# MATH2561: Probability and Statistics

Instructor: **Dr. Harsha K V**, Department of Mathematics

**GITAM Hyderabad**

## Unit 4: Practice Questions

### QUESTIONS

1) Define standard error of the sample mean $\bar{x}$ (sample size $n$) of a population with mean $\mu$ and variance $\sigma^2$.
2) Consider a random sample of size $n = 25$ taken from a population with mean $\mu = 5$ and variance $\sigma^2 = 9$. Answer the following:
   - If the sample is taken from an infinite population, then what is the mean and variance of the sample mean $\bar{X}$? calculate the standard error of $\bar{X}$? How the standard error changes when sample size $n = 25$ is increased to $n = 2500$?
   - If the sample is taken from a finite population with size $N = 100$, the what is the mean and variance of $\bar{X}$? Calculate the finite population correction factor in this case?
3) State the Central limit theorem with proper assumptions, and provide an example to elucidate the application of the central limit theorem.
4) Car mufflers are constructed by nearly automatic machines. One manufacturer finds that, for any type of car muffler, the time for a person to set up and complete a production run has a normal distribution with mean 1.82 hours and standard deviation 1.20. What is the probability that the sample mean of the next 40 runs will be from 1.65 to 2.04 hours? (*Hint: use Central limit theorem*)
5) The following is the time taken (in hours) for the delivery of 8 parcels within a city: 28, 32, 20, 26, 42, 40, 28, and 30. Use these figures to judge the reasonableness of delivery services when they say it takes 30 hours on average to deliver a parcel within the city. (*You can use 5% significance level, and assume observations are from normal distribution even if it is not given in the question if sample size is small*).

6) The tensile strength (in the unit of 1,000 psi) of a new composite can be modeled as a normal distribution. A random sample of size 25 specimens has mean $45.3$, and standard deviation $7.9$. Does this information tend to support or refute the claim that the mean of the population is $40.5$?
7) A sample of 900 members has a mean 3.4 cm, and standard deviation (S.D) 2.61 cm. Is this sample from a population of mean 3.25 cm and S.D 2.61 cm? Test at 5 % significance?
8) Consider the sample variance $S^2$ of a random sample of size $n = 19$ taken from a normal population with variance $\sigma^2 = 9$. The what is the sampling distribution of the statistic $Y = 2S^2$ ? (*Hint: the sampling distribution of* $\frac{(n-1)S^2}{\sigma^2}$ *is the chi-square distribution with* $n - 1$ *degrees of freedom*).
9) Samples of 20 thickness measurements of some product are collected regularly. A sample standard deviation exceeding 1.4 mm signals concern about the product. Find the probability that, when the population standard deviation is $\sigma = 1.35$ mm, the next sample will signal concern. (Use $5\%$ significance level, and critical values are given by $\chi^2_{19,0.05} = 30.144$, and $\chi^2_{20,0.05} = 31.41$).
10) A random sample of size $n = 100$ is taken from a population with $\sigma = 5.1$. Given that the sample mean is $\bar{x} = 21.6$, construct a 95% confidence interval for the population mean $\mu$.
11) A study measures the heights of nanopillars fabricated using a new process. A random sample of n = 50 nanopillars yields the following summary statistics: $\bar{x} = 305.58$, $s = 36.97$. construct a 99% confidence interval for the population mean of all nanopillars.
12) A research group reports the following summary statistics for the toughness (measured in MJ/m³) of a sample of processed synthetic silk fibers: $\bar{x} = 22.6$, $s = 15.7$, and sample size $n = 18$. Construct a 95% confidence interval for the mean toughness of these fibers. Assume that the population is normal.
13) Define type I error and Type II error in hypothesis testing. What is level of significance? What are the different types of alternative hypothesis and decision criteria; explain with an example?
14) A manufacturer claims that the average fatigue life of a certain type of product is 1650 cycles. A quality-control engineer tests a sample from the production line. Suppose the fatigue life $X$ is normally distributed with standard deviation $\sigma = 1$. The decision criterion is modified so that the manufacturer's claim is accepted for $X > 1640$ cycles, then find the probability of a Type I error?

15) Scientists need to be able to detect small amounts of contaminants in the environment. As a check on current capabilities, measurements of lead content (in $\mu g/L$) are taken from twelve water specimens spiked with a known concentration are : $2.4, 2.9, 2.7, 2.6, 2.9, 2.0, 2.8, 2.2, 2.4, 2.4, 2.0, 2.5$. Test the null hypothesis $\mu = 2.25$ against the alternative hypothesis $\mu > 2.25$ at 0.05 level of significance.

16) A sample of 900 members has a mean of 3.4 cm. Is the sample comes from a population of mean 3.25 cm, and standard deviation $2.61$ cm? Test at $1\%$ significance.

17) To reduce the amount of recycled construction materials entering landfills it is crushed for use in the base of roadways. Green engineering practices require that their strength, resiliency modulus (MPa), be accessed. Measurements on $n_1 = n_2 = 6$ specimens of recycled materials from two different locations produce the data as follows:
Location 1 : $707 \;\; 632 \;\; 604 \;\; 652 \;\; 669 \;\; 674$
Location 2 : $552 \;\; 554 \;\; 484 \;\; 630 \;\; 648 \;\; 610$.
Use the 0.05 level of significance to establish a difference in mean strength for materials from the two locations. (Assume that population variances are the same). Construct $95\%$ confidence interval for difference of population means of both locations?

18) One process of making green gasoline takes sucrose, which can be derived from biomass, and converts it into gasoline using catalytic reactions. This is not a process for making a gasoline additive but fuel itself, so research is still at the pilot plant stage. At one step in a pilot plant process, the product consists of carbon chains of length 3. Nine runs were made with each of two catalysts and the product volumes (gal) are
Catalyst 1 : $0.63 \;\; 2.64 \;\; 1.85 \;\; 1.68 \;\; 1.09 \;\; 1.67 \;\; 0.73 \;\; 1.04 \;\; 0.68$
Catalyst 2 : $3.71 \;\; 4.09 \;\; 4.11 \;\; 3.75 \;\; 3.49 \;\; 3.27 \;\; 3.72 \;\; 3.49 \;\; 4.26$.
A chemical engineer wants to show that the mean product volumes are different. Test at $0.05$ level?

19) The following are the average weekly losses of worker-hours due to accidents in 10 industrial plants before and after a certain safety program was put into operation:
Before: $45 \;\; 73 \;\; 46 \;\; 124 \;\; 33 \;\; 57 \;\; 83 \;\; 34 \;\; 26 \;\; 17$
After: $36 \;\; 60 \;\; 44 \;\; 119 \;\; 35 \;\; 51 \;\; 77 \;\; 29 \;\; 24 \;\; 11$.
Use the 0.05 level of significance to test whether the safety program is effective.

20) The lapping process which is used to grind certain silicon wafers to the proper thickness is acceptable only if $\sigma$, the population standard deviation of the thickness of dice cut

from the wafers, is at most 0.50 mil. Use the 0.05 level of significance to test the null hypothesis $\sigma = 0.5$ against the alternative hypothesis $\sigma > 0.5$ if the thicknesses of 15 dice cut from such wafers have a standard deviation of 0.64 mil. Also construct a $95\%$ confidence interval for $\sigma^2$?

21) A die is thrown 9000 times and a throw of $3$ or $4$ is observed 3240 times. Can we assume that die is fair at $1\%$ confidence?

22) A sample of 1000 people in a city, 540 are rice eaters, and 460 are wheat eaters. Can we assume that both rice eaters and wheat eaters are equally probable at $1\%$ level of significance? What is the $99\%$ confidence interval for the population proportion of the rice eaters? What is the standard error of the sample proportion?

23) In a sample survey conducted in a large city, 136 of 400 persons answered yes to the question of whether their city's public transportation is adequate. With 99% confidence, what can we say about the maximum error, if $\frac{136}{400} = 0.34$ is used as an estimate of the corresponding true proportion?