# Grothendieck's Galois Theory

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October 5, 2020

## 1 Section 1

**Definition 1.1.** In a category $\mathscr{C}$ an arrow $f\colon X \to Y$ is a **strict epimorphism** if it is the joint coequalizer of all the arrows it equalizes. This means that any arrow $g\colon X \to Z$ such that $g \circ x = g \circ y$ for any $x, y\colon C \to X$ such that $f \circ x = f \circ y$ there exists a unique arrow $h\colon Y \to Z$ such that $h \circ f = g$. Refer to Figure 1.1.

$$\begin{tikzcd}
C \arrow[r, shift left, "y"] \arrow[r, shift right, "x"'] & X \arrow[r, "f"] \arrow[dr, "g"'] & Y \arrow[d, dashed, "h"] \\
& & Z
\end{tikzcd}$$

Figure 1.1

**Remark 1.2.** Strict epimorphisms are coequalizers, thus epimorphisms (as the name implies).

**Remark 1.3.** If an arrow is both a stric epimorphism and a monomorphism then it is an epimorphism.

**Definition 1.4.** Let $H$ be a group, $A$ an object of $\mathscr{C}$ and $G = \mathrm{Aut}(A)$ the group of automorphisms of $A$ in $\mathscr{C}$ i.e. the group whose underlying set is the set of isomorphisms of type $A \to A$ of $\mathscr{C}$ and whose operation is composition in $\mathscr{C}$. An **action** of $H$ on $A$ is a group homomorphism $H \to G$.