Lehrstuhl für Steuerungs- und Regelungstechnik / Lehrstuhl für Informationstechnische Regelung	Einführung in die Roboterregelung (ERR)
Technische Universität München	Kurzlösung zur 6. Übung

Aufgabe 1:

1.1 kinetische Energie T:

Arm 1: $T_1 = \frac{1}{2}\int\limits_0^{l_1} v_1^2(\lambda)\frac{m_1}{l_1}d\lambda$

mit $v_1^2(\lambda) = (\dot{\Theta}_1\lambda)^2$

$\rightarrow T_1 = \frac{1}{6}m_1 l_1^2\dot{\Theta}_1^2$

Arm 2: $T_2 = \frac{1}{2}\int\limits_0^{l_2} v_2^2(\lambda)\frac{m_2}{l_2}d\lambda$

mit $v_2^2(\lambda) = \dot{x}_2^2(\lambda) + \dot{z}_2^2(\lambda)$,

$\dot{x}_2(\lambda) = \frac{d}{dt}(l_1\cos\Theta_1 + \lambda\cos(\Theta_1+\Theta_2))$,

$\dot{z}_2(\lambda) = \frac{d}{dt}(l_1\sin\Theta_1 + \lambda\sin(\Theta_1+\Theta_2))$,

$\rightarrow T_2 = \frac{1}{2}m_2(l_1^2\dot{\Theta}_1^2 + l_1 l_2\cos\Theta_2\dot{\Theta}_1(\dot{\Theta}_1+\dot{\Theta}_2) + \frac{1}{3}l_2^2(\dot{\Theta}_1+\dot{\Theta}_2)^2)$

$$\begin{aligned} T = T_1 + T_2 \quad &= \quad \left(\frac{1}{6}m_1 l_1^2 + \frac{1}{2}m_2 l_1^2 + \frac{1}{2}m_2 l_1 l_2\cos\Theta_2 + \frac{1}{6}m_2 l_2^2\right)\dot{\Theta}_1^2 \\ &\quad + \left(\frac{1}{2}m_2 l_1 l_2\cos\Theta_2 + \frac{1}{3}m_2 l_2^2\right)\dot{\Theta}_1\dot{\Theta}_2 + \frac{1}{6}m_2 l_2^2\dot{\Theta}_2^2 \end{aligned}$$

potentielle Energie V:

$V = m_1\, g\, \frac{l_1}{2}\sin\Theta_1 + m_2\, g\, (l_1\sin\Theta_1 + \frac{l_2}{2}\sin(\Theta_1+\Theta_2))$

$L = T - V$

1.2 $T = \left(\frac{5}{6} + \frac{1}{2}\cos\Theta_2\right)ml^2\dot{\Theta}_1^2 + \left(\frac{1}{3} + \frac{1}{2}\cos\Theta_2\right)ml^2\dot{\Theta}_1\dot{\Theta}_2 + \frac{1}{6}ml^2\dot{\Theta}_2^2$,

$V = \frac{3}{2}mgl\sin\Theta_1 + \frac{1}{2}mgl\sin(\Theta_1+\Theta_2)$,

$L = T - V$

in

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\Theta}_i}\right) - \frac{\partial L}{\partial\Theta_i} = U_i \qquad , \qquad i = 1,2$$

eingesetzt liefert

$$\left(\frac{5}{3} + \cos\Theta_2\right)ml^2\ddot{\Theta}_1 + \left(\frac{1}{3} + \frac{1}{2}\cos\Theta_2\right)ml^2\ddot{\Theta}_2 - ml^2\sin\Theta_2\dot{\Theta}_1\dot{\Theta}_2 - \frac{1}{2}ml^2\sin\Theta_2\dot{\Theta}_2^2 + \frac{3}{2}mgl\cos\Theta_1$$

$$+\frac{1}{2}mgl\cos(\Theta_1+\Theta_2)=U_1$$

$$\left(\frac{1}{3}+\frac{1}{2}\cos\Theta_2\right)ml^2\ddot{\Theta}_1+\frac{1}{3}ml^2\ddot{\Theta}_2+\frac{1}{2}\sin\Theta_2 ml^2\dot{\Theta}_1^2+\frac{1}{2}mgl\cos(\Theta_1+\Theta_2)=U_2$$

1.3

$$\underbrace{ml^2\begin{bmatrix}\frac{5}{3}+\cos\Theta_2 & \frac{1}{3}+\frac{1}{2}\cos\Theta_2\\ \frac{1}{3}+\frac{1}{2}\cos\Theta_2 & \frac{1}{3}\end{bmatrix}\begin{bmatrix}\ddot{\Theta}_1\\ \ddot{\Theta}_2\end{bmatrix}}_{\text{Massenkräfte}} + \underbrace{ml^2\begin{bmatrix}-\sin\Theta_2\dot{\Theta}_1\dot{\Theta}_2-\frac{1}{2}\sin\Theta_2\dot{\Theta}_2^2\\ \frac{1}{2}\sin\Theta_2\dot{\Theta}_1^2\end{bmatrix}}_{\text{Kreiselkräfte}}$$

$$\underbrace{+\,mgl\begin{bmatrix}\frac{3}{2}\cos\Theta_1+\frac{1}{2}\cos(\Theta_1+\Theta_2)\\ \frac{1}{2}\cos(\Theta_1+\Theta_2)\end{bmatrix}}_{\text{Gravitationskräfte}} = \underbrace{\begin{bmatrix}U_1\\ U_2\end{bmatrix}}_{\text{Steuerkräfte}}$$

1.4 siehe Zeichnung

1.5	kleine Verfahrgeschwindigkeiten:	Kreiselkräfte vernachlässigbar
	horizontale Ausrichtung:	Gravitationskräfte entfallen

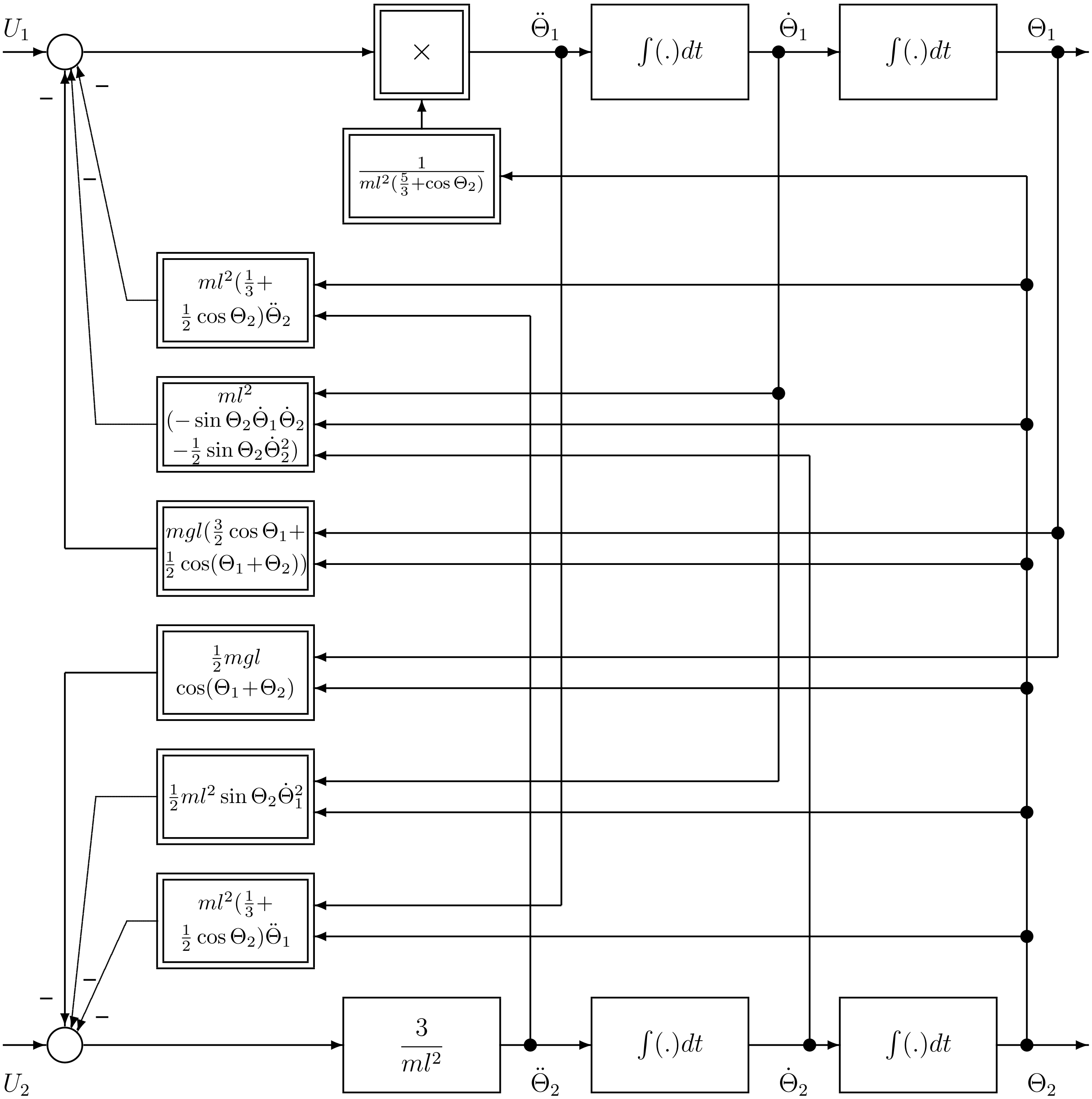
1.6 $\Theta_2 = 0$ (Arm gestreckt):

$$\underline{\underline{M}}\,(\underline{q}) = ml^2\begin{bmatrix}\frac{8}{3} & \frac{5}{6}\\ \frac{5}{6} & \frac{1}{3}\end{bmatrix}$$

$\Theta_2 = \pi$ (Arm gefaltet):

$$\underline{\underline{M}}\,(\underline{q}) = ml^2\begin{bmatrix}\frac{2}{3} & -\frac{1}{6}\\ -\frac{1}{6} & \frac{1}{3}\end{bmatrix}$$

1.4



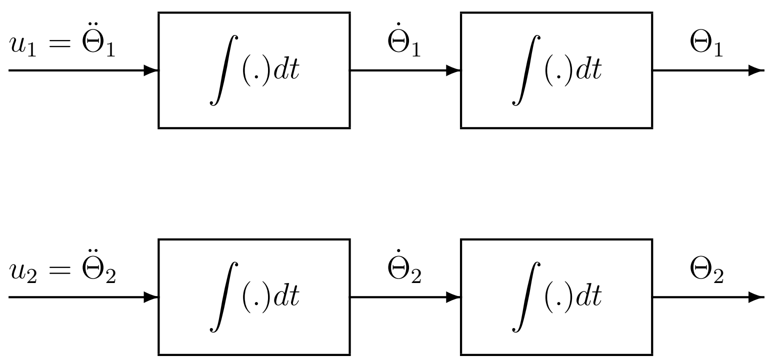
U_1
$\times$
$\ddot{\Theta}_1$
$\int(.)dt$
$\dot{\Theta}_1$
$\int(.)dt$
Θ_1
$\frac{1}{ml^2(\frac{5}{3}+\cos\Theta_2)}$
$ml^2(\frac{1}{3}+\frac{1}{2}\cos\Theta_2)\ddot{\Theta}_2$
$ml^2(-\sin\Theta_2\dot{\Theta}_1\dot{\Theta}_2-\frac{1}{2}\sin\Theta_2\dot{\Theta}_2^2)$
$mgl(\frac{3}{2}\cos\Theta_1+\frac{1}{2}\cos(\Theta_1+\Theta_2))$
$\frac{1}{2}mgl\cos(\Theta_1+\Theta_2)$
$\frac{1}{2}ml^2\sin\Theta_2\dot{\Theta}_1^2$
$ml^2(\frac{1}{3}+\frac{1}{2}\cos\Theta_2)\ddot{\Theta}_1$
U_2
$\frac{3}{ml^2}$
$\ddot{\Theta}_2$
$\int(.)dt$
$\dot{\Theta}_2$
$\int(.)dt$
Θ_2

Aufgabe 2:

2.1
$$
\begin{aligned}
\underline{U} &= \underline{U}_K + \underline{U}_R \\
&= \underline{\tilde{N}} + \underline{\tilde{G}} + \underline{\underline{\tilde{M}}}\,\underline{u} \\
&= \tilde{m}\tilde{l}^2 \begin{bmatrix} -\sin\tilde{\Theta}_2\dot{\tilde{\Theta}}_1\dot{\tilde{\Theta}}_2 - \frac{1}{2}\sin\tilde{\Theta}_2\dot{\tilde{\Theta}}_2^2 \\ \frac{1}{2}\sin\tilde{\Theta}_2\dot{\tilde{\Theta}}_1^2 \end{bmatrix} \\
&\quad + \tilde{m}\tilde{g}\tilde{l} \begin{bmatrix} \frac{3}{2}\cos\tilde{\Theta}_1 + \frac{1}{2}\cos(\tilde{\Theta}_1+\tilde{\Theta}_2) \\ \frac{1}{2}\cos(\tilde{\Theta}_1+\tilde{\Theta}_2) \end{bmatrix} \\
&\quad + \tilde{m}\tilde{l}^2 \begin{bmatrix} \frac{5}{3}+\cos\tilde{\Theta}_2 & \frac{1}{3}+\frac{1}{2}\cos\tilde{\Theta}_2 \\ \frac{1}{3}+\frac{1}{2}\cos\tilde{\Theta}_2 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}
\end{aligned}
$$

2.2 siehe Zeichnung

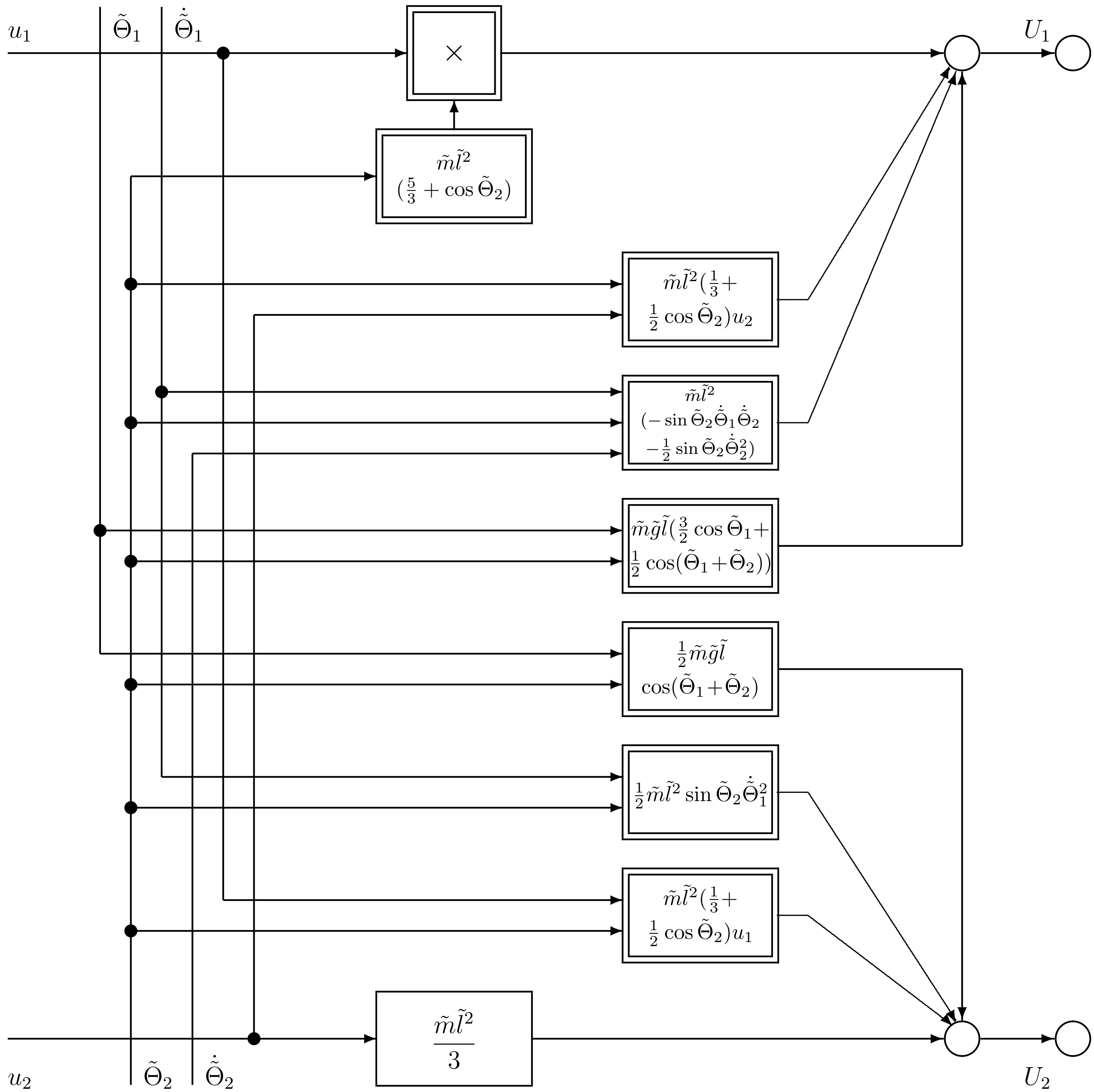
2.3 in Gelenkachsen entkoppelter Manipulator:



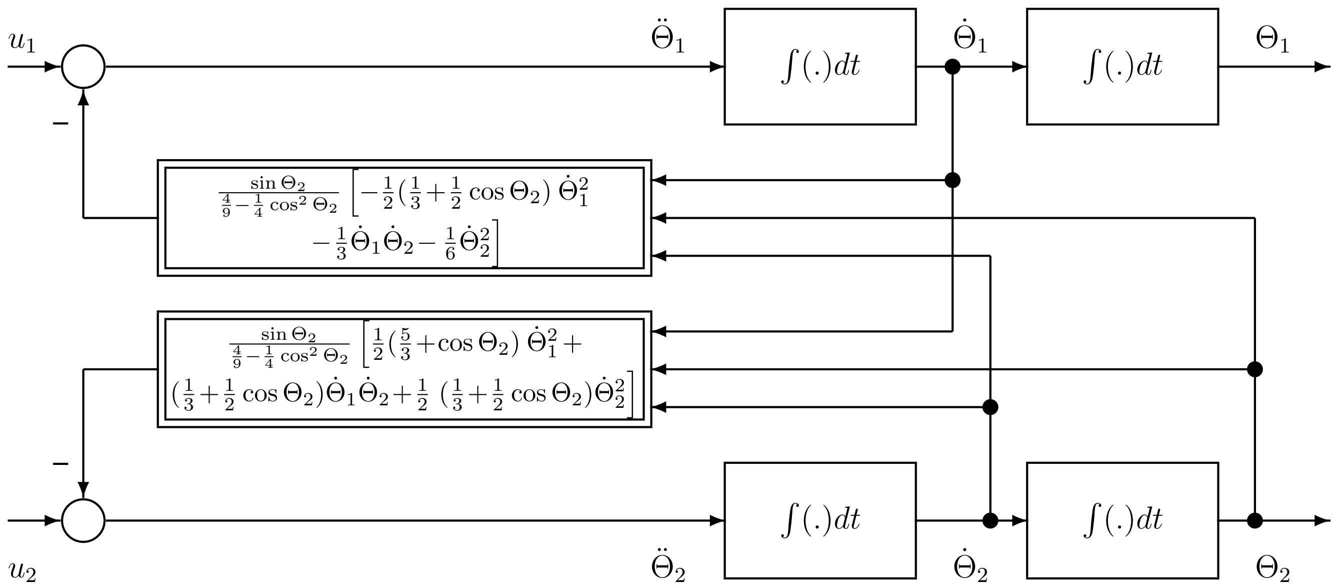
Annahmen:

- Zustandsgrößen und Parameter der Strecke exakt bekannt
- keine Beschränkung der Steuerkräfte
- Reibung vernachlässigbar

2.2


u_1
$\tilde{\Theta}_1$
$\dot{\tilde{\Theta}}_1$
$\times$
U_1
$\tilde{m}\tilde{l}^2$
$(\frac{5}{3} + \cos\tilde{\Theta}_2)$
$\tilde{m}\tilde{l}^2(\frac{1}{3}+$
$\frac{1}{2}\cos\tilde{\Theta}_2)u_2$
$\tilde{m}\tilde{l}^2$
$(-\sin\tilde{\Theta}_2\dot{\tilde{\Theta}}_1\dot{\tilde{\Theta}}_2$
$-\frac{1}{2}\sin\tilde{\Theta}_2\dot{\tilde{\Theta}}_2^2)$
$\tilde{m}\tilde{g}\tilde{l}(\frac{3}{2}\cos\tilde{\Theta}_1+$
$\frac{1}{2}\cos(\tilde{\Theta}_1+\tilde{\Theta}_2))$
$\frac{1}{2}\tilde{m}\tilde{g}\tilde{l}$
$\cos(\tilde{\Theta}_1+\tilde{\Theta}_2)$
$\frac{1}{2}\tilde{m}\tilde{l}^2\sin\tilde{\Theta}_2\dot{\tilde{\Theta}}_1^2$
$\tilde{m}\tilde{l}^2(\frac{1}{3}+$
$\frac{1}{2}\cos\tilde{\Theta}_2)u_1$
$\frac{\tilde{m}\tilde{l}^2}{3}$
u_2
$\tilde{\Theta}_2$
$\dot{\tilde{\Theta}}_2$
U_2

2.4 $\underline{\ddot{\Theta}} = \underline{u} - \underline{\underline{M}}^{-1}(\underline{\Theta}) \cdot N(\underline{\Theta}\ ,\ \underline{\dot{\Theta}})$



2.5
$$
\begin{aligned}
u_1 &= \ddot{\Theta}_{W1} + K_{V1}(\dot{\Theta}_{W1} - \dot{\Theta}_1) + K_{P1}(\Theta_{W1} - \Theta_1)\ , \\
u_2 &= \ddot{\Theta}_{W2} + K_{V2}(\dot{\Theta}_{W2} - \dot{\Theta}_2) + K_{P2}(\Theta_{W2} - \Theta_2)
\end{aligned}
$$