Information Sciences 507 (2020) 700–714

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Credit scoring using three-way decisions with probabilistic rough sets

Sebastián Maldonado[a,*], Georg Peters[b,c], Richard Weber[d]

[a] Universidad de los Andes, Facultad de Ingeniería y Ciencias Aplicadas, Mons. Álvaro del Portillo 12455, Las Condes, Santiago, Chile
[b] Department of Computer Science and Mathematics, Munich University of Applied Sciences, Munich 80335, Germany
[c] Australian Catholic University, Sydney, Australia
[d] Department of Industrial Engineering, Universidad de Chile, FCFM, Santiago, Chile

ARTICLE INFO

Article history:
Received 22 May 2017
Revised 26 July 2018
Accepted 1 August 2018
Available online 2 August 2018

Keywords:
Credit scoring
Business analytics
Three-way decisions
Probabilistic rough sets

ABSTRACT

Credit scoring is a crucial task within risk management for any company in the financial sector. On the one hand, it is in the self-interest of banks to avoid approving credits to customers who probably default. On the other hand, regulators require strict risk management systems from banks to protect their customers and, from “too big to fail institutions”, to avoid bankruptcy with negative impacts on an economy as a whole. However, credit scoring is also expensive and time-consuming. So, any possible method, like three-way decisions, to further increase its efficiency, is worth a try. We propose a two-step approach based on three-way decisions. Customers whose credit applications can be approved or rejected right away are decided in a first step. For the remaining credit applications, additional information is gathered in a second step. Hence, these decisions are more expensive than the ones in the first step. In our paper, we present a methodology to apply three-way decisions with probabilistic rough sets for credit scoring and an extensive case study with more than 7000 credit applications from Chilean micro-enterprises.

1. Introduction

In finance, credit scoring is one of the crucial challenges and a core responsibility for lenders in their risk management. For example, the financial crisis of 2007/2008 impressively shows possible impacts of faulty risk management [31]. Started as local sub-prime crisis in the housing sector in the U.S., the crisis spread around the world and severely infected the global economy as a whole. Even a decade later, the aftermaths of the crisis are still virulent, challenging economies around the world. As a lesson of the financial crisis, the rules for the assessment of credit ratings and the capital requirements for financial institutions have been tightened all over the world [14].

Credit scoring ranges from the rating of countries and global international companies to ratings of small enterprises applying for credits. Furthermore, individuals are regularly assessed regarding their creditworthiness, whether they are applying for a large home loan or just seeking for a small mobile phone contract.

The impacts of credit scoring are significant. On the one hand, approving credits to clients who fail to repay, may lead to affordable losses in the best case but may also jeopardize a lender's business or even have severe impacts on macroeconomic levels like in the financial crisis in the worst case. On the other hand, strict rules for credit approvals may damage an

* Corresponding author.
E-mail addresses: smaldonado@uandes.cl (S. Maldonado), georg.peters@cs.hm.edu (G. Peters), richard.weber@uchile.cl (R. Weber).

https://doi.org/10.1016/j.ins.2018.08.001
0020-0255/

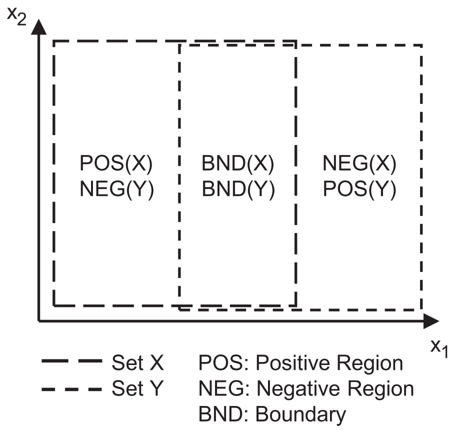

Fig. 1. Positive and negative regions and boundaries for the sets *X* and *Y*.

economy that could be strengthened by affording credits. Consumer credits may increase the demand for goods whereas investment credits may support the supply side of an economy.

When performing credit scoring three results are normally obtained. Some clients are solvent at the first sight while some other clients instantly fail to be creditworthy. The remaining clients need to be assessed in more detail before a final decision on the approval or disapproval of the credit can be made. More generally, the intensities of assessment required are not constant for all clients but they vary from client to client. They depend on the amount of information that is needed for a particular decision. The principles of such three-way decisions are widely applied in a diverse range of areas, such as medical diagnosis [28,35], computer vision [19,33], or recommender systems [50,51]. However, only recently Yao [41–43] proposed a formal framework for three-way decisions that is derived from probabilistic rough sets. Yao proposed the theory of three-way decisions to obtain cost efficient categorizations of objects into such three classes; the positive and negative ones and the objects whose final decision should or must be postponed.

The objective of this paper is to propose a methodology for credit scoring that minimizes the decision-relevant costs using three-way decisions with probabilistic rough sets. Furthermore, we underline this methodology's potential via a real-world application, where we analyze a data set that was collected by a Chilean bank. It consists of more than 7000 credit applications from small and micro-companies and their evaluations. In our analysis, we take the perspective of financial accounting as it is relevant for banking regulation.

In Section 2, we review the theory of three-way decisions and present some of their applications. In Section 3, we develop our framework for credit scoring using three-way decisions and design decision rules to take the final deterministic decision for the case of credit granting. The subsequent section contains the results of applying this framework to the before-mentioned data. The paper concludes with a summary in Section 5.

2. Three-way decisions

In Section 2.1, we summarize the basic concept of three-way decisions. Then we present selected applications of the theory of three-way decisions in Section 2.2.

2.1. Basic concept of three-way decisions

2.1.1. Rough sets

The theory of three-way decisions was proposed by Yao [41,42] as a new perspective of rough sets. Therefore, to nest three-way decisions into rough set theory, we briefly recall the basic idea of rough sets.

Like in classic set theory, in rough sets [32] an object belongs to one and only one set. However, while for some objects their membership values can be determined, this is not the case for others. To reflect this, a rough set X is defined by two approximations, viz., the lower approximation $\underline{X}$ that comprises the objects x that surely belong to the set and the upper approximation $\overline{X}$ that contains the sure objects as well as those objects that possibly belong to the set X. Consequently, the lower approximation is a subset of the upper approximation: $\underline{X} \subseteq \overline{X}$. The difference between the upper approximation and the lower approximation is called boundary of set X: $BND(X) = \overline{X} \setminus \underline{X}$. The boundary contains objects with unclear membership. Due to missing or incorrect information they may or may not belong to the set X. This implies that a boundary object belongs to at least two boundaries indicating its unclear membership (see Fig. 1 for an example of two rough sets X and Y). The lower approximation is also called positive region of X ($POS(X)$). The region not covered by the upper approximation is called negative region of X ($NEG(X)$). The negative region comprises the objects that surely do not belong to the set X.

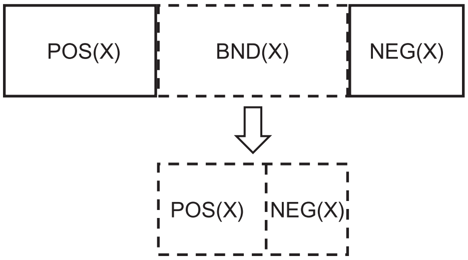

Fig. 2. Example for a two-step decision process.

Table 1
Costs λ associated with a decision.

Decision	Object actually:	
	Positive	Negative
Positive	$\lambda_{\text{POS, POS}}$	$\lambda_{\text{POS, NEG}}$
Boundary	$\lambda_{\text{BND, POS}}$	$\lambda_{\text{BND, NEG}}$
Negative	$\lambda_{\text{NEG, POS}}$	$\lambda_{\text{NEG, NEG}}$

2.1.2. Introduction to three-way decisions

The following paragraphs are based on Yao's papers on three-way decisions [41,42]. The respective theory is a generalization of classic dichotomous two-way decisions (decide 'positive' or 'negative') by adding a third decision category, the 'boundary'. The boundary contains those objects that cannot be decided at the moment ('don't know yet'), i.e., the decision is postponed until further notice.

This implies that in three-way decisions the definition of a rough set by its positive region, negative region and, in particular, the boundary is of special importance. At any point of the decision process, except the last step, it is decided if a final verdict about an object's membership can be made or not. In the affirmative case, it is decided if the object belongs to the set X (positive region of X: $POS(X)$) or if it does not belong to the set X (negative region of X: $NEG(X)$). Otherwise the final verdict about the membership of the object is postponed, i.e., the objects are assigned to the boundary ($BND(X)$). After obtaining additional information the boundary objects are evaluated again if they can be classified as positive or negative or if the final decision has to be postponed again. This decision procedure continues until all objects are assigned to the positive ($POS(X)$) or negative region ($NEG(X)$), i.e., until no further objects remain in the boundary. See Fig. 2 for an example of a two-step decision process applying three-way decisions.

It is important to note that in all steps any final verdict involves risk, i.e., that there is a certain probability that an object that is assumed to be positive turns out to be negative and vice versa. Furthermore, postponing a decision has positive as well as negative impacts. Gathering additional information may possibly lead to a better decision but is generally time consuming and expensive. Hence, postponing a decision only makes sense, when the positive effects outweigh the negative ones.

Therefore, it is crucial to apply a cost-based analysis of any possible decision to obtain optimal results. For three possible decisions ('positive', 'negative', and 'boundary') and two actual states of an object ('positive' and 'negative') a 3x2 matrix for the costs λ associated with any decision is obtained (see Table 1).

Note, that the term 'costs' used in the context of three-way decisions is more general than the term 'costs' in cost accounting. In three-way decisions costs refer to any kind of unit that needs to be minimized, including 'costs' in cost accounting but not limited to. For example, other units could be time or negative cash flows in financial accounting as in our application.

For example, in Table 1 $\lambda_{\text{POS, POS}}$ are the costs, that occur when a positive object is assigned to the positive class. The costs $\lambda_{\text{POS, NEG}}$ have to be paid, if a negative object is assigned to the positive class. Note, this categorization is similar to binary classification [12,37]: $\lambda_{\text{POS, POS}}$ are the costs associated with true positives (t_p) and $\lambda_{\text{POS, NEG}}$ is related to false negatives f_n. Similar interpretations apply for $\lambda_{\text{NEG, NEG}}$ and $\lambda_{\text{POS, NEG}}$.

In addition to this binary classification, in three-way decisions, costs of postponing a decision are considered. When an object is actually positive but the decision is postponed, it is put into the boundary and costs of $\lambda_{\text{BND, POS}}$ arise (similarly for $\lambda_{\text{BND, NEG}}$).

In the next paragraphs, we briefly summarize some decision rules as proposed by Yao [42]. They can directly be derived from the costs as given in Table 1.

General case. First, we define the probabilities that an object x belongs to the positive and negative region, respectively:

- $P(C|[x])$ is the probability that an object x belongs to set C. In our case, we assume that C represents the actually positive cases.

- $P(C^C|[x])$ is the probability that an object x does not belong to set C. In our case, we assume that C^C represents the actually negative cases.

Given these probabilities and the costs as depicted in Table 1, the expected losses $R()$ for any possible decision (*POS, NEG*, or *BND*) for object x can be calculated as follows [39,40,56]:

$$\begin{array}{lcl} R(POS|[x]) & = & P(C|[x])\lambda_{\mathrm{POS,POS}} + P(C^C|[x])\lambda_{\mathrm{POS,NEG}} \\ R(BND|[x]) & = & P(C|[x])\lambda_{\mathrm{BND,POS}} + P(C^C|[x])\lambda_{\mathrm{BND,NEG}} \\ R(NEG|[x]) & = & P(C|[x])\lambda_{\mathrm{NEG,POS}} + P(C^C|[x])\lambda_{\mathrm{NEG,NEG}} \end{array} \tag{1}$$

To assign an object x to the positive or negative region, or alternatively to postpone the final decision (boundary region), Bayesian decision rule is applied (for simplicity's sake tie-breaking rules are omitted in Eq. (2)) [42]:

$$\begin{array}{llllll} \text{If} & R(POS|[x]) \le R(NEG|[x]) & \text{and} & R(POS|[x]) \le R(BND|[x]) & \text{decide} & x \text{ as } POS \\ \text{If} & R(BND|[x]) \le R(POS|[x]) & \text{and} & R(BND|[x]) \le R(NEG|[x]) & \text{decide} & x \text{ as } BND \\ \text{If} & R(NEG|[x]) \le R(POS|[x]) & \text{and} & R(NEG|[x]) \le R(BND|[x]) & \text{decide} & x \text{ as } NEG \end{array} \tag{2}$$

Yao [42] proposed to simplify this general case when certain conditions hold. Furthermore, he developed a perspective on three-way decisions based on relative costs instead of absolute costs. In the following paragraphs, we briefly review those cases that are relevant in this paper (simplified cases 1 and 2).

Simplified Case 1. In contrast to classic rough sets, in probabilistic rough sets [39,40,56] the decision if an object is assigned to the positive or negative region or to the boundary involves risk. For an optimal decision threshold probabilities can be determined.

If conditions as given in Eq. (3):

$$\begin{array}{l} \lambda_{\mathrm{POS,POS}} \le \lambda_{\mathrm{BND,POS}} < \lambda_{\mathrm{NEG,POS}} \\ \lambda_{\mathrm{NEG,NEG}} \le \lambda_{\mathrm{BND,NEG}} < \lambda_{\mathrm{POS,NEG}} \end{array} \tag{3}$$

hold, the following decision rules are obtained:

$$\begin{array}{llllll} \text{If} & P(C|[x]) \ge \alpha & \text{and} & P(C|[x]) \ge \gamma & \text{decide} & x \text{ as } POS \\ \text{If} & P(C|[x]) \le \alpha & \text{and} & P(C|[x]) \ge \beta & \text{decide} & x \text{ as } BND \\ \text{If} & P(C|[x]) \le \beta & \text{and} & P(C|[x]) \le \gamma & \text{decide} & x \text{ as } NEG \end{array} \tag{4}$$

with the threshold probabilities α, β, and γ defined as:

$$\begin{aligned} \alpha &= \frac{\lambda_{\mathrm{POS,NEG}} - \lambda_{\mathrm{BND,NEG}}}{(\lambda_{\mathrm{POS,NEG}} - \lambda_{\mathrm{BND,NEG}}) + (\lambda_{\mathrm{BND,POS}} - \lambda_{\mathrm{POS,POS}})} \\ \beta &= \frac{\lambda_{\mathrm{BND,NEG}} - \lambda_{\mathrm{NEG,NEG}}}{(\lambda_{\mathrm{BND,NEG}} - \lambda_{\mathrm{NEG,NEG}}) + (\lambda_{\mathrm{NEG,POS}} - \lambda_{\mathrm{BND,POS}})} \\ \gamma &= \frac{\lambda_{\mathrm{POS,NEG}} - \lambda_{\mathrm{NEG,NEG}}}{(\lambda_{\mathrm{POS,NEG}} - \lambda_{\mathrm{NEG,NEG}}) + (\lambda_{\mathrm{NEG,POS}} - \lambda_{\mathrm{POS,POS}})} \end{aligned} \tag{5}$$

Simplified Case 2. The following supplementary condition Eq. (6):

$$(\lambda_{\mathrm{POS,NEG}} - \lambda_{\mathrm{BND,NEG}})(\lambda_{\mathrm{NEG,POS}} - \lambda_{\mathrm{BND,POS}}) > (\lambda_{\mathrm{BND,POS}} - \lambda_{\mathrm{POS,POS}})(\lambda_{\mathrm{BND,NEG}} - \lambda_{\mathrm{NEG,NEG}}) \tag{6}$$

implies that $0 \le \beta \le \alpha \le 1$ which makes γ obsolete. Hence, in this case, the decision rules as defined in Eq. (4) can be further simplified:

$$\begin{array}{llll} \text{If} & P(C|[x]) \ge \alpha & \text{decide} & x \text{ as } POS \\ \text{If} & P(C|[x]) \le \beta & \text{decide} & x \text{ as } NEG \\ \text{If} & \beta < P(C|[x]) < \alpha & \text{decide} & x \text{ as } BND \end{array} \tag{7}$$

2.2. Applications of three-way decisions

Three-way decisions have already been applied to a wide range of areas. In the following paragraphs we briefly discuss selected applications to other methods and applications in practice. Note, that the distinction between applications to other methods and applications in practice is not disjunctive but has arbitrary elements.

Applications to other methods. The concept of three-way decisions has already been applied intensively to clustering [46]. For example, Yu and Wang [47] proposed a clustering method that uses three-way decisions for overlapping clusters. While some objects can be assigned to some cluster right away, the decision on objects in the overlapping region are postponed; they are put into the boundary region first. Yu et al. [48] suggested an incremental clustering approach applying the concept of three-way decisions and, recently, Yu el al. [49] discussed a tree-based incremental overlapping clustering method supported by three-way decisions.

Applications of three-way decisions addressing sequential aspects of a decision process were proposed by Yao and Deng [45]. In the context of granular computing, Yao [44] discussed sequential three-way decision strategies. Li et al. [20] focused on a sequential strategy and cost-sensitive three-way decisions.

Finally, Zhang et al. [52] presented a study on the ranking of interval sets based on inclusion measures in the context of three-way decisions.

Applications in practice. The underlying concept of three-way decisions has already been applied widely in practice; it can even be considered as a kind of business wisdom. Back in the nineties, Gehrlein and Wagner [11] presented a simplified model for such kind of decisions to predict a firm's bankruptcy.

Later, Yao formalized three-way decisions with probabilistic rough sets (as presented in Section 2.1.2), which are an emerging approach for decision making in practice, as will be shown next.

Three-way decisions have been intensively applied to email filtering. For example, back in 2010, Zhou et al. [54] presented a study how to detect spam emails using this concept. Jia et al. [16] performed an empirical study on three-way decisions and spam email filtering. A multistage approach for filtering such emails using three-way decisions was proposed by Li et al. [21] in 2013. Finally, a study comparing two and three-way decision methods for detecting SPAM emails was performed by Jia and Shang [15].

Three-way decisions were also applied to government decision analysis [24]. Liu et al. [26] reported on a study where three-way decisions were used to support investment decisions.

Yao and Azam [38] applied a three-way model and game-theoretic rough sets to a web-based medical decision support system, and Zhu et al. [55] used the concept of three-way decisions and a naïve Bayes classifier to identify subjective sentences in micro-blogs. Three-way decisions were also used for linguistic assessment with support of group decision making [23]. In the field of pattern recognition, Li et al. [18] presented a three-way decision approach for cost-sensitive face recognition, whereas Savchenko [33] applied sequential three-way decisions to multi-class object recognition. Zhang and Min [50] proposed a recommender system combining random forests and three-way decisions and showed its performance for movie recommendation. Using three-way decisions for text classification has been proposed by Zhou et al. [53] providing superior results compared to alternative methods. Li et al. [22] contributed to the area of software defect prediction showing that their method based on three-way decisions can obtain a higher accuracy and lower decision costs compared to traditional approaches.

3. Applying three-way decisions to credit scoring

3.1. Credit scoring for credit granting

The notion of granting credits goes back to the early days of the banking system in ancient Greece [30]. From the very beginning it was important for the lender to discriminate between good and bad payers. However, it was not until the first half of the 20th century that quantitative techniques offered decision support for such tasks. Based on the ideas presented by Fisher [10], who showed how to discriminate between groups in a population, Durand [8] was the first to recognize that the same techniques could be used to classify loans. Following those developments, we understand credit scoring in this paper as a set of quantitative models and their underlying techniques that aid lenders in the process of granting credits.

In 1956, Fair and Isaac founded a company where they introduced a credit scoring system two years later [9,36]. In the sixties of the last century, Altman [1] developed a scoring model based on five financial ratios to predict corporate bankruptcy. The so-called Z-score is a linear combination of those ratios. From those early quantitative models up to today, many advanced techniques have been proposed for credit scoring, such as, e.g., decision trees, neural networks, support vector machines, among others (see, e.g., Louzada et al. [27]). Baesens et al. [2] compared these techniques using benchmark data sets from credit scoring.

Generally, credit scoring is important for a bank because of two different reasons [29]: cost accounting and financial accounting. From the perspective of *cost accounting*, credit scoring is important to optimize the cost structure of a bank. In this case, the bank's managers are the target group. From the perspective of *financial accounting*, however, credit scoring has impact on the cash flow, and thereby the balance sheet of a bank. Now, internal as well as external stakeholders of the bank, including shareholders and regulators, are the target group. Both areas, cost and financial accounting, are partly overlapping but also have some distinguished differences. Regarding their focus, in cost accounting outlay and opportunity costs should be considered while financial accounting includes non-operating expenses. Cost accounting, although having some well accepted standards, is not controlled by a regulatory framework. In contrast to this, financial accounting is governed by regulations such as GAAP (Generally Accepted Accounting Principles) [34] and is the approach we adopted in our paper.

Motivated by several financial crises, countries around the world have agreed on stricter regulations for the financial service industry. The Basel II Accord, released in 2004 [3], tries to align bank capital requirements with underlying risks, such as credit risk, market risk, and operational risk. Among many other issues, it requires banks to make sure "that data flows and processes associated with the risk measurement system are transparent and accessible" (Basel Committee on Banking Supervision [3], p. 156). As a consequence advanced non-linear techniques, like e.g. neural networks, are not allowed to be used for credit scoring due to their black box behavior, even if they would provide best results among all available methods. Therefore, banks need to apply methods, like logistic regression, that are transparent and deliver comprehensible results.

Table 2
Costs λ associated with credit scoring on Level 1 and Level 2.

Decision on		Object actually	
Level 1	Level 2	POS	NEG
POS	–	$\lambda_{\text{POS,POS}} = EC1$	$\lambda_{\text{POS,NEG}} = EC1$
BND	POS	$\lambda^{\text{BND}}_{\text{POS,POS}} = EC1 + EC2$	$\lambda^{\text{BND}}_{\text{POS,NEG}} = EC1 + EC2$
BND	NEG	$\lambda^{\text{BND}}_{\text{NEG,POS}} = LGD + EC1 + EC2$	$\lambda^{\text{BND}}_{\text{NEG,NEG}} = -INT + EC1 + EC2$
NEG	–	$\lambda_{\text{NEG,POS}} = LGD + EC1$	$\lambda_{\text{NEG,NEG}} = -INT + EC1$

What, however, almost all models mentioned before have in common is that they are used to support binary decisions, i.e., to accept or to reject a requested loan. Only recently, formal quantitative models have been suggested to classify borrowers into three classes as has been used for decades by experts using their business knowledge and experience. For instance, Bravo et al. [5] proposed to differentiate defaulters into two classes, generally known as 'can't pay' and 'won't pay'. Together with the class of good payers this leads to a three-class classification model which has shown to give superior results compared to the classic binary approach. The respective gain comes with the costs of several assumptions regarding the borrowers' behavior which are not always fulfilled. In this paper, we use a different approach and propose formalizing the intuitive procedure of classifying borrowers into three instead of two classes using the theory of three-way decisions [42] presented in Section 2. Consequently, this leads to a two-step decision process with the goal of minimizing the decision-relevant costs.

3.2. Framework for credit scoring applying three-way decisions

We follow the idea of Liu et al. [25] of applying logistic regression to calculate the probabilities that an object x belongs to the positive and negative region, respectively. In Section 3.2.1, we motivate why logistic regression is particularly useful for credit scoring.

Furthermore, we apply Yao's [42] general model for three-way decisions as described in Section 2.1.2. Since we use a two-step approach as depicted in Fig. 2, we distinguish two levels of decisions depending on the sets of variables we use. We refer to 'Level 1' when only a basic set of features is used for decision making. In contrast to this, we refer to 'Level 2' when all available features are considered.

In the case of three-way decisions, we discuss properties of decision making on Level 1 in Section 3.2.2. The final decision on Level 2 for the boundary objects from Level 1 will be analyzed in Section 3.2.3.

3.2.1. Logistic regression

In the case of credit scoring, the final result of a bank's evaluation process can be positive or negative: a credit application is approved or rejected. Logistic regression is a well-known and widely applied method for such cases, i.e., when the dependent variable is binary (see, e.g., [13,17] for further details on logistic regression). Since models obtained by using logistic regression are easy to explain and to implement, this approach has been widely accepted in the banking industry as the method of choice since it is compliant with the Basel II Accord [3]. Therefore, we follow this industry standard and apply logistic regression to probabilistically classify credit applications and apply it to Level 1 as well as to Level 2. We define the probabilities as follows: For an object (customer) x, we obtain a probability $P^{L_i}_{\text{LOGIT}}(C|[x])$ that it belongs to the positive class (defaulter) on Level i ($i = 1, 2$). As the negative case is complementary to the positive ($P^{L_i}_{\text{LOGIT}}(C^C|[x]) = 1 - P^{L_i}_{\text{LOGIT}}(C|[x])$) ($i = 1, 2$), we only use probabilities for the positive (defaulter) and abbreviated them as $P^{L_i}_{\text{LOGIT}} = P^{L_i}_{\text{LOGIT}}(C|[x])$ in the further course of the paper.

3.2.2. Three-way decisions for credit scoring: decision on Level 1

Basic decision process on Level 1. As we focus on a financial accounting perspective (see Section 3.1), we only consider financial cash flows as "costs" as defined within the theory of three-way decisions in our analysis (see Section 2.1.2). Table 2 depicts these costs associated with the decisions of the credit scoring process. The variables are defined as follows:

- $EC1$: Evaluation costs on the first level of assessment
- $EC2$: Evaluation costs on the second level of assessment
- INT: Interest payable by the credit debtor
- LGD: Loss given default, i.e., expected loss of the bank when a credit is not paid back.

Furthermore, in Table 2, e.g., $\lambda^{\text{BND}}_{\text{NEG,POS}}$ are the costs on Level 2, when the decision taken for an object in the boundary (BND) is negative (NEG) while it is actually positive (POS).

The evaluation costs $EC1$ and $EC2$ are identical for all customers. INT and LGD, however, are different for each requested credit: $LGD = fct(c_x)$ and $INT = fct(c_x, i_x, n_x)$, with x a particular customer requesting a credit, $c_x =$ the amount of the requested credit, $i_x =$ the interest rate, and $n_x =$ the term of the credit. The costs of the boundary region depend on the decision taken on Level 2, i.e., after the interview with the applicant has been conducted.

Table 3
Decision-relevant costs λ associated with the credit decision on Level 1.

Decision on		Object actually	
Level 1	Level 2	POS	NEG
POS	–	$\lambda_{POS,POS}=0$	$\lambda_{POS,NEG}=0$
BND	POS	$\lambda^{BND}_{POS,POS}=EC2$	$\lambda^{BND}_{POS,NEG}=EC2$
BND	NEG	$\lambda^{BND}_{NEG,POS}=LGD+EC2$	$\lambda^{BND}_{NEG,NEG}=-INT+EC2$
NEG	–	$\lambda_{NEG,POS}=LGD$	$\lambda_{NEG,NEG}=-INT$

Table 4
Decision-relevant costs λ associated with the credit decision on Level 1 with estimated boundary costs.

Decision	Object actually	
	POS	NEG
POS	$\lambda_{POS,POS}=0$	$\lambda_{POS,NEG}=0$
BND	$\lambda^{BND}_{*,POS}=(1-P^{BND}_{POS,POS})LGD+EC2$	$\lambda^{BND}_{*,NEG}=-(1-P^{BND}_{POS,NEG})INT+EC2$
NEG	$\lambda_{NEG,POS}=LGD$	$\lambda_{NEG,NEG}=-INT$

However, not all costs shown in Table 2 are relevant for the credit decision. For example, the evaluation costs $EC1$ are irrelevant for the decision on Level 1. They cannot be manipulated but occur independently of the decision that will be taken. The evaluation costs $EC2$ arising on Level 2, however, must be considered on Level 1. They can be avoided when it is decided not to proceed to Level 2. This leads to the decision-relevant costs on Level 1 as shown in Table 3.

Note, that the relevant costs on Level 1 (Table 3) depend on the decision taken on Level 2. However, this decision is taken later and, therefore, unknown when the decision takes place on Level 1. Hence, we calculate the expected costs on Level 2

$$\begin{aligned}\lambda^{BND}_{*,POS} &= P^{BND}_{POS,POS}\lambda^{BND}_{POS,POS}+P^{BND}_{NEG,POS}\lambda^{BND}_{NEG,POS}\\ \lambda^{BND}_{*,NEG} &= P^{BND}_{POS,NEG}\lambda^{BND}_{POS,NEG}+P^{BND}_{NEG,NEG}\lambda^{BND}_{NEG,NEG}\end{aligned} \tag{8}$$

with, e.g., $P^{BND}_{POS,NEG}$ the probability that on Level 2 a boundary object (*BND*) is decided to be positive (*POS*) while it is actually negative (*NEG*). The star in, e.g., $\lambda^{BND}_{*,POS}$ indicates that these are the expected costs for the boundary decisions for a *POS* object. Furthermore, $P^{BND}_{POS,POS}=1-P^{BND}_{NEG,POS}$ and $P^{BND}_{POS,NEG}=1-P^{BND}_{NEG,NEG}$ hold. Then, we obtain the decision-relevant costs for Level 1 as depicted in Table 4.

While the costs are given now, the probabilities still need to be derived from the data. We consider a reasonable estimation of the probabilities $P^{BND}_{POS,POS}$ and $P^{BND}_{POS,NEG}$ as one of the most crucial challenges when applying three-way decisions in our methodology.

Approximation of the decision on Level 2. In the following paragraphs, we discuss a strategy for estimating the probabilities $P^{BND}_{POS,POS}$, $P^{BND}_{NEG,POS}$, $P^{BND}_{POS,NEG}$, and $P^{BND}_{NEG,NEG}$ from the given data. We take the training data (see Section 4.2) and count the frequency of occurrence of any possible combination of binary decisions based on logistic regression observed on Level 1 and Level 2.

For example, a customer is positive (defaulter), i.e., its actual classification is positive (*POS*). However, the prediction obtained by the logistic regression model on Level 1 indicates that it is negative. Hence, it is a false-positive case (f_p) regarding the results of the logit model on Level 1. In contrast to the results of the logit model on Level 1, the logit model for Level 2 may correctly predict that the same customer is positive. Therefore, it is true-positive (t_p) with respect to the prediction of the logit model on Level 2.

In total, we get eight combinations depending on the prediction/decisions made by the logit models for Level 1 and Level 2. They are depicted in Table 5 together with their probabilities of occurrence P_i $(i=1,\ldots,8)$ and the associated costs on Levels 1 and 2. In case of the example above, we assign the probability P_3 to this case. The costs of Level 1 are $\lambda_{NEG,\,POS}$ indicating the actual positive classification and the negative prediction and the costs on Level 2 are $\lambda^{BND}_{POS,POS}$, respectively.

Note, that P_i does not depend on a particular customer, i.e., $P_i \neq fct(x)$ and consequently $P_{POS} \neq fct(x)$ and $P_{NEG} \neq fct(x)$. The probabilities P_i are derived from the binary classifiers on Level 1 and Level 2 using the training set and are identical for all objects. Hence, these probabilities should not be confused with $P(C|[x]) = P_{LOGIT}(C|[x])$, the probability of default for each customer.

With the following costs and associated probabilities

Table 5
Possible costs for Level 1 and Level 2 decisions.

Actually	Probability	Binary classifier		Costs	
		Level 1	Level 2	Level 1	Level 2
POS	P_1	t_p	t_p	$\lambda_{POS,\ POS}$	$\lambda^{BND}_{POS,POS}$
$P_{POS} = \sum_{i=1}^{4} P_i$	P_2	t_p	f_p	$\lambda_{POS,\ POS}$	$\lambda^{BND}_{NEG,POS}$
	P_3	f_p	t_p	$\lambda_{NEG,\ POS}$	$\lambda^{BND}_{POS,POS}$
	P_4	f_p	f_p	$\lambda_{NEG,\ POS}$	$\lambda^{BND}_{NEG,POS}$
NEG	P_5	t_n	t_n	$\lambda_{NEG,\ NEG}$	$\lambda^{BND}_{NEG,NEG}$
$P_{NEG} = \sum_{i=5}^{8} P_i$	P_6	t_n	f_n	$\lambda_{NEG,\ NEG}$	$\lambda^{BND}_{POS,NEG}$
	P_7	f_n	t_n	$\lambda_{POS,\ NEG}$	$\lambda^{BND}_{NEG,NEG}$
	P_8	f_n	f_n	$\lambda_{POS,\ NEG}$	$\lambda^{BND}_{POS,NEG}$

Table 6
Decision-relevant costs λ associated with credit scoring on Level 2.

Decision	Object actually	
	POS	NEG
POS	$\lambda_{POS,POS} = 0$	$\lambda_{POS,NEG} = 0$
NEG	$\lambda_{NEG,POS} = LGD$	$\lambda_{NEG,NEG} = -INT$

$$\begin{array}{lll|l} & \text{Cost} & \text{Occurs with probability} & \\ \hline \lambda^{BND}_{*,POS}: & \lambda^{BND}_{POS,POS} = EC2 & P_1 + P_3 \text{ and} & \\ & \lambda^{BND}_{NEG,POS} = LGD + EC2 & P_2 + P_4 & \text{we obtain} \\ \lambda^{BND}_{*,NEG}: & \lambda^{BND}_{POS,NEG} = EC2 & P_6 + P_8 \text{ and} & \\ & \lambda^{BND}_{NEG,NEG} = -INT + EC2 & P_5 + P_7, & \end{array}$$

$$\begin{aligned} \lambda^{BND}_{*,POS} &= \frac{(P_1+P_3)\lambda^{BND}_{POS,POS} + (P_2+P_4)\lambda^{BND}_{NEG,POS}}{P_1+P_2+P_3+P_4} \\ &= \frac{P_2+P_4}{P_{POS}} LGD + EC2 \\ \lambda^{BND}_{*,NEG} &= \frac{(P_6+P_8)\lambda^{BND}_{POS,NEG} + (P_5+P_7)\lambda^{BND}_{NEG,NEG}}{P_5+P_6+P_7+P_8} \\ &= -\frac{P_5+P_7}{P_{NEG}} INT + EC2. \end{aligned} \tag{9}$$

Summary of the Level 1 Decision. Now, we can specify the expected losses $R()$ on Level 1 as defined in the general model for three-way decisions (Eq. (1)). The probabilities $P(C|[x])$ in Eq. (1) are estimated from a logistic regression model $P(C|[x]) = P^{L1}_{LOGIT}(C|[x])$ (subsequently abbreviate as P^{L1}_{LOGIT}). In contrast to P_i, they are customer-specific. We get:

$$\begin{array}{ll} R(POS|[x]) = 0 & \text{(see Tab. 4)} \\ R(BND|[x]) = P^{L1}_{LOGIT} \frac{P_2+P_4}{P_{POS}} LGD - (1 - P^{L1}_{LOGIT}) \frac{P_5+P_7}{P_{NEG}} INT + EC2 & \text{(see Eq. 9)} \\ R(NEG|[x]) = P^{L1}_{LOGIT} LGD - (1 - P^{L1}_{LOGIT}) INT & \text{(see Tab. 4)} \end{array} \tag{10}$$

Then, finally, the decision rules as defined in Eq. (2) can be applied.

3.2.3. Three-way decisions for credit scoring: decision on Level 2

On Level 2, the final decision for the boundary objects on Level 1 is made. Here, both evaluation costs $EC1$ and $EC2$ are irrelevant for the decision. The evaluation costs $EC1$ already occurred on Level 1. Hence, they are sunk costs and irrelevant for the current decision. For the evaluation costs $EC2$ the same applies as already discussed above on Level 1 for $EC1$. Hence, we obtain costs that are relevant for decision making on Level 2 as depicted in Table 6.

For the expected losses we get:

$$\begin{array}{ll} R(POS|[x]) = 0 & \text{(see Tab. 6)} \\ R(NEG|[x]) = P^{L2}_{LOGIT} LGD - (1 - P^{L2}_{LOGIT}) INT & \text{(see Tab. 6)} \end{array} \tag{11}$$

and as rules for the two-way decision:

$$\begin{array}{lllll} \text{If} & R(POS|[x]) \leq R(NEG|[x]) & \text{decide} & x \text{ as} & POS \\ \text{Else} & & \text{decide} & x \text{ as} & NEG \end{array} \tag{12}$$

Thus, the boundary objects are assigned to the positive or negative region. So, finally all credit applications have been decided.

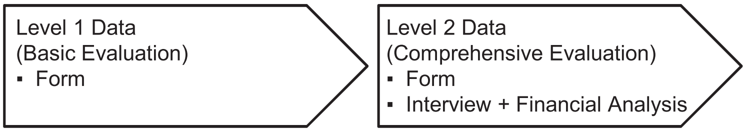

Fig. 3. Credit approval process.

Table 7
Number of loans and features.

Customers	Returning (RET)	New (NEW)
	Number of loans	
Total number	5799	1510
· Defaulters	872	629
· Non-defaulters	4927	881
	Number of features	
Form	31	32
Interview + Analysis	15	62
· Interview	2	5
· Financial analysis	13	57

4. Real-world application

In this section, we use the methodology introduced in Section 3 to a real-world data set for credit scoring. Section 4.1 provides important information on this application, such as a description of the available data set and the credit approval process. Section 4.2 presents the experimental set-up we apply. The results for returning customers are discussed in Section 4.3. Subsequently, Section 4.4 exhibits the results for new customers. Finally, managerial implications are discussed in Section 4.5.

4.1. Background to the application

Credit data set. The data we use were collected in a credit assignment project at a Chilean bank. The project consists of 7309 loans granted to small and micro-entrepreneurs, which are divided in two groups in terms of their credit history with the bank: returning customers (RET) already had a credit history with the bank and new customers (NEW) who applied for a credit at the bank for the first time. Loans were repaid in monthly installments during the period 2004–2007. If one or more installments were in arrears for more than 90 days during the first year of the loan the borrower was considered as a defaulter.

Credit approval process. The credit approval process was identical for all applicants. It consisted of a two-step decision scheme as follows (see Fig. 3). First, a basic credit evaluation was performed, which corresponds to a form that each potential borrower filled out together with an executive in one of the bank's offices. In our case, we refer to this first assessment as Level 1 evaluation.

Second, an in-depth interview of the potential borrower was conducted in her/his place of work, leading to additional information related to the micro-company. After the in-depth interview, a specialist of the bank analyzed the micro-company's financial accounting, aiming at reconstructing its financial transactions.

We refer to the comprehensive credit evaluation based on all data, i.e., the form (Level 1) as well as the interview and the financial analysis, as Level 2 evaluation. Note, the final credit decision was made for all applications after the Level 2 evaluation.

In this application, we use the paying behavior (good/bad payer) as dependent variable. The resulting data sets' metadata are summarized in Table 7. The original data set was already preprocessed by using simple filter methods for feature selection. We discarded variables with more than 30% of missing values, those concentrated in a single value in more than 99% of the cases, and those uncorrelated with the target variable using the Kolmogorov–Smirnov and χ^2-tests [7]. It can be seen in Table 7 that, after preprocessing, there are more candidate variables for the new customers than for the returning ones. This counter-intuitive result can be explained by the fact that a returning customer is usually a good payer who comes back for a new loan, and, therefore, it does not go through the thorough examination process that new customers experience. For returning customers, fewer variables were collected during the interview and financial analysis processes. However, for returning customers, some new variables were included in the form that are based on the customer history on past loans.

Associated costs. In terms of evaluation costs[1], the basic credit evaluation requires approximately one hour of an executive's time, leading to estimated costs of EUR 5 per processed application (we assumed a monthly salary of about EUR 1000 per month for an executive). The in-depth interview takes about four hours of an executive's time, leading to estimated costs between EUR 18 and EUR 22 per application ($\approx$ EUR 20 $\pm$10%). Finally, a bank's specialist needs about two hours for the subsequent financial analysis, valued in a range of EUR 18 to EUR 22 per applicant (we assumed a monthly salary of about EUR 2000 per month for the specialist).

In summary, the Level 1 costs sum up to about EUR 5. For the Level 2 decision, we need to consider additional costs for the in-depth interview (4 h leading to costs between EUR 18 and EUR 22) and the financial analysis (2 h leading to costs between EUR 18 and EUR 22). As a consequence, the additional costs on Level 2 vary between EUR 36 and EUR 44.

The benefits and losses for granting credit are computed as percentages of the amount of the loan granted. The benefit of a good payer is the interest rate (in average, 17.6% of the amount of the loan, based on historical records of good payers) multiplied by the amount granted to an approved applicant. The monetary loss of granting a credit to a defaulter is computed as the average loss given default (19% of the amount of the loan, based on historical records of defaulters) multiplied by the amount granted to the applicant.

Hypothesis. Our hypothesis for this application is that acquiring all variables (form, interview, and financial analysis) for all applicants, i.e., the bank's current practice, results in a potentially inefficient and expensive process, since the most discriminative information is already collected in the basic credit evaluation on Level 1 (filling a form). Additional information (interview and financial analysis) should be gathered only in cases where an accept/reject decision is not clear based on the basic credit evaluation on Level 1, i.e., in terms of three-way decisions the boundary cases. Therefore, the three-way decision process can possibly be a powerful way to improve credit granting.

4.2. Experimental setup

We apply the proposed three-way decision framework to the data set provided by the Chilean bank, using logistic regression as base classifier as justified above in Section 3.2.1.

The three-way decision framework fits well to the credit scoring evaluation. As discussed in Section 4.1, the bank already performs a mandatory two-steps decision process, basically consisting of the following steps (see also Fig. 3):

1. Basic credit evaluation: Credit applicants have to fill out and return a standard form.
2. Comprehensive credit evaluation: Credit applicants are interviewed by executives of the bank followed by a financial analysis. The final decision regarding the creditworthiness is made by analyzing all collected information, i.e., form, interview, and financial analysis.

Therefore, we can evaluate the efficiency of the current credit decision procedure by mapping it onto a three-way decision process:

1. All customers are requested to fill out the (cheap) standard form. This equals the basic credit evaluation.
2. Applying three-way decision, those customers with unclear evaluation (boundary cases) are identified. However, in contrast to the current process, these and only these customers are required to perform a (costly and time-consuming) interview and a subsequent financial analysis before a final decision can be made. Note, the final decision is again based on all information (form, interview, and financial analysis).

As already mentioned in Section 4.1, the bank differentiates between returning and new credit applicants. Credit-issuing companies often distinguish between these two groups. The main reason for this distinction is that the information already available from returning customers (such as, e.g., repayment behavior) is valuable for the current application scoring. To investigate if such a distinction can be justified by the data in our case, we followed the practice of the bank and performed two separate analyses, one for the returning (RET) and one for the new (NEW) customers.

For the returning and new customers we compare the performance of the following three models on our credit scoring data (see Fig. 4):

- L1 Model (L1M). The model is constructed using only variables extracted from the credit evaluation form (Level 1).
- L2 Model (L2M). The second model, which is current practice at the bank, is composed of all available variables, i.e., credit evaluation form, in-depth interview, and financial analysis.
- 3WD Model (3WDM). The model incorporates our three-way decision framework, as described in the previous section: most of the cases are decided on Level 1. Just for the boundary cases an interview followed by a financial analysis is conducted.

In all models (L1 Model, L2 Model, and 3WD Model) we use cost-benefit analysis as decider with decision-relevant costs as discussed above in Section 3.1.

[1] The original costs were recorded in Chilean Pesos. For the sake of simplicity, we use EUR.

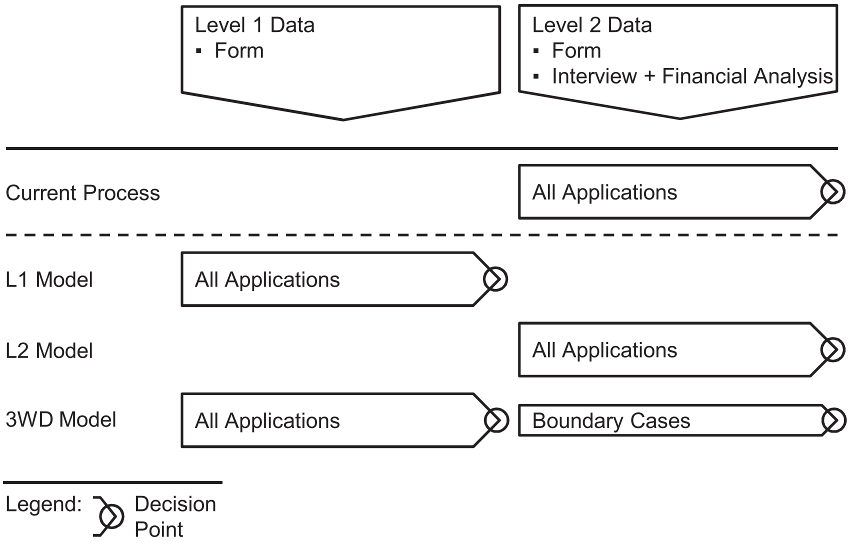

Fig. 4. Three models compared in the application.

Table 8
Number of selected features.

	RET		NEW	
	L1M	L2M	L1M	L2M
Total	21	21	14	28
· Form	21	19	14	13
· Interview + Analysis	–	2	–	15

Table 9
Accuracy and profit (returning customers).

		Accuracy (%)			Absolute profit			Difference in profit (EUR)		
L2 ΔCosts	#BND Obj.	L1M	L2M	3WDM	L1M	L2M	3WDM	3WDM-L1M	3WDM-L2M	L1M-L2M
36	0	86.77	86.94	86.77	271,955	209,902	271,955	0	62,052	62,052
37	0	86.77	86.94	86.77	271,955	208,163	271,955	0	63,791	63,791
38	0	86.77	86.94	86.77	271,955	206,424	271,955	0	65,530	65,530
39	0	86.77	86.94	86.77	271,955	204,685	271,955	0	67,269	67,269
40	0	86.77	86.94	86.77	271,955	202,946	271,955	0	69,008	69,008
41	0	86.77	86.94	86.77	271,955	201,207	271,955	0	70,747	70,747
42	0	86.77	86.94	86.77	271,955	199,468	271,955	0	72,486	72,486
43	0	86.77	86.94	86.77	271,955	197,729	271,955	0	74,225	74,225
44	0	86.77	86.94	86.77	271,955	195,990	271,955	0	75,964	75,964

We use a holdout validation process, selecting 70% of the loans for training and 30% for testing using stratified sampling. For all models, a backward feature elimination procedure is performed based on the Akaike Information Criterion (AIC) [6]. At each iteration, one variable is removed from the classifier; the one whose removal leads to the highest AIC. The numbers of selected features are shown in Table 8 (see Table 7 for the number of features originally collected by the bank):

4.3. Results for returning customers

Table 9 presents accuracy and profit obtained when applying models L1M, L2M, and 3WDM to a test set of 30% of the 5799 returning customers. The additional costs on Level 2 (L2 ΔCosts: costs for the in-depth interview and the financial analysis) vary from EUR 36 to EUR 44. Since no objects were assigned to the boundary (#BND Obj.=0), independently of L2 ΔCosts, the proposed 3WD Model melts down to L1 Model, i.e., both give identical results. Despite the fact that accuracies for 3WD Model and L2 Model are very similar (they just differ by $86.94\% - 86.77\% = 0.17\%$) the improvement in profit is considerable. The absolute gain shown in column 'Difference in Profit (EUR)' grows with L2 ΔCosts from 29% to more than 38% of the profit obtained by the L2 Model applied by the bank.

Table 10
Accuracy and profit (new customers).

L2 ΔCosts	#BND Obj.	Accuracy (%) L1M	L2M	3WDM	Absolute profit (EUR) L1M	L2M	3WDM	Difference in profit (EUR) 3WDM-L1M	3WDM-L2M	L1M-L2M
36	28	70.00	71.70	70.20	23,743	9025	23,692	−51	14,667	14,718
37	25	70.00	71.70	70.42	23,743	8572	23,977	234	15,405	15,171
38	23	70.00	71.70	70.42	23,743	8119	24,028	285	15,909	15,624
39	21	70.00	71.70	70.42	23,743	7666	24,083	340	16,417	16,077
40	19	70.00	71.70	70.20	23,743	7213	23,512	−231	16,299	16,530
41	18	70.00	71.70	70.42	23,743	6760	23,748	5	16,989	16,983
42	17	70.00	71.70	70.42	23,743	6307	23,772	29	17,466	17,436
43	15	70.00	71.70	70.42	23,743	5854	23,841	98	17,988	17,889
44	15	70.00	71.70	70.42	23,743	5401	23,826	83	18,426	18,342

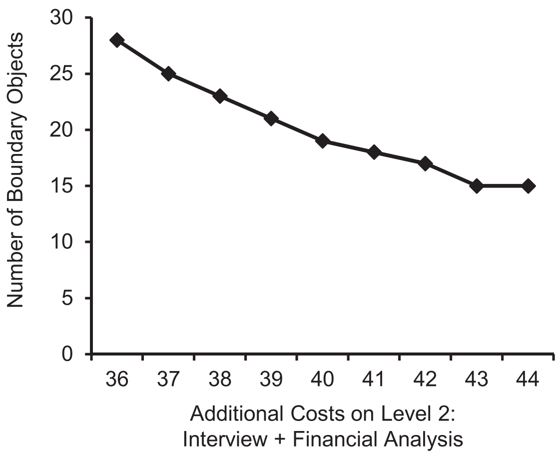

Fig. 5. Number of boundary objects (new customers).

The empty boundary indicates that Level 2 investigations (interview and financial analysis) are too expensive in trade-off for a potentially better evaluation result. As a result of our three-way methodology we can conclude that, generally, an effective approach would be to only perform L1 Model for virtually all returning customers.

4.4. Results for new customers

Table 10 summarizes the results for a test set of 30% of the 1510 new customers. If Level 2 costs grow from EUR 36 to EUR 44, the number of objects assigned to the boundary decreases from 28 to 15 (see Fig. 5).

The accuracies obtained by L1 Model and L2 Model are similar; however the range is larger in comparison to the case of returning customers. An improvement of $71.7\% - 70.0\% = 1.7\%$ for the more expensive L2 Model can be observed. Thus, the resulting three-way decision process indicates that performing an additional model on Level 2 might be advantageous in comparison to L1 Model for about $1\% - 2\%$ of the applicants - at least when the additional costs are reasonably small.

Column $3WDM - L2M$ in Table 10 shows that the three-way decision process greatly outperforms the L2 Model, applied by the bank, with a relative gain in profit of between 160% and 340%. Therefore, we refrain from any further detailed comparison.

Fig. 6 depicts the difference in profit $3WDM - L1M$. As can be seen, 3WD Model outperforms in most of the cases (7 out of 9) L1 Model leading to small profit gains over the considered range of L2 ΔCosts. On average the three-way decision process delivers EUR 88 per loan more than L1 Model. This is conform with the observation that the accuracy of 3WD Model is just marginally better in comparison to L1 Model.

4.5. Discussion

Some significant insights can be drawn from our application. The attributes collected in an expensive process of in-depth interviews and financial analyses may not be needed for all applicants. The proposed three-way decision framework allows fast decision making for most applicants with enough evidence for acceptance/rejection, limiting the effort of collecting additional, potentially expensive information for boundary cases only.

Regarding the two groups of customers analyzed in our application we obtain the following very interesting insights:

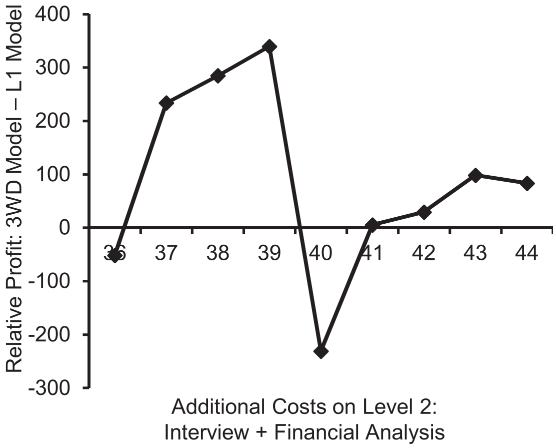

Fig. 6. Difference in profit three-way decision - level 1 decision (new customers).

- For returning customers (RET), it is sufficient to conduct the basic credit evaluation on Level 1 (filling out a form), i.e., costly, in-depth interviews should not be performed for returning customers at all.
- For most of the new customers (NEW), the basic credit evaluation on Level 1 (filling out a form) is also sufficient to decide on their credit application. However, some of the new customers need to undergo the further detailed analysis of interviewing and financial analysis.

The proposed 3WD Model methodology motivates a switch from the currently applied L2 Model to L1 Model, which leads to a very important increased profit. Even the observed gain of approximately 1% obtained by the three-way decision process over L1 Model, which seems to be marginal at the first sight, is considerable in a highly competitive business, such as, e.g., credit lending.

Furthermore, the 'threat' of a possible interview and financial analysis provides a regulating measure that may discipline new customers in a way that they deliver accurate and comprehensive information already for the base evaluation.

The data sets used in this application have particularities that are uncommon for credit risk projects. First, predictive performance is not as good as in traditional credit scoring for new customers. It seems that the variables available for the project cannot predict the behavior of micro-entrepreneurs accurately. This segment is more homogeneous than traditional credit applicants, and attributes that are usually relevant, like income, have less discriminative power in our application (see Bravo et al. [4] for a detailed discussion around this issue). Additionally, the average benefit of a good payer is roughly the same as the average loss caused by a bad payer (17.6% and 19% of the amount of the loan, respectively). On the one hand, the interest rates are unusually high due to the higher risk that represents this segment. On the other hand, recovering rates for defaulters are also very high since the bank performs numerous efforts to renegotiate debt with defaulters. These two facts make the bank profitable despite the relatively high risk of default for new customers and the fair accuracies obtained by the models, but it affects the effectiveness of the cost-benefit framework proposed for credit risk. A typical credit scoring project would have lower interest and recovery rates, making the three-way decision framework even more relevant for profitable decision making than in the application presented above.

5. Conclusion

As in many other real-world applications, experts in credit granting are used to assign a credit application to one of three classes after a first quick revision: accept, reject, acquire more information and take a better informed decision. However, such decisions are often influenced by gut feeling and experience. Therefore, we propose in this paper a way to formalize this strategy adapting the general three-way decision framework introduced by Yao to the particular decision situation of a credit granting process. We show how to calculate the costs related to the decisions on a first level using basic information as well as costs on a second level with more information.

We apply the proposed approach to a real-world data set from a bank lending credits to micro-entrepreneurs. Applying the proposed 3WD methodology justified the switch from the currently used model with many variables to a simpler one employing fewer variables. This change had a huge impact for both, returning as well as new customers. The reported gain of using the three-way decision process instead of the simpler model is considerable in highly competitive markets that are characterized by large numbers of customers with low individual margins. Beyond the monetary advantages, a more efficient credit granting process leads to faster decisions thus contributing to significant strategic advantages over competitors. Furthermore, formalizing what is common for practitioners increases their acceptance of the proposed model. On the customer side, a faster credit decision leads to higher satisfaction.

In future work, it would be interesting to apply the proposed framework to different credit granting processes. Developing similar methodologies for other application areas, where practitioners are used to think and act in three-way decisions, would also be an important contribution to the respective research agenda.

Acknowledgments

Support for Sebastián Maldonado and Richard Weber from the Complex Engineering Systems Institute (CONICYT, PIA, FB0816) and Fondecyt (projects 1160738 and 1181036, respectively) is greatly acknowledged.

References

[1] E.I. Altman, Financial ratios, discriminant analysis and the prediction of corporate bankruptcy, J. Finance 23 (4) (1968) 589–609, doi:10.1111/j.1540-6261.1968.tb00843.x.
[2] B. Baesens, T.V. Gestel, S. Viaene, M. Stepanova, J. Suykens, J. Vanthienen, Benchmarking state-of-the-art classification algorithms for credit scoring, J. Oper. Res. Soc. 54 (6) (2003) 627–635.
[3] Basel Committee on Banking Supervision, International convergence of capital measurements and capital standards a revised framework, Technical Report, Bank for International Settlements, 2004.
[4] C. Bravo, S. Maldonado, R. Weber, Granting and managing loans for micro-entrepreneurs: new developments and practical experiences, Eur. J. Oper. Res. 227 (2) (2013) 358–366.
[5] C. Bravo, L. Thomas, R. Weber, Improving credit scoring by differentiating defaulter behaviour, J. Oper. Res. Soc. 66 (5) (2015) 771–781.
[6] K.P. Burnham, D.R. Anderson, Model Selection and Multimodel Inference: A Practical Information-Theoretic Approach, Springer Science & Business Media, 2003.
[7] G.W. Corder, D.I. Foreman, Nonparametric Statistics: A Step-by-step Approach, second ed., John Wiley & Sons, Hoboken, NJ, 2014.
[8] D. Durand, Risk elements in consumer installment financing, Technical Report, National Bureau of Economic Research, New York, 1941.
[9] FICO, Fico at a Glance, Fico, 2017.
[10] R.A. Fisher, The use of multiple measurements in taxonomic problems, Ann. Eugen. 7 (III) (1936) 179–188.
[11] W.V. Gehrlein, B.J. Wagner, A two-stage least cost credit scoring model, Ann. Oper. Res. 74 (1997) 159–171.
[12] J. Han, M. Kamber, J. Pei, Data mining: Concepts and technique, third ed., Morgan Kaufmann, Waltham, MA, USA, 2011.
[13] D.W. Hosmer, S. Lemeshow, R.X. Sturdivant, Applied Logistic Regression, John Wiley & Sons, Hoboken, NJ, 2013.
[14] D. Howarth, L. Quaglia, Banking on stability: the political economy of new capital requirements in the european union, J. Eur. Integr. 35 (3) (2013) 333–346.
[15] X. Jia, L. Shang, Three-way decisions versus two-way decisions on filtering spam email, Trans. Rough Sets XVIII (LNCS 8449) (2014) 69–91.
[16] X. Jia, K. Zheng, W. Li, T. Liu, L. Shang, Three-way decisions solution to filter spam email: an empirical study, in: Rough Sets and Current Trends in Computing, 7413, Springer, 2012, pp. 287–296.
[17] D.G. Kleinbaum, M. Klein, Logistic Regression: A Self-Learning Text, Springer Science + Business Media, New York, 2010.
[18] H. Li, L. Zhang, B. Huang, X. Zhou, Sequential three-way decision and granulation for cost-sensitive face recognition, Knowl. Based Syst. 91 (2016) 241–251.
[19] H. Li, L. Zhang, X. Zhou, B. Huang, Cost-sensitive sequential three-way decision modeling using a deep neural network, Int. J. Approx. Reason. 85 (2017) 68–78.
[20] H. Li, X. Zhou, B. Huang, D. Liu, Cost-sensitive three-way decision: a sequential strategy, in: Rough Sets and Knowledge Technology, in: LNCS, 8171, Springer, 2013, pp. 325–337.
[21] J. Li, X. Deng, Y.Y. Yao, Multistage email spam filtering based on three-way decisions, in: Rough Sets and Knowledge Technology, in: LNCS, 8171, Springer, 2013, pp. 313–324.
[22] W. Li, Z. Huang, Q. Li, Three-way decisions based software defect prediction, Knowl. Based Syst. 91 (2016) 263–274.
[23] D. Liang, W. Pedrycz, D. Liu, P. Hu, Three-way decisions based on decision-theoretic rough sets under linguistic assessment with the aid of group decision making, Appl. Soft. Comput. 29 (2015) 256–269.
[24] D. Liu, T. Li, D. Liang, Three-way government decision analysis with decision-theoretic rough sets, Int. J. Uncertain. Fuzz. Knowl. Based Syst. 20 (supp01) (2012) 119–132.
[25] D. Liu, T. Li, D. Liang, Incorporating logistic regression to decision-theoretic rough sets for classifications, Int. J. Approx. Reason. 55 (2014) 197–210.
[26] D. Liu, Y.Y. Yao, T. Li, Three-way investment decisions with decision-theoretic rough sets, Int. J. Comput. Intell. Syst. 4 (1) (2011) 66–74.
[27] F. Louzada, A. Ara, G.B. Fernandes, Classification methods applied to credit scoring: systematic review and overall comparison, Surv. Oper. Res. Manag. Sci. 21 (2) (2016) 117–134.
[28] X.-A. Ma, Y. Yao, Three-way decision perspectives on class-specific attribute reducts, Inf. Sci. 450 (2018) 227–245.
[29] T.L. Miller-Nobles, B.L. Mattison, E.M. Matsumura, Horngren'S Financial & Managerial Accounting, sixth ed., Pearson Education, Harlow, UK, 2018.
[30] P. Millett, Lending and Borrowing in Ancient Athens, Cambridge University Press, 2002.
[31] F. Milne, Anatomy of the credit crisis: the role of faulty risk management systems, Technical Report, CD Howe Institute Commentary (269), 2008.
[32] Z. Pawlak, Rough Sets: Theoretical Aspects of Reasoning About Data, Kluwer Academic Publishers, Dordrecht, Netherlands, 1991.
[33] A. Savchenko, Fast multi-class recognition of piecewise regular objects based on sequential three-way decisions and granular computing, Knowl. Based Syst. 91 (2016) 252–262.
[34] D.L. Street, Gaap 2001 benchmarking national accounting standards against IAS: summary of results, J. Int. Account. Audit. Tax. 11 (1) (2002) 77–90.
[35] B. Sun, W. Ma, X. Xiao, Three-way group decision making based on multigranulation fuzzy decision-theoretic rough set over two universes, Int. J. Approx. Reason. 81 (2017) 87–102.
[36] L.C. Thomas, A survey of credit and behavioural scoring: forecasting financial risk of lending to consumers, Int. J. Forecast. 16 (2000) 149–172.
[37] I.H. Witten, E. Frank, M. Hall, Data Mining: Practical Machine Learning Tools and Techniques, Morgan Kaufmann, Burlington, MA, USA, 2011.
[38] J.T. Yao, N. Azam, Web-based medical decision support systems for three-way medical decision making with game-theoretic rough sets, IEEE Trans. Fuzzy Syst. 23 (1) (2015) 3–15.
[39] Y.Y. Yao, Probabilistic approaches to rough sets, Expert Syst. 20 (5) (2003) 287–297.
[40] Y.Y. Yao, Probabilistic rough set approximations, Int. J. Approx. Reason. 49 (2008) 255–271.
[41] Y.Y. Yao, Three-way decision: an interpretation of rules in rough set theory, in: P. Wen, Y. Li, L. Polkowski, Y. Yao, S. Tsumoto, G. Wang (Eds.), Proceedings of the International Conference on Rough Sets and Knowledge Technology, LNCS, 5589, Springer, Berlin, Heidelberg, 2009, pp. 642–649.
[42] Y.Y. Yao, Three-way decisions with probabilistic rough sets, Inf Sci (Ny) 180 (2010) 341–353.
[43] Y.Y. Yao, An outline of a theory of three-way decisions, in: J. Yao, Y. Yang, R. Slowinski, S. Greco, H. Li, S. Mitra, L. Polkowski (Eds.), Proceedings of the RSCTC, LNAI, 7413, Springer, Heidelberg, 2012, pp. 1–17.
[44] Y.Y. Yao, Granular computing and sequential three-way decisions, in: Proceedings of the Rough Sets and Knowledge Technology, in: LNCS, 8171, Springer, 2013, pp. 16–27.
[45] Y.Y. Yao, X. Deng, Sequential three-way decisions with probabilistic rough sets, in: Proceedings of the 10th IEEE International Conference on Cognitive Informatics & Cognitive Computing (ICCI* CC), IEEE, 2011, pp. 120–125.

[46] H. Yu, X. Wang, G. Wang, X. Zeng, An active three-way clustering method via low-rank matrices for multi-view data, Inf. Sci. (2018), doi:10.1016/j.ins.2018.03.009.
[47] H. Yu, Y. Wang, Three-way decisions method for overlapping clustering, in: Proceedings of the Rough Sets and Current Trends in Computing, in: LNCS, 7413, Springer, 2012, pp. 277–286.
[48] H. Yu, C. Zhang, F. Hu, An incremental clustering approach based on three-way decisions, in: Proceedings of the International Conference on Rough Sets and Current Trends in Computing, in: LNCS, 8536, Springer, 2014, pp. 152–159.
[49] H. Yu, C. Zhang, G. Wang, A tree-based incremental overlapping clustering method using the three-way decision theory, Knowl. Based Syst. 91 (2016) 189–203.
[50] H.-R. Zhang, F. Min, Three-way recommender systems based on random forests, Knowl. Based Syst. 91 (2016) 275–286.
[51] H.-R. Zhang, F. Min, B. Shi, Regression-based three-way recommendation, Inf. Sci. 378 (2017) 444–461.
[52] H.Y. Zhang, S.Y. Yang, J.M. Ma, Ranking interval sets based on inclusion measures and applications to three-way decisions, Knowl. Based Syst. 91 (2016) 62–70.
[53] B. Zhou, Y. Yao, Q. Liu, Rough sets, vol. 9920 of Lecture Notes in Artificial Intelligence, Springer, pp. 219–228.
[54] B. Zhou, Y.Y. Yao, J. Luo, A three-way decision approach to email spam filtering, in: A. Farzindar, V. Keselj (Eds.), Proceedings of the Advances in Artificial Intelligence, LNCS, 6085, Springer, Berlin, Heidelberg, 2010, pp. 28–39.
[55] Y. Zhu, H. Tian, J. Ma, J. Liu, T. Liang, An integrated method for micro-blog subjective sentence identification based on three-way decisions and naive bayes, in: Proceedings of the Rough Sets and Knowledge Technology, in: LNCS, 8818, Springer, 2014, pp. 844–855.
[56] W. Ziarko, Probabilistic approach to rough sets, Int. J. Approx. Reason. 49 (2008) 272–284.