ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Conc-Trees

Parallel Programming in Scala

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List Data Type

Let's recall the list data type in functional programming.

```
sealed trait List[+T] {
  def head: T
  def tail: List[T]
}

case class ::[T](head: T, tail: List[T])
extends List[T]

case object Nil extends List[Nothing] {
  def head = sys.error("empty list")
  def tail = sys.error("empty list")
}
```

List Data Type

How do we implement a `filter` method on lists?

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```
def filter[T](lst: List[T])(p: T => Boolean): List[T] = lst match {
  case x :: xs if p(x) => x :: filter(xs)(p)
  case x :: xs => filter(xs)(p)
  case Nil => Nil
}
```

Trees

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```
sealed trait Tree[+T]

case class Node[T](left: Tree[T], right: Tree[T])
extends Tree[T]

case class Leaf[T](elem: T) extends Tree[T]

case object Empty extends Tree[Nothing]
```

Filter On Trees

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```
def filter[T](t: Tree[T])(p: T => Boolean): Tree[T] = t match {
  case Node(left, right) => Node(parallel(filter(left)(p), filter(right)(p)))
  case Leaf(elem) => if (p(elem)) t else Empty
  case Empty => Empty
}
```

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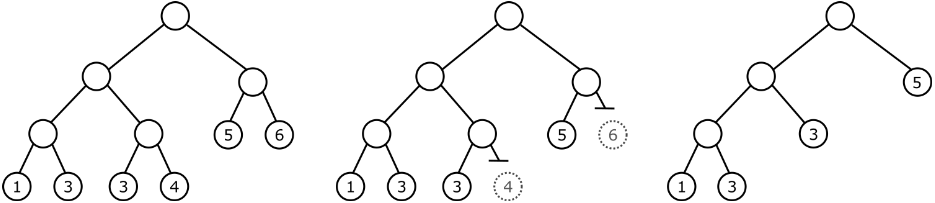

Conc Data Type

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Let's devise a data type called `Conc`, which represents balanced trees:

```
sealed trait Conc[+T] {
  def level: Int
  def size: Int
  def left: Conc[T]
  def right: Conc[T]
}
```

In parallel programming, this data type is known as the *conc-list* (introduced in the Fortress language).

Conc Data Type

Concrete implementations of the `Conc` data type:

```
case object Empty extends Conc[Nothing] {
  def level = 0
  def size = 0
}
class Single[T](val x: T) extends Conc[T] {
  def level = 0
  def size = 1
}
case class <>[T](left: Conc[T], right: Conc[T]) extends Conc[T] {
  val level = 1 + math.max(left.level, right.level)
  val size = left.size + right.size
}
```

Conc Data Type Invariants

In addition, we will define the following *invariants* for Conc-trees:

1. A `<>` node can never contain `Empty` as its subtree.
2. The `level` difference between the left and the right subtree of a `<>` node is always 1 or less.

Conc Data Type Invariants

In addition, we will define the following *invariants* for Conc-trees:

1. A <> node can never contain `Empty` as its subtree.
2. The `level` difference between the left and the right subtree of a <> node is always 1 or less.

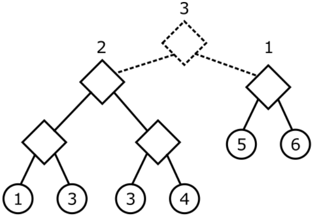
We will rely on these invariants to implement concatenation:

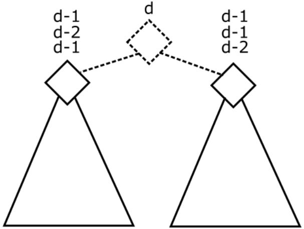
```
def <>(that: Conc[T]): Conc[T] = {
  if (this == Empty) that
  else if (that == Empty) this
  else concat(this, that)
}
```

Concatenation with the Conc Data Type

Concatenation needs to consider several cases.

First, the two trees could have height difference 1 or less:





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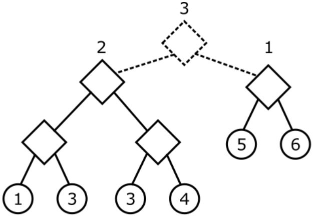

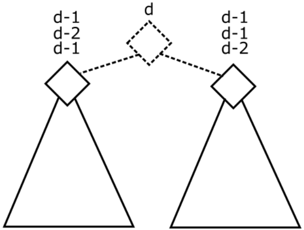


```
def concat[T](xs: Conc[T], ys: Conc[T]): Conc[T] = {
  val diff = ys.level - xs.level
  if (diff >= -1 && diff <= 1) new <>(xs, ys)
  else if (diff < -1) {
```

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Otherwise, let's assume that the left tree is higher than the right one.

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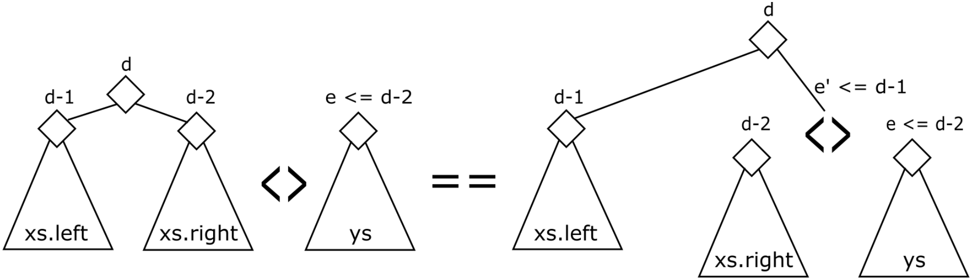
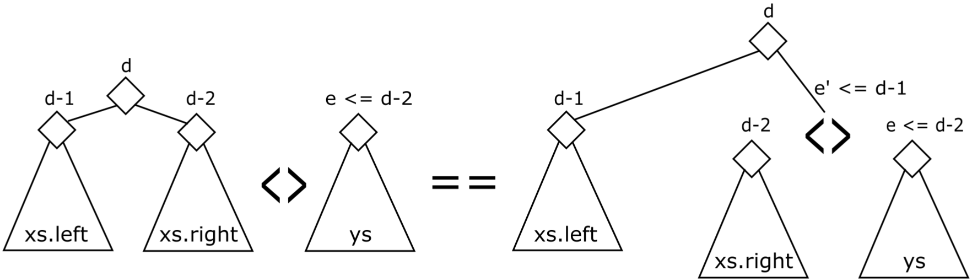


Case 1: The left tree is left-leaning.

Recursively concatenate the right subtree.

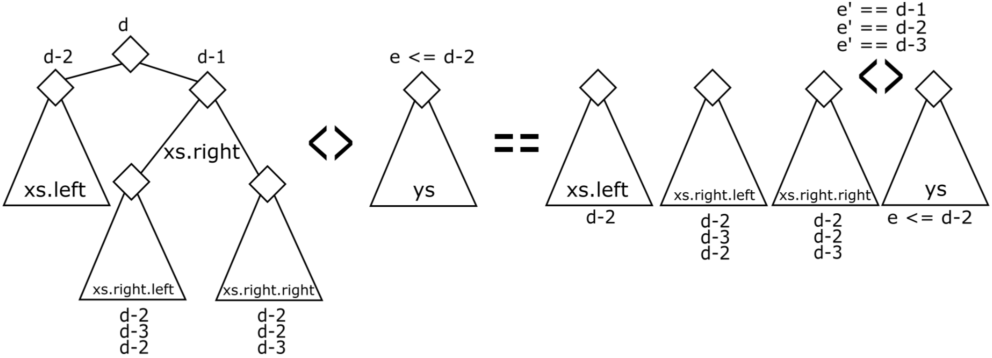
Concatenation with the Conc Data Type

```
if (xs.left.level >= xs.right.level) {
  val nr = concat(xs.right, ys)
  new <>(xs.left, nr)
} else {
```



Concatenation with the Conc Data Type

Case 2: The left tree is right-leaning.



Concatenation with the Conc Data Type

```
} else {
  val nrr = concat(xs.right.right, ys)
  if (nrr.level == xs.level - 3) {
    val nl = xs.left
    val nr = new <>(xs.right.left, nrr)
    new <>(nl, nr)
  } else {
    val nl = new <>(xs.left, xs.right.left)
    val nr = nrr
    new <>(nl, nr)
  }
}
```

Summary

Question: What is the complexity of <> method?

- $O(\log n)$
- $O(h_1 - h_2)$
- $O(n)$
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Concatenation takes $O(h_1 - h_2)$ time, where h_1 and h_2 are the heights of the two trees.