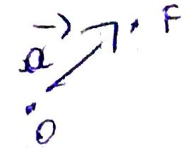
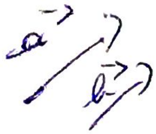
Vettori



Origine → Punto di applicazione

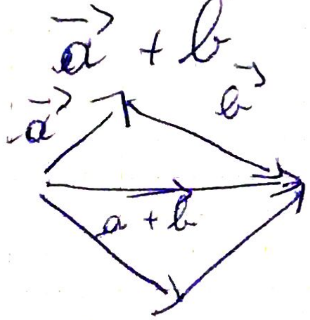
Vettore = Modulo, Direzione, Verso

Modulo$(\vec a)$ > Modulo$(\vec b)$
Direzione$(\vec a)$ = Direzione$(\vec b)$
Verso$(\vec a)$ ≠ Verso$(\vec b)$

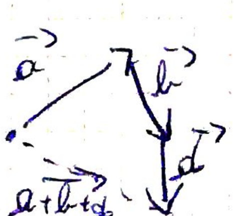


$\vec a = \vec b$ (Punto di applicazione diverso)

$\vec a + \vec b$ Metodo dei parallelogrammi



Metodo Punta-Coda

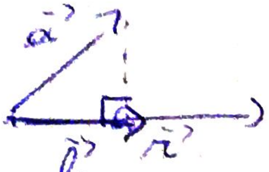


$\vec b = K \cdot \vec a$ Modulo = K · Modulo

$-\vec{b} = k\vec{b}$ $\quad k = -1$ $\quad$ Modulo uguale
Verso opposto

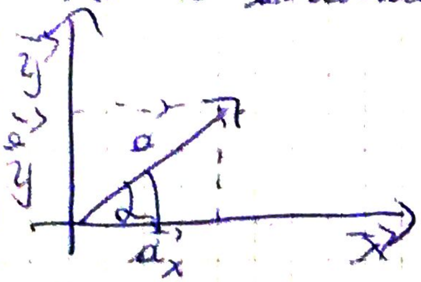
$\vec{a} - \vec{b}$

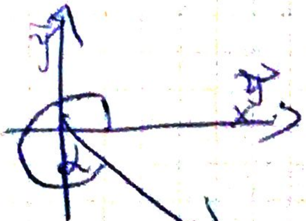
Componente di $\vec{a}$ su $\vec{r}$
La sua proiezione



Assi cartesiani

Scalari e Vettori invarianti
Componenti $\neq$ Scalari





$a_x = a \cdot \cos(\alpha)$

$a_y = a \cdot \sin(\alpha)$

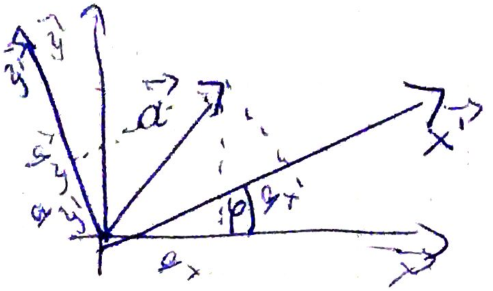
$a = \sqrt{a_x^2 + a_y^2}$

$\tan(\alpha) = \frac{a_y}{a_x}$

Direzione
Verso
$\hat{i}, \hat{j}, \hat{k}$
Vettore Unitario
Modulo 1

$\vec{a} = a_x \hat{i} + a_y \hat{j}$

$\vec{a} \neq a_x + a_y$



$a_x \neq a_{x'}$

$a_y \neq a_{y'}$

$\sqrt{a_x^2 + a_y^2} = \sqrt{a_{x'}^2 + a_{y'}^2}$

$\vec{a} + \vec{b}$

$\vec{a} = a_x \hat{i} + a_y \hat{j}$

$\vec{b} = a_x \hat{i} + a_y \vec{j}$

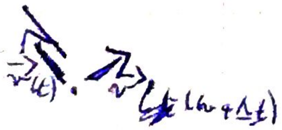
$\vec{a+b} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j}$

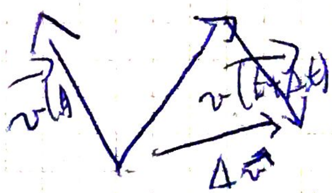
$\vec{a} - \vec{b} = (a_x - b_x)\hat{i} + (a_y - b_y)\hat{j}$

Derivata di un versore

$\hat{v}(t)$

$\hat{v}(t+\Delta t)$

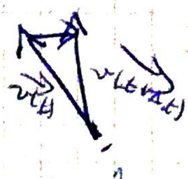




$\Delta\hat{u} = \hat{v}(t+\Delta t) - \vec{v}(t)$

$\frac{d\hat{u}}{dt} \not\approx r \approx d\alpha$

$$\lim_{\Delta t \to 0} \frac{\Delta\hat{v}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\hat{v}(t+\Delta t) - \hat{v}(t)}{\Delta t}$$



$\approx R\,d\alpha$

$r = 1$

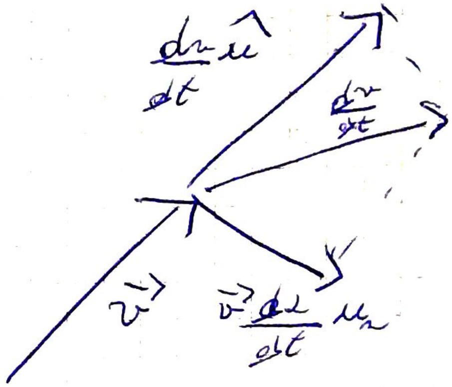
Direzione Perpendicolare $\frac{d\hat{u}}{dt} = d\alpha$

Direzione Normale

Modulo $\frac{d\alpha}{dt}$

Derivata di un versore non è un versore

Derivata di un vettore



$\vec{v} = v\hat{u}$

$$\frac{d\vec{v}}{dt} = \frac{d}{dt}(v\hat{u})$$

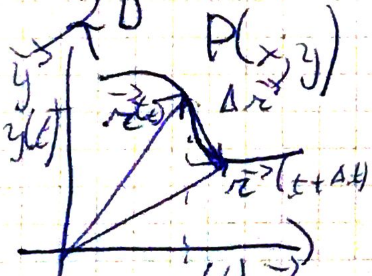
$$= \frac{dv}{dt}\hat{u} + v\frac{d\hat{u}}{dt}$$

$$= \frac{dv}{dt}\hat{u} + v\frac{d\alpha}{dt}\hat{u}_n$$

Modulo ≃ Corda ≃ Arco

$$\Delta\hat{u} \simeq \Delta l = R\,\Delta\alpha$$

$$\lim_{\Delta t \to 0}\frac{\Delta\hat{u}}{\Delta t} = \lim_{\Delta t \to 0} R\frac{\Delta\alpha}{\Delta t} = \lim_{\Delta t \to 0}\frac{\Delta\alpha}{\Delta t}$$



$\vec{r}(t)$ Vettore posizione

1D $x(t)$ $\mathbb{R} \to \mathbb{R}$

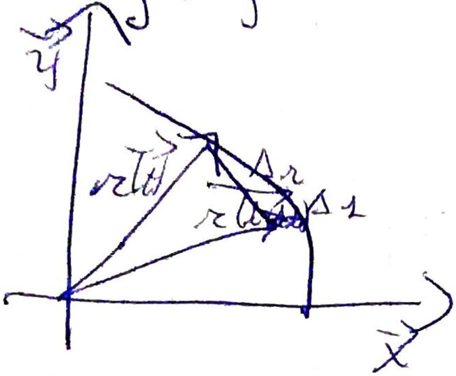
2D $\vec{r}(t)$ $\mathbb{R} \to (\mathbb{R}\times\mathbb{R})$

$$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$$

$$\Delta\vec{r}(t) = \vec{r}(t+\Delta t) - \vec{r}(t)$$

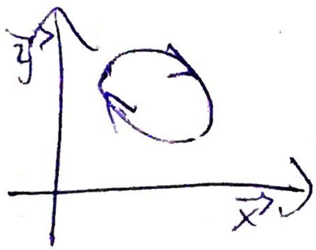
$$\Delta x = x(t+\Delta t) - x(t)$$

$$\Delta y = y(t+\Delta t) - y(t)$$

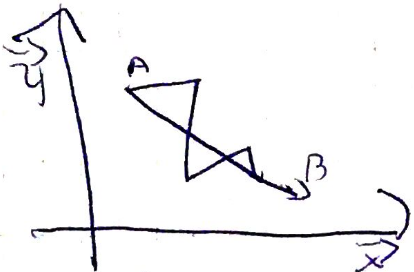


Δr Spostamento

Δs Spazio percorso



$$\Delta r = 0$$



$$\Delta r = \sum_{i=1}^{N} \Delta r_i$$

$$v_x = \frac{dx}{dt}$$

$$v_y = \frac{dy}{dt}$$

Velocità

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$a_x = \frac{dv_x}{dt}$$

$$a_y = \frac{dv_y}{dt}$$

Accellerazione

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$\vec{r}(t)$ $\quad x(t) \quad y(t)$

$\vec{v}(t)$ $\quad v_x(t) \quad v_y(t)$

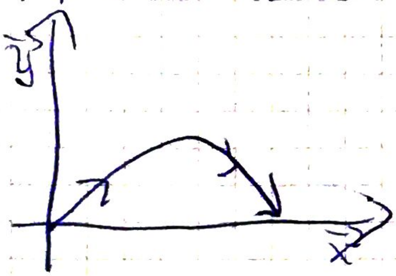
$\vec{a}(t)$ $\quad a_x(t) \quad a_y(t)$

Equazioni Vettoriali

$$\int_{x_0}^{x} dx = \int_{t_0}^{t} v_x \, dt \qquad dv_x = a_x \, dt$$

$$\int_{y_0}^{y} dy = \int_{t_0}^{t} v_y \, dt \qquad dv_y = a_y \, dt$$

Moto del proiettile



$x(t)$

$y(t)$

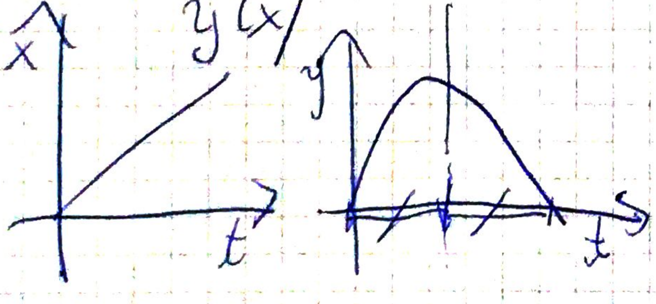
$a_x = 0 \quad v_x = v_0 \quad x(t) = x_0 v_0 t$

$a_y = -g \quad v_y = v_0 - g t \quad y(t) = y_0 - g t + \frac{1}{2} g t^2$

$t = \frac{x}{v_x} \quad y(t) = \frac{v_{y0} x}{v_{x0}} \frac{1}{2} g \frac{x^2}{v_x^2}$

$$y(t) = \tan(\alpha) x - \frac{1}{2} g \frac{x^2}{v_0^2 \cos^2(\alpha)}$$

$y(x)$



$y_{max} = y(t) = y_0 t - \frac{1}{2} g t^2$

Gittata

$$y(t) = y_0 + v_0 t - \frac{1}{2} g t^2$$

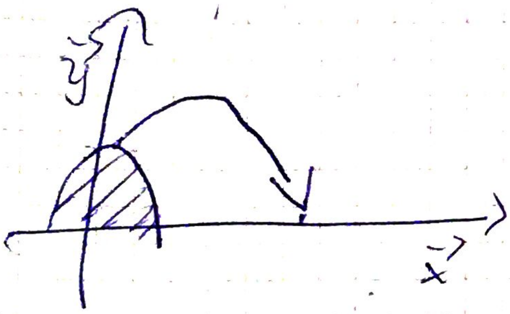
$$x(t_v) = v_x t_v = \frac{v_0 \cos(\alpha) \cdot v_0 \sin(\alpha)}{g}$$

$$= \frac{2 v_0^2 \sin(\alpha) \cos(\alpha)}{g}$$

$$= \frac{v_0^2 \sin(2\alpha)}{g}$$

$x_0 = 0$

$y_0 = 0$



$h = y_0$

$v_0 = v_{0x}$

gittata

$t_v = ?$

v impatto $= ?$