Hypothesis Testing

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we want to test if a coin is fair.

Null Hypothesis H_0: The coin is fair

Alternative Hypothesis H_1: The coin is biased.

we decide to flip the coin 100 times.

Let X be the number of heads in these 100 flips.

Then $X \sim \text{Binomial}(n, p)$ where $p =$ probability of heads.

$$P(X = i) = \binom{100}{i} p^i (1-p)^{n-i}$$

we set up a rejection region R.

- If $X \in R$ then we reject H_0.
- If $X \notin R$ we retain H_0.

Here are the outcomes.

	H_0 True	H_0 False.
retain H_0	True Negative $P = 1-\alpha$	Type-2 error $P = \beta$.
reject H_0	Type 1 error $P = \alpha$	True Positive $P = 1-\beta$.

we want a 1% chance of a type 1 error.

(want $\alpha = .01$).

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If X is indeed $\text{Binomial}(100, \frac{1}{2})$ then on the other hand.

$P(X \leq 36 \text{ or } X \geq 64) = .0066$

$P(X \leq 37 \text{ or } X \geq 63) = .012 > .01$ So we set

R to be $R = \{X : X \leq 36 \text{ or } X \geq 64\}$.

What about type II errors? (Probability of β). which is

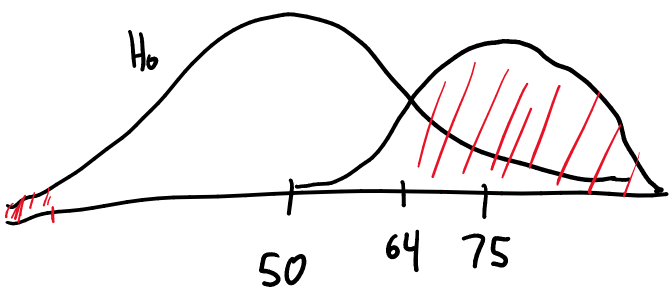
we usually phrase this in terms of $1-\beta$

the <u>power</u> of the test.

Suppose our coin flips heads 75% of the time.

Then $H_1: X \sim \text{Binomial}(100, .75)$. If we are testing against

$H_0: X \sim \text{Binomial}(100, .5)$ the two distributions look like.



If H_1 was true $P(X \notin R) = P(36 < X < 64)$

$$= \sum_{i=37}^{63} \binom{100}{i} (3/4)^i \left(\frac{1}{4}\right)^{100-i} = .005 \text{ or } .5\%$$

On the other hand if $X \sim \text{Binomial}(100, .6)$ then

$P(X \notin R) = 76\%$.

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