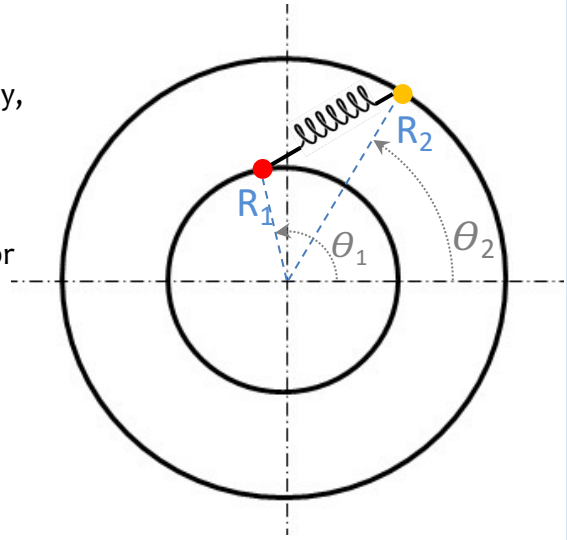
7d. Coupled Rotation

- **coupled_rotation.cpp**: Two identical masses (m=1) are constrained to move on two circles of radii, respectively, $R_1 = 1$ and $R_2 = 2$. The two masses are connected by a spring with constant k=1.2 and rest length $L_0=R_2 - R_1$.



- If θ_1 and θ_2 are, respectively, the angles formed by the two masses with the x-axis, the equations of motion for the coupled system can be written as

$$\ddot{\theta}_1 = -k\left(1 - \frac{L_0}{L}\right)\frac{R_2}{R_1}(\sin\theta_1 - \sin\theta_2)$$

$$\ddot{\theta}_2 = +k\left(1 - \frac{L_0}{L}\right)\frac{R_1}{R_2}(\sin\theta_1 - \sin\theta_2)$$

where, using the law of cosines, $L^2 = R_1^2 + R_2^2 - 2R_1R_2\cos(\theta_1 - \theta_2)$

- Solve the equations of motion using both RK4 and either Position-Verlet (PV) or Velocity-Verlet (VV) using the initial condition $\theta_1 = 0$, $\theta_2 = \pi/3$, $\omega_1 = 0$, $\omega_2 = 0.05$.
- Choose $\Delta t = 0.2$ and use a single loop to advance both solutions in time. Write the solution to disk using the standard multicolumn format.
- Break from the loop when the difference between the solutions generated by the two methods (on θ_1 or θ_2) exceeds *tol = 0.1*, i.e.:

$$\max(\epsilon_1, \epsilon_2) > tol, \quad \text{where} \quad \epsilon_j = \left|\theta_j^{\text{RK4}} - \theta_j^{\text{PV}}\right|/\pi \quad (j = 1, 2)$$

- ount the (approximate) number of inversion points for the two masses (for RK4).
- Upload your code with i) the output inserted in the comments at the beginning of the file, ii) the required library function at the end, e.g.

```
// Name: ..., Date: ...
//
// Code output:
// ***************************************************
// Loop break at nstep = ??; t = ??      # when you exit from the loop
//               eps1  = ??; eps2 = ??   # differences between the two solutions
//               ip1   = ??; ip2  = ??   # number of inversion points
// ***************************************************
#include ...
...
int main()
{
  // code here
}

void RK4Step (...){

}

void  VerletStep(...){

}
```

7d. Coupled Rotation (cont)

- Also, upload a plot (**png** screen capture is fine) of the solution for **0 < t < 50**, showing *only* the angles θ_1 and θ_2 obtained with RK4 and the Verlet algorithm (4 plots in total) as a function of time.
- If you saved the output using a multi-column format, you may use the following Gnuplot script:

```
reset

fname = "coupled_rotation.dat"

# Set column indices
th1_RK4 = 2
th2_RK4 = 3

th1_PV = 6
th2_PV = 7

# Set plot speicifications
set grid
set key    left font ",14"
set title  font ",14"
set tics   font ",14"
set xlabel "x"      font ",14"
set ylabel "theta"  font ",14"

# Plot
set xrange[0:50]
#set yrange[-0.1:1]
  plot fname using 1:th1_RK4  title "th1 (RK4)"  w lines
replot fname using 1:th2_RK4  title "th2 (RK4)"  w lines

replot fname using 1:th1_PV  title "th2 (PV)"   w lines
replot fname using 1:th2_PV  title "th2 (PV)"   w lines
```