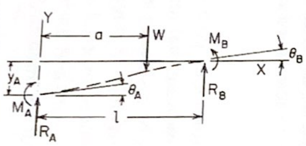
8.17 Tables

Notation: W = load (force); w = unit load (force per unit length); M_o = applied couple (force-length); θ_o externally created concentrated angular displacement (radians); Δ_o = externally created concentrated lateral displacement; T_1 and T_2 = temperatures on the top and bottom surfaces, respectively (degrees). R_A and R_B are the vertical end reactions at the left and right, respectively, and are positive upward. M_A and M_B are the reaction end moments at the left and right, respectively. All moments are positive when producing compression on the upper portion of the beam cross section. The transverse shear force V is positive when acting upward on the left end of a portion of the beam. All applied loads, couples, and displacements are positive as shown. All deflections are positive upward, and all slopes are positive when up and to the right. E is the modulus of elasticity of the beam material, and I is the area moment of inertia about the centroidal axis of the beam cross section. γ is the temperature coefficient of expansion (unit strain per degree).

1. Concentrated intermediate load



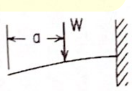
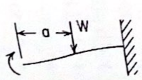
Transverse shear $= V = R_A - W\langle x-a\rangle^0$

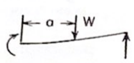
Bending moment $= M = M_A + R_A x - W\langle x-a\rangle$

Slope $= \theta = \theta + \dfrac{M_A x}{EI} + \dfrac{R_A x^2}{2EI} - \dfrac{W}{2EI}\langle x-a\rangle^2$

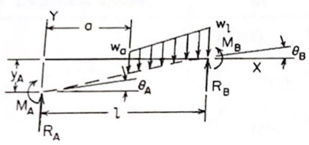
Deflection $= y = y_A + \theta_A x + \dfrac{M_A x^2}{2EI} + \dfrac{R_A x^3}{6EI} - \dfrac{W}{6EI}\langle x-a\rangle^3$

(*Note:* See page 160 for a definition of the term $\langle x-a\rangle^n$.)

End Restraints, Reference No.	**Boundary Values**	**Selected Maximum Values of Moments and Deformations**
1a. Left end free, right end fixed (cantilever) 	$R_A = 0 \quad M_A = 0 \quad \theta_A = \dfrac{W(l-a)^2}{2EI}$ $y_A = \dfrac{-W}{6EI}(2l^3 - 3l^2 a + a^3)$ $R_B = W \quad M_B = -W(l-a)$ $\theta_B = 0 \quad y_B = 0$	Max $M = M_B$; max possible value $= -Wl$ when $a = 0$ Max $\theta = \theta_A$; max possible value $= \dfrac{Wl^2}{2EI}$ when $a = 0$ Max $y = y_A$; max possible value $= \dfrac{-Wl^3}{3EI}$ when $a = 0$
1b. Left end guided, right end fixed 	$R_A = 0 \quad M_A = \dfrac{W(l-a)^2}{2l} \quad \theta_A = 0$ $y_A = \dfrac{-W}{12EI}(l-a)^2(l+2a)$ $R_B = W \quad M_B = \dfrac{-W(l^2 - a^2)}{2l}$ $\theta_B = 0 \quad y_B = 0$	Max $+M = M_A$; max possible value $= \dfrac{Wl}{2}$ when $a = 0$ Max $-M = M_B$; max possible value $= -\dfrac{Wl}{2}$ when $a = 0$ Max $y = y_A$; max possible value $= \dfrac{-Wl^3}{12EI}$ when $a = 0$

End Restraints, Reference No.	Boundary Values	Selected Maximum Values of Moments and Deformations
1f. Left end guided, right end simply supported 	$R_A = 0 \quad M_A = W(l-a) \quad \theta_A = 0$ $y_A = \frac{-W(l-a)}{6EI}(2l^2 + 2al - a^2)$ $R_B = W \quad M_B = 0$ $\theta_B = \frac{W}{2EI}(l^2 - a^2) \quad y_B = 0$	Max $M = M_A$ for $0 < x < a$; max possible value $= Wl$ when $a = 0$ Max $\theta = \theta_B$; max possible value $= \frac{Wl^2}{2EI}$ when $a = 0$ Max $y = y_A$; max possible value $= \frac{-Wl^3}{3EI}$ when $a = 0$

2. Partial distributed load



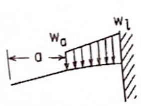
$$\text{Transverse shear} = V = R_A - w_a\langle x-a\rangle - \frac{w_l - w_a}{2(l-a)}\langle x-a\rangle^2$$

$$\text{Bending moment} = M = M_A + R_A x - \frac{w_a}{2}\langle x-a\rangle^2 - \frac{w_l - w_a}{6(l-a)}\langle x-a\rangle^3$$

$$\text{Slope} = \theta = \theta_A + \frac{M_A x}{EI} + \frac{R_A x^2}{2EI} - \frac{w_a}{6EI}\langle x-a\rangle^3 - \frac{w_l - w_a}{24EI(l-a)}\langle x-a\rangle^4$$

$$\text{Deflection} = y = y_A + \theta_A + \theta_A x + \frac{M_A x^2}{2EI} + \frac{R_A x^3}{6EI} - \frac{w_a}{24EI}\langle x-a\rangle^4 - \frac{(w_l - w_a)}{120EI(l-a)}\langle x-a\rangle^5$$

End Restraints, Reference No.	Boundary Values	Selected Maximum Values of Moments and Deformations

2a. Left end free, right end fixed (cantilever) 	$R_A = 0 \quad M_A = 0$
	$\theta_A = \frac{w_a}{6EI}(l-a)^3 + \frac{w_l - w_a}{24EI}(l-a)^3$
	$y_A = \frac{-w_a}{24EI}(l-a)^3(3l+a) - \frac{w_l - w_a}{120EI}(l-a)^3(4l+a)$
	$R_B = \frac{w_a - w_l}{2}(l-a)$
	$M_B = \frac{-w_a}{2}(l-a)^2 - \frac{w_l - w_a}{6}(l-a)^2$
	$\theta_B = 0 \quad y_B = 0$
		If $a = 0$ and $w_l = w_a$ (uniform load on entire span), then
		Max $M = M_B = \frac{-w_a l^2}{2}$ Max $\theta = \theta_A = \frac{w_a l^3}{6EI}$
		Max $y = y_A = \frac{-w_a l^4}{8EI}$
		If $a = 0$ and $w_a = 0$ (uniformly increasing load), then
		Max $M = M_B = \frac{-w_l l^2}{6}$ Max $\theta = \theta_A = \frac{w_l l^3}{24EI}$
		Max $y = y_A = \frac{-w_l l^4}{30EI}$

Case No. in Table 8.1	Load Location a/l	Multiplier Listed for	I_B/I_A 0.25	0.50	2.0	4.0	8.0
1a	0	y_A	2.525	1.636	0.579	0.321	0.171
		θ_A	2.262	1.545	0.614	0.359	0.201
	0.25	y_A	2.663	1.682	0.563	0.303	0.159
		θ_A	2.498	1.631	0.578	0.317	0.168
	0.50	y_A	2.898	1.755	0.543	0.284	0.146
		θ_A	2.811	1.731	0.548	0.289	0.149
	0.75	y_A	3.289	1.858	0.521	0.266	0.135
		θ_A	3.261	1.851	0.522	0.267	0.135
1c	0.25	R_A	1.055	1.028	0.972	0.946	0.926
		θ_A	1.492	1.256	0.744	0.514	0.330
	0.50	R_A	1.148	1.073	0.936	0.887	0.852
		θ_A	1.740	1.365	0.682	0.435	0.261
1d	0.25	R_A	1.046	1.026	0.968	0.932	0.895
		M_A	1.137	1.077	0.905	0.797	0.686
	0.50	R_A	1.163	1.085	0.915	0.837	0.771
		M_A	1.326	1.171	0.829	0.674	0.542
1e	0.25	θ_A	1.396	1.220	0.760	0.531	0.342
		$y_{l/2}$	1.563	1.301	0.703	0.452	0.268
	0.50	θ_A	1.524	1.282	0.718	0.476	0.293
		$y_{l/2}$	1.665	1.349	0.674	0.416	0.239
2a. Uniform load	0	y_A	2.711	1.695	0.561	0.302	0.158
		θ_A	2.525	1.636	0.579	0.321	0.171
	0.25	y_A	2.864	1.742	0.547	0.289	0.149
		θ_A	2.745	1.708	0.556	0.296	0.154
	0.50	y_A	3.091	1.806	0.532	0.275	0.140
		θ_A	3.029	1.790	0.535	0.278	0.142
	0.75	y_A	3.435	1.890	0.516	0.262	0.132
		θ_A	3.415	1.886	0.516	0.263	0.133
2c. Uniform load	0	R_A	1.074	1.036	0.968	0.941	0.922
		θ_A	1.663	1.326	0.710	0.473	0.296
	0.50	R_A	1.224	1.104	0.917	0.858	0.818
		θ_A	1.942	1.438	0.653	0.403	0.237

Moments of inertia vary as $(1 + Kx/l)^n$, where $n = 1.0$

Table 8.11(a) Reaction and Deflection Coefficients for Tapered Beams

Case No. in Table 8.1	Load Location a/l	Multiplier Listed for	I_B/I_A				
			0.25	0.50	2.0	4.0	8.0
1a	0	y_A	2.729	1.667	0.589	0.341	0.194
		θ_A	2.455	1.577	0.626	0.386	0.235
	0.25	y_A	2.872	1.713	0.572	0.320	0.176
		θ_A	2.708	1.663	0.588	0.338	0.190
	0.50	y_A	3.105	1.783	0.549	0.296	0.157
		θ_A	3.025	1.761	0.555	0.301	0.161
	0.75	y_A	3.460	1.877	0.525	0.272	0.140
		θ_A	3.437	1.872	0.526	0.273	0.140
1c	0.25	R_A	1.052	1.028	0.970	0.938	0.905
		θ_A	1.588	1.278	0.759	0.559	0.398
	0.50	R_A	1.138	1.070	0.932	0.867	0.807
		θ_A	1.867	1.390	0.695	0.468	0.306
1d	0.25	R_A	1.049	1.027	0.969	0.934	0.895
		M_A	1.155	1.082	0.909	0.813	0.713
	0.50	R_A	1.169	1.086	0.914	0.831	0.753
		M_A	1.358	1.177	0.833	0.681	0.548
1e	0.25	θ_A	1.509	1.246	0.778	0.586	0.428
		$y_{l/2}$	1.716	1.334	0.721	0.501	0.334
	0.50	θ_A	1.668	1.313	0.737	0.525	0.363
		$y_{l/2}$	1.840	1.385	0.692	0.460	0.294
2a. Uniform load	0	y_A	2.916	1.724	0.569	0.318	0.174
		θ_A	2.729	1.667	0.589	0.341	0.194
	0.25	y_A	3.067	1.770	0.554	0.301	0.161
		θ_A	2.954	1.737	0.563	0.311	0.169
	0.50	y_A	3.282	1.830	0.537	0.283	0.148
		θ_A	3.226	1.816	0.540	0.287	0.150
	0.75	y_A	3.580	1.906	0.518	0.266	0.136
		θ_A	3.564	1.902	0.519	0.267	0.136
2c. Uniform load	0	R_A	1.068	1.035	0.965	0.932	0.899
		θ_A	1.774	1.349	0.723	0.510	0.351
	0.50	R_A	1.203	1.098	0.910	0.831	0.761
		θ_A	2.076	1.463	0.664	0.430	0.271

Moments of inertia vary as $(1 + Kx/l)^n$, where $n = 2.0$

TABLE 8.11(*b*) Reaction and Deflection Coefficients for Tapered Beams