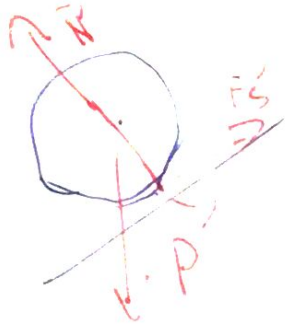
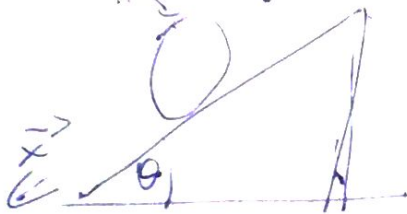


Finis Generale I

02/12/2020



Moto di puro rotolamento lungo un piano inclinato

$$\vec{F} = m \cdot \vec{a}_{cm}$$

$$x: mg \sin(\theta) - F_s = m \cdot a_{cm} \quad (1)$$

$$y: N - mg \cos(\theta) = 0$$

Polo C.M.

$$\tau_z = I_z \alpha \quad (\text{Simmetria})$$

$$\tau_P = 0 \quad b=0 \quad 2) F_s R = I_{cm} \alpha$$

$$\tau_N = 0 \quad \perp$$

$$(3) a_{cm} = d \alpha$$

$$F_s = \frac{I_{cm} a_{cm}}{R^2}$$

Polo P

$$\tau_{PY} = 0$$

$$\tau_{FS} = 0$$

$$mg \sin(\theta) - \frac{I_{cm} a_{cm}}{R^2} = m a_{cm} \quad mg \sin(\theta) = I_P \alpha$$

$$mg \sin(\theta) = \left(m + \frac{I_{cm}}{R^2} \right) a_{cm}$$

$$I_P = I_{cm} + m R^2 \quad \perp$$

$$\text{Ghera } I_{cm} = \frac{2}{5} m R^2$$

$$a_{cm} = \frac{mg \sin(\theta)}{m + \frac{I_{cm}}{R^2}}$$

$$a_{cm} = \frac{mg \sin(\theta)}{m + \frac{2}{5} m}$$

$$a_{cm} = \frac{g \sin(\theta)}{1 + \frac{2}{5}}$$

Cilindro

$$a_{cm} = \frac{2}{3} g \sin(\theta) \quad a = \frac{2}{3} \frac{g \sin(\theta)}{R}$$

$$F_s = \frac{I_{cm} a_{cm}}{R^2}$$

$$= \frac{\frac{1}{2} m R^2 \frac{2}{3} g \sin(\theta)}{R^2}$$

$$= \frac{mg \sin(\theta)}{3}$$

$$F_s \leq \mu_s N$$

$$\frac{mg \sin(\theta)}{3} \leq \mu_s mg \cos(\theta)$$

$$\tan(\theta) \leq 3\mu_s \iff \text{Mata di pura rotolamento}$$

$$K_o + U_o = K_f + U_f$$

$$K_o = 0$$

Γ h riferita a cm

$$K_f = \frac{1}{2} I_P \omega^2$$

$$\frac{1}{2} I_P \omega^2 = U_o - U_f = mgh$$

$$\frac{1}{2} \left(\frac{1}{2} m R^2 + m R^2 \right) \omega^2 = mgh$$

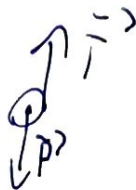
$$\frac{1}{2} \left(\frac{3}{2} R^2 \right) \omega^2 = gh$$

$$\omega^2 = \frac{4}{3} \frac{gh}{R^2}$$

$$\omega = \sqrt{\frac{4}{3} \frac{gh}{R^2}}$$

$$v_{cm} = \omega R$$

$$y_0 - x_0$$



Rolls without slipping

$$\vec{F} = m \cdot \vec{a}_{cm}$$

$$\vec{F}_x = 0$$

$$I \quad mg - T = m \cdot a_{cm}$$

II Polar C.M.

$$\tau_P = 0$$

$$T R_0 = I_{cm} \alpha$$

$$T = \frac{I_{cm} \alpha}{R_0}$$

$$R_0$$

III Polar P

$$\tau_T = 0$$

$$mg R_0 = I_P \alpha$$

$$\alpha = \frac{a_{cm}}{R_0}$$

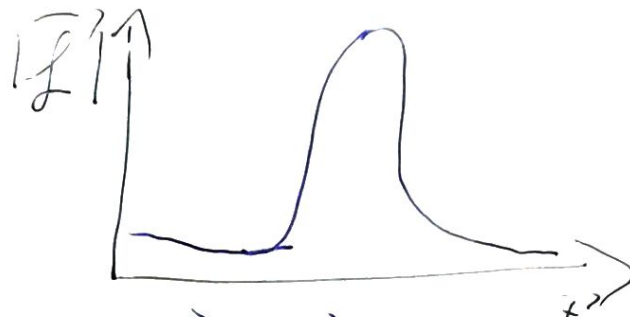
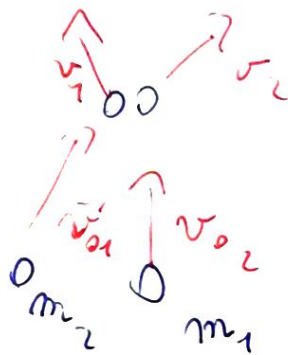
$$mg - \frac{I_{cm} \alpha}{R_0} = m a_{cm}$$

$$mg - \frac{I_{cm} a_{cm}}{R_0^2} = m a_{cm}$$

$$a_{cm} = \frac{mg}{m + \frac{I_{cm}}{R_0^2}}$$

$$I_{cm} = \frac{1}{2} m R^2$$

Urti

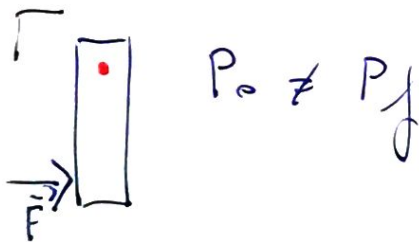


$$\vec{F}_I + \vec{F}_E = \text{Impulso}$$

$$\vec{F}_E \text{ variabile}$$

$$\vec{P}_0 = \vec{P}_f = \text{const}$$

Quadrato vincolato



$$m = 14.0 \text{ g}$$

$$v_0 = 30 \text{ m/s}$$

$$\theta = \frac{\pi}{6}$$

$$v_f = 40 \text{ m/s}$$

$$\Delta t = 2 \text{ ms}$$



$$\vec{I}_P = \Delta \vec{P} = \vec{P}_f - \vec{P}_0$$

$$I_x = \vec{P}_f \cos(\theta) - \vec{P}_0 \cos(\theta) = -m v_{0f} \sin(\frac{\pi}{6})$$

$$I_y = \vec{P}_f \sin(\theta) - \vec{P}_0 \sin(\theta) = +m v_0 \sin(\frac{\pi}{6})$$

$$I = \sqrt{I_x^2 + I_y^2}$$

$$I = F_m \Delta t$$

$$F_m = \frac{I}{\Delta t}$$

Conservazione momento angolare

$$\tau_{vincolato} = 0$$

$$\vec{L}_0 = \vec{L}_f$$

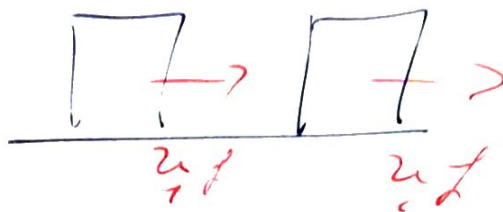
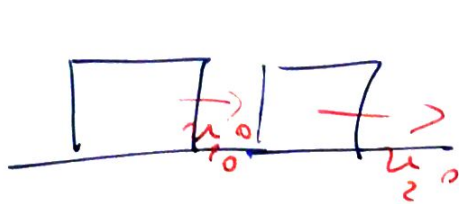
Conservazione Energia

Meccanica

\vec{F}_i Conservative

↓

Urto elastico



$$v_1 > v_2$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

l'urto è elastico e si conserva l'energia meccanica

$$K_o + U_o = K_f + U_f$$

$$K_o + K_f = \text{costante}$$

$$\frac{1}{2} m v^2 + \frac{1}{2} m v^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_2 v_{2f}^2 - m_2 v_{2i}^2 = - (m_1 v_{1f}^2 - m_1 v_{1i}^2)$$

$$m_2 (v_{2f}^2 - v_{2i}^2) = - m_1 (v_{1f}^2 - v_{1i}^2)$$

$$m_2 (v_{2f} - v_{2i}) = - m_1 (v_{1f} - v_{1i})$$

$$- m_1 (v_{1f} - v_{1i}) (v_{2f} + v_{2i}) = - m_1 (v_{1f} - v_{1i}) (v_{1f} + v_{1i})$$

$$v_{2f} + v_{2i} = v_{1f} + v_{1i}$$

$$v_{2f} - v_{1f} = - (v_{2i} - v_{1i})$$

Allo stesso tempo

$$v_{1f} = \frac{(m_1 - m_2) v_{1i} + 2 m_2 v_{2i}}{m_1 + m_2}$$

$$v_{2f} = \frac{2 m_1 v_{1i} + (m_2 - m_1) v_{2i}}{m_1 + m_2}$$

$$m_1 = 4 \text{ kg} \quad v_1^0 = 6 \text{ m/s}$$

$$m_2 = 2 \text{ kg} \quad v_2^0 = 3 \text{ m/s}$$

$$m_1 v_1^0 + m_2 v_2^0 = m_1 v_{1f} + m_2 v_{2f}$$

$$24 + 6 = 4v_{1f} + 2v_{2f}$$

$$= 30$$

$$v_{2f} = v_{1f} + 3$$

$$4v_{1f} + 2(v_{1f} - 3) = 30$$

$$6v_{1f} - 6 = 30$$

$$6v_{1f} = 24 \text{ m/s}$$

$$v_{1f} = 4 \text{ m/s}$$

$$v_{2f} = 7 \text{ m/s}$$

$$v_{2f} - v_{1f} = 3 \text{ m/s}$$

$$6 - 3 = 3 \text{ m/s} \quad \checkmark$$

Sistema di riferimento C.M.



$$C.M.$$

$$v_{cm} = 0$$

$$K_1 = \frac{1}{2} m_1 u_1^2 = \frac{p_1^2}{2m_1}$$

$$K_1^0 + K_2^0 = K_1^f + K_2^f$$

$$\frac{p_1^0}{2m_1} + \frac{p_2^0}{2m_2} = \frac{p_1^f}{2m_1} + \frac{p_2^f}{2m_2}$$

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$p_1^0 = m_1 u_1^0$$

$$p_2^0 = m_2 u_2^0$$

u_1, u_2 velocità nel sistema C.M.

v_1, v_2 sistema di laboratorio

$$v_1 = u_1 + v_{cm}$$

$$v_2 = u_2 + v_{cm}$$

$$(m_1 + m_2) v_{cm} = 0$$

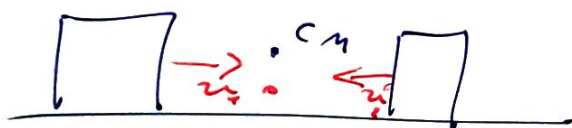
$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v_{cm} = 0$$

$$m_2 u_2 = -m_1 u_1 \quad p_1 = -p_2$$

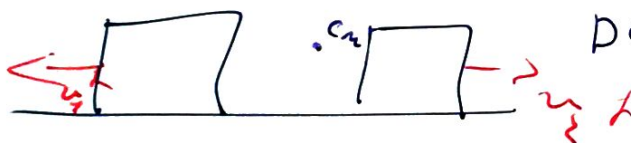
$$\frac{1}{2} p_0^2 \left(\frac{1}{m_1} + \frac{1}{m_2} \right) = \frac{1}{2} p_f^2 \left(\frac{1}{m_1} + \frac{1}{m_2} \right) \Rightarrow p_2^2 = p_1^2$$

$$p_1^2 = p_2^2$$

$$p_2 = \pm p_1 \Rightarrow \begin{aligned} p_1^0 &= -p_2^f \\ p_2^0 &= -p_1^f \end{aligned}$$

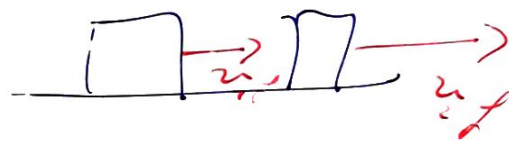


PRIMA



S. C.M.

DOPO



S. LABORATORIO

$$v_{cm} = \frac{30}{6} = 5 \text{ m/s}$$

$$= \frac{m_1 v_1}{m_2 v_2}$$

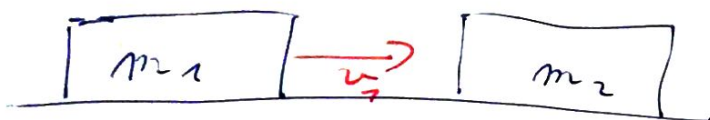
$$u_1 = v_1 - v_{cm} = 1 \text{ m/s}$$

$$u_2 = v_2 - v_{cm} = -2 \text{ m/s}$$

$$u_1^f = -1 \text{ m/s} \quad u_2^f = 2 \text{ m/s}$$

$$v_1^f = u_1 + v_{cm} = 4 \text{ m/s}$$

$$v_2^f = u_2 + v_{cm} = 7 \text{ m/s}$$



S.R. C.M.

$$v_{cm} = \frac{m_1 v_1}{m_1 + m_2}$$

$$u_1 = v_1 - v_{cm}$$

$$u_2 = v_2 - v_{cm}$$

$$u_1 = v_1 - \frac{m_1 v_1}{m_1 + m_2}$$

$$u_2 = - \frac{m_1 v_1}{m_1 + m_2}$$

$$\frac{m_1 v_1^0 + m_2 v_1^0 - m_1 v_1^0}{m_1 + m_2} = \frac{m_2 v_2^0}{m_1 + m_2}$$

$$u_2^f = -u_2^0 = + \frac{m_1 v_1^0}{m_1 + m_2}$$

$$v_1^f = u_1^f - v_{cm}$$

$$= - \frac{m_2 v_1^0}{m_1 + m_2} + \frac{m_1 v_1^0}{m_1 + m_2}$$

$$= \frac{m_1 - m_2}{m_1 + m_2} v_1^0$$

$$v_2^f = u_2^f + v_{cm}$$

$$= \frac{m_1 v_1^0}{m_1 + m_2} + \frac{m_1 v_1^0}{m_1 + m_2}$$

$$= 2 \frac{m_1 v_1^0}{m_1 + m_2}$$

$$m_1 = m_2$$

$$v_1^f = 0$$

$$v_2^f = 2 v_1^0$$

$$m_1 > m_2$$

$$v_1^f \approx \frac{m_1 - m_2}{m_1 + m_2} v_1^0$$

$$v_2^f \approx 2 v_1^0 \frac{m_1}{m_1 + m_2}$$

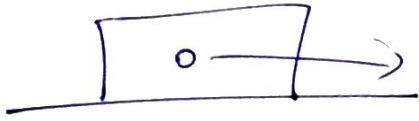
$$m_1 < m_2$$

$$v_1^f \approx -v_1^0$$

$$v_2^f \approx 0$$

Billiard entre Parete

Urto completamente anelastico



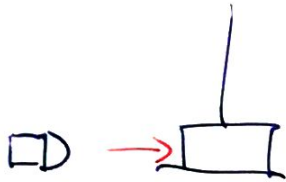
Conservazione quantità di moto

$$m_1 v_0 = (m_1 + m_2) v_{cm}$$

$$v_{cm} = \frac{m_1 v_0}{m_1 + m_2}$$

Lorpi invisibile

Pendolo Balistico

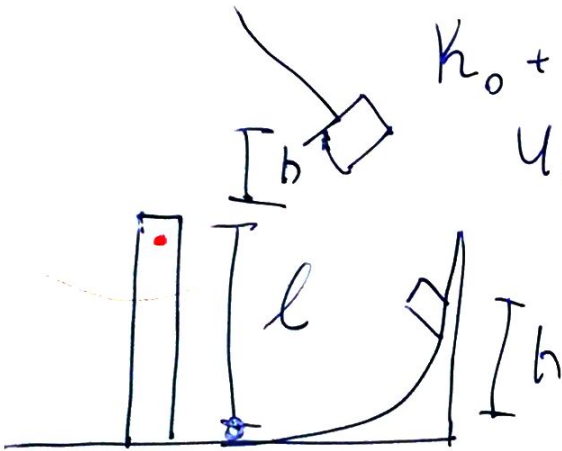


$$m v_0 = (m_1 + m_2) v_f$$

$$(m_1 + m_2) g h = \frac{1}{2} (m_1 + m_2) v_f^2$$

$$K_0 + U_0 = K_f + U_f$$

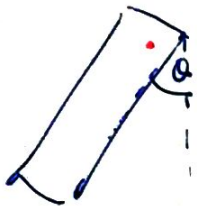
$$U_0 = K_f$$



$$P_0 \neq P_f \quad (\text{vincolato})$$

$$r_z^0 \rightarrow l \quad r_z^f = l \quad r_z^f$$

$$r_N = 0 \quad r_D \parallel r_z \quad r_z \sim 0$$



$$mgh = \frac{1}{2} m v_0^2$$

$$v_0^2 = 2gh$$

$$v_0 = \sqrt{2gh}$$

$$L_z^0 = L_z l$$

$$m v_0 l = \left(I_z^0 + m l^2 \right) \omega^2$$

$$\omega = \frac{m v_0 l}{\frac{1}{3} M l^2 + m l^2} \quad v_B = \omega l$$

$$\frac{1}{2} \left(\frac{1}{3} M l^2 + m l^2 \right) \omega^2 = \overbrace{m g \frac{l}{2} (1 - \sin(\alpha))}^{A_1 - C_1} + \underbrace{m g l (1 - \sin(\alpha))}_{\text{Pullin}}$$

$$I = P_F - P_0$$

$$= \underbrace{M v_{CM} \frac{l}{2}}_{P_f \text{ Anten Pullin}} + \underbrace{m \omega l}_{P_0 \text{ Pullin}}$$