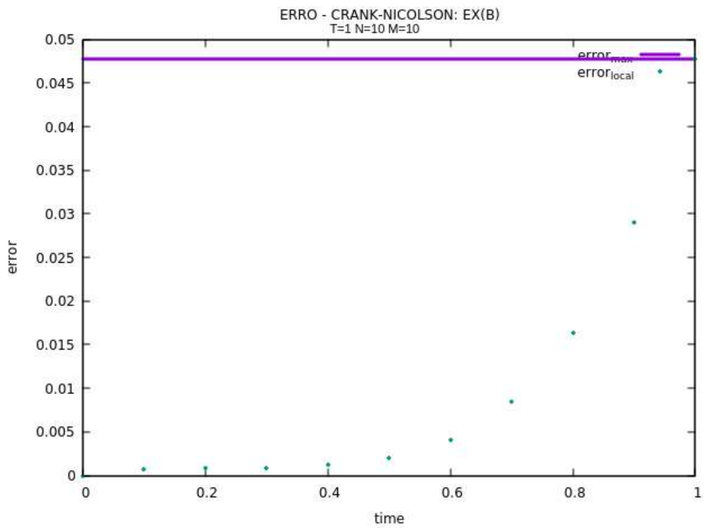
Exercício b) Função $f(t,x) = 10cos(10t)x^2(1-x)^2 - (1+sin(10t))(12x^2 - 12x + 2)$

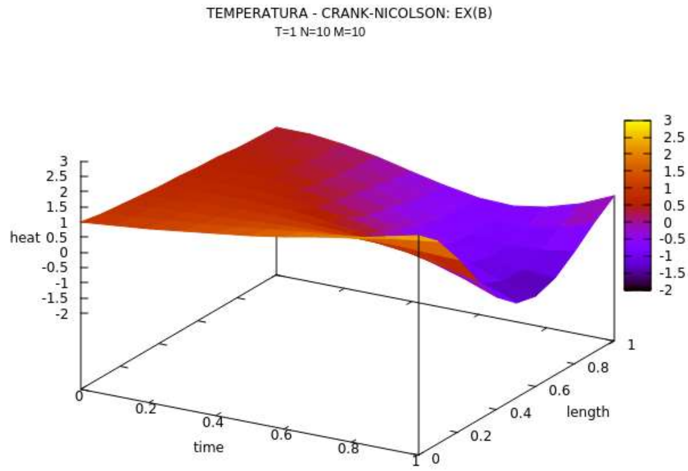
Solução exata $u(t,x) = (1+sin(10t))x^2(1-x)^2$

Valor inicial $u_0(x) = x^2(1-x)^2$

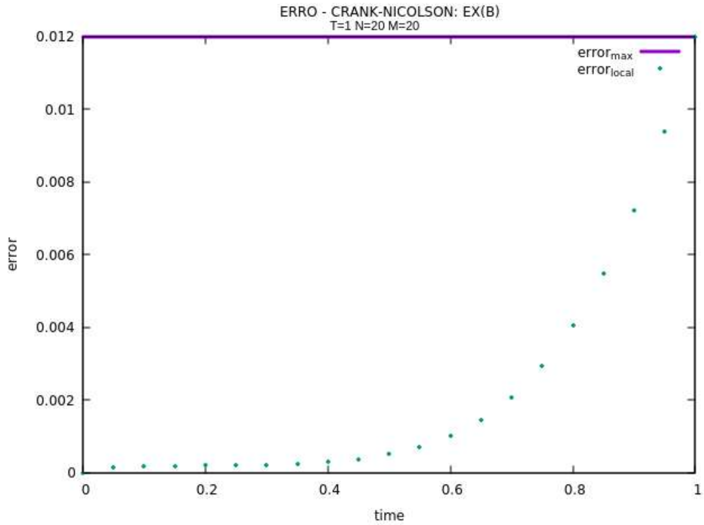
Condições nulas na fronteira.

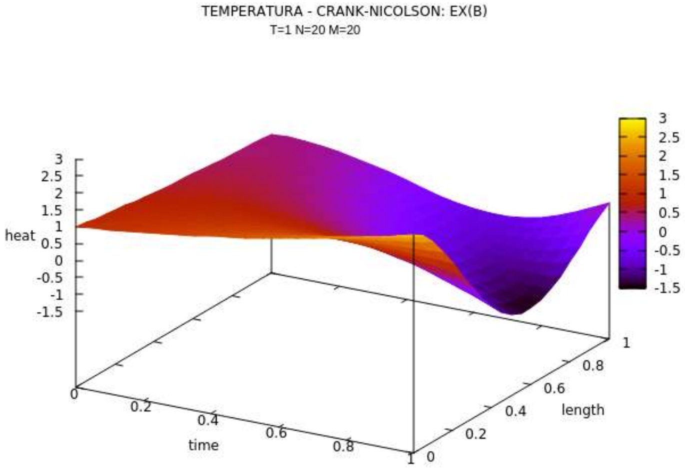
N=10



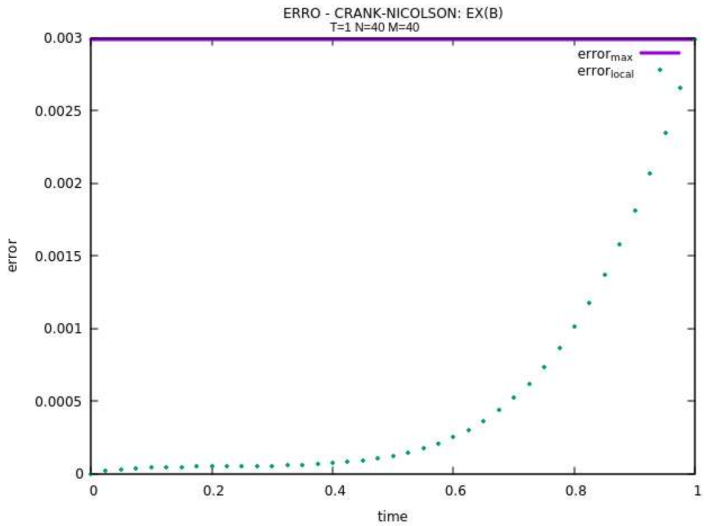


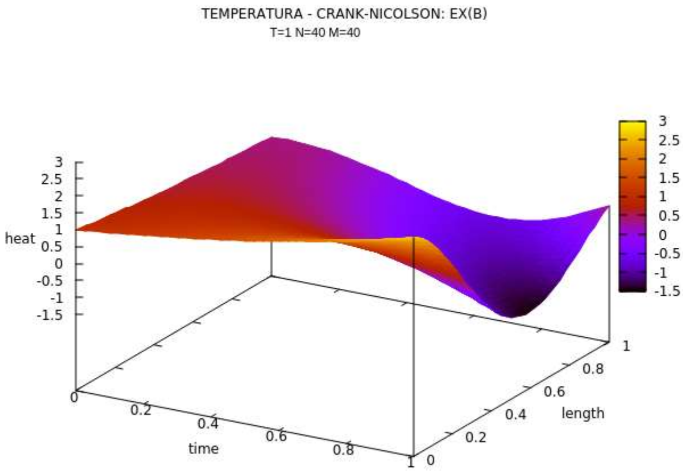
N=20


ERRO - CRANK-NICOLSON: EX(B)
T=1 N=20 M=20
error$_{max}$
error$_{local}$
error
time

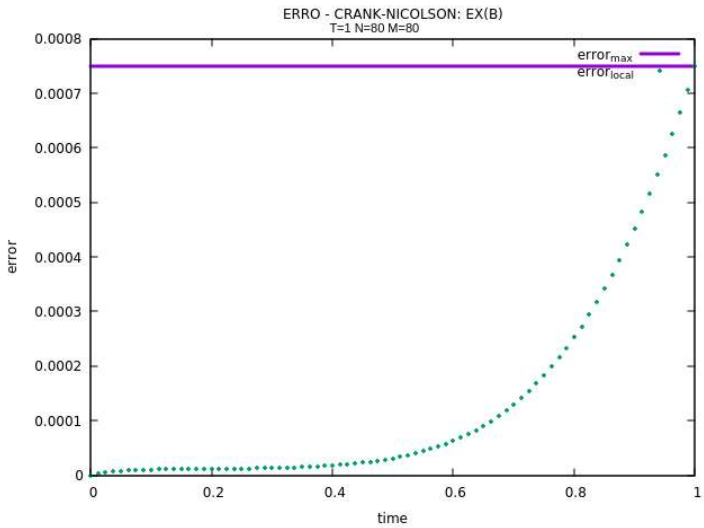

TEMPERATURA - CRANK-NICOLSON: EX(B)
T=1 N=20 M=20
heat
time
length

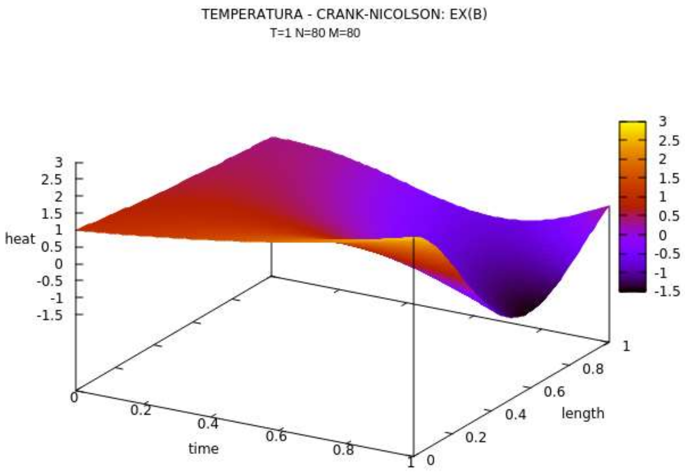
N=40


ERRO - CRANK-NICOLSON: EX(B)
T=1 N=40 M=40
error$_{max}$
error$_{local}$
error
time

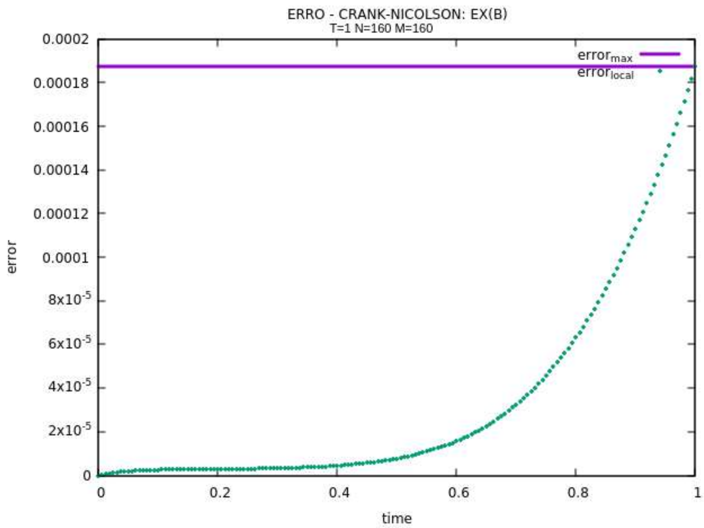

TEMPERATURA - CRANK-NICOLSON: EX(B)
T=1 N=40 M=40
heat
time
length

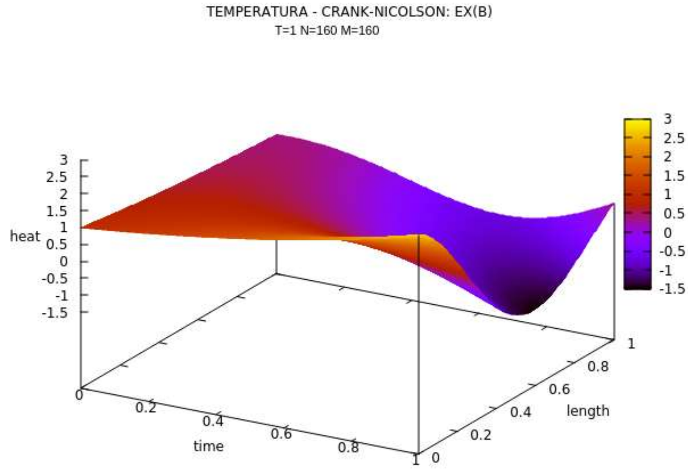
N=80


ERRO - CRANK-NICOLSON: EX(B)
T=1 N=80 M=80
error$_{max}$
error$_{local}$
error
time

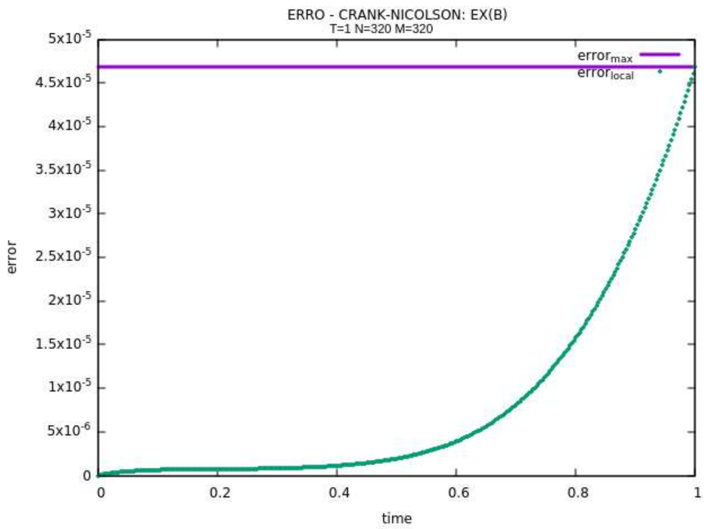

TEMPERATURA - CRANK-NICOLSON: EX(B)
T=1 N=80 M=80
heat
time
length

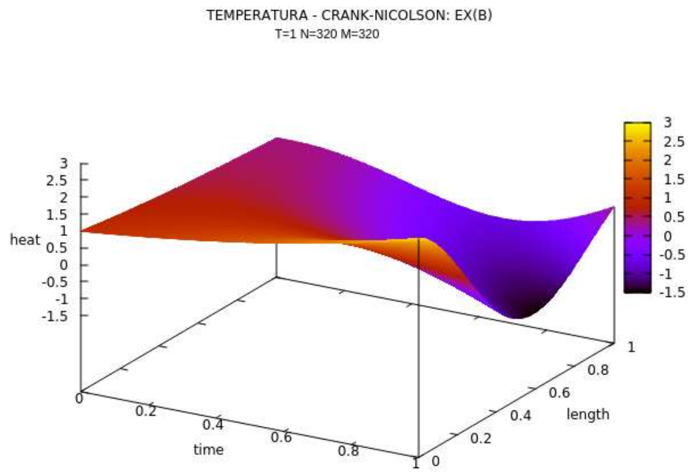
N=160





N=320





Exercício c) T=1

Condição inicial nula

Fonte pontual de intensidade $r(t) = 10000 * (1 - 2t^2)$

$f(t, x) = r(t)g_h(x)$, com $h = \Delta x$ e $p = 0.25$

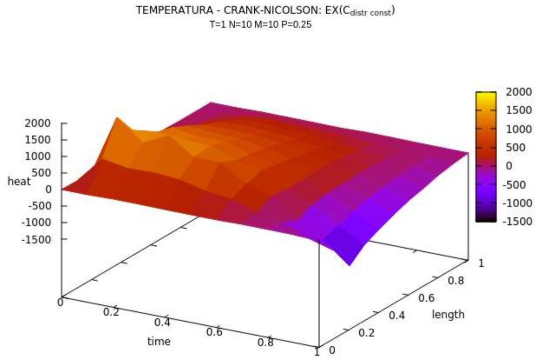
Na fronteira $g_1(t) = g_2(t) = 0$.

Exercício c.1) Distribuição constante

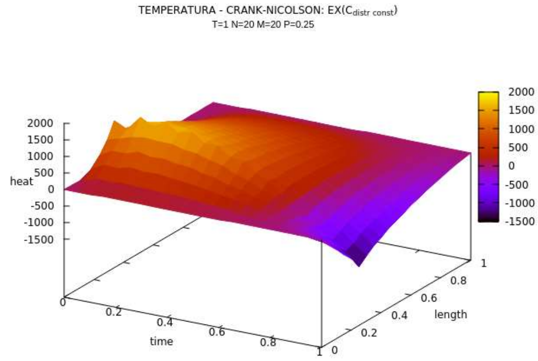
$$g_h(x) = \frac{1}{h},\ se\ p - \frac{h}{2} \leq x \leq p + \frac{h}{2}$$

$$g_h(x) = 0,\ caso\ contrário$$

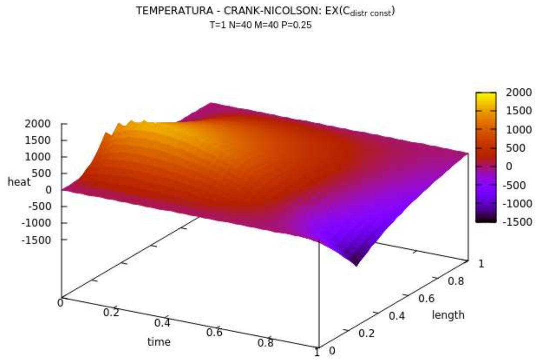
N=10



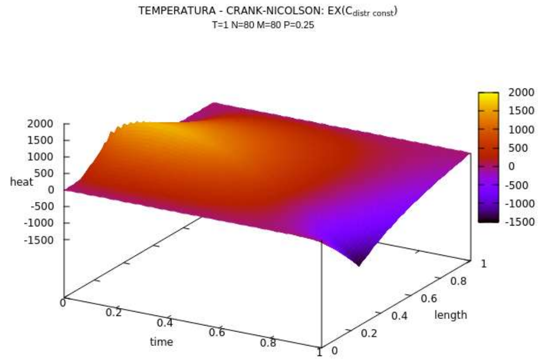
N=20



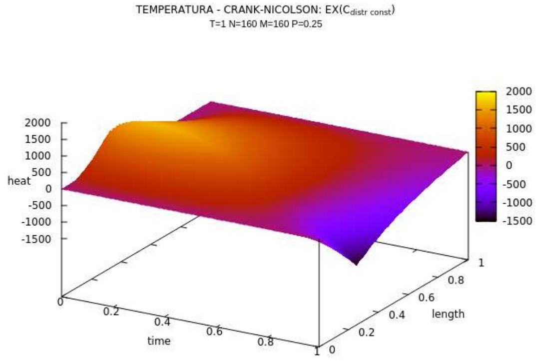
N=40



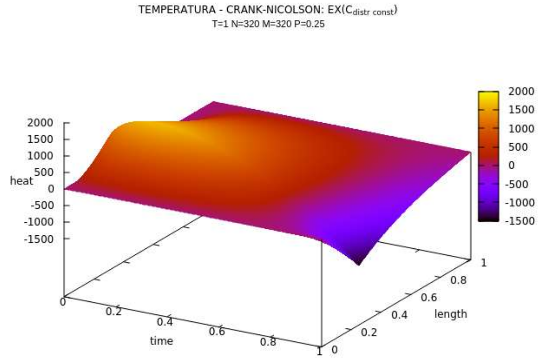
N=80



N=160



N=320



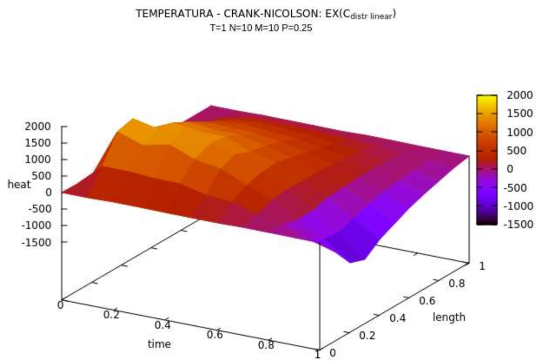
Exercício c.2) Distribuição linear

$$g_h(x) = \tfrac{1}{h^2}[x-(p-h)]\ ,\ se\ p-h \leq x \leq p$$

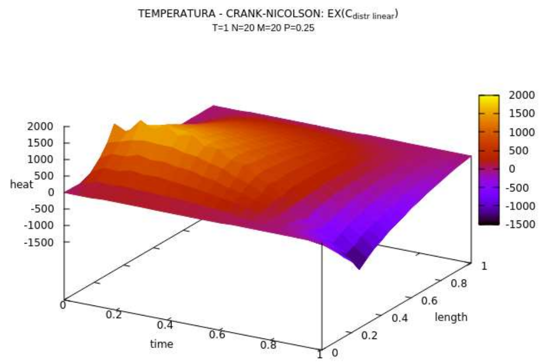
$$g_h(x) = \ -\tfrac{1}{h^2}[x-(p+h)]\ ,\ se\ p < x \leq p+h$$

$$g_h(x) = 0,\ caso\ contrário$$

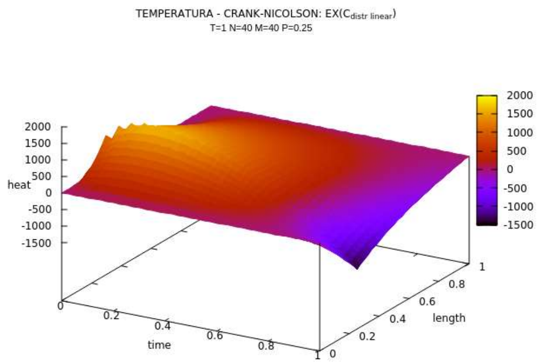
N=10



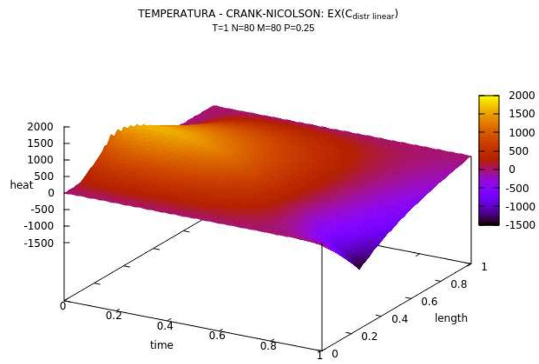
N=20



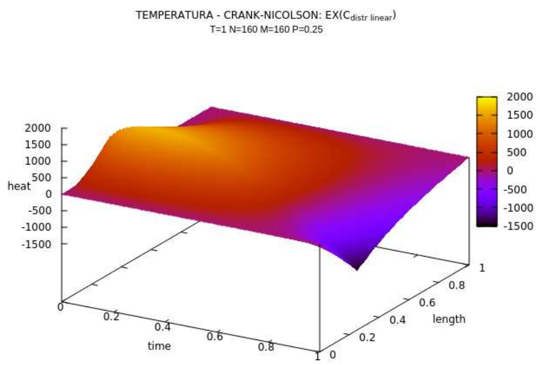
N=40



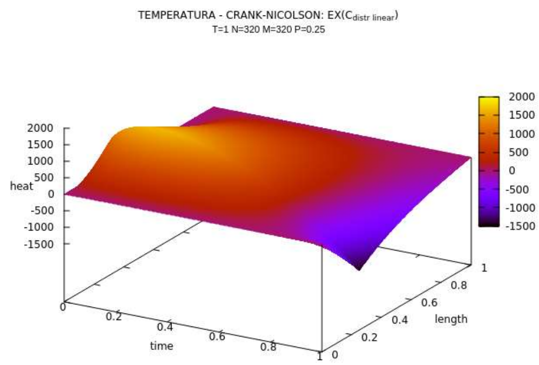
N=80



N=160



N=320



5. Resultados obtidos

Método Explícito- Item a

Método	Explícito					
Exercício	a					
N	λ = 0,25	Fator de Redução	λ = 0,50	Fator de Redução	λ = 0,51	Fator de Redução
10	0,004763		0,00529		0,005307	
20	0,001189	0,2496325845	0,001316	0,2487712665	10,76	2027,510835
40	0,0002972	0,2499579479	0,0003287	0,2497720365	1,87E+39	1,74E+38
80	7,43E-05	2,50E-01	8,22E-05	2,50E-01	3,87E+197	2,07E+158
160	1,86E-05	2,50E-01	2,05E-05	2,50E-01	NaN	NaN
320	4,64E-06	2,50E-01	5,14E-06	2,50E-01	NaN	NaN

Método Explícito- Item b

Método	Explícito					
Exercício	b					
N	λ = 0,25	Fator de Redução	λ = 0,50	Fator de Redução	λ = 0,51	Fator de Redução
10	0,05005		0,05045		0,05046	
20	0,0125	0,2497502498	0,01258	0,2493557978	5,698	112,9211256
40	0,003129	0,25032	0,003154	0,2507154213	9,88E+38	1,73E+38
80	7,82E-04	2,50E-01	7,88E-04	2,50E-01	2,04E+197	2,07E+158
160	1,96E-04	2,50E-01	1,97E-04	2,50E-01	NaN	NaN
320	4,89E-05	2,50E-01	4,93E-05	2,50E-01	NaN	NaN

Método de Euler - Itens a e b

Método	Implícito: Euler	
Exercício	a	
N	**λ = N**	**Fator de Redução**
10	0,01627	
20	0,009194	0,5650891211
40	0,00494	0,5373069393
80	2,58E-03	5,23E-01
160	1,32E-03	5,11E-01
320	6,67E-04	5,06E-01

Método	Implícito: Euler	
Exercício	b	
N	**λ = N**	**Fator de Redução**
10	0,04497	
20	0,01562	0,3473426729
40	0,007835	0,5016005122
80	3,93E-03	5,02E-01
160	1,97E-03	5,01E-01
320	9,97E-04	5,06E-01

Método de Crank-Nicolson- Itens a e b

Método	Implícito: Crank-Nicolson	
Exercício	a	
N	**λ = N**	**Fator de Redução**
10	0,006453	
20	0,001605	0,2487215249
40	0,0003994	0,248847352
80	9,97E-05	2,50E-01
160	2,50E-05	2,50E-01
320	6,24E-06	2,50E-01

Método	Implícito: Crank-Nicolson	
Exercício	b	
N	**λ = N**	**Fator de Redução**
10	0,04778	
20	0,01199	0,2509418167
40	0,002993	0,2496246872
80	7,49E-04	2,50E-01
160	1,87E-04	2,50E-01
320	4,68E-05	2,50E-01

Caso haja interesse, é possível ver o desenvolvimento do projeto no GitHub clicando aqui, ou no link https://github.com/gAkira/C-eh-a-melhor-linguagem-de-programacao