1 Introduction

The point of philosophy is to start with something so simple as not to seem worth stating, and to end with something so paradoxical that no one will believe it.
— Bertrand Russell

Things should be as simple as possible, but not simpler.
— Albert Einstein

REDUCTIONISM IS THE idea that a system can be understood by examining its individual parts. Implicit in this idea is that one may have to examine something at multiple levels, looking at the parts, then the parts of the parts, and so on. This is a natural way of attempting to understand the universe. Indeed, the hierarchy of science is recognizably organized in this manner. For example, take the so-called hard sciences. Biologists study living things at various levels ranging from the whole body, to organ systems, down to cellular structure. Beyond the cellular level lie chemical interactions and agents such as enzymes, which are organic chemical catalysts, and amino acids, which are building blocks for proteins. This is the domain of the chemist. To reduce things further, one would have to start looking into how atoms and molecules interact through chemical bonds that are dependent upon the number of electrons in the outermost electron shell of an atom. But what, exactly, are atoms? The physicist probes further into the nature of things by shattering atoms into their constituent parts, which brings us to protons, neutrons, and finally quarks. Ironically, at this level of understanding, scientists are dependent on mathematical techniques that often bear little resemblance to the reality that we are familiar with.

To be sure, there is some overlap among scientific and mathematical fields that is exemplified by disciplines that use tools common to other areas (e.g., organic chemistry, biophysics, and quantum mathematics), but even these hybrid disciplines have well-defined niches. Reductionism is a powerful way of looking at the universe. But this begs a somewhat silly question: Since everything ultimately breaks down to the quantum level, why aren't all scientists mathematicians at the core? In such a world physicians would make diagnoses based on the patient's bodily quarks, which makes about as much sense as building a house particle by particle. Nevertheless, every scientist must possess some knowledge of the level one step more fundamental than his or here specialty, but at some point reductionism must stop for science to be effective.

Now, suppose that we wished to describe how the universe works. We could take a reductionist's approach and catalog all of the different types of objects that exist—perhaps starting with galaxy clusters, hitting terrestrial life forms about midway through, and then ending with subatomic particles—but would this approach really succeed in describing the universe? In making a large list of "things" it is easy to forget that the manner in which "things" work more often than not depends on the environment in which they exist. For example, we could describe the form of a duck in excruciating detail, but this gives us only half of the story. To really appreciate what a duck is, we should look at ducks in the air, in water, in the context of what they eat or what eats them, how they court, mate, and reproduce, the social structures they form, how they flock, and their need to migrate.

Looking back at the organization of the sciences, we find that at each level of understanding, traditional scientists study two types of phenomena: agents (molecules, cells, ducks, and species) and interactions of agents (chemical reactions, immune system responses, duck mating, and evolution). Studying agents in isolation is a fruitful way of discovering insights into the form and function of an agent, but doing so has some known limitations. Specifically, reductionism fails when we try to use it in a reverse direction. As we shall see throughout this book, having a complete and perfect understanding of how an agent behaves in no way guarantees that you will be able to predict how this single agent will behave for all time or in the context of other agents.

We have, then, three different ways of looking at how things work. We can take a purely reductionist approach and attempt to understand things through dissection. We also can take a wider view and attempt to understand whole collections at once by observing how many agents, say the neurons in a brain, form a global pattern, such as human intelligence. Or we can take an intermediate view and focus attention on the interactions of agents. Through this middle path, the interactions of agents can be seen to form the glue that binds one level of understanding to the next level.

1.1 Simplicity and Complexity

Let's take this idea further by examining a single ant. By itself, an ant's behavior is not very mysterious. There is a very small number of tasks that any ant has to do in the course of its lifetime. Depending on its caste, an ant may forage for food, care for the queen's brood, tend to the upkeep of the nest, defend against enemies of the nest, or, in the special case of the queen, lay eggs. Yet when we consider the ant colony as a whole, the behavior becomes much more complex. Army ant colonies often consist of millions of workers that can sweep whole regions clean of animal life. The fungus-growing ants collect vegetable matter as food for symbiotic fungi and then harvest a portion of the fungi as food for the colony. The physical structures that ants build often contain thousands of passageways and appear mazelike to human eyes but are easily navigated by the inhabitants. The important thing to realize is that an ant colony is more than just a bunch of ants. Knowing how each caste in an ant species behaves, while interesting, would not enable a scientist to magically infer that ant colonies would possess so many sophisticated patterns of behavior.

Instead of examining ants, we could have highlighted a number of interesting examples: economic markets that defy prediction, the pattern recognition capabilities of any of the vertebrates, the human immune system's response to viral and bacterial attack, or the evolution of life on our planet. All of these examples are *emergent* in that they contain simple units that, when combined, form a more complex whole. This is a case of the whole of the system being greater than the sum of the parts, which is a fair definition of holism—the very opposite of reductionism.

We also know that agents that exist on one level of understanding are very different from agents on another level: cells are not organs, organs are not animals, and animals are not species. Yet surprisingly the interactions on one level of understanding are often very similar to the interactions on other levels. How so? Consider the following:

- Why do we find self-similar structure in biology, such as trees, ferns, leaves, and twigs? How does this relate to the self-similarity found in inanimate objects such as snowflakes, mountains, and clouds? Is there some way of generalizing the notion of self-similarity to account for both types of phenomena?
- Is there a common reason why it's hard to predict the stock market and also hard to predict weather? Is unpredictability due to limited knowledge or is it somehow inherent in these systems?
- How do collectives such as ant colonies, human brains, and economic markets self-organize to create enormously complex behavior that is much richer than the behavior of the individual component units?
- What is the relationship between evolution, learning, and the adaptation found in social systems? Is adaptation unique to biological systems? What is the relationship between an adaptive system and its environment?

The answers to all of these questions are apparently related to one simple fact: Nature is frugal. Of all the possible rules that could be used to govern the interactions among agents, scientists are finding that nature often uses the simplest. More than that, the same rules are repeatedly used in very different places. To see why, consider the three attributes below that can be used to describe the interactions of agents.

Collections, Multiplicity, and Parallelism Complex systems with emergent properties are often highly parallel collections of similar units. Ant colonies owe much of their sophistication to the fact that they consist of many ants. This is obvious, but consider the implications. A parallel system is inherently more efficient than a sequential system, since tasks can be performed simultaneously and more readily via specialization. Parallel systems that are redundant have fault tolerance. If some ants die, a task still has a good chance of being finished since similar ants can substitute for the missing ones. As an added bonus, subtle variation among the units of a parallel system allows for multiple problem solutions to be attempted simultaneously. For example, gazelles as a species actively seek a solution to the problem of avoiding lions. Some gazelles may be fast, others may be more wary and timid, while others may be more aggressive and protective of their young. A single gazelle cannot exploit all of the posed solutions to the problem of avoiding lions simultaneously, but the species as a whole can. The gazelle with the better solution stands a better chance of living to reproduce. In such a case, the species as a whole can be thought of as having found a better solution through natural selection.

Iteration, Recursion, and Feedback For living things, iteration corresponds to reproduction. We can also expand our scope to include participants of an economic system, antibodies in an immune system, or reinforcement of synapses in the human brain. While parallelism involves multiplicity in a space, iteration involves a form of persistence in time. Similarly, recursion is responsible for the various types of self-similarity seen in nature. Almost all biological systems contain self-similar structures that are made through recurrent processes, while many physical systems contain a form of functional self-similarity that owes its richness to recursion. We will also see that systems are often recurrently coupled to their environment through feedback mechanisms. While animals must react according to their surroundings, they can also change this environment, which means that future actions by an animal will have to take these environmental changes into account.

Adaptation, Learning, and Evolution Interesting systems can change and adapt. Adaptation can be viewed as a consequence of parallelism and iteration in a competitive environment with finite resources. In this case the combination of multiplicity and iteration acts as a sort of filter. We see this when life reproduces because it is fit, companies survive and spawn imitations because they make money, antibodies are copied because they fight infections, and synapses are reinforced because of their usefulness to the organism. With feedback mechanisms in place between an agent and an environment, adaptation can be seen as forming a loop in the cause and effect of changes in both agents and environments.

There are certainly many more ways to describe the interactions of agents; however, multiplicity, iteration, and adaptation by themselves go a long way in describing what it is about interactions between agents that makes them so interesting. Moreover, multiplicity, iteration, and adaptation are universal concepts in that they are apparently important attributes for agents at all levels—from chemical reactants to biological ecosystems.

Looking back at our original goal of attempting to describe the universe, we find that there are a few generalizations that can be made regarding agents and interactions. Describing agents can be tedious, but for the most part it is a simple thing to do with the right tools. Describing interactions is usually far more difficult and nebulous because we have to consider the entire environment in which the agents exist. The simplest type of question that we can ask about an interaction is what will X do next, in the presence of Y? Notice that this question has a functional, algorithmic, or even *computational* feel to it, in that we are concerned not with "What is X?" but with "What will X do?" In this respect, describing an interaction is very similar to discovering nature's "program" for something's behavior. The goal of this book is to highlight the computational beauty found in nature's programs.

1.2 The Convergence of the Sciences

In a way, this book is also a story about scientific progress in the last part of the twentieth century. In the past, and even today, there is a worrisome fragmentation of the sciences. Specifically, scientists' areas of expertise have become so specialized that communication among scientists who are allegedly in the same field has become difficult, if not impossible. For example, the computer sciences can be subdivided into a short list of subdisciplines: programming languages, operating systems, software engineering, database design, numerical analysis, hardware architectures, theory of computation, and artificial intelligence. Most computer scientists can comfortably straddle two or three of the subdisciplines. However, each subdiscipline can be further divided into even more specialized groups, and it is fair to say that a recursion theorist (a subset of theory of computation) will usually have little to talk about with a connectionist (a subset of artificial intelligence). To make things worse, computer science is a new science; that is, the situation is much worse in the older sciences, such as physics and biology.

Traditionally, there has also been a subdivision in most scientific disciplines between theorists and experimentalists. Again, some notable scientists (such as Henri Poincaré, quoted in the preface) have dabbled in both areas, but as a general rule most scientists could be safely classified in one of the two classes.

It is no coincidence that the recent renaissance in the sciences also marks the introduction and proliferation of computers. For the first time, computers have blurred the line between experimentation and theory. One of the first uses of computers was to simulate the evolution of complicated equations. Someone who creates and uses a simulation is simultaneously engaging in theory and experimentation. As computers became more affordable and easy to use, they became a general-purpose tool for all of the sciences. Thus, the line between experimentation and theory has been blurred for all of the disciplines wherever computer simulations provide some benefit.

And just what sort of computer simulations have been built? Meteorologists build weather simulations. Physicists study the flow of plasma and the annealing of metals. Economists have modeled various types of economies. Psychologists have examined cognitive models of the brain. In all cases, scientists have found beauty and elegance in modeling systems that consist of simple, parallel, iterative, and/or adaptive elements. More important, scientists have been making discoveries that have been relevant to multiple fields. Thus, in many ways, the computer has reversed the trend toward fragmentation and has helped the sciences converge to a common point.

1.3 The Silicon Laboratory

Where do we begin? This book is in five parts, with the first part acting as a general introduction to the theory of computation. The remaining four parts highlight what I believe are the four most interesting computational topics today: fractals, chaos, complex systems, and adaptation. Each topic has had popular and technical books devoted to it alone. Some few books deal with two or three of the topics. I hope to convince you that the combination of the five parts is far more interesting taken together than alone. Figure 1.1 illustrates this point further by showing an association map of the book parts. The line segment between any two parts is labeled by a topic that straddles both of the joined parts. Many of the labels in the figure may be unfamiliar to you at this point, but we will eventually see how these topics not only are casually related because of the computational aspect of each but also intricately bounded together into a powerful metaphor for understanding nature's more beautiful phenomena.

An overview of the book's contents follows. Each part is relatively self-contained and can be appreciated on its own. However, this book is also designed so that each part acts as a rough introduction to the next part.

Computation What are the limits of computers and what does it mean to compute? We will examine this question with a bottom-up approach, starting with the properties of different types of numbers and sets. The key point of the first part of this book is that the theory of computation yields a surprisingly simple definition of what it means to compute. We will punctuate this fact by showing how one can construct higher mathematical functions with only a very small set of primitive computational functions as a starting place.

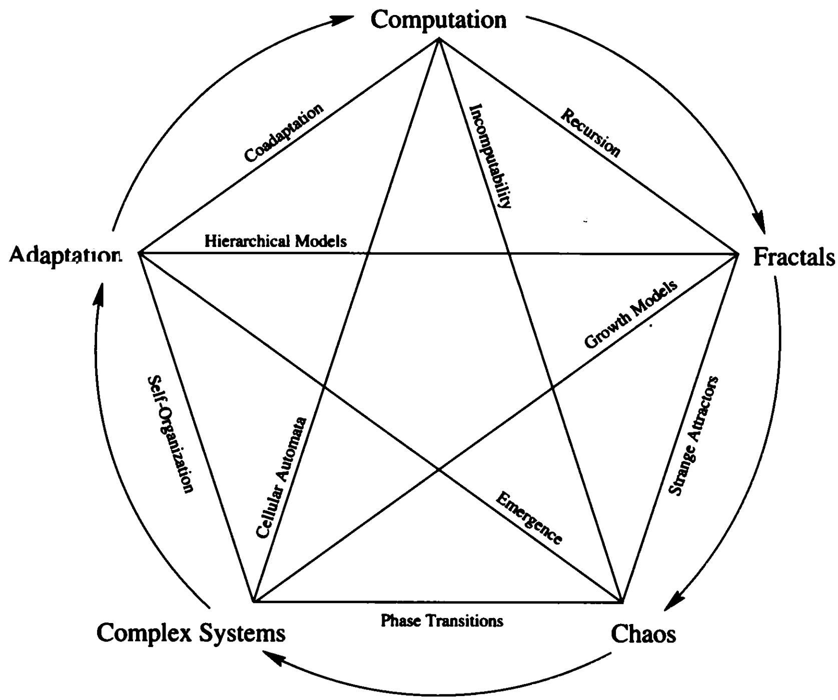

Figure 1.1 An association map of the contents of this book

However, even though the notion of computability is easy to define, it turns out that the process of computation can be extremely rich, complex, and full of pitfalls. We will examine what it means for a function to be incomputable and also see that there are more incomputable functions than computable functions.

Fractals In Part II, we will study various types of *fractals*, which are beautiful images that can be efficiently produced by iterating equations. Since fractals can be computationally described, it is interesting to see that they are often found in natural systems, such as in the way trees and ferns grow or in the branching of bronchial tubes in human lungs. Curiously, the last type of fractal that we will examine in Part II has the same sort of pathological quality that the incomputable programs in Part I have.

Chaos In Part III, we will examine a special type of fractal, known as a *strange attractor*, that is often associated with chaos. Chaos makes the future behavior of deterministic systems such as the weather, economies, and biological systems impossible to predict in the long term but predictable in the short term. Chaos can be found when nonlinear systems are iterated repeatedly but is also found in multi-agent complex systems such as ecosystems, economies, and social structures. Ironically, the inherent sensitivity of chaotic systems can make them easier to control than one would think, since their sensitivity can be used to make large changes with only small control forces.

Complex Systems In Part IV, we will study complex systems consisting of many very simple units that interact. The amount of interaction among agents partially determines the overall behavior of the whole system. On one extreme, systems with little interaction fall into static patterns, while on the other extreme, overactive systems boil with chaos. Between the two extremes is a region of criticality in which some very interesting things happen. A special type of cellular automata known as the Game of Life, which is in the critical region, is able to produce self-replicating systems and roving creatures, but it is also capable of universal computation. We will also study the Iterative Prisoners' Dilemma, which may explain why cooperation in nature is more common than one would expect. Afterward, competition and cooperation among agents will be highlighted as a natural method of problem-solving in nature. We will see how an artificial neural network with fixed synapses can solve interesting problems seemingly non-algorithmically.

Adaptation Finally, in Part V, we will allow our complex systems to change, adapt, learn, and evolve. The focus of these chapters will include evolutionary systems, classifier systems, and artificial neural networks. Genetic algorithms will be used to evolve solutions to a wide variety of problems. We will also see how a simple form of feedback coupled with evolutionary mechanisms can be used to mimic a form of intelligence in classifier systems. We will then examine how artificial neural networks can be trained by example to solve pattern classification and function approximation problems. At the end of this part, we will see how in many ways one can view learning, evolution, and cultural adaptation as one process occurring on varying time scales.

Throughout this book we will talk about physics, biology, economics, evolution, and a host of other topics, but the prevailing theme will be to use the computer as a laboratory and a metaphor for understanding the universe.

I Computation

Any discrete piece of information can be represented by a set of numbers. Systems that compute can represent powerful mappings from one set of numbers to another. Moreover, any program on any computer is equivalent to a number mapping. These mappings can be thought of as statements about the properties of numbers; hence, there is a close connection between computer programs and mathematical proofs. But there are more possible mappings than possible programs; thus, there are some things that simply cannot be computed. The actual process of computing can be defined in terms of a very small number of primitive operations, with *recursion* and/or *iteration* comprising the most fundamental pieces of a computing device. Computing devices can also make statements about other computing devices. This leads to a fundamental paradox that ultimately exposes the limitations not just of of machine logic, but all of nature as well.

Chapter 2 introduces some important properties of different types of numbers, sets, and infinities. Chapter 3 expands on this by introducing the concepts behind computation and shows how computation can be seen to operate over sets of integers. Chapter 4 ties together some of the paradoxes seen in the earlier chapters to show how they are applicable to all of mathematics.