



## Full length article

## Value-at-risk methodologies for effective energy portfolio risk management



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## ABSTRACT

Research has shown that the prediction of future variance through advanced GARCH type models is essential for an effective energy portfolio risk management. Still there has been a failure to provide a clear view on the specific amount of capital that is at risk on behalf of the investor or any party directly affected by the price fluctuations of specific or multiple energy commodities. Thus, it is necessary for risk managers to make one further step, determining the most robust and effective approach that will enable them to precisely monitor and accurately estimate the portfolio's value-at-risk (VaR) which by definition, provides a good measure of the total actual amount at stake. Nevertheless, despite the variety of the variance models that have been developed and the range of various methodologies, most researchers have concluded that there is no model or specific methodology that outperforms all others. We find the best approach to minimize risk and accurately forecast the future potential losses is to adopt a methodology which takes into consideration the particular features which characterize the trade of energy products.

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## 1. Introduction

Under normal circumstances, in a competent and balanced energy market the dimension of discrepancy among supply and demand would simply be indicated by the change in the price level. But there are a number of other aspects disturbing energy market regularity, the most crucial which drive energy price volatility can be structural and have a long term impact. Following this line of reasoning, Athukorala and Wilson (2010) have investigated the short run (SR) dynamics and long run (LR) equilibrium relationship between residential electricity demand and factors affecting demand. They find that raising the electricity price is not the most effective way to reduce electricity consumption.

Additionally, unexpected events such as geopolitical instability and distortion and severe environmental problems can seriously affect the global economic climate and consequently necessitate enhanced supply capability for some of the most extensively used energy products, such as oil and natural gas. These events may raise considerably the level of risk carried by energy market investors.

Clements et al. (2015) have developed a model for dealing with price spikes across connected regions which form part of the Australian national electricity market. They find that improved risk estimates are derived when inter-regional connections are taken into account. Similarly, Clements et al. (2017) investigate the role of price volatility in

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the Queensland electricity market due to transmission constraints. From this empirical study it is found that transmission constraints are a highly significant influence on the price level and its variability. As a result the performance of a price forecasting model can be improved by taking into account the available information on transmission constraints. They emphasize the significance to practitioners of such effects in computing summary measures such as value-at-risk.

As energy commodity producers heavily depend on industrial firms, traders and refiners, energy investors focus on developing the essential technical tools to regularly monitor and minimize their overall price risk market exposure. At the same time, they have sought to build an optimal strategy which would allow them to maximize their profitability given a certain acceptable amount of risk. As a result, many financial consulting firms and researchers have been motivated to get closely involved in finding ways to manage risk.

The basic approach behind the research conducted in this field has been to appropriately modify traditional financial risk management tools, in order to take into account the unique characteristic features of the energy commodities' market. A fundamental concern in this respect was to find an accurate and scientifically approved way to measure energy price risk exposure for a certain strategy and portfolio. In this case, risk analysts based their research on minimum variance, using it as a key indicator of the total price risk of a portfolio containing energy assets. Following this approach, a large number of studies took a further step by seeking to obtain an estimate of the actual potential loss, using the well-known VaR methodology.

VaR was originally introduced by [Markowitz \(1952\)](#) and [Roy \(1952\)](#) in an attempt to optimize profit for corresponding specific levels of risk. In its current form, VaR was developed in 1989 by JP Morgan in the form of their risk management tool called the RiskMetrics. Since then, it has been widely used in finance especially by financial control institutions, investment and commercial banks and private investment funds, to estimate current risk exposure. Based on the degree of acceptable level of risk exposure, VaR and its alternations can be estimated for a 1%, 5% or 10% confidence level. Depending on the examined time horizon (daily, weekly, monthly or even yearly) VaR can be calculated indicating the probability of suffering a certain loss given the mixture of a certain investment portfolio.

The aim of the present study is to extend the work of [Halkos and Tsirivis \(2018\)](#), by offering a complete presentation of the most representative models and methodologies that have been developed to help investors in the energy market obtain valuable information and precise monitoring of the total amount of capital which is at risk based on their portfolio assets, through the most scientifically appropriate and accurate estimation of VaR.

## 2. The VaR approach as a risk management tool in the energy commodity market

Economic growth during the 1970s and 1980s was accompanied by a very rapid expansion of financial markets and offerings of financial products and services. These developments made risk management and risk quantification tools an absolute necessity for those involved in financial markets. The VaR methodology was particularly welcomed by regulatory authorities in the banking sector, as it enabled financial institutions to examine and report their overall risk exposure daily or for a predetermined time period.

A key event in the future development and implementation of the VaR methodology by financial market participants occurred in 1996 with agreement on the Basel II accord by the Basel Committee on Banking Supervision (BCBS) and the presidents of national central banks of the most powerful global economies together with a number of other countries. Importantly, this accord established the VaR as the official measurement of risk exposure for banking institutions. Basel II also encouraged financial institutions to estimate their minimum capital requirements based on their own VaR calculations for their total market risk. This development was brought about by the collapse of the British bank Barings in the same year — which came as a shock to the global banking sector creating severe turbulence in financial markets and an increase in uncertainty for investors.

By 1989 the private investment bank JP Morgan started to incorporate VaR in its risk exposure valuations. In 1996 it developed its own econometric risk management tool called risk metrics. This was created by integrating the VaR methodology into their own risk management concept. Another important event in the establishment of the VaR methodology as one of the most important and scientifically approved methods for monitoring risk exposure, was when the US Securities and Exchange Commission in 1997 allowed public traded corporations to use VaR in order to report any potential losses regarding their earnings or cash flows.

Where risk management in the energy sector is concerned, the special features of this particular market include complex returns and price distributions, non-normality and outliers, as well as strong mean reversion and high and unstable volatility and correlations. In light of these characteristics, a need has been created for extra risk management tools and methodologies outside traditional financial market approaches. In this respect the VaR methodology has become the most popular example in this domain of economics.

Despite its simplicity, the VaR concept can be a valuable tool for all banks, corporations and investors to monitor their portfolios' and investments' total market risk exposure. It provides, providing important guidance for investors' current accepted level of risk providing them the opportunity to either to reduce their exposure to speculative-high risk investments or become less conservative if allowed by their given position. Based on [Jorion \(2001\)](#), VaR represents the worst possible expected loss for an examined time horizon and a pre-specified confidence interval under normal market conditions. According to [Hendricks \(1996\)](#), [Saunders and Allen \(2002\)](#) and [Holton \(2003\)](#) VaR is able to reveal the market price risk exposure of a financial asset or portfolio in the event of a statistically bad day. Thus, if a corporation reports

**Table 1**  
Basel II committee's penalty zones.

Zone	Number of exceedances	Multiplier k	Cumulative probability assuming $q^* = 0.99$
Green	0	3.00	0.0811
	1	3.00	0.2858
	2	3.00	0.5432
	3	3.00	0.7581
	4	3.00	0.8922
Yellow	5	3.40	0.9588
	6	3.50	0.9863
	7	3.65	0.9960
	8	3.75	0.9989
	9	3.85	0.9997
Red	10 or more	4.00	0.9999

a 1% one-day-VaR of a million euro, it means that for the corporation – given its current investment portfolio mix and under normal conditions – 99% of the time it will suffer a loss of a million euro or less in a single day, while there is a 1% chance that the corporation will lose more than a million euro in a single day.

Assuming that the financial asset's returns are normally distributed the VaR can be estimated by the following formula:

$$\text{VaR} = a^* \sigma^* V_0 \sqrt{\Delta t} \quad (1)$$

The VaR for a portfolio of assets, again assuming that the returns are normally distributed, can be estimated as follows:

$$\text{VaR} = a^* \sqrt{X' \sum X} \quad (2)$$

Due to the assumed normal distribution of returns the  $\alpha$  variable corresponds to the examined level of confidence (2.33 for 99% VaR and 1.65 for 95% VaR respectively),  $\sigma$  is the converted standard deviation for the specific examined VaR (1-day-VaR, 5-day-VaR, 20-day-VaR ETC.),  $V_0$  is the initial market value of the financial asset and  $\Delta t$  denotes the examined time horizon, while  $X$  and  $\sum X$  are the diagonal matrix of the asset's returns and the covariance matrix respectively.

This basic methodology for estimating VaR corresponds to the 'delta-normal' approach, which, because of its simplicity, is popular among risk management analysts as it requires a low computational effort, while providing a quick and rough approximate of the firm's risk exposure.

The Basel II regulations (see [Committee on Banking Supervision, 1996](#)) regarding the necessary capital requirements of banks was based on the aforementioned concept of VaR, as the Basel II Accord imposes on banks the need to report their 99% daily-VaR estimate to the relative supervision authority which is then compare it to their actual risk exposure calculated at the end of the same trading day. The capital framework of the Basel II agreement specifies that a bank's capital requirement is equal to the highest amount recorded of the previous day's actual VaR or the average of actual VaR values over the 60 previous trading days, times a scaling factor accounting and penalizing the bank for the number of VaR violations during the past 250 trading days.

The daily capital need for a banking institute is calculated as follows:

$$\text{DCC}_{t+1} = \text{Max}\{(3 + k)\overline{\text{VaR}}_{60}, \text{VaR}_t\} \quad (3)$$

where  $\text{DCC}_{t+1}$  is the designated capital charges at time  $t + 1$ ;  $\text{VaR}_t$  represents the 99% VaR at day  $t$  estimated with use of GARCH class models and long-position trading data, and 60 denotes the average of the past 60 VaR values. The scaling factor, depending on the number of violations of the 99% daily-VaR during the past 250 trading days, is represented by the values  $3 \leq (3 + k) \leq 4$  ([Table 1](#)). As a result, the most appropriate GARCH model for the calculation of VaR is the one that provides VaR estimates with the lowest possible violations according to the Basel II capital regulations.

The proposed approach of the Basel II regulatory framework forms the basis behind the established risk management methodologies incorporating a GARCH-VaR procedure. However, instead of using the scaling factor, several tests have been developed which penalize the examined GARCH type models whenever they under or over-estimate VaR thus revealing an appropriate model for the proposed risk management research.

### 2.1. Alternations of the basic VaR approach

In addition to the previously analyzed traditional VaR concept, several other approaches have been developed by researchers and risk management analysts in order to meet the need for examining the potential risk concerning the expected earnings or cash flows of a financial institution or corporation. Such work also related to the need to identify the actual amount of the expected loss in cases where the loss surpasses the estimated VaR value.

One of these variations of VaR which focus on the uncertainty of future earnings, is the earnings-at-risk approach (EAR), which is widely used for risk management analysis of energy commodities (Dorris and Dunn, 2001; Denton et al., 2003). The EAR places an emphasis on measuring the variability in the accumulated earnings with regard to both physical deliveries and financial contracts over a defined time period. The EAR is mostly applied to a full fiscal year in order to gain a comprehensive view of any potential effects on earnings due to changes in energy commodity prices, exchange rates and investments. EAR is a valuable tool for managers to plan their firm's management, investment and hedging strategies over an extended time horizon.

Another popular variation of VaR incorporated in several risk management studies of the energy sector such as that of Guth and Sepetys (2001) and Stein et al. (2001), is the cash-flow-at-risk (CFaR). This is used to measure the variability in the expected cash flows regarding both physical deliveries and financial contracts taking into consideration the cash-flow timing of the relative settled trades or deliveries. Both EAR and CFaR can prove to be useful management tools. However, in order to obtain accurate estimates, a meticulously regular monitoring of price processes and a detailed analysis of all contracts is necessary.

Finally, despite all the important advantages arising from VaR, there are also a number of limitations which apply to this methodology. First, a serious drawback is that it does not provide the researcher with any information relevant to the potential losses exceeding the VaR estimates. Additionally, using VaR for portfolio optimization may lead to a stretching of the loss distribution tail, allowing the possibility of even greater losses that exceed the VaR value. Furthermore, VaR can prove to be difficult to optimize unless an assumption for normal distributions of the underlying market variables is made. As a result of the aforementioned limitations, an alternation of VaR was developed with the aim of providing a substitute for the use of VaR in risk management and portfolio optimization analysis. The specific methodology is referred as conditional VaR (CVaR) first introduced by Artzner et al. (1999). The CVaR was developed as a risk management tool for estimating the downside risk of an investment or portfolio, whenever the VaR value is exceeded.

According to Rockafellar and Uryasev (2000), CVaR is mathematically defined as follows:

$$\text{For } \alpha \in ]0, 1[, \quad \text{CVaR}_\alpha(X) = \int_{-\infty}^{+\infty} z dF_x^\alpha(z), \quad (4)$$

$$\text{where } F_x^\alpha(z) \begin{cases} 0 & \text{when } z < \text{VaR}_\alpha(X) \\ \frac{F_x(z) - \alpha}{1 - \alpha} & \text{when } z \geq \text{VaR}_\alpha(X) \end{cases} \quad (5)$$

By definition  $\text{CVaR} \geq \text{VaR}$ , hence CVaR is a more conservative version of VaR.

CVaR is basically the interval of the loss function, providing valuable information regarding the actual amount of loss in cases where the losses cross the estimated boundary set by VaR for the specific examined confidence level. Hence, CVaR offers more specific and accurate estimations for relatively less computational effort than VaR. Therefore CVaR is widely used by academic researchers in portfolio optimization with some of the most characteristic examples being the papers of Rockafellar and Uryasev (2000), Bertsimas et al. (2004) and Harris and Shen (2006).

## 2.2. Quantification methodologies for VaR

While one of the most important advantages of the VaR methodology is that it is based on a simple theoretical concept, practical estimation of VaR can prove to be a quite challenging statistical problem. As a result, several researches have made an attempt to develop a methodology that addresses this issue by ensuring both the accuracy and efficiency of the process and providing the researcher with an analytical tool that will produce robust results. The developed methodologies vary in terms of input data requirements, conditions they impose, the way they can be applied and the degree of complexity of the necessary computations.

But, even though all the models used for the calculation of VaR employ a variety of different methodologies, they all share a common general procedure consisting of a three stage structure. The first-stage includes the market valuation of the current price of the portfolio (market-to-market). The second stage refers to the estimation or selection of the distribution of the portfolio returns, in which is revealed the major difference between the various VaR methodologies. Finally, the third stage follows the actual calculation of VaR.

Traditionally, depending on the approach that is incorporated to estimate or select the portfolio return distribution during the second stage – and hence predict the potential changes of its value – economists have categorized the developed methodologies for calculating VaR into three main groups. These are the historical-simulation approach, the analytical methodology and the Monte-Carlo simulation method. However in more recent years researchers have tended to identify the following four different VaR calculation methodologies:

1. The *parametric*, including the risk metrics, GARCH and Markov-switching GARCH approaches
2. The *non-parametric*, including the historical simulation approach
3. The *semi-parametric*, including the historical simulation ARMA forecasting (HSAF) approach and the extreme value theory (EVT) approaches
4. *Monte-Carlo simulation* methodologies

### 2.2.1. Parametric methodologies

**2.2.1.1. Risk metrics.** All the developed parametric methodologies are based on selecting the most appropriate probability curve that will best fit the examined data sample and from this curve extracting the VaR. One of the oldest, simplest and perhaps most representative parametric methodologies to calculate VaR is the risk metrics model introduced by J. P. Morgan (1996). It uses a normal distribution to fit the portfolio returns and return innovations and where the conditional variance is calculated, uses an exponentially weighted moving average (EWMA):

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) r_{t-1}^2, \quad (6)$$

where  $\lambda$  denotes the decay factor and has values from 0 to 1. Risk metrics is basically equal to an IGARCH model with an assumed normal distribution. However it is not necessary to calculate any unknown parameters in the conditional variance equation as J. P. Morgan proposes a pre-determined value for  $\lambda$  of either 0.94 or 0.97 depending on the research purpose. Fleming et al. (2001) support a value of 0.97 for the decay parameter to enable the model to produce sufficient predictions of the one-day VaR. Using the risk metrics model for a pre-specified time period and confidence level  $\alpha$  and under the normality assumption, the VaR value is obtained by multiplying  $\sigma_t$  with the  $1-\alpha$  quantile of the standard normal distribution.

In general, although the risk metrics approach is a relatively crude way to calculate VaR, it is popular among risk managers as a simple and quick method to get satisfactory volatility predictions for short-time horizons. However, the risk metrics methodology carries all the disadvantages of a normal IGARCH model, as it assumes a normal distribution for the returns and the innovations distribution. There is strong empirical evidence from both financial and energy markets, which reveals persistent signs of kurtosis and skewness and which are contrary to the properties to be expected of a symmetric normal distribution.

Additionally, contrary to other GARCH type models, use of risk metrics does not allow for asymmetry and hence account for phenomena such as the leverage volatility effect (Halkos and Tsirivis, 2018). Finally, as one of the most characteristic of the early parametric approaches, it contains the false assumption of independent and identically distributed (IID) return data, which is far from the evidence coming from real financial markets (e.g. Brooks et al., 2005; Bali and Weinbaum, 2007) and especially the energy commodity market.

**2.2.1.2. The GARCH methodology.** In an attempt to overcome the disadvantages of the traditional parametric approach, researchers focused on more sophisticated models to capture the various volatility effects observed in both the returns and prices of financial and energy products. The most popular models, especially in energy commodities research, are the GARCH family volatility models. The most easily implemented and widely used GARCH model for estimating VaR is the basic GARCH(1,1) model with normal distribution.

The VaR value using a GARCH model with an assumed normal distribution can be estimated as follows:

$$\text{VaR}_t = \hat{\mu}_t + z^* \hat{\sigma}_t, \quad (7)$$

where  $\hat{\mu}_t$  and  $\hat{\sigma}_t$  represent the conditional mean and volatility of the financial product or energy commodity returns, which are calculated with the use of a GARCH class model. In order to estimate the VaR for a long market position,  $z$  is denoted as  $z_\alpha$ , which represents the left end quantile corresponding to the  $\alpha$  per cent confidence level of the assumed distribution — which, in this case, is the typical Gaussian (Normal) distribution. Similarly, when estimating VaR for a short market position,  $Z$  denotes  $z_{1-\alpha}$ , corresponding to the right end quantile of the  $\alpha$  per cent confidence level of the assumed model distribution.

It is important to note that energy commodity returns exhibit far more marked signs of skewness and kurtosis than the vast majority of financial products. Many researchers, among them Clewlow and Strickland (2000), have found that, by assuming a normal return and innovations distribution, forming a firm's strategy becomes problematic for energy risk management. This is particularly so when trying to obtain a robust and accurate estimate of VaR as it relates to individual or a portfolio of energy commodities. As a result, by assuming a normal return distribution it is high likely that VaR calculations will substantially underestimate the potential loss.

Therefore, in trying to model effectively energy data and an accurately measure of VaR based on GARCH family models, academic researchers have experimented with replacing the traditional normal distribution of the basic model with a variety of density functions. They include the student-t, skewed student-t, heavy tailed distributions and others<sup>1</sup>. Additionally, to overcome some of the weaknesses of the basic GARCH model researchers have instead used other GARCH type models such as EGARCH, GJR-GARCH, FIGARCH, FIAPARCH and HYGARCH (and others) to account for asymmetric and long memory volatility effects.

Aside from the research using single-regime GARCH models, there is interest in capturing changes in the market behavior of energy products under the influence of extreme economic and geopolitical events. In such conditions there is a high likelihood that these volatile assets will behave in a completely different way than under normal conditions. For this reason, researchers have adopted models that combine the Markov-switching regime methodology with several GARCH family models. In this alternative approach, by incorporating a regime-switching variable, the original GARCH model used

<sup>1</sup> See Halkos and Tsirivis (2018).



is modified to account for a probable regime switch in the estimated parameters of the variance process (Halkos and Tsirivis, 2018).

Overall then, by combining different GARCH type models with various return distributions and the use of a number of evaluation methodologies to obtain the most accurate and efficient model, researchers have sought to find a successful approach for obtaining trustworthy approximations of VaR. However, these procedures still suffer from the fact that a fixed random distribution is selected in advance in order to predict the future distribution of risk factors.

### 2.2.2. Non parametric methodologies

**2.2.2.1. Standard historical simulation methodology.** The standard historical simulation is by far the fastest and simplest methodology to estimate VaR, as it uses past time series return data for the current portfolio asset mixture. In this way a hypothetical return distribution is created on which future possible returns can be predicted. The VaR value in this case is obtained by sorting in an ascending order, the asset or portfolio returns of the considered past time framework. The observation that fulfills the requirement of having X% of the observations both beneath and (1-X%) and above represents the VaR value for the initially determined confidence level (95%, 99%, etc.).

In contrast with the traditional parametric approaches, in an historical simulation no assumptions are made about the return distribution being similar to a normal distribution or other fixed form distribution. Thus while the procedure to estimate VaR by ascending observations would lead someone to reasonably think that this leads to a normal return distribution – in reality it produces a random distribution. This allows the historical simulation to account for fat tails, skewness and other phenomena that are frequently observed in financial markets and which are particularly present in the energy commodity markets.

Nevertheless, as for other methods, the historical simulation has some important weaknesses. Namely, it requires a large sample of past historical data to ensure a reliable result. As well, the sample may contain the effect of rare and unexpected events which are not likely to take place again in the future or may lack the effect of events that researchers might want to consider in their research. Lastly historical simulation gives the same weight to each observation regardless of whether they are in the distant past or more recent, so the final result is equally affected by both very distant and recent returns.

Nevertheless, although historical simulation may be computationally demanding and bring with it a number of disadvantages, the basic concept still remains very popular in risk management research which requires a VaR calculation. Over the years, many researchers have included historical simulation in their studies and made important contributions to the academic literature. This has been by way of modifying and extending the original methodology to overcome its shortcomings and make it more suitable for risk management analyses in highly volatile economic environments such as the financial and energy markets. A characteristic examples of such studies is the paper of Boudoukh et al. (1998), in which historical simulation is combined with the risk metrics model thereby putting less weight on distant observations and more weight on recent ones. In the paper of Hull and White (1998) the authors create a link between the historical return and volatility changes. Thus they update the historical returns by multiplying them by a volatility ratio of historical and current volatility – both estimated through a GARCH model.

### 2.2.3. Semi-parametric methodologies

**2.2.3.1. Historical simulation ARMA forecasting (HSAF) approach.** It can be said that in the field of risk management applied to energy commodities, the most important development in the original concept of historical simulation is the paper of Cabedo and Moya (2003), in which a modification of the initial methodology is presented. This involves the use of an Autoregressive Moving Average (ARMA) model, on which is based the return distribution. This is instead of using the past returns' distribution as in the original historical simulation method or making a normal or other type of distributional assumption for the return distribution as in the GARCH framework. The HSAF as presented by Cabedo and Moya (2003) consists of a four stage process, during which the sample of past asset or portfolio returns is initially tested for both stationarity and autocorrelation using an appropriate statistical test such as the augmented Dickey Fuller and the Ljung–Box test. In cases where the assumption of stationarity is rejected by the relative test and the examined return sample is characterized by non-stationarity, then the first difference or the lowest possible difference that exhibits stationarity is used. On the other hand, if the autocorrelation test reveals that the hypothesis for autocorrelation in the sample is not statistically significant, then the HSAF becomes equal to the standard historical simulation methodology. Nevertheless, to proceed to the next stages requires a statistically significant sample autocorrelation to be determined.

The second step includes the estimation of an ARMA model for the original past return sample. As part of this stage, the Ljung–Box test is used to check for autocorrelation, and if present, is used to control for the number of lags that are needed to remove it.

During the third stage, the forecasting errors that are produced from the relative forecasts for the estimated coefficients of the previous stage are used to form the statistical distribution upon which the percentile associated with the desired likelihood level will be determined after conducting the necessary statistical analysis.

Finally, in the fourth and last stage of the procedure the quantification of VaR is made. Based on the ARMA model used in the second step of the method, forecasts for the possible future returns are made t, adjusted for the percentile that was estimated previously. These forecasts provide the VaR value for a statistical likelihood level equal to the percentile decided earlier.

The HSAF is an improved methodology compared to the traditional historical simulation, providing the researcher with a more flexible and efficient VaR estimate, which takes into account the persistent return fluctuations. As a result, the HSAF has been used and further developed in a number of studies over the years such that of [Sadeghi and Shavvalpour \(2006\)](#). Here they concluded that the HSAF is the most appropriate methodology, among a number of others, to estimate VaR using a high confidence level (e.g. 99%) to manage risk in the oil market. The HSAF is often used in the academic literature as one of the competing alternative methodologies for determining the most appropriate methodology to estimate VaR for specific energy markets or energy commodities.

**2.2.3.2. The extreme value theory (EVT) approach.** The EVT approach is mainly used by researchers to deal with the fact that the standardized residuals in the majority of volatility models used to estimate VaR in portfolios containing financial assets – and especially assets related to energy products – exhibit strong signs of fat tails and asymmetry. Calculating VaR following the traditional procedure through a volatility model, is most likely to lead to an underestimated VaR value for high confidence levels. EVT accommodates the extreme returns in the tail of the return distribution, which can be considered and examined as a smaller independent distribution.

Nevertheless, the need to take into consideration a long series of data makes forecasting the near future difficult as the model does not have the capability to allow sufficient weight to be placed on recent market fluctuations. In addition, the EVT approach is based on the assumption that the examined extreme returns in the tail distribution are IID. In this case this assumption is most likely to be proved wrong, as the stochastic volatility and the long memory volatility effect present in the returns of highly volatile assets suggest that an extreme and rare event is most likely to be followed by another.

[McNeil and Frey \(2000\)](#) try to address all the above issues by proposing a two stage process. During the first stage, they suggest using an appropriate GARCH class model as a filter for the examined return series in order for the GARCH residuals that are produced to appear as close as possible to a match with IID observations. In the second stage the authors employ the EVT methodology to the residuals previously estimated. Hence this hybrid model manages to incorporate both time-varying volatility as well as the important characteristic of fat tails in the return distribution, as the GARCH residuals will still maintain this characteristic from initial returns. Moreover, [McNeil and Frey \(2000\)](#) point out that both conditional volatility and marginal distribution are individually modeled for the left tail. This provides the advantage of limiting their studies only to the left tail of the return distribution as this is the only important part of the distribution for estimating VaR.

Empirical results in numerous academic papers confirm the success of the [McNeil and Frey \(2000\)](#) two stage approach to deal with the previously mentioned shortcomings of implementing the EVT in the returns of portfolios containing highly volatile assets. Particularly in the case of risk management analysis involving VaR estimation of energy products, most researchers – such as [Byström \(2004\)](#) and [Chan and Gray \(2006\)](#) – have based their studies on the aforementioned paper including an EVT implementation similar to the one proposed by [McNeil and Frey \(2000\)](#).

The EVT methodology incorporates alternative distributions used to investigate the behavior of the returns belonging to the tail of the return distribution. For the implementation of EVT two main basic models have been developed based on a variety of different distributions: the block maxima (BM) model using the generalized extreme value (GEV) distribution and the peaks over threshold (POT) model using the general Pareto distribution.

**The block maxima (BM) model** In the BM model the Frechet, Weibull and Gumbel extreme value distributions are combined in constructing a GEV distribution. The BM concept relies on the idea of dividing the examined data series into fixed size blocks focusing on the extreme values that appear during the individual time periods. These extreme observations represent extreme events forming what is called a block maximum. The BM model guides the researcher to make critical decisions regarding the time interval  $n$  and the data block which is contained in that interval.

Given that  $X_1, \dots, X_n$  represent a group of IID observations forming a cumulative distribution  $F(x)$  with maximum loss equal to  $M_n = \max \{ X_1, \dots, X_n \}$ , the cumulative distribution function of  $M_n$  in this case can be described as follows [McNeil et al. \(2005\)](#)

$$P(M_n \leq x) = P(X_1 \leq x, \dots, X_n \leq x) = \prod_{t=1}^n P(X_t \leq x) = F^n(x), \quad (8)$$

Concerning  $F^n(x)$  the asymptotic approach suggests that it relies on the maximum standardized value:

$$Z_n = \frac{M_n - \mu_n}{\sigma_n} \quad (9)$$

with  $\mu_n$  denoting the location and  $\sigma_n$  a positive constant parameter. According to the theorem of [Fisher and Tippet \(1928\)](#) if  $Z_n$  approaches a non-degenerated distribution then this is the GEV distribution, which can be mathematically represented as:

$$H_{\xi, \mu, \sigma}(x) = \begin{cases} \exp \left( -1 + \frac{\xi (x - \mu)}{\sigma} \right)^{-1/\xi} & \xi \neq 0 \text{ with } \left( -1 + \frac{\xi (x - \mu)}{\sigma} \right) > 0 \\ \exp(-e^{-x}) & \xi = 0, \end{cases} \quad (10)$$

for which  $\sigma > 0$ ,  $-\infty < \mu < +\infty$  and  $-\infty < \xi < +\infty$ , with all three  $\xi$ ,  $\sigma$  and  $\mu$  parameters being estimated using the maximum likelihood approach. Nevertheless, the above GEV distribution is basically a generalized representation of the subsequent three distributions:

$$\text{Frechet : } \Phi_{\alpha}(x) = \begin{cases} 0, & x \leq 0 \\ e^{-x^{-\alpha}}, & x > 0 \end{cases} \text{ with } \alpha > 0, \quad (11)$$

$$\text{Weibull : } \Psi_{\alpha}(x) = \begin{cases} e^{-(-x)^{\alpha}}, & x \leq 0 \\ 1, & x > 0 \end{cases} \text{ with } \alpha > 0, \quad (12)$$

$$\text{Gumbell : } \Lambda(x) = e^{-e^{-x}}, \text{ with } x \in \mathbb{R}, \quad (13)$$

The  $\xi$  parameter defines the shape of the distribution, contingent on the value of the parameter  $H$  distribution becoming a generalization of the above distributions for  $\xi > 0$  of the Frechet type distribution, for  $\xi < 0$  of the Weibull type distribution and for  $\xi = 0$  of the Gumbell distribution. The VaR estimation based on the Frechet and Gumbell distributions can be as follows:

$$\text{VaR} = \begin{cases} \mu_n - \frac{\sigma_n}{\xi_n} 1 - (-n \ln(a))^{-\xi_n}, & \text{for } \xi > 0, \text{ Frechet} \\ \mu_n - \sigma_n \ln(-n \ln(a)), & \text{for } \xi = 0, \text{ Gumbell} \end{cases} \quad (14)$$

It is usual in the BM model approach that the blocks are chosen in such an order that they match a full economic year representing the number of observations during that year. Nevertheless, the BM model is not regularly used when examining samples containing time series data of financial returns as a result of the intense presence of volatility clustering.

**Peaks over threshold (POT) model** The peaks over threshold (POT) model is by far the most popular for implementing the EVT approach to estimate VaR in risk management analyses concerning financial assets and energy commodity assets. The main reason is that the POT model has proved to be more practically applicable in such studies providing a more efficient use of the available data regarding extreme prices or returns. The POT model enables a comprehensive use of the entire data sample exceeding a significant threshold. This contrasts with the BM model which uses only the maximum from a fixed size 'block' to estimate the extreme value distribution. Hence, the POT model exhibits an obvious advantage over the BM model.

The POT model is largely based on the general Pareto distribution (GPD) and seeks to fit the distribution of the data exceeding the predetermined threshold to a GPD distribution. Specifically, given that  $(X_1, X_2, \dots, X_n)$  is a sequence of IID observations denoting financial returns which form an unknown distribution  $F$ , and  $u$  is a predetermined threshold, the POT model examines all the  $(Y_1, Y_2, \dots, Y_{N_u})$  values exceeding this threshold ( $Y_i = X_i - u$ ), with  $N_u$  representing the  $N$  number of sample observations which exceed  $u$ .

In this case, the distribution of losses exceeding the threshold  $u$  can be specified as the following conditional probability:

$$F_u(Y) = P(X - u/X > u), \quad (15)$$

$$F_u(Y) = \begin{cases} \frac{F(u + Y) - F(u)}{1 - F(u)}, & Y \geq 0 \\ 0, & Y < 0 \end{cases} \quad (16)$$

Based on the theorem of [Balkema and Haan \(1974\)](#) and [Pickands \(1975\)](#), for large values of  $u$  the excess distribution function  $F_u$ , can resemble a generalized Pareto distribution. This GPD distribution is described as follow:

$$G_{\xi, \sigma}(Y) = \begin{cases} 1 - \left(1 + \frac{\xi Y}{\sigma}\right)^{-\frac{1}{\xi}}, & \xi \neq 0 \\ 1 - \exp\left(-\frac{Y}{\sigma}\right), & \xi = 0 \end{cases} \quad (17)$$

in which  $Y \geq 0$  for  $\xi \geq 0$  and  $Y \in [0, -\frac{\sigma}{\xi}]$  for  $\xi < 0$ . Again, similar to the BM model, the  $\xi$  parameter defines the shape of the extreme value distribution,  $\sigma$  is a scale parameter, while the decided threshold  $u$  represents the location parameter.

Furthermore, as for the GEV distribution, the main GPD distribution is a combination of three other distributions. As a result, when  $\xi$  becomes 0 the estimated GPD distribution is approximated by the normal distribution with the tails decreasing at an exponential rate. For negative values of the  $\xi$  parameter the GPD distribution is approximated by a beta distribution with finite tails. For  $\xi$  values exceeding 0 the particular GPD distribution resembles a student-t type distribution. By making the assumption that the distribution of extreme values can be fitted to a GPD distribution, the VaR value for a predetermined probability  $p$  can be estimated as follows:

$$\text{VaR}_p = u + \frac{\sigma}{\xi} \left( \left( \frac{N}{N_u} (1-p) \right)^{-\xi} - 1 \right), \quad (18)$$

Nevertheless, all the previously discussed EVT methodologies lack the ability to take into consideration the phenomenon of volatility clustering, which is highly prevalent in financial and energy markets. To account for this, a



conditional VaR estimation is required using the conditional EVT methodology. This methodology is based on the work of [McNeil and Frey \(2000\)](#) and combines the above unconditional EVT method with GARCH type volatility models. The main advantage that arises from this hybrid approach is that it initially enables the researcher to account for conditional heteroskedasticity in the data sample as well as measure and predict future volatility with the use of a single-regime GARCH or a Markov-Switching GARCH model. For a second stage model the extreme values at the distribution tail are accommodated using the EVT concept.

In this case the filtering of the sample using a determined GARCH type volatility model produces IID time series data on which the EVT can be directly implemented. Given a stationary return sample and assuming that the residuals ( $\varepsilon_t$ ) can be approximated by a GPD distribution ( $G_{\xi, \sigma}$ ), then the conditional VaR for a specific  $\alpha$  quantile can be estimated as follows:

$$\text{VaR}_\alpha = \mu_t + \sigma_t G_{\xi, \sigma}^{-1}(\alpha), \quad (19)$$

where,  $\mu_t$  denotes the conditional mean and  $\sigma_t^2$  the conditional variance, while the  $G_{\xi, \sigma}^{-1}(\alpha)$  is the  $\alpha$  quantile of the specific GPD distribution which can be obtained based on Eq. (17).

#### 2.2.4. The Monte Carlo methodology

A rather popular methodology for estimating VaR when conducting risk management analyses involving assets from financial and energy markets is the Monte Carlo methodology. This is based on the widely used Monte Carlo simulation and relies on the hypothesis that prices or returns track a particular stochastic path. As a result, by incorporating these processes in the Monte Carlo simulation it is possible to form the distribution of a specific asset or portfolio value for an examined time frame. Furthermore, simulating a series of important and representative market variables and obtaining their possible future value paths, enables the researcher to take account of factors influencing the future performance of the market. Hence any possible volatility jumps or extreme events can be included in the research. Specifically, in the case of VaR studies, the necessary quantile in the tail of the distribution is being produced straightforwardly from the random paths.

In general, academic researchers consider the Monte Carlo approach as a useful alternative model among others to estimate VaR relative to highly volatile markets and assets such as energy commodities. This is because it can account for several unique features of the market such as volatility clustering and non-normal return distribution. It is exactly this methodological flexibility that led many researchers to use it, despite the fact that, in some cases, it has proved computationally intensive.

The most simplified version of the Monte Carlo approach used to calculate VaR for a specific time horizon and confidence level, involves simulating  $N$  draws from the return distribution at time  $t + 1$  and ranking them from the lower to the highest. Then it is necessary to locate the price for the  $\alpha\%$  lowest percentile that corresponds to the initial confidence level for which the VaR is estimated. This means that there is  $\alpha\%$  probability that the asset value could diminish from this value ( $S_{t+1}^{\alpha\%}$ ) to even lower levels. Finally, by deducting the above future asset value from the current value ( $S_t - S_{t+1}^{\alpha\%}$ ), the potential loss that corresponds to the VaR for the specific time interval and confidence level is calculated. The VaR value in the Monte Carlo approach therefore represents the maximum loss from the random return distribution for a specific and predetermined time interval and confidence level.

For applications involving a dynamic model with multiple risk factors affecting the asset or portfolio returns and taking into account phenomena such as volatility clustering and non-normality, it is necessary to first define the dynamics of the fundamental processes. To do so  $N$  sample paths are generated illustrating variations in the asset or portfolio value during the examined time frame, and in which all the details included in the probability distribution must be integrated. Finally, based on the generated sample paths and according to the hypothesized processes, the value of every individual risk factor is estimated and used to define the asset or portfolio value at the specific examined time.

**2.2.4.1. Hybrid Monte Carlo and historical simulation approach.** A rather interesting variation of the traditional Monte Carlo approach is the hybrid model proposed by [Andriosopoulos and Nomikos \(2013\)](#). The authors combined Monte Carlo with the historical simulation approach in an attempt to build a model that would be capable of providing a more accurate estimation of VaR relative to other competitive models. The main focus of the researchers is to create an appropriate risk management tool suitable to investigate risk particularly in the energy markets, by bridging the gap that is left by other studies which sought to present improved variations of these two methodologies.

Specifically, this hybrid approach provides an ability for the researcher to account for volatility jumps and fat tails in the return distribution. This is in contrast with most variations of historical simulation which focus mostly on capturing any volatility drifts which the original method underestimates or fails to consider. The hybrid Monte Carlo-historical simulation takes advantage of the flexibility given by the Monte Carlo simulation.

[Andriosopoulos and Nomikos \(2013\)](#) suggest generating an extremely large number  $N$  of sample paths for the underlying processes for the purpose of forecasting the spot price for a particular time period in the future. The daily VaR is estimated using the average sample path and is based on the rolling window method as in the historical simulation which rolls the simulated observations forward one by one until the last at the end of the initially determined future time period. This VaR using this hybrid approach can be mathematically represented as follows:

$$\text{VaR}_{t+1} = \text{Perccrntile} \left\{ \{\bar{r}_i\}_{i=t-T}^t, \alpha \right\}, \quad (20)$$

where  $T$  represents the total return sample including the observations both from the original sample as well as those simulated; while  $\bar{r}_t^s = \sum_{\omega=1}^N \frac{r_{t,\omega}^s}{N}$  is the average simulated return at time  $t$ ,  $r_{t,\omega}$  is the simulated price value  $\omega$  at time  $t$  and  $N$  the number of simulations.

### 2.3. Model evaluation and statistical accuracy of VaR

The wide variety of methodologies and approaches to estimates VaR creates the need to compare and determine which models are appropriate and provide accurate results for a specific risk management analysis. In the relevant academic literature, researchers base their examination for the most suitable VaR models on two main statistical tests: the unconditional coverage test and the conditional coverage test developed by Kupiec (1995) and Christoffersen (1998). The models that are not rejected and fulfill the requirements of both tests for the specific VaR confidence level are then compared using the regulatory loss function (Lopez, 1999; Sarma et al., 2003).

In general, the backtesting analysis of VaR involves comparing the predicted values of the competing VaR models with the actual losses in the following period. Next, according to the results from the unconditional and the conditional coverage tests, an examination is made of whether the number of violations exceeds the expected number based on the selected confidence level of VaR, as well as whether the violations are independent and randomly distributed.

#### 2.3.1. The Kupiec's unconditional coverage test

The concept of the Kupiec's (1995) test relies on estimating the probability of observing a loss exceeding the predicted VaR amount. In an attempt to examine the accuracy and performance of the various VaR models, Kupiec developed a likelihood ratio test ( $LR_{uc}$ ) which investigates if the failure rate of a particular model is statistically equal to the one expected. For this purpose, the following exception indicator needs to be determined:

$$I_{t+1} = \begin{cases} 1, & \text{if } r_{t+1} < VaR_{(\alpha)} \\ 0, & \text{if } r_{t+1} \geq VaR_{(\alpha)}, \end{cases} \quad (21)$$

In the above context  $x = \sum I_{t+1}$  is a variable following a binomial distribution  $x \sim B(N, \alpha)$  and denotes the number of exceptions in a sample consisting of  $N$  observations and  $p = \sum I_{t+1}/N$  represents the expected exception frequency or ratio of violations. Then in order for  $VaR_{(\alpha)}$  to provide an unconditional coverage of  $\alpha\%$ , the null hypothesis that  $H_0: p = \alpha$  needs to be confirmed or else the alternative will apply  $H_1: p \neq \alpha$ . In this case the likelihood ratio statistic of the unconditional coverage test ( $LR_{uc}$ ) for the specific null hypothesis is approximated by an  $\chi^2$  asymptotic distribution and is described as follows: LR

$$LR_{uc} = -2 \log\{\alpha^x (1 - \alpha)^{N-x}\} + 2\{\log p^x (1 - p)^{N-x}\}, \quad (22)$$

#### 2.3.2. The Christoffersen's conditional coverage test

The unconditional coverage test may be able to reject a model that under or over-estimates VaR, however it is unable to detect those VaR models which, while providing sufficient unconditional coverage, do not identify VaR violations that are not independent. Christoffersen (1998) in trying to overcome this weakness of the Kupiec's test, developed the conditional coverage test ( $LR_{cc}$ ). This enables the researcher to examine simultaneously whether the sum of actual VaR violations match with that predicted by the model and whether these violations are correlated through time. This allows a model to be discarded where it produces either a very high or very low number of clustered exceptions. In risk management studies of highly volatile markets and assets such as the energy markets and energy commodities, it is important to examine for conditional coverage as in most cases they exhibit strong signs of volatility clustering.

The unconditional coverage test examines the joint hypothesis of both unconditional and independence tests, with an appropriate model providing both sufficient unconditional coverage and serial independence of  $I_{t+1}$ . Combining the two properties the test statistic can be described as  $LR_{cc} = LR_{uc} + LR_{ind}$ , with  $LR_{cc}$  under the null hypothesis following a  $\chi^2(2)$  asymptotic distribution and  $LR_{ind}$  representing the likelihood ratio statistic for the null hypothesis of serial independence. Specifically, for  $\pi_{i,j} = P(I_t = j | I_{t-1} = i)$  denoting the transition probability and  $n_{i,j} = \sum_{t=1}^N P(I_t = j | I_{t-1} = i)$ , with  $n_{i,j}$  representing the number of observations with value  $i$  being followed by value  $j$ , where  $i, j = 0$  or  $1$ , the hypothesis of the independence test is described as follows Aloui and Mabrouk (2010)

$$H_{0ind} : \pi_{00} = \pi_{10} = \pi, \pi_{01} = \pi_{11} = 1 - \pi \quad (23)$$

Finally, the combined null hypothesis for both unconditional coverage and independence of the failure process is tested based on the following likelihood ratio statistic:

$$LR_{cc} = -2 \log\{\hat{\pi}^{n_0} (1 - \hat{\pi}^{n_1})\} + 2 \log\{\hat{\pi}_{00}^{n_{00}} \hat{\pi}_{01}^{n_{01}} \hat{\pi}_{10}^{n_{10}} \hat{\pi}_{11}^{n_{11}}\}, \quad (24)$$

$$\text{where } \hat{\pi}_{i,j} = (n_{ij}/(n_{ij} + n_{i,1-j})), n_{ij} + n_{i,1-j} n_j n_j = n_{0j} + n_{1j} \text{ and } \hat{\pi} = (n_0/(n_0 + n_1)). \quad (25)$$

### 2.3.3. The regulatory loss function criterion

It is most common both at the business but especially at the academic level that when a risk management analysis is conducted – including the estimation of an individual or multiple VaR values – more than one model is used for this purpose. The coverage tests help researchers reject some or the majority of the models for lacking the necessary accuracy to provide a reliable VaR estimate. However, there is a high probability that there will remain several competing models at the end of this procedure. In that case, researchers use an extra model evaluation criterion in order to conclude which of the available models performs better than the others producing a final stage assessment of the regulatory or magnitude loss function. This test was initially developed by Lopez (1999) as an additional tool to compare the models that satisfy the conditions of the coverage tests. It primarily takes into account the recommendation of the Basel Banking Supervision Committee, that both the number of VaR exceptions as well as the magnitude or size of these exceptions should be of equal importance for individual researchers or institutes when pursuing a risk management evaluation. The regulatory loss function is generally described as follows:

$$\text{Loss}_{i,t+1} = \begin{cases} 1 + (r_{i,t+1} - \text{VaR}_{i,t})^2, & \text{if } r_{i,t+1} < \text{VaR}_{i,t} \\ 0, & \text{otherwise,} \end{cases} \quad (26)$$

The proposed quadratic loss function has the advantage of incorporating the extra information regarding the magnitude of the VaR exceptions, creating a more reliable model with greater accuracy together with a performance criterion which penalizes in a more severe way, large exceptions.

## 3. Literature review

### 3.1. Energy commodity risk management studies based on the VaR concept

Due to the various advantages and unique characteristics of the VaR approach which were presented in detail earlier in this paper, it is considered as a powerful analytical tool to measure and manage risk both at the academic level as well as in business and financial institutions circles. In particular, in the field of risk management of the energy commodity market, an increasing number of researchers incorporate VaR in their analyses, with the aim of creating precision as to the amount that could be lost as a result of the risk exposure to an individual or a portfolio of assets in this highly volatile market.

Academic researchers in energy economics have long ago realized the advantages of VaR, hence a wide range of studies have been published especially over the last 15 years (see Table A.1), where the interest for energy commodities has dramatically increased. This has been due to the high level of economic importance of the various types of energy products along with the extreme levels of uncertainty in energy markets. Researchers have compared the different methodologies and approaches in order to arrive at the most appropriate way to estimate VaR for a specific energy commodity or portfolio for a specific data sample.

#### 3.1.1. Academic papers using historical simulation, and GARCH based models for VaR forecasting

As already mentioned, the standard historical simulation approach and its variations have been a popular methodology used by a large number of researchers as one of the competing methods used to provide the most accurate and efficient VaR estimation. Cabedo and Moya (2003) present a development of the traditional historical simulation method – the historical simulation ARMA forecast (HSAF) – which was based on an 8-year sample (1992–1999) of daily Brent oil prices. They compare the VaR estimates provided by a basic GARCH model assuming a normal return distribution, with those produced by historical simulation and the HSAF. The authors concluded that the HSAF is the most appropriate methodology of the three to estimate VaR, as it provides a more flexible VaR quantification than historical simulation. It also better fits the extremely high price volatility, while the standard normal GARCH model tends to overestimate VaR.

Sadeghi and Shavvalpour (2006) further confirmed the findings of Cabedo and Moya (2003), by using the HSAF methodology employing a 6-year sample (1997–2003) of weekly OPEC oil price data. This produced the most accurate of forecasted VaR values, and represented a substantial improvement on the results of the various basic normal ARCH and GARCH models to which it was compared with. They noted that both ARCH and GARCH models again systematically overestimated VaR. Nevertheless they argued that no matter which method is used, VaR is a valuable and trustworthy tool to quantify risk regarding oil prices. Fan and Jiao (2006) developed another form of the standard historic simulation – the exponential decreased frequency with ARMA forecasts approach (EDFAAF) – which was based on the HSAF concept. Their approach was tested against the HSAF approach using a 12-year sample consisting of weekly observations relating to Brent spot oil prices. They found that at all times their model performed better at forecasting VaR than the HSAF approach.

Žiković et al. (2015) compare the predicting ability of the standard historic simulation approach together with other historic simulation based approaches such as the mirrored historic simulation<sup>2</sup> (MHS), the filtered historic simulation<sup>3</sup>

<sup>2</sup> See Holton (1998).

<sup>3</sup> See Barone-Adesi et al. (1999).

(FHS), as well as that of the BRW<sup>4</sup> simulation, a basic GARCH model, risk metrics and that of an unconditional EVT (GPD) model and an EVT-GARCH model. The researchers gathered a 9-year sample (1995–2014) of daily return data of one month futures contracts for several key energy commodities such as WTI crude oil, Brent oil, natural gas and heating oil, in order to test the accuracy of the forecasted VaR values of the competing models. The results of the relative tests revealed that the rather simplistic methodology of FHS performed better than any other competing model, noting that the simpler non-parametric models were better able to capture the large number of extreme events included in the sample.

In line with the previous study, [Huisman et al. \(2015\)](#) using a 5-year sample (2008–2013) of daily futures returns for crude oil, gas oil, natural gas and coal, also found that among other models such as historic simulation and risk metrics, the FHS model produced the best VaR estimates. Finally, [Andriosopoulos and Nomikos \(2013\)](#) based on a 9-year sample of daily price data concerning a wide range of energy commodities, such as heating oil, crude oil, gasoline, natural gas propane and electricity, discovered that for the vast majority of the examined energy commodities, the models that relied on the combination of the hybrid Monte Carlo and HS approach and the standard Monte Carlo approach, significantly outperform the various competing GARCH type models, in terms of accurately forecasting the 1% daily VaR.

On the other hand, [Costello et al. \(2008\)](#) argue that the findings of [Cabedo and Moya \(2003\)](#) and [Sadeghi and Shavvalpour \(2006\)](#), are mainly based on certain assumption. That is, in the examined ARCH and GARCH models, the return distribution is approximated by the normal distribution – an assumption which is empirically rejected by numerous studies for both financial and especially for the more volatile energy commodity markets.

To address this problem the authors incorporate the methodology of [Barone-Adesi et al. \(1999\)](#) who modify the basic procedure under which the VaR is estimating using a GARCH based model. In this way, Barone-Adesi et al. try to improve the poor VaR forecasts of GARCH models which assume a normal return distribution, by advocating the use of historical simulation to determine the future return distribution. In their results it is revealed that the VaR estimates from their suggested hybrid GARCH-HS approach significantly outperform those from standard GARCH models. [Costello et al. \(2008\)](#) adopt the main idea of the aforementioned semi-parametric procedure using a 23 year sample (1992–2003) containing daily Brent oil prices. They conclude that the semi-parametric GARCH approach performs better than the HSAF of [Cabedo and Moya \(2003\)](#), as it manages to capture changing volatility.

The inability of GARCH family models to provide trustworthy VaR forecasts when assuming a normal innovations distribution, triggered the interest of many researchers in modifying the standard normal GARCH model in such way that it would outperform other competing VaR estimation methodologies. Researchers have also sought to determine which alternation of the basic model will outperform the other GARCH based models. [Fan et al. \(2008\)](#) found that GARCH models based on the generalized error distribution (GED) are better able to capture phenomena like fat tails and skewness which are generally accepted to be present in the highly volatile energy commodity returns. The authors using a 20-year (1987–2006) daily spot price sample for WTI oil and Brent oil, concluded that both the GED-GARCH and GED-TGARCH models are better able to take into consideration the volatility characteristics of these two commodities, while they outperform the HSAF model at forecasting the daily VaR for a 95% confidence level.

Other researchers have largely focused on determining which specific innovation distribution would alter the standard normal ARCH and GARCH models and produce a better model in terms of VaR forecasting ability. [Giot and Laurent \(2003\)](#) assessed the risk metrics, skewed-student-APARCH and skewed-student-ARCH models based on their predicted VaR values. Their sample consisted of daily spot prices for rare precious metals and WTI oil and Brent oil, for a 15-year (1987–2002) period, with the researchers determining that the skewed-student-APARCH model is the more appropriate model to deal with the fat tailed and skewed return distribution of WTI oil and Brent oil and produce the best 1% VaR forecast.

Similarly, [Hung et al. \(2008\)](#) compare the basic GARCH model with normal distribution, the student-GARCH and the GARCH-HT model which incorporates the heavy-tailed distribution proposed by [Politis \(2004\)](#). A sample of 10-year (1996–2006) daily spot prices for WTI oil, Brent oil, heating oil, propane and gasoline was used, with the relative accuracy and efficiency tests revealing that GARCH-HT model is by far the best model to forecast VaR. It provided precise estimates at very high confidence levels thus making it suitable for more conservative risk management analyses.

[Aloui and Mabrouk \(2010\)](#) using a 21-year (1986–2007) sample of daily spot prices for WTI oil, Brent Oil and gasoline, compare the ability of the FIGARCH, FIAPARCH and HYGARCH models to provide accurate future VaR estimations taking into account three different innovation distributions – the student, the skewed-student and the normal distribution. The relative tests indicated that the FIAPARCH model incorporating the student-t distribution outperforms the other models in terms of out of sample VaR estimates.

Additionally, [Cheng and Hung \(2011\)](#) compared the performance of three GARCH type models they developed. These were based on the skewed-generalized-t (SGT) distribution, the generalized-error-distribution (GED) and the normal distribution respectively. Cheng and Hung found that, according to the relative accuracy tests, the GARCH-SGT model outperforms rival models, as it is more capable of taking into account the exhibited fat-tailed and negatively skewed return distribution of the examined energy commodities when forecasting VaR. This result is rather important as the authors used for their analysis a 7-year (2002–2009) sample. This included the years of financial crisis and which contained observations of daily spot and futures prices for WTI oil, gasoline and heating oil, among other commodities.

<sup>4</sup> See [Boudoukh et al. \(1998\)](#).

Finally, Chkili et al. (2014) using a large sample containing 14-year (1997–2011) data of daily spot and 3-month futures returns for WTI oil, natural gas and precious metals, examined the future VaR estimates of a group of seven linear and non-linear GARCH family models including GARCH, IGARCH, EGARCH, risk metrics, FIGARCH, FIAPARCH and HYGARCH. The accuracy tests used show that the FIAPARCH model clearly performs better than the rest of the models of the group, with the authors pointing that the key reason is the presence of the asymmetric and the long memory volatility effect in the return data.

### 3.1.2. Academic papers using EVT based models for VaR forecasting

Another popular methodology for estimating and forecasting VaR is the EVT approach, which is either used individually or, in most cases is combined with a GARCH family model in an attempt to lift the restrictions of the standard methodology and accomplish a more precise prediction of VaR. Specifically, regarding energy commodity risk management there has been a number of studies incorporating EVT most of which have compared EVT models with other models which rely on other competitive methodologies.

Ren and Giles (2010) after finding significant evidence of fat tails and negative skewness in their examined data sample (consisting of 8-years (1998–2006) of daily returns from the Canadian oil market) – determined that the peaks-over-thresholds (POT) approach with generalized Pareto distribution (GPD) is the most appropriate methodology to employ to effectively model their data and estimate VaR, after ruling out the existence of conditional heteroskedasticity and hence the necessity of employing a combined GARCH-EVT approach.

Byström (2005) using a 5-year (1996–2000) sample of hourly electricity returns, determined that both the basic GARCH model with normal distribution as well as the GARCH model with student-t distribution tend to both underestimate and overestimate the probability that an extreme return is observed, even though the electricity return distribution was found to be less fat tailed and negatively skewed than for other energy commodities. The author argues that this is largely due to the fact that the GARCH family models are built to model the behavior of the whole return distribution and as a result, are less effective in capturing the extreme returns in the tails of the distribution. On the contrary, EVT models focus on the tail of the distribution and hence are more suitable.

Furthermore, the presence of skewness in the return distribution makes the models which use symmetrical distributions less appropriate for making tail quantile estimations. For all these reasons, Byström (2005) employing the methodology of McNeil and Frey (2000), develops a combined GARCH-EVT model based on the POT approach with GPD to better forecast the probability of extreme returns. This was found to be slightly more accurate than the GARCH-EVT model based on the BM approach.

Similarly, Krehbiel and Adkins (2005) examining the returns for both spot and futures prices for WTI oil, Brent oil, heating oil, gasoline and natural gas, also find evidence of fat tails and negative skewness. Applying the same procedure as Byström (2005) they further agree that a GARCH-EVT model based on the POT method outperforms GARCH family models using symmetrical distribution when forecasting the probability of the occurrence of an extreme price change. This finding was the same when measuring risk exposure as in the case of VaR, while it also provides more accurate results than a GARCH-EVT model using the BM method.

Chan and Gray (2006) rely on an AR-EGARCH model to successfully account for seasonality and leverage effects in the conditional volatility in electricity prices. The authors, using a large sample of spot prices coming from five major international electricity markets, compare the ability of several competing models – such as the historic simulation model, the AR-EGARCH model with normal distribution, the AR-EGARCH model with student-t distribution and the AR-EGARCH-EVT model based on the POT approach – to provide precise predictions of VaR. The results clearly indicate that the AR-EGARCH-EVT model is by far the most appropriate to model data coming from markets which exhibit high volatility, skewness and kurtosis as well in performing tail quantile estimates and predicting VaR.

Marimoutou et al. (2009) rely on an AR-GARCH model to model a 20-year (1987–2006) sample of daily Brent oil spot returns and perform future projections of VaR. The authors incorporate an AR-GARCH model but modify traditional methods, such as the historical simulation and the conditional EVT approach. In this way they develop models which are then tested according to their ability to predict VaR. Results in this case show that the AR-GARCH-EVT model relying on the POT approach as well as the FHS employing the AR-GARCH, outclass the traditional historical simulation model and the normal AR-GARCH model. Nevertheless, it is found that the AR-GARCH model with student-t distribution is equally able to provide VaR estimates that capture changes of volatility dynamics and appropriately adjust to them.

However, Paraschiv et al. (2016) testing a sample of 5-year (2009–2014) hourly electricity returns, reach the conclusion that the AR-GARCH-EVT relying on the conditional GPD approach, better estimates the probability of observing extreme returns when compared to the basic AR-GARCH model with either normal or student-t distribution. Additionally, Youssef et al. (2015) following the same concept of the aforementioned studies, examine the out of sample VaR estimates of a FIAPARCH-EVT model with GPD distribution relative to a similar GARCH-EVT model used as a benchmark. The backtesting results in this occasion reveal that the FIAPARCH-EVT model outperforms its benchmark in terms of estimating the one day ahead VaR values. Moreover, when the authors incorporated a combination of bootstrap and GPD approach it was found that the model continued to provide reliable VaR estimates for even longer time periods. The two models in this study were built and tested using a 10-year (2003–2012) daily return sample for WTI oil, Brent oil and gasoline.

Finally, an interesting finding regarding the use of the GARCH-EVT methodology to estimate VaR in energy portfolios is that presented in the research paper of Nomikos and Pouliasis (2011). The evidence presented based on a data sample containing daily futures prices from 1991 to 2008 for WTI oil, Brent, heating oil and gasoline, revealed that the GARCH-EVT approach consistently overestimated the VaR values. Such an error could produce inefficiency in terms of estimating the capital cost for investor groups with an average risk aversion and a valuable tool for more conservative investors.



**Table A.1**

Proposed VaR estimating models for portfolios containing energy commodity assets.

Year	Author	Examined data set	Examined energy commodity	Outperforming method
2003	Cabedo and Moya	Daily spot	Brent oil	HS-ARMA Forecast (HSAF)
2003	Giot and Laurent	Daily spot	WTI oil, Brent oil	Skewed-Student-APARCH
2005	Byström	Hourly spot	Electricity	GARCH-EVT-POT with GPD
2005	Krehbiel and Adkins	Daily spot, daily futures	WTI crude oil, Brent oil, heating oil, gasoline, natural gas	GARCH-EVT-POT with GPD
2006	Sadeghi and Shavvalpour	Weekly spot	OPEC oil	HS-ARMA Forecast (HSAF)
2006	Chan and Gray	Daily spot	Electricity	AR-EGARCH-EVT-POT
2006	Fan and Jiao	Weekly spot	Brent oil	HS-Exponential Decreased Frequency ARMA forecasts (EDFAAF)
2008	Costello et al.	Daily spot	Brent oil	Semi-parametric GARCH
2008	Hung et al.	Daily spot	WTI oil, Brent oil, heating oil gasoline, propane	GARCH-HT
2009	Marimoutou et al.	Daily spot	Brent oil	AR-GARCH-EVT-POT
2010	Ren and Giles	Daily spot	WTI oil, Brent oil	GARCH-EVT-POT with GPD
2011	Nomikos and Pouliaxis	Daily futures	WTI oil, Brent oil, heating oil, gasoline	GARCH-EVT
2011	Cheng and Hung	Daily spot, daily futures	WTI oil, heating oil, gasoline	GARCH-SGT
2013	Andriosopoulos and Nomikos	Daily spot	WTI oil, heating oil, gasoline, natural gas propane, electricity	Monte Carlo — historical simulation
2014	Chkili et al.	Daily spot, 3-month Futures	WTI oil, natural gas	FIAPARCH
2015	Zikovic et al.	Daily futures	WTI crude oil, Brent oil, heating oil, natural gas	Filtered historical simulation
2015	Huisman et al.	Daily futures	WTI oil, natural gas, gasoline, coal	Filtered historical simulation
2015	Youssef et al.	Daily spot	WTI oil, Brent oil, gasoline	FIAPARCH-EVT with GPD
2017	Paraschiv et al.	Hourly spot	Electricity	GARCH-EVT-POT with GPD

#### 4. Conclusion

The energy market is characterized by extreme ambiguity and price instability due to the widespread effect of geopolitical and ecological influences, in addition to the level of worldwide demand, increasing competition and market deregulation. These extremely unbalanced market circumstances demand the consideration and determination of the most suitable approaches and instruments to administer the energy commodities' disproportionate price risk.

Extending the work of [Halkos and Tsirivis \(2018\)](#), the current paper emphasizes the importance of VaR as a tool for a more effective risk management and the need for a deeper understanding of investor's risk of those holding a portfolio of energy assets. The various VaR based methodologies which are comprehensively presented here, can provide researchers conducting risk management analysis relating to an energy commodity, with vital information regarding the risk arising from price volatility.

Due to the nature of VaR, models based on this particular methodology can offer a deeper understanding of the total risk involving the investigated or forecasted price shifts of a particular energy product, as they have the ability to quantify this risk and express it in currency units. To be noted is that selecting the most appropriate return distribution still remains of critical importance in the use of VaR. In general, then, a VaR model using the correct return distribution with the lowest possible number of violations, constitutes a reliable risk management tool in the hands of energy economists, corporate managers and policy makers.

Another important finding of his study is that – as emphasized by the most recent research – there is no particular individual model or methodology that outweighs the others in modeling and accurately predicting the total amount of capital which is at risk in portfolios containing energy products. Additionally, we find as do other volatility centered researchers, that the appropriateness and therefore the performance of a unique model or approach firmly depends on the precise sample considered together with any particular distinguishing characteristics that influence the trade of the particular energy product.

However, particularly in the case of VaR research regarding a peculiar and high risky group of assets such as energy commodities, it is found that there is a key factor that can determine which specific methodology is the most appropriate for estimating the VaR value. This is the attitude of the portfolio owner towards risk and its development through time. Different models have been developed which are variously more suitable for investors who are becoming more risk averse or risk friendly. They therefore provide both more conservative and less conservative values relative to the amount of money which is at risk based on the specific portfolio.

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## Appendix

(See Table A.1).

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