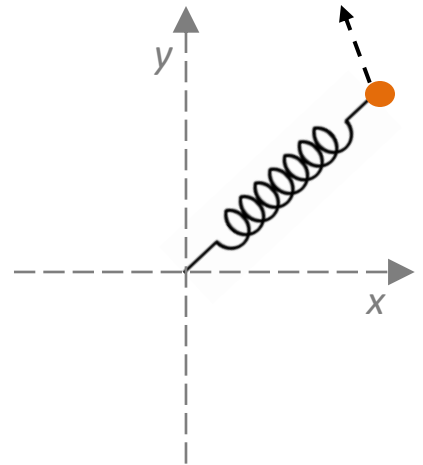


7f. Central Spring

- **A point mass** is attached to a spring of rest length $L_0 = 1$ and stiffness $k = 3$. The other side of the spring is fixed at the origin.
- In a 2D plane ($z = 0$) the motion of the mass is regulated by the following system of ODEs:

$$\ddot{x} = -k(R - L_0) \frac{x}{R}$$
$$\ddot{y} = -k(R - L_0) \frac{y}{R}$$



where $R = \sqrt{x^2 + y^2}$ is the distance from the origin.

- Solve the previous system for $t \in [0, 200]$ with initial condition given by $\{x, y\} = \{1, 0.5\}$ and velocity $\{v_x, v_y\} = \{0.4, 0.6\}$. Integrate with the 4th-order Runge-Kutta scheme using 1200 time steps. Repeat the simulation using the position (or velocity) Verlet method, again with 1200 time steps.
- During each simulation:
 - Determine the time between two successive revolutions of the mass around the origin and estimate the orbital period;
 - At each revolution, print the current period and the magnitude of the angular momentum ($\vec{L} = \vec{r} \times \vec{v}$).

7f. Central Spring (cont)

- Upload your code with your name and the output inserted in the comment at the beginning of the file and the necessary library function at the end:

```
// Name: ..., Date: ...
//
// *****
// [RK4] Revolution #1, period = ?; L = ?
// [RK4] Revolution #2, period = ?; L = ?
// ...
// [PV] Revolution #1, period = ?; L = ?
// [PV] Revolution #2, period = ?; L = ?
// *****
#include ...
...
int main()
{
    // code here
}

void RK4Step (...){
    ...
}

void PositionVerlet (...){
    ...
}
```

- Also, upload a png (or jpeg) plot showing the orbit in the $x - y$ plane for the two methods. If you're using gnuplot, here's a script to produce the plot:

```
reset

# Set column indices
ix = 2
iy = 3

# Set plot specifications
set grid
set key font ",14"
set title font ",14"
set tics font ",14"
set xlabel "x" font ",14"
set ylabel "y" font ",14"

# Plot
filename = "central_spring.dat"
plot filename using ix:iy index 0 title "RK4" with lines
replot filename using ix:iy index 1 title "PV" with lines
```