



# WATER WAVE OPTIMIZATION ALGORITHM FOR SOLVING COMBINED ECONOMIC AND EMISSION DISPATCH PROBLEM

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## ABSTRACT

The main focus in recent power system engineering practices is to get an optimal balance between the cost and emission reduction in thermal power plants. The combined economic emission dispatch (CEED) optimization is performed for allocating the committed units for generation such that fuel cost and emission level are simultaneously optimized while satisfying the generator constraints. CEED problem is formulated by considering both the economy and emission objectives. This bi-objective CEED problem is then converted into a single objective function using a modified price penalty factor approach. In this paper, Water Wave Optimization Algorithm (WWOA) is implemented to solve the CEED problems including the system transmission losses. WWOA is inspired by the shallow water wave theory. The capabilities of the proposed approach are well demonstrated using the numerical results of four test systems. The results are compared to those obtained from other solution techniques like GA, PSO, NSGA-II, FCGA, MABC, MODE, PDE, BSA, GSA and FPA for different cases.

**Keywords:** water wave optimization algorithm, combined economic emission dispatch, transmission losses, cost minimization.

## 1. INTRODUCTION

The main objective of Economic Dispatch (ED) is to minimize the fuel cost while satisfying the system load demand. There are different algorithms available to solve an ELD problem for different loads with or without considering transmission loss [1]. Other methods such as gradient, Newton, linear programming and interior point have also been applied to solve ED, problem [2]. The generation cost is minimized with the help of ED, now a day's the world concentrates on reducing the pollutants during power generation in thermal power plants. The main components of pollutants from the fossil fuels are Carbondioxide ( $\text{CO}_2$ ), Oxides of Nitrogen ( $\text{NO}_x$ ) and Oxides of Sulphur ( $\text{SO}_x$ ). So, for the benefit of the environment, the Economic Emission Dispatch (EED) was introduced to minimize the emissions for a certain load demand. On considering both the fuel cost economy and the emission objectives simultaneously, the problem thus gets modified in to a Combined Economic and Emission Dispatch (CEED) problem by using the price penalty factor. At hand there are many methods available to find the price penalty factor like 'Max-Max', 'Min-Max', 'Min-Min', 'Avg. penalty factor', etc. Here a different approach is used by modifying the price penalty factor to obtain the exact value of CEED cost by the method of linear interpolation [3].

A number of strategies have appeared in the literature over the years for solving Economic Emission Dispatch (EED). Lagrange relaxation methods, weighted sum method, linear programming method are used to solve the EED problem [4]. As these methods took much computational time for medium to large-scale economic dispatch problems, many nature inspired algorithms are being developed in search for high reliability, less computational time and highly efficient.

Recently, many evolutionary algorithms like Real Coded Genetic Algorithm (RCGA) [5], Ant Colony Optimization (ACO) [6], Multi Objective Particle Swarm

optimization (MOPSO) [7] and Fire Fly Algorithm (FFA) [8] are used to eliminate many difficulties in the traditional and classical methods to solve non-linear CEED problem with no change in the shape of fuel and emission cost curves.

The emphasis on direct search and stochastic methods like nature inspired techniques is due to the observation that mathematical programming approaches are often not suitable for tackling such problems due to the non-convexity of the search space.

The Environmental Economic Dispatch problem is of either a single objective or multi-objective and is solved using various stochastic algorithms. Techniques like Neural network, Fuzzy system and Lagrange's algorithm (LA) [9], Dispatch problem on different power system using Stochastic algorithm [10], Penalty factor based approach [11,12], Opposition based learning approach [13], WSM technique [14], AI technique [15] are employed to find the optimal solution for the combined economic and emission dispatch problems.

Although there are several methods available for the environmental economic dispatch problem, the larger the system, greater is the complexity which necessitates developing efficient algorithms to stably find an optimal solution. In this context, the focus of this work is to demonstrate efficiency of a nature inspired approach for solving CEED problems.

Initially water wave theory was related to gravitational force and other forces dating back to Newton's work in 1687 [16] and later by the development of mathematical models like Laplace. Lagrange, Poisson made the linear wave theory advanced along with non-linear waves as considered by Stokes, Gerstner and Kelland [17].

In this paper a new nature inspired meta-heuristic optimization technique called Water Wave Optimization Algorithm (WWOA) initially proposed by Zheng [18] is implemented for obtaining the solutions for CEED



problems. The idea is from the wave motions, which is controlled by the wave-current-bottom interactions [17] in the search mechanism designs for high dimensional global optimization problems and the wave turbulence theory by Zakharov *et al.* [19]. The WWOA maintains the population of solutions, each of which is analogous to a "Wave" with a height (or amplitude) 'h' and a wavelength 'λ'. For the case of maximization problem with an objective function  $f(x)$ , the solution space  $X$  is analogous to the seabed area, and the fitness of a point 'x' is inversely proportionally to its seabed depth: the shorter the distance to the still water level, the higher the fitness  $f(x)$  is. The 3D space of the seabed is made analogy to the 'n' dimensional solution search space. For the minimization problem, the objective function is taken as  $\max [1 / (1+f(x))]$  (or)  $\max [-f(x)]$ . To show the very competitive nature and efficiency of WWOA, it is compared with some of the familiar meta-heuristics techniques that were proposed in the recent years.

The remaining of the paper is organized as follows: Section 2 describes different cases of combined environmental economic dispatch problems. Section 3 describes the shallow water wave model and the three phases of WWO Algorithm with their mathematical models. Section 4 deals with the implementation part of the WWO Algorithm to the CEED problems. The results of the WWOA implementation to test systems with 3, 6, 10, 40 units are compared with a few popular techniques which are presented in Section 5.

The capability of the WWOA algorithm is emphasized by applying the technique to various environmental economic dispatch problems having the convex and non-convex characteristics considering the existence of power system losses.

## 2. PROBLEM FORMULATION

The main objective of the CEED problem is to minimize two different objective functions simultaneously, fuel cost and emission, while satisfying various practical constraints.

### 2.1 Objective function of ED

The main objective of the ED problem is to determine the optimal combination of power generations that minimizes the total generation cost satisfying both equality and inequality constraints. The traditional objective function of the ED problem that has to be minimized can be approximately represented as a single quadratic function.

$$\text{Max CEED} = -\text{Min} \left[ C = \sum_{i=1}^{N_g} F_i(P_i) + h_m \sum_{i=1}^{N_g} E_i(P_i) + [(P_D + P_L) - \sum_{i=1}^{N_g} P_{Gi}]^2 \right] (\$/h) \quad (6)$$

$$\text{Where } F_i(P_i) = a_i + b_i P_i + c_i P_i^2 + |d_i \sin\{e_i(P_i^{\min} - P_i)\}| \quad (\$/h)$$

$$E_i(P_i) = \alpha_i + \beta_i P_i + \gamma_i P_i^2 + \eta_i \exp(\delta_i P_i) \quad (\text{Ton/h})$$

where,  $a_i, b_i, c_i$  are the fuel cost coefficients.  $d_i$  and  $e_i$  are the fuel cost coefficients modelled for the valve point effect for the  $i^{\text{th}}$  generator.

$$\text{Min } F_T =$$

$$\min \sum_{i=1}^{N_g} F_i(P_{Gi}) = \min \sum_{i=1}^{N_g} a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad (\$/h) \quad (1)$$

Thus in the case of economic dispatch problem the fitness function to be maximized considering the equality constraint is,

$$\text{Max } F_T =$$

$$- \left[ \min \sum_{i=1}^{N_g} F_i(P_{Gi}) + \psi [(P_D + P_L) - \sum_{i=1}^{N_g} P_{Gi}]^2 \right] (\$/h) \quad (2)$$

where,  $F_T$  is the total generation cost ( $\$/h$ ),  $F_i$  is the cost function of the  $i^{\text{th}}$  generator;  $a_i, b_i, c_i$  are the cost coefficients of the  $i^{\text{th}}$  generator,  $P_{Gi}$  is the power output (MW) of the  $i^{\text{th}}$  generator and  $N_g$  is the number of generators,  $\psi$  is the penalty factor that takes care of the power balance equality constraint as shown in Equation. (2).

The minimization is performed subject to the equality constraint that the total generation must equal to the total power demand ( $P_D$ ) in addition to system transmission losses ( $P_L$ ) as given by,

$$\sum_{i=1}^{N_g} P_i = P_D + P_L \quad (3)$$

Where,  $P_D$  is the total power demand and  $P_L$  is the real power loss, both in MW.

The total transmission line power loss can be calculated using B-matrix loss coefficients by Kron's formula as,

$$P_L = \sum_{i=1}^{N_g} \sum_{j=1}^{N_g} P_i B_{ij} P_j + \sum_{i=1}^{N_g} B_{0i} P_i + B_{00} \quad (4)$$

The generated power output ( $P_i$ ) of each generator should vary within its minimum ( $P_{i,\min}$ ) and maximum ( $P_{i,\max}$ ) limits. This inequality constraint is represented as,

$$P_{i,\min} \leq P_i \leq P_{i,\max}, i = 1, 2, \dots, N_g \quad (5)$$

### 2.2 Objective function of CEED

The addition of environmental issues to the solution of the economic dispatch problem adds complexity due to the nonlinear characteristics of the mathematical models used to represent the emissions. The CEED objective function is formulated by combining the fuel cost and emission rate into a single objective using a price penalty factor  $h_m$  to combine the emission rate with the fuel cost, which is formulated as given by the Equation. (6) as follows,

$\alpha_i, \beta_i, \gamma_i, \eta_i$  and  $\delta_i$  are the emission coefficients of the  $i^{\text{th}}$  generating unit.



The main difficulty with such a type of optimization problem is the conflicts between the two features. By introducing a price penalty factor “ $h_e$ ”, this bi-objective problem is converted into a single objective problem. The price penalty factor blends the emission cost with the normal fuel cost and it is the ratio between the maximum fuel cost and maximum emission rate of the corresponding generator. This method gives only the approximate value of ( $h_e$ ) [20]. Hence, a modified price penalty factor ( $h_m$ ) is used to get an exact value for the particular load demand by interpolating the values of  $h_e$  corresponding to the load demand values [3]. This value indicates the exact significance between the two objectives in a CEED problem.

### 2.2.1 Modified price penalty factor method (MPPF)

The steps to determine the Modified Price Penalty Factor ( $h_m$ ), for a particular load demand are given below:

1. Find the ratio between maximum fuel cost and the maximum emission of each generator.

$$h_i = \frac{F_i(P_{imax})}{E_i(P_{imax})} \text{ ($/kg)}, \text{ where } i = 1, 2, \dots, N_g \quad (7)$$

2. Arrange the values of price penalty factor  $h_i$  in ascending order.

3. For a power demand of  $P_D$  (MW), let  $P_m$  be the vector having the maximum generation values of each unit for the respective  $h$  values.  $P_m = [P_{m1}, P_{m2}, \dots, P_{mn}]$ .

Let ‘ $g$ ’ be the vector having  $g = [g_1, g_2, \dots, g_n]$ , where  $g_{i+1} = g_i + P_{mi+1}$

4. Now add the  $P_{mi}$  of each unit until,

**Case1:** If the load demands  $P_D = g_i$ , then  $h_m = h_i$  is the modified price penalty factor (Rs/Ton) for the given load  $P_D$ .

**Case2:** If the load demands  $P_D$  is in between  $g_i$  and  $g_{i+1}$ , then by interpolation,

$$h_m = h_i + \frac{h_{i+1} - h_i}{g_{i+1} - g_i} \times (P_D - g_i) \quad (8)$$

where  $h_m$  is the modified price penalty factor (MPPF) found by the above interpolation formula which is fixed for a load demand. It is clear that the value of ( $h_m$ ) is dependent on the total power demand ( $P_d$ ) and hence it will be different for different power demand. Whatever may be the problem type (ED, EED or CEED), the modified price penalty factor ( $h_m$ ) is same for a particular load demand.

## 3. WATER WAVE OPTIMIZATION ALGORITHM

### 3.1 Inspiration

Majority of the population-based stochastic optimization techniques are inspired by from the nature. Optimization is performed by these techniques are random in nature. The optimization process is usually started by creating a set of random solutions. These initial solutions are then combined, moved, or evolved over a predefined number of steps called iterations or generations to generate new feasible solutions. This is almost the main framework of all population-based algorithms.

In WWOA, the Wave propagation, Breaking and Refraction phase are the three important phases for finding solution to the problem at hand. In wave propagation, the wave is propagated to a random position exactly once in an iteration. If a wave attains a lower sea depth (best fitness), it breaks into solitary waves which are formed in the Breaking phase. Thus breaking is used for the intensive search (exploitation) in search spaces by producing random solitary waves around the current best position. While in the Refraction phase, the algorithm explores the search space for any other best solution and avoids search inactiveness (stagnation). Overall, these three phases plays a vital role in the finding optimal or near optimal solution for the problem. Here, each solution is represented as a Wave, with corresponding height ( $h$ ) and wavelength ( $\lambda$ ).

### 3.2 Mathematical model of WWOA

The mathematical model of propagation, breaking and refraction of WWOA is given below [18].

#### 3.2.1 Propagation

Each original wave from the wave population is allowed to propagate only once in each iteration. Here the propagation operator shifts the original wave  $x$  in each dimension ( $d$ ) to produce a new propagated wave  $x'$ . The new wave is modelled by the following equation:

$$x'(d) = x(d) + R \cdot \lambda \cdot L(d) \quad (9)$$

where,  $R$  is a uniformly distributed random number within the range  $[-1, 1]$  and  $L(d)$  is the length of the  $d^{\text{th}}$  dimension of the search space ( $1 \leq d \leq n$ ). The  $\lambda$  is the wavelength of wave  $x$ , which is updated after each iteration, as follows:

$$\lambda = \lambda' \cdot \alpha_c^{-(f(x) - f_{min} + \epsilon) / (f_{max} - f_{min} + \epsilon)} \quad (10)$$

where,  $\alpha_c$  is the wavelength reduction coefficient, where  $f_{max}$  and  $f_{min}$  are respectively the maximum and minimum fitness values among the current population and  $\epsilon$  is a very small positive number to avoid division-by-zero. The Equation. (10) ensures that the waves with higher fitness value have lower wavelengths and thus propagate with smaller ranges.



### 3.2.2 Breaking

The breaking operation in WWOA is performed only on a wave  $x$  that finds a new best solution (i.e.,  $x$  becomes the new  $x^*$ ) and conduct a local search around  $x^*$  using  $k$  solitary waves to simulate wave breaking using Equation (11).

$$x'(d) = x(d) + N(0,1) \cdot \beta_c \cdot L(d) \quad (11)$$

where,  $\beta_c$  is the breaking coefficient.  $N$  is the Gaussian random number. If none of the solitary waves are better than  $x^*$ ,  $x^*$  is retained; otherwise  $x^*$  is replaced by the fittest one among the solitary waves. Totally  $k$  number of solitary waves  $x'$  are generated at each dimension  $d$  and the value of  $k$  is generated randomly between 1 and  $k_{max}$  (predefined value).

### 3.2.3 Refraction

During wave propagation, if the wave ray is not perpendicular to the isobaths the direction of the wave gets deflected and it is observed that the waves converge in shallow regions and diverge in deeper regions. Refraction operation is performed on the wave whose height reduces to zero. The wave position after refraction is calculated as,

$$x'(d) = N\left(\frac{x^*(d)+x(d)}{2}, \frac{|x^*(d)-x(d)|}{2}\right) \quad (12)$$

Where,  $x^*$  is the best solution found so far and  $N$  is a Gaussian random number. So the new position of the wave is a random number midway between the original and the current best known position. Once the refraction phase is ended, the wave height of  $x'$  is reset to its maximum value  $h_{max}$  and its wavelength is updated by,

$$\lambda' = \lambda \frac{f(x)}{f(x')} \quad (13)$$

### Control parameters

The four main parameters that control the WWOA apart from the population size are: the maximum wave height  $h_{max}$ , the wavelength reduction coefficient  $\alpha_c$ , the breaking coefficient  $\beta_c$ , and the maximum number  $k_{max}$  of breaking directions. In all our test system the parameters used are  $\alpha_c = 1.01$ ,  $\beta_c = 0.001$ , and  $h_{max} = 6$  are used for the study of combined economic emission dispatch problems and the maximum number of iterations is considered as the stopping criteria. The following are the parameter selection range as recommended by Zheng [18].

### Parameter selection range

Wavelength reduction coefficient ( $\alpha_c$ ) = (1.001 to 1.01)  
 Breaking coefficient ( $\beta_c$ ) = (0.001 to 0.01)  
 Maximum wave height ( $h_{max}$ ) = 6  
 Maximum no. of breaking directions ( $k_{max}$ ) = min (12,  $D/2$ ), where  $D$  is the problem dimension.  
 Initial Wavelength ( $\lambda$ ) = 0.5

## 4. IMPLEMENTATION OF WWOA FOR CEED PROBLEM

The following steps are involved in optimizing the CEED problem using WWOA:

1. Initialize the parameters for WWOA like  $\alpha_c$ ,  $\beta_c$ ,  $\lambda$ ,  $h_{max}$ ,  $k_{max}$  and read the system cost, emission and loss coefficients, maximum no. of iterations, power demand and the generator limits.

2. Create initial random wave population (Solutions) ( $x_i$ ) within the generation limits  $lb_j$  and  $ub_j$ . The wave heights ' $h$ ' initialized to ' $h_{max}$ ' and wavelengths ' $\lambda_i$ ' initialised with 0.5 each.

$$x_i^j = lb_j + rand(ub_j - lb_j) \quad , i = 1, 2, 3, \dots, n, j = 1, 2, 3, \dots, d. \quad (14)$$

where,  $lb_j$  and  $ub_j$  are the lower and upper limits of power generation of the  $i^{th}$  generator. ' $n$ ' is the total number of waves ( $popsiz$ ), ' $d$ ' is the total number of variables.

3. Calculate modified price penalty factor ( $h_m$ ) using the MPPF algorithm detailed in Section 2.2.1.

4. Evaluate the initial wave population fitness  $f(x)$  using Equation. (6) and select the Best and Worst wave with minimum ( $f_{min}$ ) and maximum ( $f_{max}$ ) fitness respectively.

5. While  $iter \leq Maxiter$ , perform step 6.

6. For each wave ' $x'$ ' in the population

Perform *propagation* on wave  $x$  only once per iteration to get  $x'$  using Equation. (9). Check the waves  $x'$  in the population that goes beyond the search space limits and bring it within the limits and find the propagated wave's fitness  $f(x')$ .

6.1 If  $f(x')$  is greater than  $f(x)$ ,

Replace the original wave  $x$  with  $x'$  and the fitness  $f(x)$  with  $f(x')$ .

Update height  $h$  of the original wave to  $h_{max}$ .

Go to step 6.2.

Else,

Go to step 6.3

6.2 If  $f(x')$  is greater than the current global fitness  $f(x^*)$ , Perform *Wave Breaking* on wave ( $x'$ ) using Equation. (11)

Replace the best wave  $x^*$  found so far with  $x'$  got after *Wave Breaking*.

Else,

do  $h = h - 1$  and if  $h = 0$ ,

Perform *Wave Refraction* using Equation. (12) and Equation. (13).

Go to step 6.3

6.3  $wave = wave + 1$

7. At the end of each iteration, update original population wavelengths  $\lambda$ , based on Equation. (10) and update the iteration,

$iter = iter + 1$ .

8. If the stopping criteria are satisfied, print the results.

Else, go to step 5





The flow chart of the proposed WWOA applied to the combined economic emission dispatch problem is shown in Figure-1.

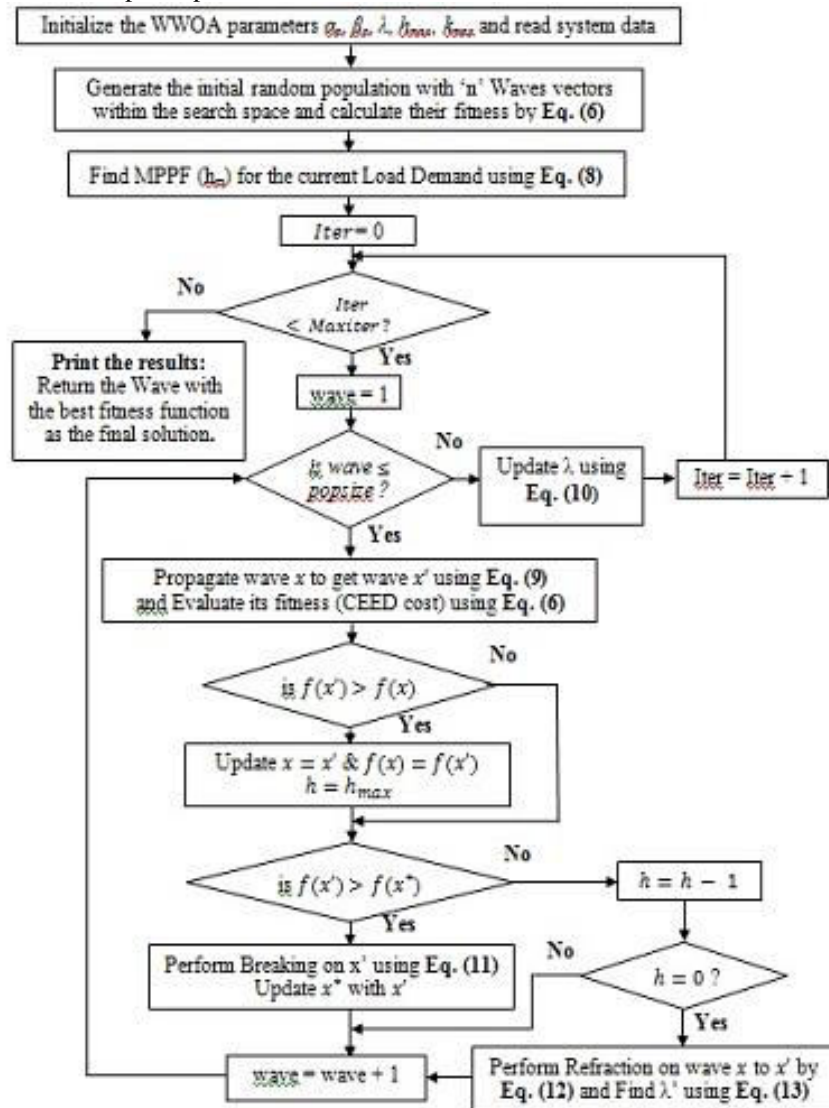


Figure-1. Flow chart of WWOA implementation for CEED problem.

## 5. NUMERICAL SIMULATION RESULTS AND DISCUSSION

The proposed WWOA has been applied to four test systems with different problem scales for investigating the optimization capability: The four test systems considered are 3, 6, 10 unit systems with network losses and 40 units system with valve point effect. In order to demonstrate the feasibility and efficiency performance of the proposed algorithm, it has been compared to various population based optimization techniques like NR [10], GA [28], PSO [28], PDE [21], FPA [21], MODE [22], GSA [23], MABC [24], BSA [25], NSGA-II [26] and FCGA [27].

### 5.1 Test System 1: 3-unit system

The test system consists of three thermal units considering the impact of emissions including transmission loss whose characteristics are given in Table-

1 and the loss coefficients are taken from [21]. The system load is 400 MW. The objective is to optimize the total generation cost that includes the fuel cost as well as the emission in Thermal power plants. A modified price penalty factor by the method of linear interpolation is used to get the exact value of total CEED cost. The optimal results obtained using WWOA are presented in Table-4. From the comparison the WWOA obtained lower fuel cost, emission release than GA, PSO and FPA for the modified price penalty factor of 43.5598 (\$/Ton). The total CEED cost obtained by WWOA was 29557.0438 (\$/h) when compared to 29559.81(FPA), 29559.9 (\$/h) (PSO), 29563.2 (\$/h) (GA). The loss is calculated to be 7.3792 MW which is also less than the other compared methods. The convergence characteristics is as shown in Figure-2 and the comparison of fuel cost, emission rate with other reported algorithms are shown in Figure-3.

**Table-1.** Data for 3-units system ( $P_d = 400$  MW).

Unit	$P_{i,min}$	$P_{i,max}$	Cost coefficients			Emission coefficients		
			$a_i$	$b_i$	$c_i$	$\alpha_i$	$\beta_i$	$\gamma_i$
1	35	210	1243.5311	38.30553	0.03546	40.2669	-0.54551	0.00683
2	130	325	1658.5696	36.32782	0.02111	42.895553	-0.5116	0.00461
3	125	315	1356.6592	38.27041	0.01799	49.89553	-0.5116	0.00461

**Table-2.** Data for 6-units systems ( $P_d = 700$  MW).

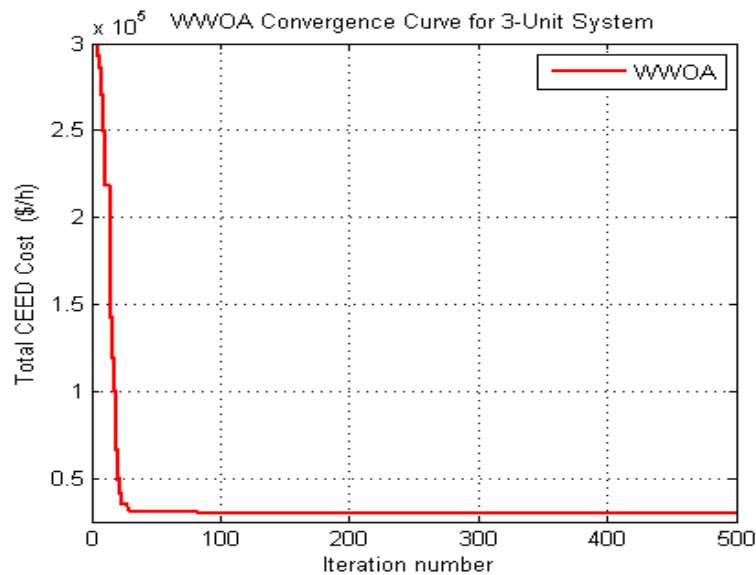
Units	$P_{i,min}$	$P_{i,max}$	$a_i$	$b_i$	$c_i$	$\alpha_i$	$\beta_i$	$\gamma_i$
1	10	125	756.800	38.540	0.1525	13.860	0.3300	0.0042
2	10	150	451.325	46.160	0.1060	13.860	0.3300	0.0042
3	35	225	1050.000	40.400	0.0280	40.267	-0.5455	0.0068
4	35	210	1243.530	38.310	0.0355	40.267	-0.5455	0.0068
5	130	325	1658.570	36.328	0.0211	42.900	-0.5112	0.0046
6	125	315	1356.660	38.270	0.0180	42.900	-0.5112	0.0046

**Table-3.** Data for 10-units system ( $P_d = 2000$  MW).

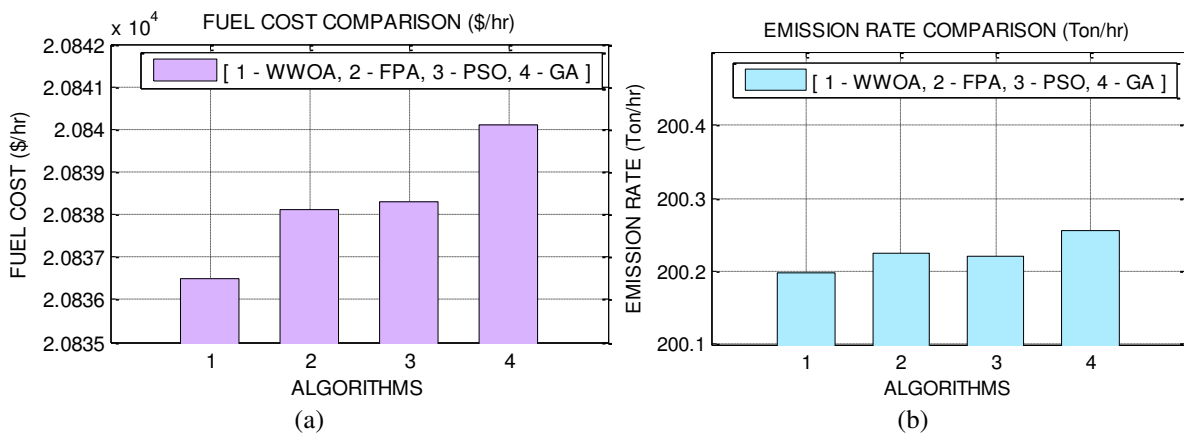
Units	$P_{i,min}$	$P_{i,max}$	$a_i$	$b_i$	$c_i$	$\alpha_i$	$\beta_i$	$\gamma_i$
1	10	55	1000.403	40.5407	0.12951	360.0012	-3.9864	0.04702
2	20	80	950.606	39.5804	0.10908	350.0056	-3.9524	0.04652
3	47	120	900.705	36.5104	0.12511	330.0056	-3.9023	0.04652
4	20	130	800.705	39.5104	0.12511	330.0056	-3.9023	0.04652
5	50	160	756.799	38.5390	0.15247	13.8593	0.3277	0.00420
6	70	240	451.325	46.1592	0.10587	13.8593	0.3277	0.00420
7	60	300	1243.531	38.3055	0.03546	40.2669	-0.5455	0.00680
8	70	340	1049.998	40.3965	0.02803	40.2669	-0.5455	0.00680
9	135	470	1658.569	36.3278	0.02111	42.8995	-0.5112	0.00460
10	150	470	1356.659	38.2704	0.01799	42.8995	-0.5112	0.00460

**Table-4.** Comparison of results for three-units system ( $P_d=400$  MW).

Unit (MW)	FPA [21]	PSO [28]	GA [28]	WWOA (Proposed)
P1	102.4468	102.612	102.617	102.3839
P2	153.8341	153.809	153.825	153.7743
P3	151.1321	150.991	151.011	151.221
$\sum P_i$	407.4126	407.4117	407.4132	407.3792
$P_{loss}$	7.4126	7.4117	7.41324	7.3792
Fuel Cost (\$/h)	20838.1	20838.3	20840.1	20836.4864
Emission (Ton/h)	200.2238	200.221	200.256	200.1972



**Figure-2.** Convergence characteristics of WWOA for 3 units system.



**Figure-3.** (a) Fuel cost, (b) Emission rate comparison of WWOA with other algorithms for a three-units system.

## 5.2 Test System 2: 6-unit system

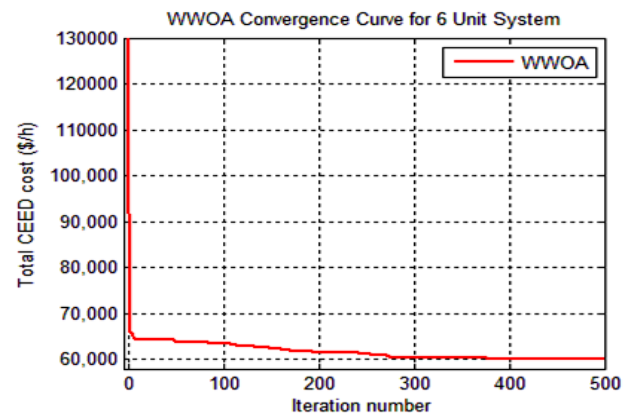
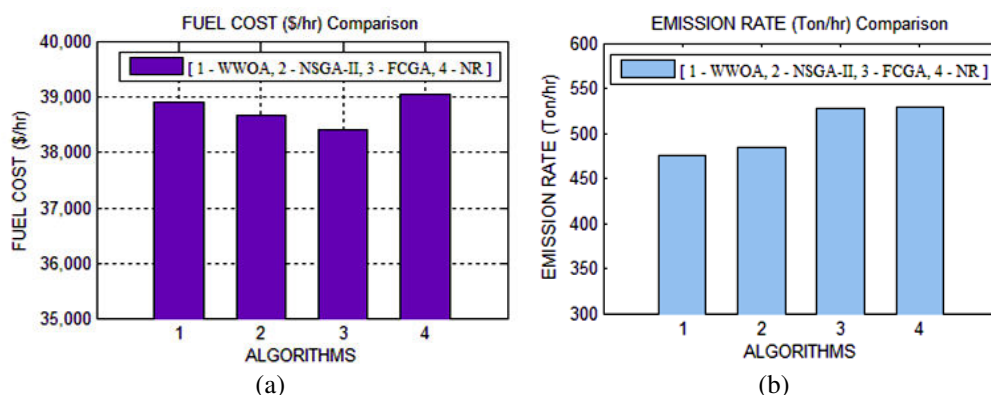
The test system has six-thermal units. The cost coefficients, emission coefficients, minimum and maximum generation limits of the units are given in Table-2. The system load demand is 700 MW. The B loss coefficients used to calculate the transmission loss is taken

from [22]. The test results obtained by solving the 6-unit test system are given in Table-5. Figure-4 shows the convergence characteristics of the system. The fuel cost and emission rate are compared with other reported algorithms are shown in Figure-5.

**Table-5.** Comparison of results for six-units system (Pd=700 MW).

Unit (MW)	NSGA-II [26]	FCGA [27]	NR [10]	WWOA (Proposed)
P1	86.286	80.16	85.924	91.2235
P2	60.288	53.71	60.963	64.7522
P3	73.064	40.93	53.909	84.5232
P4	109.036	116.23	107.124	103.2023
P5	223.448	251.20	250.503	211.4939
P6	184.111	190.62	176.504	182.9675
$\sum P_{Gi}$	736.234	732.85	734.927	738.1625
$P_{loss}$	36.234	32.85	34.927	38.1625
Fuel Cost (\$/h)	38671.813	38408.82	39070.74	38912
Emission (Ton/h)	484.931	527.46	528.447	475.6253

Modified price penalty factor of 44.3219 (\$/Ton) for the load demand of 700 MW is applied to get the exact value of total CEED cost. The proposed algorithm without any roundup values of the computed output power produced the best compromise CEED solution as 59992.6169 (\$/h). Table-5 best describes the fuel cost and emission rate comparison between the approximate and exact modified price penalty factor method. The loss coefficients used in references cited in the comparison table are four decimal places rounded up from the original loss coefficients.

**Figure-4.** Convergence characteristics of WWOA for six-unit system.**Figure-5.** (a) Fuel cost (\$/hr), (b) Emission rate (Ton/hr) comparison of WWOA with other algorithms for a six-unit system.

### 5.3 Test system 3: 10 generator units

A ten thermal unit test system with network transmission losses is considered for solving the CEED problem of medium size power system for demonstrating the efficiency of the proposed method. The input data for this system is provided in Table-3 and the B-loss coefficients of the above system are taken from [21]. Total

of 50 test runs were conducted to ensure global optimum solution, by varying the total number of objective function evaluations. The total load demand of the system is 2000 MW. The optimum generations of individual units, fuel cost and emission rate by the proposed algorithm and their comparison with other methodologies are reported in Table-6. The price penalty factor is calculated from the



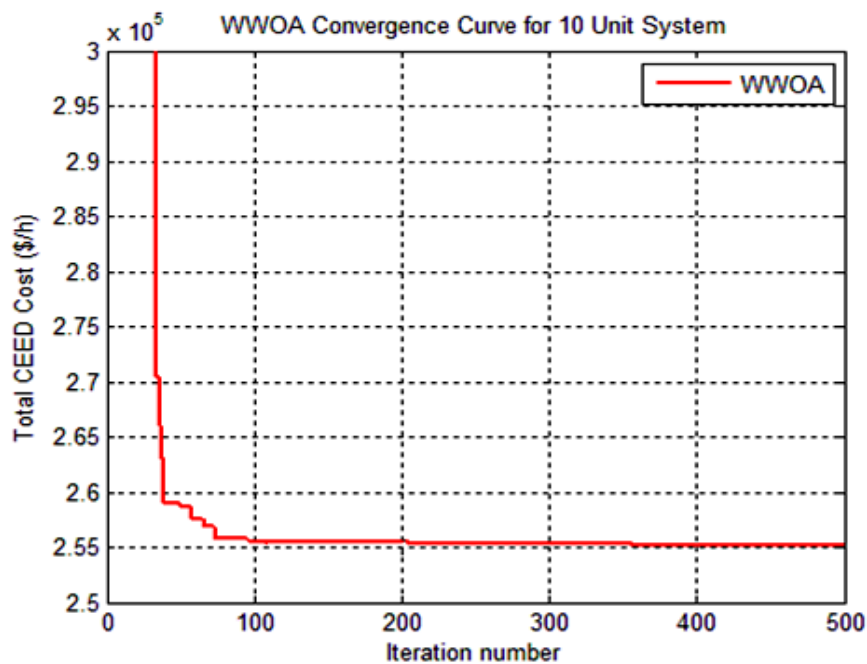


values obtained by the 'Max-Max' penalty method. The modified price penalty factor ( $h_m$ ) is an improvement of the above mentioned price penalty factor which is found by linear interpolation for the above mentioned demand is 34.786 (\$/Ton). The total CEED cost obtained is 255961

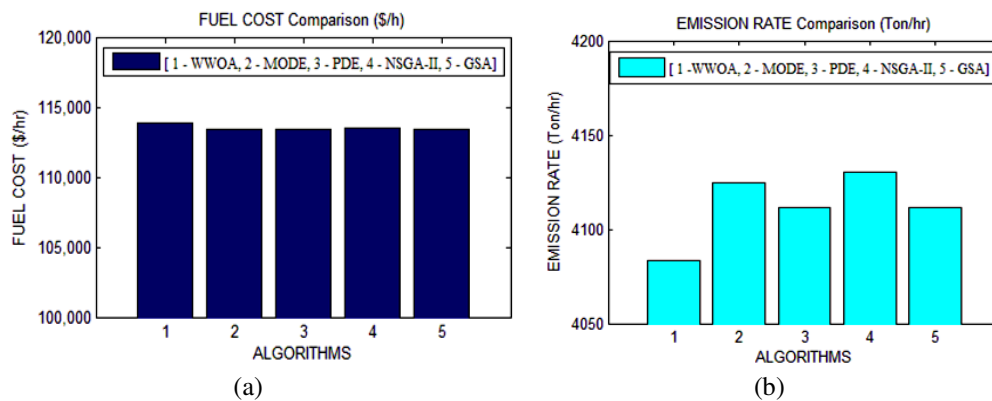
(\$/h). The convergence characteristic plot for the ten units system is shown in Figure-6. The fuel cost and emission rate are compared with other reported algorithms are shown in Figure-7.

**Table-6.** Comparison of results for ten-unit system ( $P_d = 2000$  MW).

Unit (MW)	MODE [21]	PDE [21]	NSGA-II [21]	GSA [23]	WWOA (Proposed)
P1	54.9487	54.9853	51.9515	54.9992	55
P2	74.5821	79.3803	67.2584	79.9586	74.62347
P3	79.4294	83.9842	73.6879	79.4341	81.65139
P4	80.6875	86.5942	91.3554	85	85.119
P5	136.8551	144.4386	134.0522	142.1063	160
P6	172.6393	165.7756	174.9504	166.5670	174.6812
P7	283.8233	283.2122	289.4350	292.8749	267.785
P8	316.3407	312.7709	314.0556	313.2387	301.0342
P9	448.5923	440.1135	455.6978	441.1775	426.0056
P10	436.4287	431.8054	431.8054	428.6306	458.07
$\sum P_{Gi}$	2084.327	2083.9	2084.25	2083.9869	2083.9698
$P_{loss}$	84.33	83.9	84.25	83.9869	83.9698
Fuel Cost (\$/h)	113484	113510	113539	113490	113914
Emission (Ton/h)	4124.9	4111.4	4130.2	4111.4	4083.4603



**Figure-6.** Convergence characteristics of WWOA for ten-units system.



**Figure-7.** (a) Fuel cost (\$/hr), (b) Emission rate (Ton/hr) comparison of WWOA with other algorithms for a ten-units system

#### 5.4 Test System 4: 40 generator units

A system with 40 thermal units including the effect of valve-point loading and  $\text{NO}_x$  emission has been considered. The unit cost and emission coefficients, operating limits for the above system is taken from [23]. The simulation results of the test system for a load demand of 10500 MW after 50 trials are presented in Table-7 and Table 8. The Table-8 shows that both the fuel cost and emission rate are very much reduced than the other

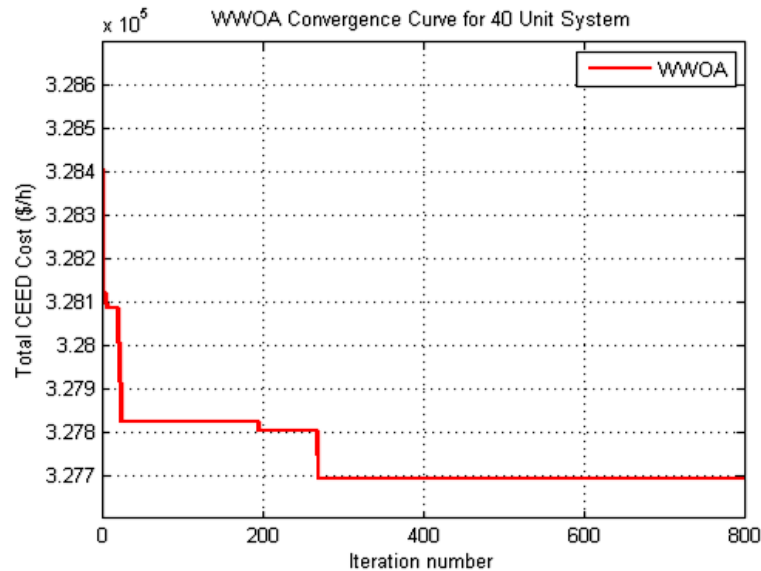
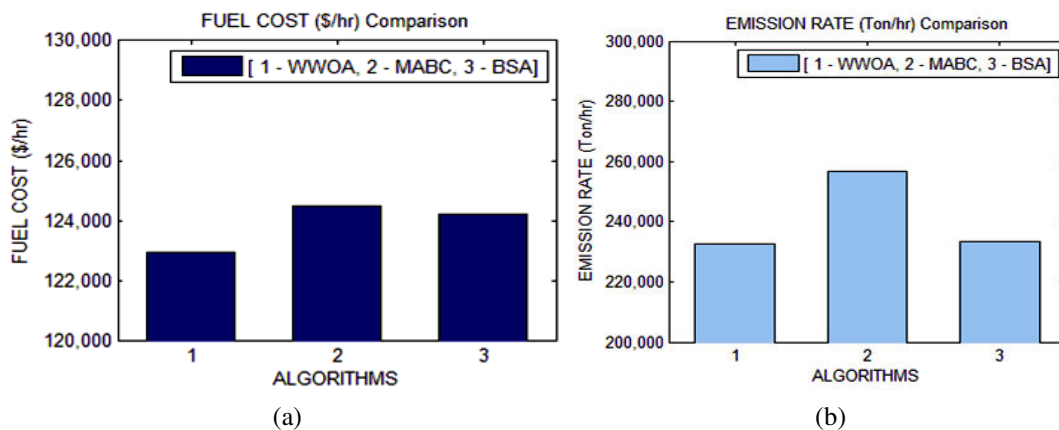
presented techniques. The modified price penalty factor for the 10500 MW load demand is 0.88011 (\$/Ton) giving the exact best total CEED cost as 327634.7954 (\$/h). The convergence characteristic is shown in Figure-8 and the cost comparisons are illustrated in Figure-9. From the results it is seen that the WWOA provides a high quality solution better than those obtained by MABC and BSA with fast convergence and robustness for solving complex large combined economic emission dispatch problems.

**Table-7.** Comparison of generator values for forty-unit system including v. p. effect (Pd =10500 MW).

Unit	BSA [25]	MABC/D/Cat [24]	WWOA (Proposed)	Unit	BSA [25]	MABC/D/Cat [24]	WWOA (Proposed)
P1	111.0281	110.7998	111.5281	P21	433.4452	514.1472	433.9452
P2	110.9843	110.7998	111.4843	P22	433.5131	514.1455	433.1036
P3	97.5185	97.3999	97.0185	P23	433.5401	514.5237	434.0401
P4	179.6235	174.5504	179.1235	P24	521.7719	514.5386	521.2719
P5	87.9014	87.7999	88.4014	P25	433.6479	433.5195	433.6515
P6	139.9362	105.3999	139.4362	P26	433.6179	433.5196	434.1179
P7	299.9973	259.5996	299.8521	P27	10.0987	10	10.5987
P8	284.8178	284.5996	284.3178	P28	10.0804	10	10.0000
P9	284.6970	284.5996	284.2214	P29	10.0058	10	10.2420
P10	130.0010	130	130.1773	P30	88.0451	87.8042	88.5451
P11	243.5997	318.2129	244.0944	P31	189.9999	159.733	189.4999
P12	243.5861	243.5996	244.0861	P32	189.9849	159.7331	189.4848
P13	394.3575	394.2793	394.8575	P33	189.9779	159.733	190.0000
P14	394.0843	394.2793	394.5843	P34	199.9723	200	200.0000
P15	394.2627	394.2793	394.7628	P35	199.9822	200	199.4822
P16	394.3353	394.2793	394.0027	P36	199.9999	200	199.4999
P17	489.2358	399.5195	488.7358	P37	89.2085	89.1141	89.7085
P18	489.2747	399.5195	488.7747	P38	109.9998	89.1141	109.4997
P19	511.2517	506.1985	510.7517	P39	109.9719	89.1141	109.4719
P20	421.4733	506.1985	421.9733	P40	511.1699	506.1951	510.6699

**Table-8.** Cost comparison results for forty-unit system including v. p. effect ( $P_d = 10500$  MW).

Unit	BSA [25]	MABC/D/Cat [24]	WWOA (Proposed)
Fuel Cost (\$/h)	124187.8724	124490.903	122947.7452
Emission (Ton/h)	233544.8777	256560.267	232569.5649

**Figure-8.** Convergence characteristics of WWOA for forty-unit system.**Figure-9.** (a) Fuel cost, (b) Emission rate comparison of WWOA with other algorithms for a forty-units system.

## 6. CONCLUSIONS

In this paper, Water Wave Optimization Algorithm has been presented to solve the combined economic emission dispatch (CEED) problem on four different test systems. The problem has been subjected to various constraints. The modified price penalty factor employing linear interpolation corresponding to the load demand is used to obtain the exact best solution. The analysis of the numerical simulation results emphasizes the performance of the proposed algorithm. The convergence property and the computational efficiency of WWOA are also demonstrated. From the simulation results it is observed that the performance of WWOA is

excellent for small, medium and large scale systems and has the ability to converge to a better quality solution in comparison with other presented techniques. Further, it takes lesser number of iterations and time. Therefore WWOA is a reliable and promising technique to solve the CEED problems.

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