

# Groups and their actions: algebraic, geometric and combinatorial aspects

3.06.2024 - 7.06.2024 — Levico Terme (Italy)



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## List of Speakers

## Some things I learned about permutation representations

9:15  
10:05Martin Gallauer  
Warwick University

Say I pick a field  $k$  and a finite group  $G$ , and I ask you to exhibit  $k$ -linear  $G$ -representations. The catch is: I don't tell you which  $k$  nor  $G$  that I picked. There is one family you can always exhibit: the permutation representations  $k(G/H)$  for subgroups  $H$  of  $G$ . Arguably it is this universality that makes permutation representations so ubiquitous in mathematics. In this talk I will explain some of the things I learned about them over the last few years, notably in collaboration with Paul Balmer. In the spirit of the conference, these facts will be drawn from algebra (modular representation theory), (tensor-triangular) geometry, and combinatorics. Naturally, I'll also mention things I don't but would like to know.

## The Module structure of a Group Action on a Ring

10:15  
10:40Peter Symonds  
University of Manchester

Consider a finite group  $G$  acting on a graded Noetherian  $k$ -algebra  $S$ , for some field  $k$  of characteristic  $p$ ; for example  $S$  might be a polynomial ring. Regard  $S$  as a  $kG$ -module and consider the multiplicity of a particular indecomposable module as a summand in each degree. We show how this can be described in terms of homological algebra and how it is linked to the geometry of the group action on the spectrum of  $S$ .

## Minimal degenerations for quiver varieties

11:15  
12:05Gwyn Bellamy  
University of Glasgow

Minimal degenerations between nilpotent orbits in a simple Lie algebra have long played an important role in understanding the singularities of nilpotent orbit closures, or more generally Slodowy slices. The minimal degenerations label the edges in the Hasse diagram of each nilpotent orbit closure. These notions makes sense much more generally for any symplectic singularity. In this talk, I'll describe the classification of minimal degenerations for Nakajima quiver varieties. I'll also explain how one can combinatorially compute the associated Hasse diagram just from the root system of the underlying quiver. To set the scene, the first third of the talk will be a recollection of the Kraft-Procesi Theorem describing the minimal degenerations in the nilpotent cone of  $\mathfrak{sl}_n$ . The talk is based on joint work in progress with Travis Schedler.

## Component groups of the stabilizers of nilpotent orbit representatives

15:00  
15:25Emanuele Di Bella  
University of Trento

The theory of nilpotent orbits of simple Lie algebras has seen tremendous developments over the past decades. In this context an important role is played by the component group of the stabilizer of a nilpotent element. In this talk, the aim is to show computational methods to obtain explicit generators of the component group of the centralizer of a nilpotent element in a simple Lie algebra over  $\mathbb{C}$ . In some cases such generators had already been determined by impressive hand calculations but these often use the specific form of the chosen representative and are thus not immediately applicable to different representatives of the same orbit. It is then interesting to show how to overcome this issue constructing specific algorithms: for the classical types there is a straightforward method that directly translates well-known theoretical constructions; for the exceptional types we devise a method using the double centralizer of an  $\mathfrak{sl}_2$ -triple. In particular, it gives an independent construction of the component group, that does not depend on the prior knowledge of its isomorphism type.

For our purposes, we needed to construct many algorithms which have been mainly implemented in the computational algebra system GAP.

15:30  
15:55

## The lower central series of the unit group of an integral group ring

Sugandha Maheshwary

Indian Institute of Technology Roorkee

For a group  $G$ , denote by  $\mathcal{V}(\mathbb{Z}G)$ , the group of normalized units, i.e., units with augmentation one in the integral group ring  $\mathbb{Z}G$ . The study of  $\mathcal{V}(\mathbb{Z}G)$  and its center attracts a varied set of questions and one naturally seeks the understanding of central series of  $\mathcal{V}(\mathbb{Z}G)$ . While the upper central series of  $\mathcal{V}(\mathbb{Z}G)$  has been well explored, at least for a finite group  $G$ , apparently, not much is known about its lower central series  $\{\gamma_n(\mathcal{V})\}_{n \geq 1}$  where  $\mathcal{V} := \mathcal{V}(\mathbb{Z}G)$  and

$$\gamma_1(\mathcal{V}) = \mathcal{V}, \gamma_2(\mathcal{V}) = \mathcal{V}', \gamma_i(\mathcal{V}) = [\gamma_{i-1}(\mathcal{V}), \mathcal{V}], i \geq 2$$

In this talk, I will try to draw attention towards certain fundamental problems associated to the study of the lower central series of  $\mathcal{V}(\mathbb{Z}G)$  and present some recent advancements. In particular, I will present some results on the abelianisation of the  $\mathcal{V}(\mathbb{Z}G)$ . I would also like to discuss a natural filtration of the unit group  $\mathcal{V}(\mathbb{Z}G)$  analogous to the filtration of the group  $G$  given by its dimension series, leading to results on residual nilpotence of  $\mathcal{V}(\mathbb{Z}G)$ .

16:10  
17:00

## Isomorphisms, automorphisms and torsion units of integral group rings of finite groups

Wolfgang Kimmerle

University of Stuttgart

The talk presents a survey on the isomorphism problem of integral group rings of finite groups. The interplay between automorphisms and isomorphisms will be considered. This leads to the  $F^*$ -theorem. It will become transparent that in some sense  $\mathbb{Z}G$ ,  $G$  finite, determines the group  $G$  "almost" up to isomorphism.

The second part gives an overview on open problems concerning torsion units of  $\mathbb{Z}G$ . Applications of the  $F^*$ -theorem are given, in particular for Sylow like theorems in the units of  $\mathbb{Z}G$  and related problems concerning ordinary character tables of finite groups.

## Blocks of group algebras over local rings

9:00  
9:50

Florian Eisele

University of Manchester

For many questions in the modular representation theory of finite groups it is important to work over a local ring  $\mathcal{O}$  (some extension of the  $p$ -adic integers), rather than just a field of characteristic  $p$ . This has become particularly apparent in recent work on Donovan's conjecture. In this talk I will give an overview of the structure of block algebras as orders over  $\mathcal{O}$ , and properties that set them apart from other  $\mathcal{O}$ -algebras and  $\mathcal{O}$ -orders.

## Some questions on modular group algebras of finite $p$ -groups

10:00  
10:25

Diego García-Lucas

University of Murcia

We address some questions relating the algebra structure of the group algebra  $kG$  of a finite  $p$ -group  $G$  with coefficients in the field  $k$  of  $p$  elements to the structure of the group  $G$  itself. A paradigmatic example of such questions is the modular isomorphism problem, which asks whether the isomorphism type of  $G$  can be read from  $kG$ , and to which we give positive answer provided that  $G$  belongs to some specific classes of  $p$ -groups. Another example is a question of Carlson and Kovacs about whether the group algebra of an indecomposable  $p$ -group (as a direct product of proper subgroups) must be indecomposable as a tensor product of proper subalgebras. We are able to show that, if such decomposition exists, none of these subalgebras can be commutative, and as a consequence we give positive answer to this question for  $p$ -groups that are at most 3-generated.

## From extension of groups to realization-obstruction of graded algebras and back

10:50  
11:40

Yuval Ginosar

University of Haifa

We survey the theory of group extensions- the obstruction for realizing outer actions in the third cohomology, and the torsor role of the second cohomology. We pass to the parallel world of strongly graded algebras, establishing a generalization of the well-known 7-term exact sequence. Finally, we "forget" the addition once again in order to fill in the gap between the two theories.

## Graded relations of crossed products

11:50  
12:15

Ofir Schnabel

Braude college of engineering

We classify crossed product gradings for arbitrary groups and fields up to several equivalence relations in terms of group actions and their orbits and as a particular case we classify all graded division algebras up to graded isomorphism. Joint work with Yuval Ginosar.

14:30  
14:55Do we need all Sylow  $p$ -subgroups to cover the  $p$ -elements of a group?Juan Martínez Madrid  
University of Valencia

Let  $G_p$  be the set of  $p$ -elements of a finite group  $G$ . Do we need all Sylow  $p$ -subgroups of  $G$  to cover? Although this question does not have an affirmative answer in general, our work indicates that the answer is yes more often than one could perhaps expect.

This is a joint work with professors Attila Maróti and Alexander Moretó [1].

- [1] A. Maróti, J. Martínez, A. Moretó, Covering the set of  $p$ -elements in finite groups by Sylow  $p$ -subgroups, *J. Algebra* 638 (2024), 840-861.
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15:00  
15:25

## Multiplicity-free induced characters of symmetric groups

Pavel Turek  
Royal Holloway, University of London

Let  $n$  be a sufficiently large positive integer. Wildon in 2009 and independently Godsil and Meagher in 2010 have found all multiplicity-free permutation characters of the symmetric group  $S_n$ . In this talk, we focus on a more general problem when the permutation characters are replaced by induced characters  $\rho \uparrow^{S_n}$  with  $\rho$  irreducible. Despite the nature of the problem, I explain why this problem may be feasible and present some of my (often surprising) results to combinatorial questions, which naturally arise when solving the problem. Some of the main results, such as the complete classification of subgroups  $G$  of  $S_n$ , which have an irreducible character which stays multiplicity-free when induced to  $S_n$ , will be presented at the end.

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16:00  
16:25

## Continuum Braid group

Margherita Paolini  
Sapienza University of Rome

In the foundational manuscript [1] Emil Artin has introduced the sequence of Braid Group  $B_n$ .  $B_n$  is a group whose elements are equivalence classes of  $n$ -braids up to isotopy. The Braid Group admits different equivalent definitions, in particular, we will introduce the Birman-Ko-Lee presentation [2] whose generators are  $a_{l,m}$  (the  $a_{l,m}$  braid is the elementary interchange of the  $l$ -th and the  $m$ -th strand of the braid with all the other strands held fixed). A classical result done by Lusztig [3] shows that there exists an action of the Braid Group over the Drinfel-Jimbo Quantum group  $(U_q^{DJ})$ ; this action plays a central role in order to understand the structure of  $U_q^{DJ}$ . In recent years Appel, Sala and Schiffmann [3], [4] introduced a continuum analogue Quantum Group  $U_q^{DJ}(X)$ , that is an appropriate colimit of DJ Quantum Groups and their Cartan datum  $X$  can be thought of as a generalization of a quiver, where vertices are replaced by intervals. In order to study these continuum Quantum Groups, we define a continuum analogue of Braid Groups  $B_X$  mean by the BKL generators. We show that these groups preserve the colimit structure, we show that the Theorem of Hiwatori and Matsumoto holds [6] for the BKL presentation of  $B_n$  and it is compatible with the colimit structure.

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  - [6] N. Iwahori and H. Matsumoto, On some Bruhat decomposition and the structure of the Hecke rings of  $p$ -adic Chevalley groups, Inst. Hautes Etudes Sci. Publ. Math. (1965), no. 25, 5–48
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## Linear degenerations of Schubert varieties via quiver Grassmannians

16:30  
16:55

Giulia Iezzi  
RWTH Aachen

Quiver Grassmannians are projective varieties parametrising subrepresentations of quiver representations. Their geometry is an interesting object of study, due to the fact that many geometric properties can be studied via the representation theory of quivers. For instance, this method was used to study linear degenerations of flag varieties, obtaining characterizations of flatness, irreducibility and normality via rank tuples. We realise Schubert varieties as quiver Grassmannians and define their linear degenerations, giving a combinatorial description of the correspondence between their isomorphism classes and the B-orbits of certain quiver representations.

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9:00  
9:50

## Representations of $G$ -posets and canonical Brauer induction

Robert Boltje

University of California, Santa Cruz

We consider finite posets with an action of a finite group  $G$  via poset automorphisms. A typical example is the set of all subgroups of  $G$  endowed with the conjugation action. There are three different equivalent ways to define representations of a  $G$ -poset. The category of such representations turns out to be a highest weight category. Using this set-up for a particular  $G$ -poset associated with  $G$ , we are able to categorify the canonical Brauer induction formula to a finite projective resolution in the category of representations of this  $G$ -poset. Parts of this talk are joint and ongoing work with Nariel Monteiro.

10:00  
10:25

## The Complex Stable Representations of the General Linear Group over Finite Local Rings

Nariel Monteiro

University of California Santa Cruz

In this talk, we will give a survey of the representation theory of the general linear group over finite local rings. We will explain how Clifford's theory plays an important role in the construction of the irreducible representations of such groups. We will emphasize the construction of a class of irreducible representations called stable representations. The study of such a class of representations is motivated by constructions of strongly semisimple representations, introduced by the work of Hill.

10:50  
11:40

## Infinite friezes of affine type D

Léa Bittmann

University of Strasbourg

Conway-Coxeter friezes are staggered arrays of integers satisfying a local  $SL_2$ -rule. They arise in particular from triangulations of certain marked Riemann surfaces. In this talk, we will focus on triangulations of the disk with two marked points. Three infinite friezes are associated to each of these triangulations, they correspond to the evaluation of the cluster-character map of the three non-homogeneous tubes of the cluster categories of affine type D. We show that for each triangulation, these three infinite friezes have the same growth coefficient, extending a result known in affine type A.

11:50  
12:15

## Affine Bruhat order and Kazhdan-Lusztig polynomials for $p$ -adic Kac-Moody groups

Paul Philippe

Institut Camille Jordan

The Iwahori-Hecke algebra is a crucial tool in the study of a reductive group over local fields, it admits a basis indexed by the associated affine Weyl group. In the general Kac-Moody setting, an equivalent was constructed by N. Bardy-Panse, S. Gaussent and G. Rousseau in 2016, defined by generators and relations over a basis indexed by a semi-group  $W^+$  which plays the role of the affine Weyl group. Unlike in the reductive case,  $W^+$  is no longer a (extended) Coxeter group, which makes classical Kazhdan-Lusztig theory inapplicable in this context. However in 2018 D. Muthiah and D. Orr have managed to define an order and a length function on  $W^+$  analogous to the Bruhat order and the Bruhat length. Moreover, Muthiah gave a strategy to define Kazhdan-Lusztig polynomials for these algebras, using measures. We present several properties recently obtained on this  $W^+$ -order, and their implications. This is a joint work with A. Hébert.



## Nichols algebras over finite groups: old and new results

9:00  
9:50

Giovanna Carnovale  
University of Padova

Nichols algebras are graded algebras associated with a solution of the Yang-Baxter equation. Notable examples are the symmetric algebra, the exterior algebra, the positive part of quantized enveloping algebras, and Fomin-Kirillov algebras for  $n = 3, 4, 5$ . Although they can be defined by generators and (infinitely-many) relations, it is usually very hard to detect the finite-dimensional ones, or to state when a Nichols algebra is finitely-presented. Focusing on solutions of the Yang-Baxter equations arising from finite groups, I will describe some techniques that we have developed to attack the problem, and their outcomes. The work is based on collaborations with N. Andruskiewitsch and G. Garcia, with M. Costantini, with F. Esposito and Ll. Rubio y Degraffi, and with G. Maret.

## The Procesi bundle over the $\Gamma$ -fixed points of the punctual Hilbert scheme in $\mathbb{C}^2$

10:00  
10:25

Raphaël Paegelow  
University of Montpellier

During my thesis, which I have done with Cédric Bonnafé and that I will defend on June 19, I have studied the fixed point locus of the punctual Hilbert scheme on  $\mathbb{C}^2$  under the group action of the finite subgroups of  $SL_2(\mathbb{C})$ . This has been done using quiver varieties on the McKay quiver attached to the finite subgroups. Let  $\Gamma$  be a finite subgroup of  $SL_2(\mathbb{C})$  and  $S_n$  be the symmetric group on  $n$  letters. Denote by  $\Gamma_n$  the wreath product of  $S_n$  with  $\Gamma$ . In this setting, I have shown that one can also retrieve all the projective and symplectic resolutions of the singularity  $(\mathbb{C}^2)^n/\Gamma_n$ , classified by Gwyn Bellamy and Alastair Craw, as irreducible components of the  $\Gamma$ -fixed points of  $k$  points in  $\mathbb{C}^2$  where  $k$  depends on the resolution. Moreover, the indexing set of the irreducible components of the  $\Gamma$ -fixed point locus leads to interesting combinatorics in type  $A$  and  $D$  in terms of cores of partitions. Another direction has been to study the Procesi bundle over these irreducible components. To be more precise, I have studied the action of the group  $S_n \times \Gamma$  on the fibers of the Procesi bundle over the  $\Gamma$ -fixed points of the Hilbert scheme of  $n$  points in  $\mathbb{C}^2$ . A joint work with Gwyn Bellamy using the geometry of the Procesi bundle and the geometry of the isospectral Hilbert scheme can be interpreted as a reduction to "cuspidal" fibers of the Procesi bundle. A first conjecture into the study of these cuspidal fibers seems to be using the Fock representation of the affine Kac-Moody algebra attached to  $\Gamma$ .

## Surfaces with involutions and skew-group $A_\infty$ categories

10:50  
11:40

Pierre-Guy Plamondon  
University of Versailles Saint-Quentin

The combinatorics of triangulations and dissections of surfaces are closely related to subjects such as cluster algebras (for triangulations) and derived categories of gentle algebras (for dissections). The latter case can be studied by means of the topological Fukaya category defined by Haiden, Katzarkov and Kontsevich, which provides an " $A_\infty$  enhancement" of gentle algebras.

In this talk, we will look at the case of a surface with an involution, whose quotient is a surface with orbifold points. We will see how algebraic constructions of "skew-group algebras" can be extended to the  $A_\infty$  setting. As an application, we will give a representation-theoretical interpretation of the "tagged arcs" which appeared both in the study of cluster algebras and skew-gentle algebras from surfaces. This is a joint work with Claire Amiot.

11:50  
12:15

## Minimal monomial lifting of cluster algebras and applications

Luca Francone

University Claude Bernard Lyon 1

The minimal monomial lifting is a homogenisation technique of geometric and combinatorial nature. Its goal is to identify a cluster algebra structure on some schemes "suitable for lifting", compatibly with a base cluster algebra structure on a distinguished subscheme and a torus action. We will present this technique, along with some applications to the study of branching problems in representation theory of complex reductive groups and Cox rings of algebraic varieties. (Time permitting).

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14:30  
14:55

## Rationality of finite groups: groups with quadratic field of values

Marco Vergani

University of Florence

The main topic of this seminar is families of groups that have a characterization of their integral central units inside the rational group algebra. Using representation theory it is possible to consider groups as acting over vector spaces in a natural way, relating the irreducible actions to the field generated by the trace of the representation. Those fields give us a lot of information about the group itself. In this seminar we will focus on groups with field of values that are quadratic extensions of the rationals and we will define tools that allow us to detect how far the group is from a rational action.

## References

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## The classification of the prime graphs of finite solvable rational/cut groups

15:00

15:25

Sara C. Debón

University of Murcia

Graphs defined on groups constitute a powerful tool in the study of finite groups. The object of interest is to understand which -and up to which extend- graph-theoretical notions can be translated to group properties. Examples of graphs associated to groups are the commuting graph, the cyclic graph or the prime graph. An exposition of different graphs defined on groups can be found in [2]. Let  $G$  be a finite group. The prime graph or Gruenberg-Kegel graph of  $G$  is the undirected graph whose vertices are the primes dividing the order of  $G$  and an edge connects a pair of different vertices  $p$  and  $q$  if and only if  $G$  contains an element of order  $pq$ . The prime graph reflects interesting properties of the base group, for instance, a graph is isomorphic to the prime graph of a finite solvable group if and only if its complement is 3-colorable and triangle-free [3]. Due to this result, several mathematicians have been dedicated to the study of the prime graphs of some classes of solvable groups. In particular, we are interested in the prime graphs of finite solvable groups which are cut or rational. A group  $G$  is cut if for every  $g$  in  $G$  each generator of  $\langle g \rangle$  is conjugate to  $g$  or  $g^{-1}$ . A group  $G$  is rational if for every  $g \in G$  all generators of  $\langle g \rangle$  are conjugate. The classification of the prime graphs of finite solvable cut/rational groups was initiated in [1]. A graph is left to complete the classification in the rational case and four in the cut case. The aim of this talk is to share recent advances in the classification and to discuss what is remaining.

## References

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Chiralisation of reduction by stages

16:00

16:25

Thibault Juillard

University of Paris-Saclay

The dual space of a simple Lie algebra is a well-known example of a Poisson variety. This variety admits a “chiralisation” (i.e. a quantisation by a vertex algebra) by some Kac-Moody vertex algebra. Given a nilpotent element in the simple Lie algebra, using Hamiltonian reduction, one can construct a new Poisson variety: the Slodowy slice associated to this nilpotent element. This variety is itself chiralised by an affine  $W$ -algebra, which is the BRST reduction of the Kac-Moody vertex algebra. Indeed, this BRST reduction is a cohomological analogue of the Hamiltonian reduction.

I will present a joint work with Naoki Genra about the problem of reduction by stages of Slodowy slices (arXiv:2212.06022) and a more recent work about the analogue problem for affine  $W$ -algebras. Given a suitable pair of nilpotent elements in the Lie algebra, it is possible to reconstruct one of the two Slodowy slices (resp. affine  $W$ -algebras) as the reduction of the other slice (resp. algebra). As an application, I will explain how this reduction by stages allows us to prove vertex algebra analogues of classical results coming from the study of nilpotent singularities by Kraft and Procesi in the 80's.

16:30  
16:55

## Symmetric polynomials over finite fields

Botond Miklósi

Eötvös Loránd University

Consider the action of the symmetric group  $S_n$  on the  $n$ -dimensional vector space over the finite field  $\mathbb{F}_q$  of  $q$  elements, where  $q$  stands for a prime power  $p^k$ . We have an induced action on  $\mathbb{F}_q[x_1, \dots, x_n]$  the coordinate ring of  $\mathbb{F}_q^n$ . Kemper, Lopatin and Reimers proved that the elementary symmetric polynomials of degree  $2k$  form a separating set of minimal size in the invariant ring over the 2-element finite field. Based on their paper we have managed to exploit this result: over an arbitrary finite field  $\mathbb{F}_q$  the set of elementary symmetric polynomials of degree  $jp^k$  (with  $j \in \{0, \dots, q-1\}$ ,  $k \in \mathbb{Z}_{>0}$  and  $jp^k$  smaller or equal to  $n$ ) form a separating set. Moreover, this separating set is not far from being minimal when  $q = p$  and the dimension is large compared to  $p$ . In the talk I will present the main ideas and the outline of the proof.

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## On the Isomorphism Problem for group rings

9:00  
9:50

Ángel del Río  
University of Murcia

The Isomorphism Problem for group rings asks whether the isomorphism type of a group is completely determined by that of its group algebra. In general the answer is negative, but many positive answers are known under some assumptions on the group or the coefficient ring. We will present a panorama of the state of the art, including some recent results and open questions.

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## On endotrivial complexes and the generalized Dade group

10:00  
10:25

Sam Miller  
UC Santa Cruz

Endotrivial chain complexes may be thought of as a chain complex-theoretic analogue of endotrivial modules, a class of modules of interest to group and representation theorists. These complexes induce splendid autoequivalences, providing a connection to Broué's abelian defect group conjecture. In this talk, we will introduce these complexes and describe how to classify them completely. We do so by highlighting a surprising connection with the Dade group of a finite group, which parameterizes capped endopermutation modules.

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## Division and Localization on Groupoid Graded Rings

10:50  
11:15

Caio Antony  
University of Sao Paulo

A groupoid is a small category in which every morphism is invertible, and as such, generalizes the concept of groups. The concept of a groupoid graded ring is similar to that of group graded ring. That is, there exists additive subgroups for each arrow in the groupoid, whose multiplication makes sense with the composition law of the arrows if they can be composed, or is equal to 0 otherwise. Different from group graded rings, groupoid graded rings do not need to be unital. We suppose that our graded rings are object unital, that is, for every object in the groupoid, there exists an idempotent in the ring which acts as unity for products with homogeneous elements of compatible degrees.

Division is studied with respect to the aforementioned idempotents. In this talk, we'll discuss recent progress regarding division and localization on object unital groupoid graded rings. In particular, we'll discuss a generalization of P.M. Cohn's results which characterize homomorphisms from a ring to a division ring, previously generalized by D. E. N. Kawai and J. Sanchez to the context of group graded rings.

This research has been developed under the supervision of professors Javier Sanchez, from Universidade de Sao Paulo, and Angel del Rio, from Universidad de Murcia, and has been supported by FAPESP grants 2022/11166-6 and 2023/11994-9.

11:20  
11:45

## Seperating Noether number of finite abelian groups

Barna Scheffler

Eötvös Loránd University

The separating Noether number  $\beta_{sep}(G)$  of a finite group  $G$  is the minimal positive integer  $d$  such that for any finite dimensional complex representation of  $G$ , the homogeneous polynomial  $G$ -invariants of degree at most  $d$  form a separating set. This is modeled on the Noether number  $\beta(G)$ . For a finite abelian group  $G$ , we have  $\beta(G) = D(G)$ , where  $D(G)$ , the Davenport constant of  $G$  is the maximal length of an irreducible zero-sum sequence over  $G$ . An open question concerning the Davenport constant is whether the equality  $D(C_n^r) = 1 + r(n - 1)$  (or  $\beta(C_n^r) = 1 + r(n - 1)$ ) holds (where  $C_n^r$  stands for the direct sum of  $r$  copies of the cyclic group of order  $n$ ). The analogous question on the separating Noether will be a main point of this presentation. The most common families of finite abelian groups for which the exact value of the Davenport constant is known are the finite abelian groups of rank two and the finite abelian  $p$ -groups. Hence it is natural to compute  $\beta_{sep}(G)$  for rank two abelian groups. This case will also appear in our talk.

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# Groups and their actions: algebraic, geometric and combinatorial aspects

Schedule 3.06.2024 - 7.06.2024 – Levico Terme

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9:00-9:10 Welcome				
9:15-10:05 Gallauer	9:00-09:50 Eisele	9:00-09:50 Boltje	9:00-09:50 Carnovale	9:00-09:50 del Río
10:15-10:40 Symonds	10:00-10:25 García-Lucas	10:00-10:25 Monteiro	10:00-10:25 Paegelow	10:00-10:25 Miller
Coffee Break	Coffee Break	Coffee Break and Photo	Coffee Break	Coffee Break
11:15-12:05 Bellamy	10:50-11:40 Ginosar	10:50-11:40 Bittmann	10:50-11:40 Plamondon	10:50-11:15 Antony
	11:50-12:15 Schnabel	11:50-12:15 Philippe	11:50-12:15 Francone	11:20-11:45 Scheffler
12:30-13:30 Lunch	12:30-13:30 Lunch	12:30-13:30 Lunch	12:30-13:30 Lunch	12:30-13:30 Lunch
15:00-15:25 Di Bella	14:30-14:55 Martínez Madrid	Free afternoon	14:30-14:55 Vergani	
15:30-15:55 Maheshwary	15:00-15:25 Turek		15:00-15:25 Cebellán Debón	
16:10 Kimmerle	Coffee Break		Coffee Break	
17:30 Aperitivo	16:00-16:25 Paolini		16:00-16:25 Juillard	
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