Analisi matematica I   ## 02/12/2020 ##

$f(x) = 2e^{x} \dfrac{x+1}{\sqrt{2x+1}}$

$g(x) = \sqrt{\dfrac{\arctan\left(\frac{1}{x-2}\right) - \arctan(x) - \arcsin\left(\right.}{\left(\frac{1}{4}\right)^{x} - 1}}$

Dominio

$D(g) = \arctan\left(\frac{1}{x-2}\right) - \arctan(x) - \arcsin\left(\left(\frac{1}{4}\right)^{x} - 1\right) \geq 0$

$\frac{1}{x-2} \neq 0$

$\left(\frac{1}{4}\right)^{x} - 1 \geq -1 \qquad \frac{1}{4^x} \geq 0 \quad \forall x \in \mathbb{R}$

$\left(\frac{1}{4}\right)^{x} - 1 = 1 \qquad \frac{1}{4^x} \leq 2 \quad x \geq 0$

$\frac{1}{4^x} \leq 2$

$4^x \cancel{\geq} \frac{1}{2}$

$2 \cdot 4^x \geq 1$

$2 \cdot 2^{2^x} \geq 1$

$2^{x+1} \geq 1$

$2^{2^x+1} \geq 2^0$

$2^x + 1 \geq 0$

$2^x \geq -1$

$2^x \geq -2^0$

$x \geq 0$

C.E.

$x > 0$

$\arctan(x) > 0 \ \forall x \in D(g)$

$\arctan(x) > \arctan\left(\frac{1}{x-2}\right)$

$\forall x \in D(g)$

$x > \frac{1}{x-2}$

$x^2 - 2x > 1$

$x^2 - 2x - 1 > 0$

$\frac{\Delta}{4} = 1 + 1 = 2$

$x = \frac{2 \pm \sqrt{2}}{1}$

$x > 2 + \sqrt{2}$

$x < 2 + \sqrt{2}$

Teorema dell' Hopital

I. $f(x), g(x)$

1) $f(x), g(x) \in$ Derivabili $_{I[x_0]}$

2) $f(x), g(x) \in$ Continua $_{I[x_0]}$

$g'(x) \neq 0$

3) $\lim_{x \to x_0} \frac{f(x)}{g(x)} = 0$

$$\Downarrow$$

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x_0)}{g'(x_0)}$$

II. $f(x), g(x)$ 1) 2) 3)

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \frac{f'(x)}{g'(x)}$$

$$\lim_{x \to 0} \frac{sen(x)}{x} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{sen(x)}{x} = \lim_{x \to 0} \frac{\frac{d}{dx} sen(x)}{\frac{d}{dx} 1} = \lim_{x \to 0} \frac{cos(x)}{1} = cos(0) = 1$$

$$\lim_{x \to -3} \frac{x^2 \; 9}{(x+3)\cancel{(x-3)}} = \frac{9-9}{-3+3} = \frac{0}{0}$$

$$\lim_{x \to -3} \frac{2x}{1} = 2x = -6$$

$$\lim_{x \to 0} (cos(x))^{\frac{1}{x^2}} = 1^{\infty}$$

$$\lim_{x \to 0} e^{ln\left(cos(x)^{\frac{1}{x^2}}\right)}$$

$$= \lim_{x \to 0} e^{\frac{1}{x^2}} \cdot e^{ln(cos(x))} = \infty \cdot e^0 = +\infty$$

$$\lim_{x \to 0} \frac{e^x - sen(x) - cos(x)}{e^{x^2} - e^{x^3}} = \frac{1 - 0 - 1}{1 - 1} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{e^x - cos(x) + sen(x)}{2xe^{x^2} - 3xe^{x^3}} = \frac{1 - 1 + 0}{2 \cdot 0 \; 3 \cdot 0} = \frac{0}{0}$$

Sviluppo in serie di Taylor

$$\lim_{x \to 0} \frac{e^x - sen(x) - cos(x)}{e^{x^2} - e^{x^3}}$$

$f(x)$ $\qquad x_0 \in D(f)$

$f(x)$ Derivabile n volte

$$f(x) = \underbrace{f(x_0)}_{\text{Punto Iniziale}} + \frac{f'(x_0)}{1!}(x-x_0)^1 + \frac{f''(x_0)}{2!}(x-x_0)^2$$

$$+ \frac{f'''(x_0)}{3!}(x-x_0)^3$$

$$f(x) = \sum_{i=0}^{N} \frac{f^n(x_0)}{i!}(x-x_0)^i$$

Resto di Peano $\qquad$ o piccolo di $x - x_0$

$f(x) = \dots + o(x-x_0)^n$

$x_0 = 0$
Mc Laren

Mc Larin

$$f(x) = f(0) + \frac{f'(0)}{1!}(x) + \frac{f''(0)}{2!}(x)$$

$$+ \frac{f'''(0)}{3!} x + \dots + o(x)^n$$

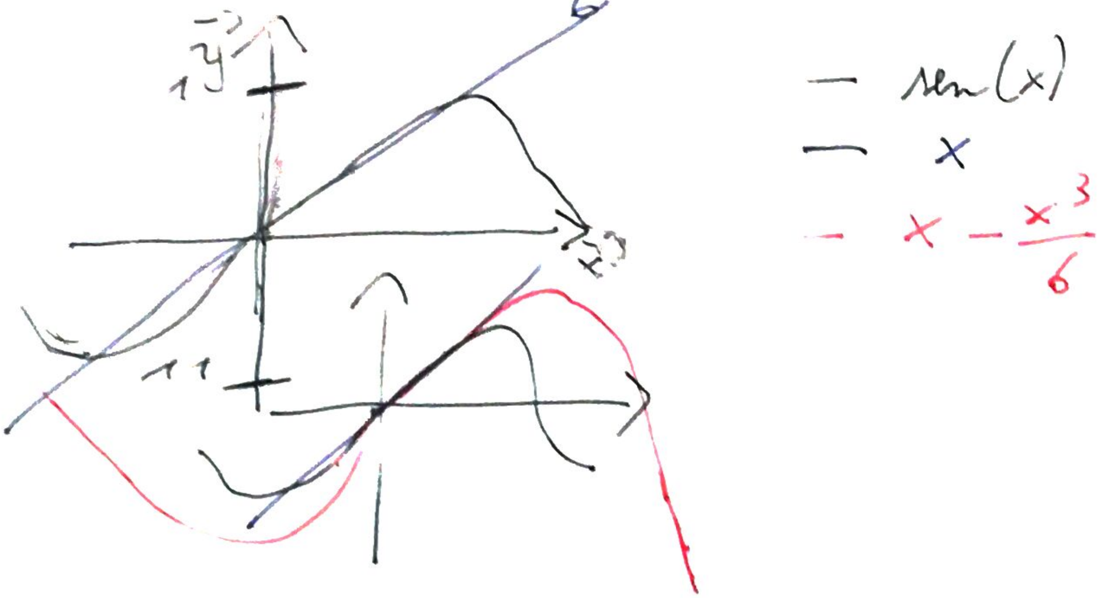
$$f(x) = \sum_{i=1}^{N} \frac{f^{i}(0)}{i!} x + o(x)^N$$

$$\operatorname{sen}(x) = \operatorname{sen}(0) + \cos(0)x - \operatorname{sen}(0)\frac{x^2}{2} + \cos(0)\frac{x^3}{6}$$

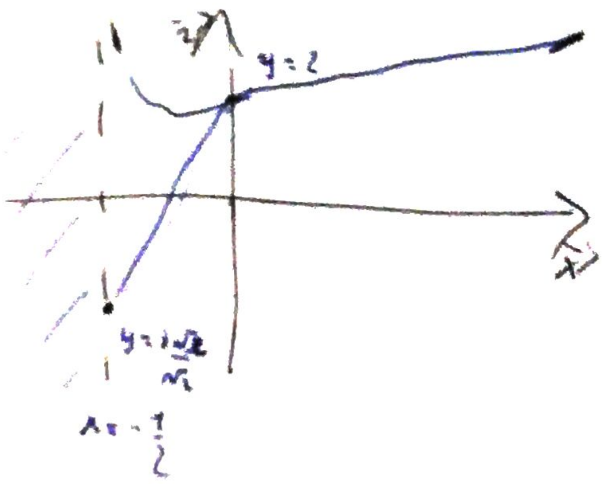
$$+ o(x)^3$$

$$= 0 + 1 \cdot x - 0 - 1 \cdot \frac{x^3}{6} + o(x^3)$$

$$= x - \frac{x^3}{6} + o(x^3)$$



$$f(x) = 2e^{x}\,\frac{x+1}{\sqrt{2x+1}}$$



$D(f) = \sqrt{2x+1} \neq 0$

$2x+1 > 0$

$x > -\frac{1}{2}$

Cas

$f(x) = f(-x)$

$f(x) = -f(-x)$

$f(x) \cap y$

$f(0)$

$2e^{0}\,\frac{0+1}{\sqrt{0+1}}$

$2\,\frac{1}{1}$

2

$f(x) \cap x$

$f(x) = 0$

$2e^{x}\,\frac{x+1}{\sqrt{2x+1}} = 0$

$x+1 = 0$

$x = -1$

$-1 \notin D(f)$

$$\lim_{x\to+\infty} 2e^{x}\cdot\frac{x+1}{\sqrt{2x+1}}$$

$= +\infty \cdot \frac{+\infty}{\sqrt{+\infty}}$

$= +\infty$

$$\lim_{x\to-\frac{1}{2}} 2e^{\frac{1}{2}}\,\frac{\frac{1}{2}+1}{\sqrt{2\frac{1}{2}+1}}$$

$= 2\sqrt{e}\cdot\frac{3}{2}\,\frac{1}{\sqrt{2}}$

$= \frac{3\sqrt{e}}{\sqrt{2}}$

$= +\infty$

$$f'(x) = 2e^x \frac{x+1}{\sqrt{2x+1}} + 2e^x \frac{1}{\sqrt[3]{(2x+1)^3}}$$

$$f'(x) > 0$$

$$2e^x \frac{x+1}{\sqrt{2x+1}} + 2e^x \frac{1}{\sqrt{(2x+1)^3}} > 0$$

$$\frac{2e^x}{\sqrt{2x+1}} \quad x+1 \quad + \frac{2e^x}{\sqrt{(2x+1)^3}} > 0$$

$$\forall x \in D(f)$$

$$f'(x) = 0$$

$$2e^{\cancel{x}} \frac{x+1}{\sqrt{2x+1}} = -2\cancel{e^x} \frac{1}{\sqrt{(2x+1)^3}}$$

$$\frac{x+1}{\sqrt{2x+2}} = -\frac{1}{\sqrt{(2x+1)^3}}$$

$$\begin{cases} x+1 = 1 \\ \sqrt{2x+1} = \sqrt{(2x+1)^3} \end{cases} \qquad x = 0$$

$$2x+2 = (2x+1)^3$$

$$+1 = +2)$$

$$+2 = 1)$$

$$x = 0$$