

Grothendieck's Galois Theory

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1 Section 1

Definition 1.1. In a category \mathcal{C} an arrow $f: X \rightarrow Y$ is a **strict epimorphism** if it is the joint coequalizer of all the arrows it equalizes. This means that any arrow $g: X \rightarrow Z$ such that $g \circ x = g \circ y$ for any $x, y: C \rightarrow X$ such that $f \circ x = f \circ y$ there exists a unique arrow $h: Y \rightarrow Z$ such that $h \circ f = g$. Refer to Figure 1.1.

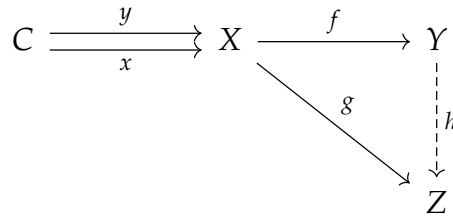


Figure 1.1

Remark 1.2. Strict epimorphisms are coequalizers, thus epimorphisms (as the name implies).

Remark 1.3. If an arrow is both a strict epimorphism and a monomorphism then it is an epimorphism.

Definition 1.4. Let H be a group, A an object of \mathcal{C} and $G = \text{Aut}(A)$ the group of automorphisms of A in \mathcal{C} i.e. the group whose underlying set is the set of isomorphisms of type $A \rightarrow A$ of \mathcal{C} and whose operation is composition in \mathcal{C} . An **action** of H on A is a group homomorphism $H \rightarrow G$.