Analisi Matematica I 04/12/2020

$$\lim_{x\to 0} \frac{\operatorname{sen}\left(\sqrt[3]{x^2}\right) - \sqrt[3]{x^2} - \ln\left(\cos(x)\right)}{x \operatorname{sen}(x)} = \frac{0}{0}$$

$$\sqrt[3]{x^2} = x^{\frac{2}{3}}$$

$$\operatorname{sen}(x) = x - \frac{x^3}{6} + \frac{x^5}{120}$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3}$$

$$\lim_{x\to 0} \frac{\left(x^{\frac{3}{2}} - \frac{x^{2}}{3}\right) - x^{\frac{3}{2}} - \left(-\frac{x^2}{2} - \frac{\left(-\frac{x^2}{2}\right)^2}{2}\right)}{x\cdot\left(x - \frac{x^3}{6}\right)}$$

$$\lim_{x\to 0} \frac{\cancel{x^{\frac{3}{2}}} - \frac{x^{2}}{3} - \cancel{x^{\frac{3}{2}}} + \frac{x^2}{2} + \frac{x^4}{8}}{x\left(x - \frac{x^3}{6}\right)}$$

$$\lim_{x\to 0} \frac{x^{2}\cdot\frac{1}{3} + \frac{x^2}{2} + \frac{x^4}{8}}{x^2 - \frac{x^4}{6}}$$

$$\lim_{x\to 0} \frac{\frac{1}{3}x^2}{x^2} = \frac{1}{2}$$

$$\lim_{x\to 0} \frac{\frac{1}{3}x^2 + \frac{x^4}{8}}{x^2 - \frac{x^4}{6}} = \lim_{x\to 0} \frac{\cancel{x^2}\left(\frac{1}{3} + \frac{x^2}{8}\right)}{\cancel{x^2}\left(1 - \frac{x^2}{3!}\right)}$$

$$= \frac{\frac{1}{3} + 0}{1 - 0} = \frac{1}{3}$$

$$f(x) = \frac{f^{0}(x_0)}{0!} + \frac{f'(x_0)}{1!}(x-x_0)^1 + \frac{f''(x_0)}{2!}(x-x_0)^2$$

$$= \sum_{i=0}^{N} \frac{f^{i}(x_0)}{i!}(x-x_0)^i + \underbrace{o\left((x-x_0)^N\right)}_{\text{Resto di Peano}}$$

$$x_0 = 0$$

$$f(x) = \sum_{i=0}^{N} \frac{f^{i}(0)}{i!}(x)^i + o\left(x^N\right)$$

Significato geometrico della derivata di un punto

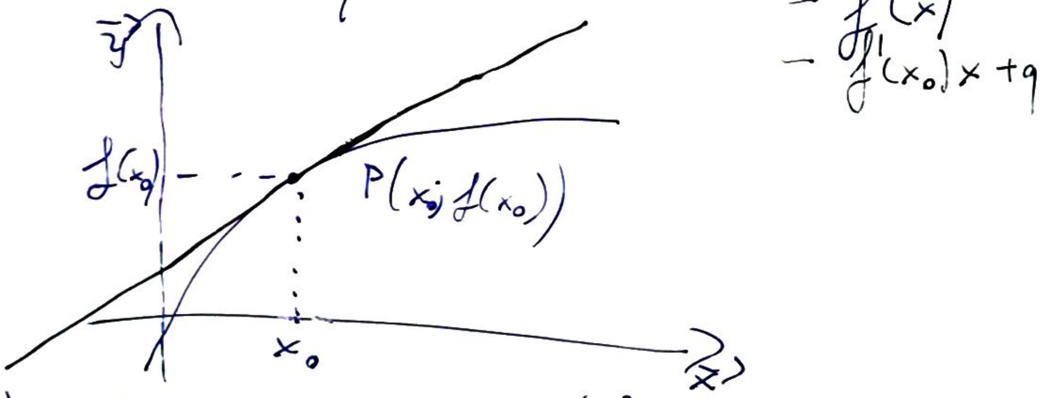
$f(x)$

$D(f)$

$\frac{d}{dx} f(x)$

$\frac{d}{dx} f(x_0) = $ Numero

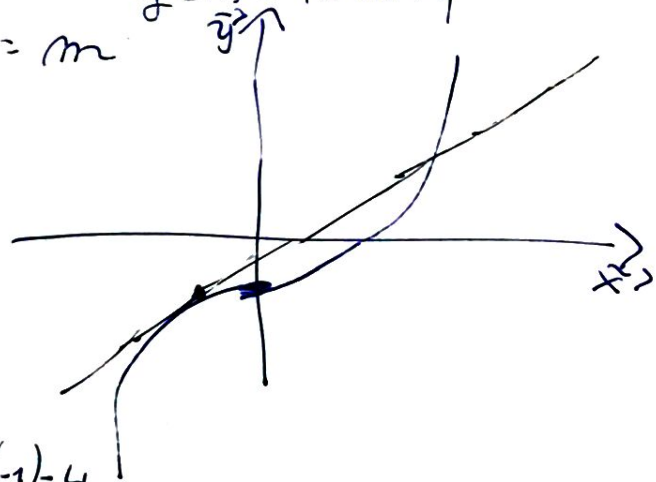
Coefficiente della retta tangente al punto



$f'(x_0) \in \mathbb{R}$

$f'(x_0) = m$

$g(x) = mx + q$



$f(x) = x^3 - 2x^2 + x - 1$

$x = -1$

$y - y_0 = m(x - x_0)$

$f'(x) = 3x - 6x + 1 \Rightarrow f'(-1) = 4$

$f(-1) = -1 - 2 - 1 = -3$

$m = 4$ $\quad y - y_0 = m(x - x_0)$

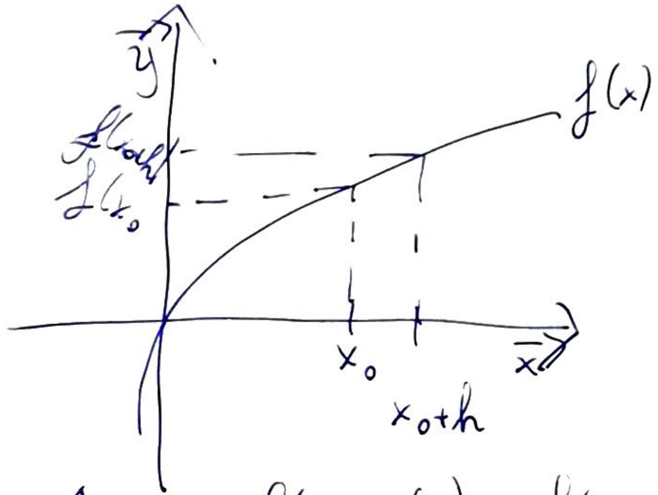
$y_0 = -5$

$x_0 = -1$

Rapporto incrementale di una funzione in un punto

$y = f(x)$

$x_0 \in D(f)$



$\dfrac{\Delta y}{\Delta x}$

$\Delta y = f(x_0 + h) - f(x_0)$

$\Delta x = -\cancel{x} + x_0 + h$

h

$$\frac{f(x_0 + h) - f(x_0)}{h} = \frac{\Delta y}{\Delta x}$$

$f(x) = x^2$

$x_0 = 2$

$f(x_0 + h) = (2 + h)^2 = 4 + h^2 + 4h$

$f(x_0) = 4$

$$\frac{\cancel{4} + h^2 + 4h - \cancel{4}}{h}$$

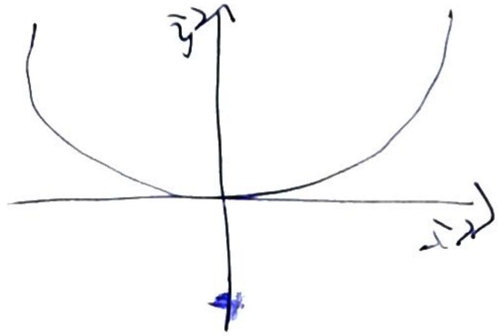
$$\frac{\cancel{h}(h+4)}{\cancel{h}}$$

$h + 4$

$\lim_{h \to 0} h + 4 =$

$= 4$

$f(x) = x^2$



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + h^2 + 2hx - \cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 2hx}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(h + 2x)}{\cancel{h}}$$

$$= \lim_{h \to 0} h + 2x$$

$$= 2x$$

$$\left\lceil \lim_{x \to 0} \frac{e^x - 1}{x} = 1 \right\rfloor$$

$f(x) = e^x$

$$\lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$\lim_{h \to 0} \frac{e^x e^h - e^x}{h}$$

$$\lim_{h \to 0} \frac{e^x (e^h - 1)}{h}$$

$$\lim_{h \to 0} e^x \frac{e^h - 1}{h}$$

$$e^x$$

$$\frac{d}{dx} sen(x) = cos(x)$$

$$sen(a+b) = sen(a) \cdot cos(b) + sen(b) \cdot cos(a)$$

$$= \lim_{h \to 0} \frac{sen(x+h) - sen(x)}{h}$$

$$= \lim_{h \to 0} \frac{2 sen(x) sen(h) - sen(x)}{h}$$

$$= \lim_{h \to 0} \frac{sen(x) cos(h) + sen(h) \cdot cos(x) - sen(x)}{h}$$

$$= \lim_{h \to 0} \frac{sen(x) cos(h)}{h} + cos(x) \frac{sen(h)}{h} - \frac{sen(x)}{h} \qquad \lim_{x \to 0} \frac{sen(x)}{x} = 1$$

$$= \lim_{h \to 0} \frac{sen(x) cos(h)}{h} + cos(x) - \frac{sen(x)}{h}$$

$$= \lim_{h \to 0} \frac{sen(x) cos(h) - sen(x)}{h} + cos(x)$$

$$= \lim_{h \to 0} \frac{sen(x)(cos(h) - 1)}{h} + cos(x)$$

$$= \lim_{h \to 0} cos(h) - 1 + cos(x)$$

$$= \lim_{h \to 0} cos(x) + 1 - 1$$

$$= cos(x)$$

$$f(x) = \frac{[1-3\ln(x)+2\ln(x)^2]^{\sqrt{3}}}{3-27^x}$$

$$f(x) = \sqrt{\log_{\frac{1}{2}}(1-2\sin(x)^2}$$

$$f(x) = \ln\left(\frac{2x-3}{x^2-4x+3}\right)$$

$$f(x) = \frac{x^2}{\sqrt{\ln(x)}}$$