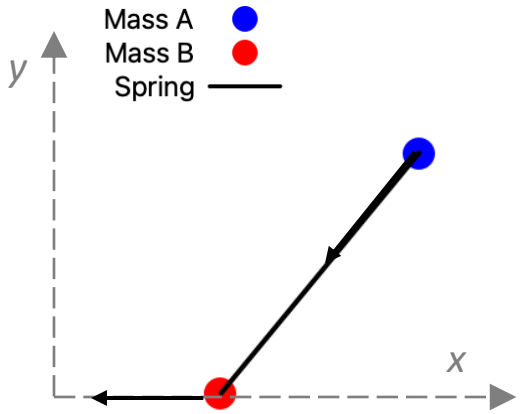
7h. Coupled Springs

- **Two point masses** A and B are connected by an **ideal spring** of rest length $L_0 = 1$ and stiffness $k = 6$. Mass B is an elastic body subject to a non-linear Hook's law acting only along the x- direction and its motion is **not affected** by the motion of mass A (one-way coupling).



- In a 2D plane (z = 0) the dynamics of such system can be described by the following system of ODEs:

$$\ddot{x} = -k(R - L_0)\frac{x - x_B}{R}$$

$$\ddot{y} = -k(R - L_0)\frac{y}{R}$$

$$\ddot{x}_B = -k_B x_B + \alpha_B x_B^2$$

where (x, y) are the coordinates of mass A, x_B is the coordinate of B, while $R = \sqrt{(x - x_B)^2 + y^2}$ is the distance between the two masses. The last equation accounts for the non-linear Hook's law with $k_B = 1, \alpha_B = 0.1$.

- Solve the previous system for $t \in [0,30]$ with initial condition given by $\{x, y, x_B\} = \{1, 0.5, 1\}$ and zero initial velocity. Integrate with the 4^{th} -order Runge–Kutta scheme using 800 time steps. Repeat the simulation using the Position (or Velocity) Verlet method, again with 800 time steps.

- For each of the two methods, during the computation:

  - Count the number of times in which mass A crosses the x axis;
  - Compute the minimum value of R during the integration time window;

7h. Coupled Springs (cont)

- Upload your code with your name and the output inserted in the comment at the beginning of the file and the necessary library function at the end:

```
// Name: ..., Date: ...
//
// ***************************************************
// Number of crossing[RK4]: ??
// Rmin[RK4:                ??
// Number of crossing[Verlet]: ??
// Rmin[Verlet]:               ??
// ***************************************************
#include ...
...
int main()
{
  // code here
}

void RK4Step (...){
...
}

void PositionVerlet (...){
...
}
```

- Also, upload a png (or jpeg) plot showing $x(t), y(t), R(t)$ for one of the two method (of your choice). If you're using gnuplot, here's a script to produce the plot:

```
reset

# Set column indices
ix  = 2
iy  = 3
ixB = 4
iR  = 5

# Set plot specifications
set grid
set key    font ",14"
set title  font ",14"
set tics   font ",14"
set xlabel "t"      font ",14"
set ylabel "x,y,R(t)"  font ",14"

# Plot
plot   "coupled_springs.dat" using 1:ix   title "x"  w lines
replot "coupled_springs.dat" using 1:iy   title "y"  w lines
replot "coupled_springs.dat" using 1:iR   title "R"  w lines
```