Analisi Matematica I ###03/12/2020###

$$f(x) = \sqrt{\arctan\left(\frac{1}{x-2}\right) - \arctan(x) - \arcsin\left(\left(\frac{1}{4}\right)^x - 1\right)}$$

$$\begin{cases} \arctan\left(\frac{1}{x-2}\right) - \arctan(x) \geq 0 \\ x - 2 \neq 0 \qquad x \neq 0 \\ -1 < \left(\frac{1}{4}\right)^x - 1 \leq 1 \end{cases}$$

$\arctan\left(\frac{1}{x-2}\right) \geq \arctan(x)$

$\frac{1}{x-2} \geq x$

$1 \geq x^2 - 2x$

$x^2 - 2x - 1 \leq 0$

$\frac{\Delta}{4} = 1 + 1 = 2$

$x = \frac{2 \pm \sqrt{2}}{1}$

$\frac{2-\sqrt{2}}{1} \leq x \leq \frac{2+\sqrt{2}}{1}$

$-1 \leq \left(\frac{1}{4}\right)^x - 1 \qquad 0 \leq \left(\frac{1}{4}\right)^x \quad \forall x \in D(f)$

$\left(\frac{1}{4}\right)^x - 1 \leq 1 \qquad \left(\frac{1}{4}\right)^x \leq 2$

$0 < \left(\frac{1}{4}\right)^x \leq 2$

$1 \leq \ln\left(\left(\frac{1}{4}\right)^x\right) \leq \ln(2)$

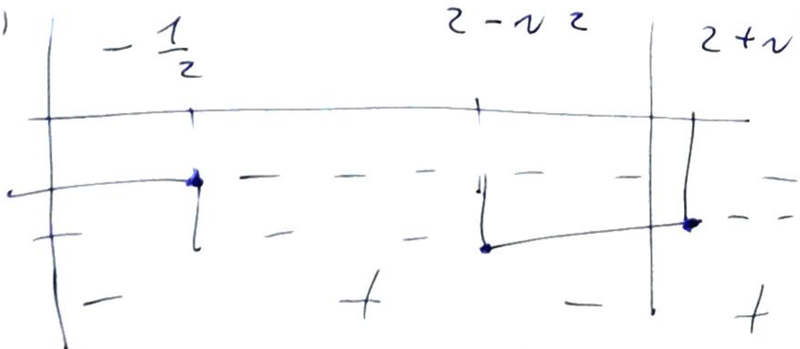
$1 < x\ln\left(\frac{1}{4}\right) \leq \ln(2)$

$x \ln\left(\frac{1}{4}\right) \leq \ln(2)$

$x \leq \frac{\ln(2)}{\ln\left(\frac{1}{4}\right)} - 1 \qquad \ln(x): \log_2(x)$

$x \leq \frac{1}{-2}$

$x \leq -\frac{1}{2}$



$\cancel{\left[\frac{1}{2}; 2-\sqrt{2}\right]}$

$\left]\frac{1}{2}; 2-\sqrt{2}\right] \cup \left[2+\sqrt{2}; +\infty\right[$

$\cap \left[-1; 1\right]$

$\cancel{\left[-\frac{1}{2}; 2-\sqrt{2}\right]}$

Algebra di o

$o(x^n)$ è un infinitesimo di ordine superiore rispetto a x^n

$$\lim_{x \to +\infty} \left(\frac{1}{5}\right)^x = 0$$

$\left(\frac{1}{5}\right)^x$ è un infinitesimo $x \to +\infty$

$$\lim_{x \to 1} \left(\frac{1}{5}\right)^x = \frac{1}{5}$$

$$\lim_{x \to -\infty} \left(\frac{1}{5}\right)^x = +\infty$$

$\left(\frac{1}{5}\right)^x$ è un infinito per $x \to -\infty$

⌜Ordine superiore = "più rapido"⌟

$$\lim_{x \to 0} \frac{o(x^n)}{x^n} = \frac{0}{0} = 0$$

$$\lim_{x \to 0} \frac{2x^2+5x^3+6x^4}{3x^2+7x^4-9x^5} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{x^2(2+5x+6x^2)}{x^2(3+7x-9x^2)} = \frac{2}{3}$$

$$\lim_{x \to +\infty} \frac{x^4(2x^{-2}+5x^{-1}+6)}{x^5(3x^{-3}+7x^{-1}-9)} = -\frac{6}{9} = -2 \cdot \frac{1}{3}$$

$$\lim_{x \to 0} \frac{e^x - \operatorname{sen}(x) - \cos(x)}{e^{x^2} - e^{x^3}}$$

Taylor

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$$

$$\operatorname{sen}(x) = x - \frac{x^3}{6}$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$$

$$e^{x^2} = 1 + x^2 + \frac{x^4}{2}$$

$$e^{x^3} = 1 + x^3 + \frac{x^6}{2}$$

$$\lim_{x \to 0} \frac{\cancel{1}+\cancel{x}+\frac{x^2}{2}+\frac{x^3}{6} - \cancel{x} + \frac{x^3}{6} - \cancel{1} + \frac{x^2}{2} + \frac{x^4}{24}}{1 + x^2 + \frac{x^4}{2} - 1 - x^3 - \frac{x^6}{2}}$$

$$\lim_{x \to 0} \frac{x^2 + \frac{x^3}{3} - \frac{x^4}{24}}{x^2 - x^3 + \frac{x^4}{2} - \frac{x^6}{2}}$$

$$\lim_{x \to 0} \frac{x^2}{x^2} = \frac{1}{1} = 1$$

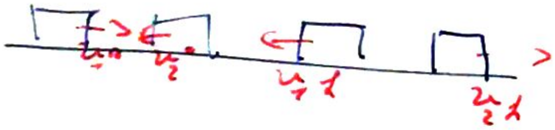
$$\lim_{x \to 0} \frac{x^2}{x^2} = 1$$

$m_1 = 200g \quad m_2 = 180g \quad p_0 = p_F$

$v_1^0 = 4\,m/s \quad v_2^0 = -3\,m/s \quad p_1^0 + p_2^0 = p_1^f + p_2^f$

$v_1^f = -2{,}63\,m/s \quad v_2^f$

$m_1 v_1^0 + m_2 v_2^0 = m_1 v_1^f + m_2 v_2^f$



$m_2 v_2^f = m_1 v_1^0 + m_2 v_2^0 - m_1 v_1^f$

$$v_2^f = \frac{m_1 v_1^0 + m_2 v_2^0 - m_1 v_1^f}{m_2}$$

$$v_2^f = \frac{m_1}{m_2}(v_1^0 - v_1^f) + v_2^0$$

$$v_2^f = -\Delta v_1 \frac{m_1}{m_2} + v_2^0 = -10{,}37\,m/s \quad 4{,}3\cancel{7}\,m/s$$

$p_0 = m_1 v_1^0 + m_2 v_2^0$

$= 4 \cdot 0{,}2 + (-3) \cdot 0{,}18\,m/s$

$= 0{,}26\,m/s$

$p_F = m_1 v_1^F + m_2 v_2^F$

$= -2{,}63 \cdot 0{,}2 + (-10{,}37) \cdot 0{,}18$

$= -2{,}39\,m/s$

$= 4{,}36\,m/s$

$= 0{,}26\,m/s$

$m_1 v_1^0 + m_2 v_2^0 = m_1 v_1^F + m_2 v_2^F$

$$v_2^F m_2 = \frac{m_1 v_1^0 + v_2^0 m_2 - m_1 v_1^F}{1}$$

$$v_2^F = \frac{m_1 v_1^0 + m_2 v_2^0 - m_1 v_1^F}{m_2}$$

$$v_2^F = \frac{m_1}{m_2} v_1^0 + v_2^0 - \frac{m_1}{m_2} v_1^F$$

$$= \frac{m_1}{m_2}(v_1^0 - v_1^F) + v_2^0$$

$m_1 = m_2$

$v_1^0 = 5\,m/s \quad v_2^0 = 0\,m/s$

$$p_0 = p_F$$

$$\cancel{m} v_1^0 + \cancel{m} v_2^0 = \cancel{m} v_1^F + \cancel{m} v_2^F$$

$$v_1^0 + v_2^0 = v_1^F + v_2^F$$

$$v_1^0 = v_1^F + v_2^F$$

$$K_0 = K_F$$

$$K_1^0 + K_2^0 = K_1^F + K_2^F$$

$$\cancel{\tfrac{1}{2} m}\, v_1^{0\,2} = \cancel{\tfrac{1}{2} m}\, v_1^{F\,2} + \cancel{\tfrac{1}{2} m}\, v_2^{F\,2}$$

$$v_1^{0\,2} = v_1^{F\,2} + v_2^{F\,2}$$

$$\begin{cases} v_1^F = v_1^0 - v_2^F \\ v_1^{0\,2} = (v_1^0 - v_2^F)^2 + v_2^F \end{cases}$$

$$v_1^{0\,2} = v_1^{0\,2} + v_2^{F\,2} - 2 v_1^0 v_2^F + v_2^F$$

$$\cancel{v_1^{0\,2}} = \cancel{v_1^{0\,2}} + 2 v_2^F - 2 v_1^0 v_2^F$$

$$-v_1^{0\,2} + v_1^0 = 2 v_2^F - 2 v_1^0 v_2^F$$

$$v_1^0 - v_1^{0\,2} = 2 v_2^F (1 - 2 v_1^0)$$

$$\frac{v_1 - v_1^{0\,2}}{2(1 - 2 v_1^0)} = v_2^F$$

$$v_2^F = \frac{v_1^{0\,2} \cancel{(v_1^0)^2}}{2 - 4 v_1^0}$$

$$2 v_2^F (-2 v_1^0 + 1) = 0$$

$$2 v_2^F = 0$$

$$-2 v_1^0 + 1 = 0$$

$$v_1^0 = +\frac{1}{2}$$