Thermodynamics of the primordial fluid and BAO

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In this review, we will cover the theoretical groundwork of baryon acoustic oscillations (BAO), focusing mainly on the statistical description of the matter and radiation in the early universe just before recombination. We will see that this can be treated as a baryon-photon fluid that can be described with standard thermodynamic theory. Using this description, we will see how initial density perturbations resulted in acoustic waves that traveled until the time of recombination, creating an over density that would result in the BAO feature.

I. INTRODUCTION

The early universe prior to recombination, $z = 1070$, $t = 370000$ years [1], was characteristic of extreme energies and densities far exceeding that of today. It is remarkable, therefore, that the matter in the early universe is well understood based on rudimentary quantum mechanical and statistical theory.

Modern cosmology exploits this well understood groundwork to test general relativity and the standard model at the largest possible stage. Many advancements in cosmology have been made exploiting the properties of the early primordial fluid and the observational signatures that the theory implies. The cosmic background radiation (CMB), for example, has proven to be a wealth of information, providing a way to study the universe at one of its earliest epochs.

Baryon acoustic oscillations (BAO), similar to the CMB, relies on an early description of universe as a relativistic fluid composed of dark matter, dark energy, neutrinos, electrons, and the coupled baryon-photon fluid to describe acoustic oscillations, or sound waves, caused by initial overdensities.

These waves were then frozen at the time of recombination and result in the characteristic over density of galaxies separated by the crossing sound horizon [2].

These features are proving instrumental in future and current wide-field red shift surveys, allowing the study of growth and evolution of the universe. Understanding the underlying mechanism using statistical means, is thus of upmost importance.

In Section II, we cover a rudimentary statistical description of the early universe, focusing mostly on the baryon-photon fluid, and discuss the essential thermodynamic properties (mainly density and pressure) and other features of the baryon-photon fluid. We also touch on the main assumptions and simplifications made.

In section III, we describe the density perturbations, the following acoustic oscillation and how they form the BAO feature.

In section IV, we conclude with a description of some future developments and refinement in the theory beyond what we cover in this review as well as some exciting uses of BAO along with other information extracted from the theory of the early universe.

II. STATISTICAL DESCRIPTION

A. State of The Early Universe

The energy-matter content prior to recombination was dominated by radiation, but can be broken up into four distinct parts that this review will focus on:

- dark matter (currently the majority of matter at 22.7 percent [3])
- baryons (in this context referring to protons and neutrons)
- photons
- neutrinos
- electrons (or fermions)

It is assumed that all other matter to be not present or in small enough fraction to be negligible. The reader may notice that dark energy is not included in this list. This is because, as we will see, the early universe dynamics is dominated by radiation at this epoch.

Dark matter was the largest source of density perturbations, but since it only interacts gravitationally, it does not lend itself to a statistical description. We will return to its role in BAO in Section III. Neutrinos are also negligible in our description as although they might be capable of being described statistically, the density perturbations smoothed out too quickly to be a significant driving force of the BAO feature [4]. We are therefore left with baryons, photons and electrons.

The energy density of a blackbody scales as a^{-4} for blackbody radiation and a^{-3} for non-relativistic matter [3]. Therefore, the radiation at this point dominates the density, and thus the early universe at this point can be treated as a plasma of relativistic particles. This is because the baryon-to-photon ratio was exceedingly small.

B. Baryon-photon ration

One of the important parameters determining the behavior of the primordial fluid is the baryon-to-photon ratio:

$$n = \frac{n_B}{n_\gamma}$$

where $n_B = n_{protons} + n_{neutrons}$. The number density of photons (as we will see) is temperature dependent as [3]:

$$n_\gamma(T) = \frac{2\zeta(3)}{\pi^2}\left[\frac{kT}{hc}\right]^3$$

This ratio and the density ratio are important for the evolution of the universe and for the BAO feature. This will be covered in more detail in Section IV.

C. Relativistic Regime

For the purposes of this review, we assume that the following treatment follows for all particles as at high enough temperatures and early enough times, matter species are a small fraction of the universe and relativistic. For relativistic particles, the energy momentum relation is:

$$\epsilon = hcp$$

The density of states is then given by:

$$\nu(\epsilon) = s\frac{1}{h^3}\int (d\epsilon)\delta(\epsilon - hcp)$$

where s is the spin. The density of states is then:

$$\frac{s\epsilon^2}{2\pi^2(h_{bar}c)^3}$$

We can now derive the major thermodynamic values.

The main values that are of interest for this review are the pressure and number density. The pressure is given by

$$P(T) = \int \nu(\epsilon) ln(1^{-1}e^{-\beta\epsilon}d\epsilon)$$

$$= \frac{s(kT)^4}{2\pi(h_{bar}c)^3}\int_0^\infty x^2 ln(1^{-x}dx)$$

And the number density is given by:

$$n(T) = \int \nu(\epsilon)\frac{1}{z^{-1}e^{-\beta\epsilon} \pm 1}d\epsilon)$$

$$= \frac{s(kT)^3}{2\pi^2(h_{bar}c)^3}\int_0^\infty \frac{x^3}{e^x \pm 1}dx)$$

where (+) is for fermions and (-) is for bosons. For the photons (and other purely relativistic species), $z = \beta\mu =$

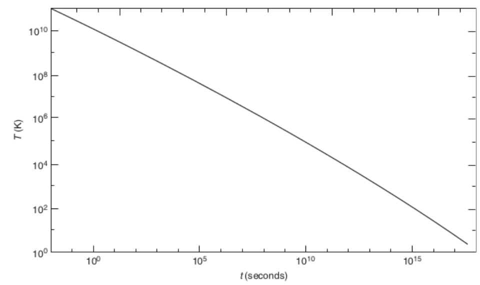


FIG. 1. Temperature of the Expanding Universe[3]

0. In this regime, we take The pressures for photons and electrons are then:

$$P_\gamma = \frac{\pi^2 kT^4}{45(h_{bar}c)^3}$$

$$P_e = (1 - \frac{1}{2^{n-1}})I^+_{n-1}(0)\frac{\pi^2 kT^4}{45(h_{bar}c)^3}$$

The number densities are:

$$n_\gamma(T) = \frac{2\zeta(3)}{\pi^2}\left[\frac{kT}{hc}\right]^3$$

$$n_e = (1 - \frac{1}{2^{n-1}})I^+_{n-1}(0)\frac{2\zeta(3)}{\pi^2}\left[\frac{kT}{hc}\right]^3$$

where I is the bose/fermi integral. The protons and neutrons at this time can be treated as an ideal gas with $\mu = mc^2 + kTln(n\lambda^3) - kTln2$ [3].

D. Expansion and Recombination

As the early universe expanded, it cooled. This rate is $T\ \frac{1}{sqrt(t)}$ at the early, relativistic dominated era and as $T\ \frac{1}{t^{2/3}}$ during the non-relativistic time [3]. This relation can be seen in Figure 1.

When the temperature fell to $T\ 2000K$, electrons combined with protons to form hydrogen.

$$\rho + e > H + \gamma$$

. When this occurred, the electrons fell out of the plasma and the photons were able to pass freely through the plasma [3]. This made the universe transparent. This radiation is the radiation detected in the CMB.

E. Properties of the primordial fluid

The main properties of the baryon-photon fluid are as folllows:

- The temperature at this time was very high and dominated by radiation (i.e. photons) which can be treated using Bose-Einstein statistics in the ultra-relativistic regime.
- The pressure scales as T^4 and the number density as T^3
- The protons/neutrons cannot be treated as ultra relativistic at later times, but can be trated as an ideal gas to to the high temperature and low number density.
- The fluid was at equilibrium due to the scattering of photons off the electron plasma. This continued until the expansion of the universe lowered the temperature and recombination occurred.

III. BARYON ACOUSTIC OSCILLATIONS

Now that the basic properties of the primordial fluid have been derived, a description of acoustic oscillations is possible. Random fluctuations in the primordial fluid caused small overdensities. Gravity then caused the surrounding matter to fall towards these overdensities [2]. As we saw in Section III, the pressure exerted by the photon gas is enormous and will cause any initial overdensities to immediately propagate away from the initial source as a wave (Figure 2a). These waves traveled outward before freezing at 150 Mpc (Figure 2b). This causes an overdensity at this location (Figure 2c).

A. Acoustic Oscillation

The acoustic perturbation travels outward at the speed of sound through the fluid. The speed of sound is given by[2]:

$$c_s = \frac{c}{\sqrt{3}(1+R)}$$

where $R = 3\rho_b/4\rho_\gamma$ (i.e. the baryon-to-photon fraction).

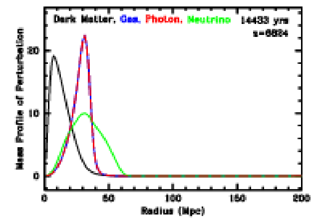
The maximum this wave could have traveled before the wave was frozen at recombination is called the crossing sound horizon and is given by integrating the speed of sound over the expansion parameter a(t) [2]:

$$r_s = \int_0^{t_{recom}} \frac{c_s dt}{a(t)} = \int_0^{t_{recom}} \frac{cdt}{\sqrt{3}a\sqrt{1+(3\Omega_b)/(4\Omega_\gamma)a}}$$

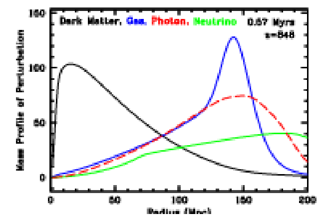
$$= \frac{c}{\sqrt{3}H_0} \int_0^{t_{recom}} \frac{da}{\sqrt{\Omega_r + a\Omega_m}\sqrt{1+(3\Omega_b)/(4\Omega_\gamma)a}}$$

For the accepted values $\Omega_b = .0492$, $\Omega_m = .3156$, $\Omega_\gamma = 5.45x10^{-5}$, $\Omega_r = 9.16x10^{-5}$, $H_0 = 67.3kms^{-1}Mpc^{-1}$ and $z_dec = 1090$ [2]:

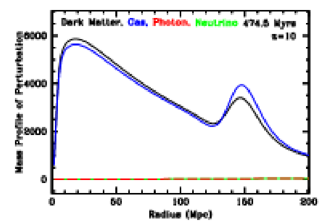
$$= \frac{3x10^8 m/s}{\sqrt{3}67.3kms^{-1}Mpc^{-1}} \int_0^{t_{recom}} (\frac{1}{\sqrt{9.16x10^{-5} + a(.3156)}} \frac{da}{\sqrt{1+(3(.0492))/(4(5.45x10^{-5}))a}}) = 144.7Mpc$$



(a) Propagation



(b) Recombination



(c) Gravitational Infall

FIG. 2. The Formation of the Baryon Acoustic Oscillation [4]. Fig. 2a illustrates the three main waves (baryon-photon fluid in red, neutrinos in green, and dark matter in black). Fig. 2b shows the time of recombination at 150 Mpc. Fig. 1c shows the time after recombination where dark matter has accumulated at the BAO bump due to gravitational infall. The distances here are distances today at z=0 and the densities are normalized.

B. Recombination and overdensity

A the time of recombination, the baryon-photon fluid became transparent and the photons were free to propagate. This propagation is what we observe as the cosmic background radiation (CMB) and the BAO feature has been detected using the CMB [3]. More important for the purposes of this review is that without the pressure from the photon gas, the acoustic wave became essentially frozen. A series of "snapshots" of this process is illustrated in Figure 1.

This created an initial overdensity of baryons at the crossing sound horizon. Dark Matter then fell into this overdensity and eventually caused an overdensity of galaxies. This can be seen in figure 3.

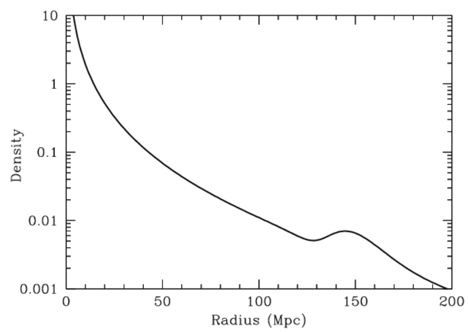


FIG. 3. This figure illustrates the BAO bump from a single acoustic oscillation [4]

IV. CONCLUSION

The BAO feature outlined in Section III and the statistical description outlined in Section II is of great importance in modern cosmology. As already mentioned, the existence of the CMB is a prediction of the well understood thermodynamics of the early universe. The BAO feature can be observed in the CMB and it, along with other clues left in the picture, are capable of providing a detailed picture of the evolution of the universe [5].

Using these methods, we can measure various cosmological parameters and use them to constrain general relativity and hopefully find a clue to new physics.

Several simplifications have been made in this cursory review which should be mentioned here. The simplistic description of acoustic oscillations is not the complete one. The neutrinos and other particles had a small but non-negligible effect on the location of the BAO feature and more rigorous analysis can better constrain this picture [5].

The single wave illustrated in Figure 2 is also not the complete picture as density perturbations occurred in many places and this caused these waves to overlap. They are not visible. However, we can still see a characteristic overdensity of galaxies at the location of the crossing sound horizon using statistical methods.

The BAO feature is an exciting area for development in cosmology. It was predicted long before it was detected, but it wasn't until the 21st century that this phenomena was detected. Recent and future large scale, deep field surveys will allow cosmologists to exploit this feature much more than in the past. This, along with the CMB, are responsible for much of the successes in recent cosmological work. Further research into the primordial fluid is being carried out, in the hopes that this might lead to possible confirmations of theory.

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[1] V. A. Rubakov, Cosmology (2015), arXiv:1504.03587 [astro-ph.CO].
[2] M. Ishak, Testing general relativity in cosmology, Living Reviews in Relativity **22**, 10.1007/s41114-018-0017-4 (2018).
[3] R. K. Pathira and P. D. Beale, *Statistical Mechanics*, 3rd ed. (Elsevier, Academic Press, 2011) pp. 211–292.
[4] D. Eisenstein, What is the acoustic peak?, `https://lweb.cfa.harvard.edu/~deisenst/acousticpeak/acoustic_physics.html` ().
[5] D. Eisenstein, Baryon acoustic oscillations ().