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MODULE - II

Probability Distribution

Definition (Discrete distribution) :- A distribution which involves discret random variable according to some probability law is known as discrete distribution (or) discrete probability distribution.

Ex :- In single coin tossing exp, the distribution is obtained head is a discrete distribution.

i.e

X	0	1
$P(X=x)$	$1/2$	$1/2$

Clearly $P(X=x) \geq 0$; $x = 0, 1$.

$$\sum_{x=0}^{1} P(X=x) = 1$$

→ In general the discrete probability distributions are studied to find probability of the events in a random experiment. and to find the expected / theoretical frequencies to the given experimental or observed data

→ The following are the good examples for discrete probability distributions

(i) BERNOULLI DISTRIBUTION $(n=1)$

(ii) BINOMIAL DISTRIBUTION $(1 < n < 30)$
n: small

(iii) POISON DISTRIBUTION (n →: large)

BINOMIAL DISTRIBUTION :-

Definition :- A random experiment which contains two possible outcomes is known "binomial expt".

→ These outcomes can be treated as success (p) and failure (q)

Definition :- A discrete random variable X is said to follow binomial distribution with parameters (n, p) if its pmf is given by

$$P(X=x) = \binom{n}{x} p^x . q^{n-x} \quad ; \quad x = 0, 1, 2, \ldots n$$

$$0 \leq p \leq 1$$

$$q = 1-p$$

$$= 0 \quad , \quad \text{otherwise}$$

Pmf verification :

$$\left.\begin{array}{l} x \geq 0 \\ p \geq 0 \\ q \geq 0 \\ n \geq 0 \end{array}\right\} \quad \begin{array}{l} \binom{n}{x} \geq 0 \\ p^x \geq 0 \\ q^{n-x} \geq 0 \end{array} \quad \Rightarrow \binom{n}{x} . p^x . q^{n-x} \geq 0$$

$$\boxed{P(X=x) \geq 0}$$

$$\sum_{x=0}^{n} P(X=x) = \binom{n}{x} p^0 . q^{n-0} + \binom{n}{1} p^1 q^{n-1}$$

$$= + \cdots + \binom{n}{x} p^x . q^{n-x}$$

$$= (p+q)^n$$

$$= 1^n$$

$$= 1$$

$\therefore$ $P(X=x)$ is a pmf.

$\therefore$ X follows binomial distribution with parameters (n, p).

→ Here n = no of trials.

p = probability of success.

q = probability of failure

x = no of ~~po~~ success.

CONDITIONS

(i) The Binomial distribution is applicable under the follo physical conditions.

(a) Number of trials are finite

(b) All the trials are independent.

(c). Events are mutually disjoint.

(d) Probability for success event is always constant.

Examples :-

→ Coin tossing expts, dice throwing expts, true or false examination, multiple choice questionare etc.

Mean, Variance, S.D of Binomial distribution :

Let $X \sim B(n, p)$

then $\text{Mean} = np$

$\text{Var} = npq$

$SD = \sqrt{npq}$

Additive property

If $X \sim B(n, p)$

$Y \sim B(n, p)$

then $X + Y \sim B(n_1 + n_2, p)$

But $X + Y \nsim B(n_1 + n_2, p + p)$

1. Ten coins are tossed at a time. Then find the probability that

(i) atleast 7 heads appear.

(ii) exactly 4 heads appear.

(iii) no head appears.

Sol :- $n(S) = 2^{10} = 1024$

n = no of trials = 10

X = Head appears.

$p = P(\text{success}) = P(H) = 1/2$

$q = 1 - p = 1/2$

$x = 0, 1, 2, \dots 10.$

Binomial distribution

$P(X=x) = ({}^{n}C_{x}) \cdot p^{x} \cdot q^{n-x}$

$= ({}^{10}C_{x}) \left(\frac{1}{2}\right)^{x} \cdot \left(\frac{1}{2}\right)^{10-x}$

$= {}^{10}C_{x} \cdot \frac{1}{\cancel{2^{x}}} \cdot \frac{1}{2^{10}} \times \cancel{2^{x}}$

$= \frac{1}{2^{10}} \cdot {}^{10}C_{x}.$

(i) P (atleast 7 heads appears)

$= P(X \geq 7)$

$= P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$= \frac{1}{2^{10}} \binom{10}{7} + \frac{1}{2^{10}} \binom{10}{8} + \frac{1}{2^{10}} \binom{10}{9} + \frac{1}{2^{10}} \binom{10}{10}$

$$\frac{1}{2^{10}}\left({}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right)$$

$$\Rightarrow \frac{1}{2^{10}}\left(\frac{10!}{3!\,7!} + \cancel{\frac{10!}{}}\right.$$

$$\Rightarrow \frac{1}{1024}\left(120 + 45 + 10 + 1\right) = 0.17.$$

(ii) P(exactly 4 heads)

$$= P(X=4)$$

$$\frac{1}{2^{10}}\cdot\left({}^{10}C_4\right)$$

$$= \frac{210}{1024} = 0.20.$$

(iii) P(no head appear)

$$= P(X=0)$$

$$= \frac{1}{2^{10}}\left({}^{10}C_0\right)$$

$$= \frac{1}{1024} = 0.0009$$

2. The mean & variance of binomial distribution is 4 & 4/3 resp.

(i) Find parameters of dist'n

(ii) $p(x=0)$; $p(x=1)$

(iii) $p(x>1)$

sol: $np = 4$; $npq = \frac{4}{3}$

$4q = 4/3$

$\boxed{q = 1/3}$

$\cancel{np(\frac{1}{3})} = \frac{4}{3}$

$\cancel{p = 1}$

$q = 1-p$

$np(1-p) = 4/3$

$\cancel{np - np^2 = 4/3}$

$p = 1 - 1/3$

$\boxed{p = 2/3}$

$n\left(\frac{2}{9}\right) = \frac{4}{3}$

$n = \frac{4 \times 9}{2 \times 3}$

$\boxed{n = 6}$

$\therefore$ $n = 6$; $p = 2/3$; $q = 1/3$

(ii) $P(X=0)$, $P(X=1)$

$$\Rightarrow \left({}^6C_0\right) \cdot \left(\frac{2}{3}\right)^0 \cdot \left(\frac{1}{3}\right)^{6-0}$$

$$= {}^6C_0 \cdot \frac{1}{3^6}$$

$$\Rightarrow \frac{1}{3^6} = \frac{1}{729} //$$

$$P(X=1) \Rightarrow \left({}^6C_1\right) \cdot \left(\frac{2}{3}\right)^1 \cdot \left(\frac{1}{3}\right)^{6-1}$$

$$\Rightarrow \cancel{\frac{6!}{5!\,1!}} \quad 6 \times \frac{2}{3} \cdot \frac{1}{3^5}$$

$$\Rightarrow \frac{12}{3^6} = \frac{4}{3^5} // = \frac{4}{243} //$$

(iii) $P(X > 1)$

$$\Rightarrow P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6)$$

$$\Rightarrow \left[\left({}^6C_2\right)\left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^{6-2}\right] + \left[\left({}^6C_3\right)\left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^{6-3}\right]$$

$$+ \left[\left({}^6C_4\right)\left(\frac{2}{3}\right)^4 \cdot \left(\frac{1}{3}\right)^{6-4}\right] +$$

$$\left[\left({}^6C_5\right)\left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)\right] + {}^6C_6 \left(\frac{2}{3}\right)^6 \cdot \left(\frac{1}{3}\right)^0]$$

$$\Rightarrow \left[{}^6C_2 \cdot \frac{4}{9} \times \frac{1}{81}\right] + \left[{}^6C_3 \cdot \frac{8}{27} \times \frac{1}{27}\right] +$$

$$\left[{}^6C_4 \cdot \frac{16}{81} \times \frac{1}{9}\right] + {}^6C_5 \left[\frac{32}{243} \times \frac{1}{3}\right]$$

$$+ 6$$

(or)

$1 - P(X=0) + P(X=1)$

$1 - (0.001 + 0.016)$

$1 - 0.017$

$\approx \underline{\underline{0.983}}$

$$\begin{array}{r} 0.001 \\ 0.016 \\ \hline 0.017 \\ \hline \end{array}$$

$$\begin{array}{r} 1.000 \\ 0.017 \\ \hline 983 \\ \hline \end{array}$$

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3. In a true, false examination test which consists 10 questions, a student is completely unprepared and attended the examation. Find the probability that

(i) exactly 6 answers are correct.

(ii) atleast 4 answers are correct.

(iii) atmost 2 answers are correct.

(iv) no answer is correct.

Sol: X follows binomial distribution

X = Answers which are correct.

$x = 0, 1, 2, 3, \dots 10$

$p = 1/2$ (success) → correct ans

$q = 1/2$ (failure) → wrong ans.

$n = 10$

$\therefore P(X=x) = \binom{n}{x} (p)^x (q)^{n-x}$

$= (10c_x) \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x}$

(i) $P(X=6)$

$\Rightarrow \left({}^{10}C_6\right)\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^{10-6}$

$= {}^{10}C_6 \left(\frac{1}{2^6}\right)\left(\frac{1}{2^4}\right)$

$= {}^{10}C_6 \; \frac{1}{64} \; \frac{1}{16}$

(ii) $P(x \geq 4)$

$P(X=4) + P(X=5) + P(X=6) + \cdots P(X=10)$

$\Rightarrow \left({}^{10}C_4 \cdot \frac{1}{2^4} \cdot \frac{1}{2^6}\right) + \left({}^{10}C_5 \cdot \frac{1}{2^5} \cdot \frac{1}{2^5}\right) + \left({}^{10}C_6 \cdot \frac{1}{2^6} \cdot \frac{1}{2^4}\right)$

$+ \left[\left({}^{10}C_7\right) \cdot \frac{1}{2^7} \cdot \frac{1}{2^3}\right] + \left({}^{10}C_8 \cdot \frac{1}{2^8} \cdot \frac{1}{2^2}\right) +$

$\left({}^{10}C_9 \cdot \frac{1}{2^9} \cdot \frac{1}{2}\right) + \left({}^{10}C_{10} \cdot \frac{1}{2^{10}}\right)$

$\Rightarrow \frac{1}{2^{10}}\left({}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}\right)$

$\Rightarrow \frac{1}{2^{10}}\left(\frac{10!}{6! \times 4!} + \frac{10!}{5!5!} + \frac{10!}{4!6!} + \frac{10!}{3!7!} + \frac{10!}{8!2!} + \frac{10!}{9!} + 1\right)$

$\Rightarrow \frac{1}{2^{10}}\left(\frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}\right)$

$P(X=0) = \frac{1}{2^{10}} \left({}^{10}C_0\right) = \frac{1}{1024} = 9.765 \times 10^{-4}$

4. The mean and variance of 4 & 3. Find probability that

(i) $X = 0$

(ii) $X = 1$

(iii) $X \geq 1$

(iv) $X \leq 2$

Sol: $np = 4$ and $npq = 3$

$npq = 3$

$4q = 3 \Rightarrow \boxed{q = 3/4}$

$\boxed{q = 1 - p}$

$p = 1 - q \Rightarrow 1 - 3/4$

$\boxed{p = 1/4}$

$npq = 3 \Rightarrow n\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) = 3$

$n\left(\frac{3}{16}\right) = 3$

$n = \frac{16 \times \cancel{3}}{\cancel{3}}$

$\boxed{n = 16}$

(i) $X = 0$

$\Rightarrow \left({}^{16}C_0\right)\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{16-0} \Rightarrow \left(\frac{3}{4}\right)^{16}$ //

(ii) $X = 1$

$$({}^{16}C_1)\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^{16-1}$$

$$\cancel{\frac{16!}{15!}} \quad 16 \times \frac{3^{15}}{4^{15}} = \cancel{0.01002}$$

$$0.0534$$

(iii) $X \geq 1$

$$P(X \geq 1) = 1 - [P(X=1) + P(X=0)]$$

$$= 1 - \left(16 \times \left(\frac{3}{4}\right)^{15} + \left(\frac{3}{4}\right)^{16}\right)$$

$$= 0.9365$$

(iv) $X \leq 2$

$$P(X=0) + P(X=1) + P(X=2)$$

$$= ({}^{16}C_0)\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{16-0} + ({}^{16}C_1)\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^{16-1} +$$

$$^{16}C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^{16-2}$$

$$\Rightarrow \left(\frac{3}{4}\right)^{16} + 16\left(\frac{3}{4}\right)^{15} + \left(\frac{\cancel{16}^{8} \times 15}{\cancel{2}}\right)\left(\frac{1}{16}\right)\left(\frac{3}{4}\right)^{14}$$

$$\Rightarrow \left(\frac{3}{4}\right)^{14}\left[\frac{9}{16} + 16\left(\frac{3}{4}\right) + \cancel{120}^{\cancel{60}\ \cancel{30}\ 15}\left(\frac{1}{\cancel{16}_{\cancel{8}\ \cancel{4}\ 2}}\right)\right]$$

$$\Rightarrow \left(\frac{3}{4}\right)^{14}\left[\frac{9}{16} + 12 + \frac{15}{2}\right] \Rightarrow 0.19961$$

$$\Rightarrow \left(\frac{3}{4}\right)^{14}\left[\frac{9 + 192 + 120}{16}\right] = \frac{321}{16} \times \left(\frac{3}{4}\right)^{14}$$

16
12
32
16
192
120
9
321

5. A fair coin is tossed 6 times. Find the probability of getting 4 heads.

Sol :- X : getting heads.

$n : 6$

$p = \frac{1}{2}$

$q = \frac{1}{2}$

$x = 0, 1, \ldots 6$

$\therefore P(X = 4) \Rightarrow ({}^6C_4)\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2$

$= \left(\frac{6!}{2! \times 4!}\right)\left(\frac{1}{16}\right)\left(\frac{1}{4}\right)$

$= \left(\frac{\cancel{6}^{3} \times 5}{\cancel{2}}\right)\left(\frac{1}{64}\right)$

$= \frac{15}{64}$

$= 0.2344$ //

6 Two dice are thrown five times. Find the probability of getting 7 as sum

(i) atleast once

(ii) exactly two times

(iii) $(1 < x < 5)$

Sol:- X : getting 7

$n : 5$

~~p~~ :

getting sum as 7 :. Cases

(4, 3) (3, 4) (1, 6) (6, 1) (2, 5) (5, 2)

$$p = \frac{6}{36}$$

$$p = \frac{1}{6}$$

$$q = \frac{5}{6}$$

$$X = \cdot$$

~~(i) P(atleast one)~~ :- $({}^{n}C_{x})(p)^{x}(q)^{n-x}$

(i) P (atleast one) $\Rightarrow P(X \geq 1)$

$$P(X \geq 1) \Rightarrow P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$$

$$\Rightarrow \cancel{({}^{5}C_{1})}($$

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(X=0)$$

$$= 1 - ({}^{5}C_{0})\left(\frac{1}{6}\right)^{0}\left(\frac{5}{6}\right)^{5}$$

$$\Rightarrow 1 - \left(\frac{5}{6}\right)^{5}$$

$$\Rightarrow$$

(ii) exactly two times = $P(X=2)$

$$= {}^{5}C_{2} \cdot \left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{3}$$

(iii) $P(1 < x < 5) = P(x=2) + P(x=3) + P(x=4)$

$$= ({}^5C_2)\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 + {}^5C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 +$$

$${}^5C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)$$

6. Two dice are thrown 120 times. Find the average number of times in which the number on first dice exceeds the number on the second dice. Then find mean

sol:- mean = np

favourable cases $(2,1)\ (3,1)\ (3,2)$

$(4,1)\ (4,2)\ (4,3)\ (5,1)$

$(5,2)\ (5,3)\ (5,4)\ (6,1)\ (6,2)$

$(6,3)\ (6,4)\ (6,5)$

$\Rightarrow \frac{15}{36}$

$\Rightarrow \frac{5}{12}$

$n = 120$

Mean $\Rightarrow np = \overset{10}{\cancel{120}} \times \frac{5}{\cancel{12}}$

Mean $= 50$

7. If the independent random variables x, y follows binomial distributions as

$X \sim B(3, 1/3)$

$Y \sim B(5, 1/3)$. Then find probability that

(i) $X+Y=0$

(ii) $X+Y=1$

(iii) $X+Y \geq 1$

sol:- $X \sim B(3, 1/3)$

$Y \sim B(5, 1/3)$

$X+Y \sim B(5+3, 1/3)$

$X+Y \sim B(8, 1/3)$

$n = 8$; $p = 1/3$; $q = \frac{2}{3}$

$P(X+Y=x) = ({}^{n}C_{x})(p)^{x}(q)^{n-x}$

$= ({}^{8}C_{x})\left(\frac{1}{3}\right)^{x}\left(\frac{2}{3}\right)^{8-x}$

(i) $P(X+Y=0) = \left(\frac{2}{3}\right)^{8}$ //

(ii) $P(X+Y=1) = {}^{8}C_{1}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{7}$ //

(iii) $P(X+Y \geq 1) = 1 - P(X+Y=0) + P(X+Y=1)$

$1 - \left[\left(\frac{2}{3}\right)^{8} + 8 \times \left(\frac{2}{3}\right)^{7}\right]$

$= 1 - \left(\frac{2}{3}\right)^8 + 8 \cdot \left(\frac{2}{3}\right)^7$

8. Find the probability that atmost 5 defective components will be found in a lott of 200. Experience shows that 2% of such components are defective. Also find probability of more than 5 defective components.

~~(i) $P(X \leq 5)$~~

$p = 0.02$

$q = 1 - 0.02$

$q = 0.98$

Sol:-

(i) $P(X \leq 5)$

$$\begin{array}{r} \overset{10}{1.\cancel{0}} \\ \underline{0.2} \\ 9.8 \end{array}$$

$\Rightarrow P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$\Rightarrow ({}^{200}C_0)(0.02)(0.98)^{200} +$

${}^{200}C_1 \; (0.02)(0.98)^{200-1} +$

${}^{200}C_2 \; (0.02)^2 (0.98)^{200-2} + {}^{200}C_3 \cdot (0.02)^3 (0.98)^{200-3}$

$+ {}^{200}C_4 (0.02)^4 (0.98)^{200-4}$

$+ {}^{200}C_5 (0.02)^5 (0.98)^{200-5}$

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Mean, Variance, S.D of Binomial Distribution :-

Let $X \sim B(n, p)$:-

Then its Pmf is given by

$$P(X=x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x} \; ; \; x = 0, 1, 2, \ldots n$$

$$0 \leq p \leq 1 \; ; \; q = 1-p$$

(i) Mean($\bar{x}$) ; Mean = $E(X)$

$$= \sum_{x=0}^{n} x \cdot P(X=x)$$

$$= \sum_{x=0}^{n} x \cdot {}^{n}C_{x} \, p^{x} \cdot q^{n-x}$$

$$= \sum_{x=0}^{n} x \cdot \frac{n!}{(n-x)!\,x!} \cdot p^{1} \cdot p^{x-1} \cdot q^{n-x}$$

$$= p \sum_{x=1}^{n} x \cdot \frac{n(n-1)!}{x \cdot (x-1)!\,(n-x)!} \cdot p^{x-1} \cdot q^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(n-x)!\,(x-1)!} \cdot p^{x-1} \cdot q^{n-x}$$

$$= np \sum_{x=1}^{n} {}^{n-1}C_{x-1} \cdot p^{x-1} \cdot q^{n-x}$$

$$= np \left[{}^{n-1}C_{0} \, p^{0} q^{n} + {}^{n-1}C_{1} \, p^{1} q^{n-2} + \ldots + {}^{n-1}C_{n-1} \, p^{n-1} \right]$$

$$= np \, (p+q)^{n-1} \quad \text{(by binomial expansion)}$$

$= np \cdot 1^{n-1}$ $\quad$ ★ $(x+y)^n = {}^nC_0 x^0 y^n + {}^nC_1 x^1 y^{n-1} + \dots {}^nC_n x^n \cdot y^0)$

$= np \quad (\because p+q=1)$

Variance (Proof) :-

$$\text{Variance} = E(X^2) - [E(X)]^2 \longrightarrow (1)$$

We know that $E(X) = \text{mean} = np \longrightarrow (2)$

$$\text{Now } E(X^2) = \sum_{x=0}^{n} x^2 \cdot P(X=x)$$

$$= \sum_{x=0}^{n} [x(x-1)+x] \, {}^nC_x \cdot p^x \cdot q^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) \, {}^nC_x \cdot p^x \cdot q^{n-x} + \sum_{x=0}^{n} x \, {}^nC_x \, p^x \, q^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) \cdot \frac{n!}{(n-x)!\,x!} \, p^2 \, p^{x-2} \cdot q^{n-x} + \sum_{x=0}^{n} x \cdot P(X=x)$$

$$= p^2 \sum_{x=2}^{n} x(x-1) \cdot \frac{n(n-1)(n-2)!}{x(x-1)(n-x)!(x-2)!} \cdot p^{x-2} \cdot q^{n-x} + E(X)$$

$$= p^2 \cdot n(n-1) \sum_{x=2}^{n} {}^{n-2}C_{x-2} \cdot p^{x-2} \cdot q^{n-x} + np$$

$$= (n^2p^2 - np^2)\left[{}^{n-2}C_0\, p^0 q^{n-2} + {}^{n-2}C_1\, p^1 q^{n-3} + \dots + {}^{n-2}C_{n-2}\, p^{n-2} q^0 \right] + np$$

$$= (n^2p^2 - np^2)\,(p+q)^{n-2} + np$$

$$= n^2p^2 - np^2 \; 1^{n-2} + np$$

$$\Rightarrow \quad n^2p^2 - np^2 + np \rightarrow (3)$$

$\therefore$ (2) and (3) in 1.

Variance $= n^2p^2 - np^2 + np - (np)^2$

$= np - np^2$

$= np(1-p)$

$= npq \qquad (\because q = 1-p)$

and $S.D = (\sigma) = \sqrt{var}$

$= \underline{\underline{\sqrt{npq}}}$

Fitting of Binomial Distribution :- (Direct Method)

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Let $X \sim B(n, p)$

Then ~~pdf~~ pmf is given by

$$\boxed{P(X=x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}} \rightarrow F_1 \quad ; \quad x = 0, 1, 2, \ldots n$$

$$0 \le p \le 1$$

$$q = 1 - p$$

(or) if $n \to \infty$; $p = 0$ $\Rightarrow$ $P(X = x) = \dfrac{e^{-\lambda} \cdot \lambda^{x}}{x!}$; $x = 0, 1, \ldots$

Thus P may be known / unknown

If P is unknown, then Mean $\bar{x} = np$

$$p = \frac{\bar{x}}{n}$$

where $\bar{x}$ = ~~ps~~ arithmetic mean $= \dfrac{\Sigma f_i x_i}{N}$

where $N = \Sigma f_i$

$\therefore$ $\boxed{\text{Expected frequency} = N \times P(X = x)} \rightarrow F_2$

Using formulae 1 & 2 computing expected frequencies is known as fitting of the distribution.

1. Seven coins are tossed and number of heads are noted, the experiment is repeated 128 times and the following distribution is obtained.

No of heads (x)	0	1	2	3	4	5	6	7	Total
Frequency (f)	7	6	19	35	30	23	7	1	Total 128

Fit a binomial distribution, assuming

(i) the coin is unbiased. i.e the nature of the coin is known.

(ii) If the nature of the coin is unknown

Sol:- Let $X \sim B(n, p)$

PMF $\Rightarrow$ $P(X=x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$

where $x = 0, 1, 2, \dots$

$0 \leq p \leq 1$

$q = 1-p$

$N = \Sigma f_i = 128$

$n = 7$

(i) If nature of coin is known (unbiased)

$p = p(\text{success}) = P(H) = \frac{1}{2}$

$q = 1/2$

$\therefore p(X=x) = {}^7C_x \left(\frac{1}{2}\right)^x \cdot \left(\frac{1}{2}\right)^{7-x}$

$= {}^7C_x \cdot \left(\frac{1}{2}\right)^7$

Expected frequency

$E.F = N \times P(X=x)$

$= 128 \times {}^7C_x \left(\frac{1}{2}\right)^7$

$= {}^7C_x$ //

$X=x$	$P(X=x) = P(X)$ $= {}^nC_x p^x q^{n-x}$	$E.F = f(x) = N \times P(X=x) = 128 \times {}^7C_x \left(\frac{1}{2}\right)^7$
0	$\left(\frac{1}{2}\right)^7 \times {}^7C_0 = \frac{1}{128}$	$f(0) = 128 \times \frac{1}{128} = 1$
1	$\left(\frac{1}{2}\right)^7 \times {}^7C_1 = \frac{7}{128}$	$f(1) = 128 \times \frac{7}{128} = 7$
2	$\left(\frac{1}{2}\right)^7 \times {}^7C_2 = \frac{21}{128}$	$f(2) = 128 \times \frac{21}{128} = 21$
3	$\left(\frac{1}{2}\right)^7 \times {}^7C_3 = \frac{35}{128}$	$f(3) = 128 \times \frac{35}{128} = 35$
4	$\left(\frac{1}{2}\right)^7 \times {}^7C_4 = \frac{35}{128}$	$f(4) = 128 \times \frac{35}{128} = 35$
5	$\left(\frac{1}{2}\right)^7 \times {}^7C_5 = \frac{21}{128}$	$f(5) = 128 \times \frac{21}{128} = 21$
6	$\left(\frac{1}{2}\right)^7 \times {}^7C_6 = \frac{7}{128}$	$f(6) = 128 \times \frac{7}{128} = 7$
7	$\left(\frac{1}{2}\right)^7 \times {}^7C_7 = \frac{1}{128}$	$f(7) = 128 \times \frac{1}{128} = 1$
	$\Sigma P(x) = \frac{128}{128} = 1$	$\Sigma f(x) = 128$

(ii) If the nature of the coin is unknown.

$$P = \frac{\bar{x}}{n} = \frac{\bar{x}}{7} = \bar{x} = \frac{\sum f(x)}{N} = N = \sum f_i$$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} \quad ; \quad N = 128.$$

$\frac{433}{128}$

$\therefore \bar{x} = \frac{433}{128}$

$P = \left(\frac{440}{128}\right)/7 = \cancel{0.49}\ \frac{3.38}{7} = 0.48$

$q = 1 - 0.48 = 0.52$

1.00
0.49
0.51

$X = x$	f_i	$f_i . x_i$	$P(X=x) = {}^nC_x . p^x . q^{n-x}$	$E(X) = N \times P(X=x)$
0	7	0	${}^7C_0 \times (0.48)^0 . (0.52)^7 = 0.0102$	$1.3056 = 1$
1	6	6	${}^7C_1 \times (0.48)^1 . (0.52)^6 = 0.0664$	$8.4992 = 8$
2	19	38	${}^7C_2 \times (0.48)^2 . (0.52)^5 = 0.1839$	$23.53 = 24$
3	35	105	${}^7C_3 \times (0.48)^3 . (0.52)^4 = 0.2830$	$36.224 = 36$
4	30	120	${}^7C_4 \times (0.48)^4 . (0.52)^3 = 0.26124$	$33.43 = 33$
5	23	115	${}^7C_5 \times (0.48)^5 . (0.52)^2 = 0.1446$	$18.50 = 19$
6	7	42	${}^7C_6 \times (0.48)^6 . (0.52) = 0.0445$	$5.69 = 6$
7	1	7	${}^7C_7 \times (0.48)^7 = 0.00587$	$0.7424 = 1$
		433		128

235
105
340
93
433

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Poisson Distribution :- (Limiting Case of Binomial) :-

Conditions / Assumptions :-

(i) Number of trials are indefinitely large (i.e $n \to \infty$)

(ii) Probability of success event is very small (i.e $p \to 0$)

(iii) Mean of the distribution is always some constant (i.e $\bar{x} = np = \lambda$ (say))

Limiting Case Proof :-

From binomial distribution, we have

$$P(X=x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x} \quad ; \; x = 0, 1, 2, \dots n \quad \longrightarrow (1)$$

$$0 \le p \le 1$$

$$q = 1 - p$$

$$= 0 \quad ; \; \text{otherwise}$$

As $n \to \infty$, $p \to 0$, $np = \lambda$

in (1) gives

$np = \lambda \Rightarrow p = \lambda/n$; $q = 1 - \lambda/n$

Now $P(X=x) = {}^nC_x \left(\lambda/n\right)^x \left(1-\lambda/n\right)^{n-x}$

$$= \frac{n!}{(n-x)!\,x!} \times \frac{\lambda^x}{n^x} \cdot \frac{(1-\lambda/n)^n}{(1-\lambda/n)^x}$$

$$= \frac{\lambda^x}{x!\,n^x} \cdot \frac{n(n-1)\cdots(n-(x-1))(n-x)!}{(n-x)!} \frac{(1-\lambda/n)^n}{(1-\lambda/n)^x}$$

$$= \frac{\lambda^x}{x!\,\cancel{n^x}} \cancel{n^x}(1-1/n)(1-2/n)\cdots\left(1-\frac{x-1}{n}\right) \cdot \frac{(1-\lambda/n)^n}{(1-\lambda/n)^x}$$

Let $n \to \infty$; we get

$$P(X=x) = \lim_{n\to\infty} \frac{\lambda^x}{x!} (1-1/n)(1-2/n)\cdots\left(1-\frac{(x-1)}{n}\right) \cdot \frac{(1-\lambda/n)^n}{(1-\lambda/n)^x}$$

$$= \frac{\lambda^x}{x!} \cdot 1 \cdot \frac{\lim_{n\to\infty} (1-\lambda/n)^n}{\lim_{n\to\infty} (1-\lambda/n)^x}$$

$$= \frac{\lambda^x}{x!} \frac{e^{-\lambda}}{1}$$

$$\Rightarrow \frac{\lambda^x \cdot e^{-\lambda}}{x!}$$

$\therefore$ i.e For $n \to \infty$; $p \to 0$; $np = \lambda$ $\Big\}$ gives $\boxed{P(X=x) = \dfrac{\lambda^x \cdot e^{-\lambda}}{x!}}$

for $x = 0, 1, \ldots\ldots$

$= 0$, otherwise.

Definition :- A discrete random variable X is said to have follows Poisson distribution with the parameter λ ($\lambda > 0$), if its PMF is given by

$$\boxed{\begin{aligned} P(X=x) &= \frac{\lambda^x \cdot e^{-\lambda}}{x!} \quad ; \quad x = 0, 1, \dots \quad ; \lambda > 0 \\ &= 0 \quad ; \quad \text{otherwise} \end{aligned}}$$

here X = number of successes, $\lambda > 0$

clearly, $e^{-\lambda} = \frac{1}{e^{\lambda}} > 0$

$\lambda^x > 0$; $x! > 0$

$$\therefore \quad \frac{e^{-\lambda} \cdot \lambda^x}{x!} \geq 0$$

i.e $P(X=x) \geq 0$

and $\Sigma P(X=x) = \Sigma \frac{\lambda^x \cdot e^{-\lambda}}{x!} = \cancel{e^{-\lambda}}$

$= \cancel{e^{-\lambda} + \frac{\lambda^2}{2!}}$

$= e^{-\lambda}\left(1 + \lambda + \frac{\lambda^2}{2!} + \dots\right) = e^{-\lambda} \cdot e^{\lambda} = 1$

Examples :-

(i) Number of accidents occurs in a busy city at a particular time.

(ii) Number of telephone calls received from an office, switch board at a telephone exchange.

(iii) Number of patients diagonises at a clinic at particular time.

(iv) Number of misprints | typographical errors per page in a text book.

(v) Number of defective items produced by a machine in a company

Mean, Variance, of Poisson distribution :- ~~We know~~

~~the~~ Let X follows Poisson distribution i.e

Let $X \sim P(\lambda)$ where λ is a parameter, then

$$P(X=x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} \quad ; \quad x = 0, 1, \ldots \quad ; \lambda > 0$$

$$= 0 \quad ; \text{ otherwise}$$

$$\text{Mean } (\bar{x}) = E(X)$$

$$= \sum_{x=0}^{\infty} x \cdot P(X=x)$$

$$= \sum_{x=0}^{\infty} x \cdot \frac{\lambda^{x} \cdot e^{-\lambda}}{x!}$$

$$= e^{-\lambda} \cdot \sum_{x=1}^{\infty} \frac{\cancel{x} \cdot \lambda (\lambda^{x-1})}{\cancel{x} (x-1)!}$$

$$= \lambda e^{-\lambda} \left(1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots\right)$$

$$= \lambda \cdot e^{-\lambda} \cdot e^{\lambda}$$

$$= \lambda$$

$$\therefore \boxed{\text{Mean } (\bar{X}) = \lambda}$$

(iii) Variance (σ^2):-

$$\text{variance} = E(X^2) - (E(X))^2 \longrightarrow (1)$$

but $E(X) = \text{Mean} = \lambda \longrightarrow (2)$

$$= E(X^2) - \lambda$$

Now $E(X^2) = \sum_{x=0}^{\infty} x^2 \cdot P(X=x)$

$$= \sum_{x=0}^{\infty} x^2 \cdot \frac{\lambda^{x} \cdot e^{-\lambda}}{x!}$$

$$\cancel{= e^{-\lambda} \sum_{x=0}^{\infty} x^2 \cdot}$$

$$= \sum_{x=0}^{\infty} x(x-1) + x \, P(X=x)$$

$$= \sum_{x=0}^{\infty} x(x-1) \cdot P(X=x) + \sum_{x=0}^{\infty} x \, (P(X=x))$$

$$= \sum_{x=0}^{\infty} x(x-1) \cdot \frac{e^{-\lambda} \cdot \lambda^{x}}{x!} + E(X)$$

$$= \sum_{x=0}^{\infty} \frac{\cancel{x}\cancel{(x-1)} \cdot e^{-\lambda} \cdot \lambda^{2} \cdot \lambda^{x-2}}{\cancel{x}\cancel{(x-1)}(x-2)!} + \lambda \longrightarrow (2)$$

$$= e^{-\lambda} \cdot \sum_{x=2}^{\infty} \lambda^{2} \left(1 + \frac{\lambda}{1} + \frac{\lambda^{2}}{2!} + \frac{\lambda^{3}}{3!} + \cdots\right) + \lambda$$

$$\lambda^{2} \cdot e^{-\lambda} \cdot e^{\lambda} + \lambda$$

$$= \lambda^{2} + \lambda \longrightarrow (3)$$

$\therefore$ ~~Variance~~ $\Rightarrow$ (2) & (3) in 1 gives

$$\text{Variance} = \lambda^{2} + \lambda - \lambda^{2}$$

$$\sigma^{2} = \lambda$$

$$\sigma = \sqrt{\lambda} = (S.D)$$

★ **NOTE :-** If $x \sim$ Poisson distribution, then mean is same as variance.

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Additive Property: Let $X \sim P(\lambda_1)$; $\lambda_1 > 0$

$Y \sim P(\lambda_2)$; $\lambda_2 > 0$, then $X+Y \sim P(\lambda_1 + \lambda_2)$

1. The number of arrivals at a bus stop follows poisson distribution with an average of 4.5 arrivals. Find the probability that no arrivals

(ii) ~~At~~ one arrivals

(iii) two arrivals

(iv) fewer than 3 arrivals.

Sol:- Let $X \sim P(\lambda_1)$ $\qquad$ X: no of arrivals

$\lambda = 4.5$

$$P(X=x) = \frac{\lambda^x \cdot e^{-\lambda}}{x!}$$

$$= \frac{(4.5)^x \cdot e^{-4.5}}{x!}$$

$$P(X=0) = e^{-4.5}$$

(ii) one arrival

Let $X \sim P$

$\lambda = 4.5$

$$\Rightarrow P(X=1) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} \Rightarrow \frac{e^{-4.5} \cdot 4.5}{1} = 4.5\, e^{-4.5}$$

(iii) two arrivals.

$$P(X=2) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$$P(X=2) = \frac{e^{-4.5} \cdot (4.5)^2}{2!}$$

$$= \frac{4.5 \times 4.5 \; e^{-4.5}}{2}$$

$$\begin{array}{r} 4.5 \\ 4.5 \\ \hline 225 \\ 180+ \\ \hline 20.25 \\ \hline \end{array}$$

$$= \frac{20.25 \; e^{-4.5}}{2}$$

(iv) $P(X<3) = 0.011 + 0.04 + 0.11 = 0.161$

2. A text book contains 520 pages and 390 typographical errors, assume that the number of errors ~~fo~~ follows poisson law, Find the chance that,

(i) A page contains no error

(ii) A page contains one error.

(iii) A page contains atleast one error

(iv) A page contains exactly two errors.

Also find

(i) P(5 pages contains no error)

(ii) Expected number of pages which contains exactly 2 errors.

Sol:- Let $X \sim P(\lambda)$

$$\lambda = \text{Mean} = \frac{390}{520} = 0.75$$

(i) $$P(X=0) = \frac{e^{-0.75} \cdot (0.75)^0}{0!}$$

$$= e^{-0.75} = 0.47$$

(ii) $$P(X=1) = \frac{e^{-0.75} \times (0.75)^1}{1!}$$

$$= e^{-0.75} \times 0.75 = 0.35$$

(iii) $$P(\cancel{X} \geq 1 - \cancel{P(X)}) \; P(X=0) \quad \{1 - P(X<1)\}$$

$$= 1 - 0.\cancel{25}47$$

$$= \cancel{0.65} \; 0.53$$

(iv) $$P(X=2) \Rightarrow \frac{(0.75)^2 \times e^{-0.75}}{2!} = 0.13$$

(v) P(5 pages contains no error) = ~~5×0.47 = 2.35~~

$$(0.47)^5 = 0.02$$

(vi) E.F.

$$\Rightarrow N \times P(X = 2)$$

$$= 520 \times 0.13$$

$$= 68 \text{ pages}$$

$$\begin{array}{r} 520 \\ \times\ 0.13 \\ \hline 1560 \\ 520 \\ \hline 67.60 \end{array}$$

3. A discrete random variable X is a Poisson Variant given that $P(X=1) = P(X=2)$. Find

(i) parameter of distribution.

(ii) $P(X=0)$ $P(X=1)$

(iii) $P(X \leq 1)$

(iv) $P(X < 2)$

Sol :- given $P(X=1) = P(X=2)$

$$\Rightarrow \frac{e^{-\lambda} \cdot \cancel{\lambda}}{1} = \frac{e^{-\lambda} \cdot \cancel{\lambda^2}}{2}$$

$$\Rightarrow 2\cancel{e^{-\lambda}} = \cancel{e^{-\lambda}} \cdot \lambda$$

$$\boxed{\lambda = 2}$$

(ii) $P(X=0) \Rightarrow \dfrac{e^{-2} \cdot (2)^0}{0!}$

$= e^{-2}$ $= 0.135$

$P(X=1) = \dfrac{e^{-2} \cdot (2)^1}{1!}$

$= 2e^{-2}$ $= 0.27$

(iii) $P(X \leq 1) \Rightarrow 1 - (P(X=0) + P(X=1))$

$= 1 - e^{-2} + 2e^{-2}$

$= 1 + e^{-2}$

(iv) $P(X < 2) = P(X=0) + P(X=1)$

$= e^{-2} + 2e^{-2}$

$= 3e^{-2}$

Fitting of Poisson Distribution :- The fitting of poisson distribution is used to find the theoretical (or) expected frequencies for the given observed / experimental data.

Its pmf is

$$P(X=x) = \frac{\lambda^x \cdot e^{-\lambda}}{x!} \longrightarrow (1)$$

from the above, the parameter λ may be known / unknown.

→ If λ is unknown, then its estimated by

$\lambda = \text{Mean} = \frac{\Sigma f_i x_i}{N}$ where $N = \Sigma f$

$\boxed{E.F = f(x) = N \times P(X=x)}$ Expected frequency → (2)

∴ By using (1) & (2) we get expected or theoretical frequencies for the data.

→ This procedure is known as fitting of Poisson distribution by direct method.

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1. A text book contains 200 pages and on an observation, it was found that the following errors.

no of errors	0	1	2	3	4
no of pages	109	65	22	3	1

Fit the Poisson distribution from the given data. Also give your conclusion.

Sol: Let $X \sim P(\lambda)$

The pmf $= \dfrac{\lambda^x . e^{-\lambda}}{x!}$; $x = 0, 1, 2, \ldots$

$\lambda = \dfrac{\Sigma f_i x_i}{N}$ N: total observed frequency

$\lambda = \dfrac{\Sigma f_i x_i}{200} = \dfrac{\cancel{122}\ 61}{\cancel{200}\ 100} = 0.61$

E.F $= N \times P(X = x)$

$x = x$	f	fx	$P(X = x) = P(x)$	Expected frequency $= N \times P(X=x)$ $f(x) = 200 \times P(x)$
0	109	0	$P(0) = \frac{e^{-0.61} \cdot (0.61)^0}{0!} = 0.543$	200×0.543 = 108.6 ≈ 109
1	65	65	$P(1) = \frac{e^{-0.61} \cdot 0.61}{1} = 0.331$	= 66.2 ≈ 66
2	22	44	$P(2) = \frac{e^{-0.61} \cdot (0.61)^2}{2} = 0.1010$	= 20.2 ≈ 20
3	3	9	$P(3) = \frac{e^{-0.61} (0.61)^3}{6} = 0.020$	= 4 ≈ 4
4	1	4	$P(4) = \frac{e^{-0.61} \cdot (0.61)^4}{24} = 0.003$	= 0.6 ≈ 0
	N = 200	122		199.6 ≃ 200

∴ Poisson distribution is the best fit to the given observed data.

Problems On POISSON approximation to the Binomial Distribution :-

1. A manufacturer of cotter pins knows that 5% of his product is defective. Pins are sold in boxes of 100. He guarantees that not more than 10 pins will be defective. What's the approximate probability that a box will fail to meet the guaranteed quality.

Sol :- X : defective item

P : 5% = 0.05

n : 100

λ Mean $= np$

$= 100 \times \frac{5}{100}$

$\lambda = 5$

Let $X \sim P(\lambda)$

$\frac{e^{-\lambda} \cdot \lambda^x}{x!} \Rightarrow \frac{e^{-5} \cdot 5^x}{x!}$

P(a box will fail to meet guaranteed quality) $= P(X > 10)$

$= 1 - P(X \leq 10)$

$1 - \{P(X=0) + P(X=1) + \cdots + P(X=10)\}$

$\Rightarrow 1 - [e^{-5} + e^{-5} \cdot 5 + \frac{e^{-5} \cdot 25}{2} + \frac{e^{-5} \cdot 125}{6} + \cdots \frac{5^{10} e^{-5}}{10!}]$

$\Rightarrow 1 - 0.9863$

$= 0.0137$

2. 6 coins are tossed 6,400 times using the poisson distrib'n. find the approx probability of getting 6 heads ~~f tails~~ r times.

Sol:

X = getting 6 heads

Let $X \sim P(\lambda)$

~~$P = P($6 coins heads~~

$P = P($getting 6 heads in one throw$)$

$= \left(\frac{1}{2}\right)^6$

$n = 6,400$

$\lambda = $ Mean $\Rightarrow 6400 \times \frac{1}{64}$

$= 100$

Let $X \sim P(\lambda)$

$$\frac{e^{-\lambda} \cdot \lambda^x}{x!} = P(X = x)$$

$$\Rightarrow \frac{e^{-100} \cdot (100)^x}{x!}$$

$\therefore$ P(getting 6 Heads r times) $= P(X = r)$

$$= \frac{100^r \cdot e^{-100}}{r!}$$

CONTINOUS DISTRIBUTION :- A distribution which involves c.r.v according to some probability law is known as continous probability distribution.

NORMAL DISTRIBUTION :-

(i) It is also known as Gaussian Distribution.

(ii) This is another limiting form of the Binomial distribution under follo condition

(a) n, the no of trials are indefinitely large i.e $n \longrightarrow \infty$.

(b) neither p nor q is very small.

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Definition :- A continous random variable X is said to have follows Normal distribution with parameters mean & variance, if its Pdf is given by

$$f_X(x) = f(x : \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$= 0 \quad , \text{ otherwise}$$

<u>Standard Normal Distribution</u> : A continous r.v X is said to be S.N.D with its pdf given by

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad ; -\infty < x < \infty$$

$\mu = 0$

$\sigma = 1$

since W.K.T

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

If $\mu = 0$; $\sigma = 1$; $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2}$.

Also if $\frac{x-\mu}{\sigma} = z$

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \cancel{z} \cdot e^{-\frac{z^2}{2}}$$ is pdf of S.N.V z.

$\therefore z = \frac{x-\mu}{\sigma}$ is S.N.V

<u>Verification for Pdf</u> :-

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}; \quad -\infty < x < \infty$$

$-\infty < \mu < \infty$

$\sigma^2 > 0$

(i) since

$$\left.\begin{array}{l}-\infty < x < \infty \\ -\infty < \mu < \infty \\ \sigma^2 > 0\end{array}\right\} \Rightarrow \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} \geq 0$$

is $f_X(x) \geq 0$

(ii) $$\int_{-\infty}^{\infty} f_X(x)\,dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} \cancel{\sigma}\, dz \qquad \text{(even funcin)}$$

$$= \frac{1}{\sqrt{2\pi}}\, 2 \cdot \int_{0}^{\infty} e^{-\frac{z^2}{2}} dz$$

$= \cancel{\frac{2}{\sqrt{2\pi}}}$ Lets assume $\frac{z^2}{2} = t$

$\cancel{2z}$ $z^2 = 2t$

$\cancel{2z\,dz}$

$z = \sqrt{2t}$

$$dz = \frac{1}{\cancel{2}\sqrt{2t}} \cdot \cancel{2}\,dt$$

$$dz = \frac{1}{\sqrt{2}\cdot\sqrt{t}}\,dt$$

$$= \frac{2}{\sqrt{2\pi}} \cdot \int_{0}^{\infty} e^{-t} \cdot \frac{1}{\sqrt{2}\cdot\sqrt{n}}\,dt$$

$= \frac{1}{\sqrt{\pi}} \int_0^\infty e^{-t} \cdot t^{-1/2} \, dt$

$= \frac{1}{\sqrt{\pi}} \int_0^\infty t^{1/2 - 1} e^{-t} \, dt = \frac{1}{\sqrt{\pi}} \cdot \Gamma 1/2$

$= \frac{1}{\sqrt{\pi}} \cdot \sqrt{\pi}$

$= 1$

$\therefore f_X(x)$ is Pdf

$\therefore X \sim N(\mu, \sigma^2)$ //

Mean of N.D :- Let $X \sim N(\mu, \sigma^2)$

$\text{Mean}(\bar{x}) = E(X) = \int_{-\infty}^{\infty} x \cdot f_X(x) \, dx$

$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x \cdot e^{-1/2 \left(\frac{x-\mu}{\sigma}\right)^2} \, dx$

$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu + \sigma z) \, e^{-\frac{z^2}{2}} \, \cancel{\sigma} \, dz$

put $\frac{x-\mu}{\sigma} = z$

$\Rightarrow x = z\sigma + \mu$

$dx = \sigma \cdot dz$

x z : $-\infty \to \infty$

$$= \frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} dz + \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z \cdot e^{-\frac{z^2}{2}} dz$$

$$= \frac{\mu}{\sqrt{2\pi}} \, 2 \int_{0}^{\infty} e^{-z^2/2} dz + \frac{\sigma}{\sqrt{2\pi}} \cdot 0$$

$$= \frac{2\mu}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-t} \cdot \frac{1}{\sqrt{2t}} dt$$

$$= \frac{\mu}{\sqrt{\pi}} \int_{0}^{\infty} t^{-1/2} e^{-t} dt$$

$$= \frac{\mu}{\sqrt{\pi}} \int_{0}^{\infty} t^{1/2 - 1} e^{-t} dt$$

$$= \frac{\mu}{\sqrt{\pi}} \Gamma\tfrac{1}{2} = \frac{\mu}{\sqrt{\pi}} \sqrt{\pi} = \mu$$

$\therefore$ $\boxed{\text{Mean } (\bar{x}) = \mu}$

2) Variance :-

Variance $= E(x^2) - [E(x)]^2 \rightarrow (1)$

W.K.T. $E(x) =$ Mean $= \mu \rightarrow$ (2)

Now $E(x^2) = \int_{-\infty}^{\infty} x^2 \cdot f_x(x) \, dx$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu + \sigma z)^2 \cdot e^{-\frac{z^2}{2}} \cdot \sigma \, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mu^2 + 2\sigma\mu z + \sigma^2 z^2) \, e^{-\frac{z^2}{2}} \, dz$$

$$= \frac{\mu^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} dz + \frac{2\sigma\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z \, e^{-z^2/2} dz +$$

$$\frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 \cdot e^{-\frac{z^2}{2}} \cdot dz$$

NOW $\frac{z^2}{2} = t$

$z^2 = 2t$

$z = \sqrt{2t}$

$$dz = \frac{1}{\cancel{2}\sqrt{2t}} \cdot \cancel{2} \, dt$$

$$dz = \frac{1}{\sqrt{2t}} \, dt$$

$$= \frac{\mu^2}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-t} \cdot \frac{1}{\sqrt{2t}} \, dt + \frac{2\sigma\mu}{\sqrt{2\pi}} \int_{0}^{\infty} \cancel{\sqrt{2t}} \cdot e^{-\frac{t}{2}} \cdot \frac{1}{\cancel{\sqrt{2t}}} \, dt$$

$$+ \frac{\sigma^2}{\sqrt{2\pi}} \int_{0}^{\infty} 2t \cdot e^{-t} \cdot \frac{1}{\sqrt{2t}} \, dt$$

$$= \frac{\mu^2}{2\sqrt{\pi}} \int_{0}^{\infty} t^{-1/2} \cdot e^{-t} \, dt + \frac{2\sigma\mu}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-t/2} \, dt +$$

$$\frac{\cancel{2}\sigma^2}{\cancel{2}\sqrt{\cancel{2}}\cdot\sqrt{\pi}}\int_0^\infty t\cdot e^{-t}\cdot t^{-1/2}\,dt$$

$$= \frac{\mu^2}{2\sqrt{\pi}}\int_0^\infty e^{-t}\cdot t^{-1/2}\,dt + \frac{2\sigma\mu}{\sqrt{2\pi}}\cdot\int_0^\infty e^{-t}\,dt +$$

$$\frac{\sigma^2}{\sqrt{\pi}}\int_0^\infty e^{-t}\,\cancel{dt},\ t^{-1/2}\,dt$$

$$= 2\left(\int_0^\infty e^{-t}\cdot t^{-1/2}\,dt\right)\left(\frac{\mu^2}{2\sqrt{\pi}}+\frac{\sigma^2}{\sqrt{\pi}}\right)$$

$$= \cancel{\frac{2\mu^2}{\sqrt{2\pi}}\int_0^\infty}$$

$$= \frac{\mu^2}{\sqrt{\pi}}\cdot\int_0^\infty t^{-1/2}\cdot e^{-t}\,dt + \frac{2\sigma^2}{\sqrt{\pi}}\int_0^\infty t^{1/2}\cdot e^{-t}\,dt$$

$$= \frac{\mu^2}{\sqrt{\pi}}\int_0^\infty e^{1/2-1}\,e^{-t}\,dt + \frac{2\sigma^2}{\sqrt{\pi}}\int_0^\infty t^{3/2-1}\,e^{-t}\,dt$$

$$= \frac{\mu^2}{\sqrt{\pi}}\Gamma\tfrac{1}{2} + \frac{2\sigma^2}{\sqrt{\pi}}\Gamma\tfrac{3}{2}$$

$$\frac{\mu^2}{\sqrt{\pi}}\cdot\sqrt{\pi} + \frac{2\sigma^2}{\sqrt{\pi}}\cdot\frac{1}{2}\Gamma\tfrac{1}{2}$$

$$\mu^2 + \sigma^2$$

$=$

$E(x^2) = \mu^2 + \sigma^2 \longrightarrow$ (3)

(2) & (3) in 1 gives

$\therefore$ variance $= \mu^2 + \sigma^2 - (\mu)^2$

i.e ~~variance~~

i.e variance $= \sigma^2$

& S.D $= \sigma$

<u>Mean Deviation of N.D (from mean):</u>

Mean Deviation $= E|x - E(x)|$

$= E|x - \mu| \quad (\because E(x) = \mu)$

$= \int_{-\infty}^{\infty} |x - \mu| \cdot f_x(x)\, dx$

$= \cancel{\int_{-\infty}^{\infty} |x-\mu| \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2}(\sigma}$

$= \int_{-\infty}^{\infty} |x - \mu| \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$

$= \frac{1}{\sigma\sqrt{\pi}} \int_{-\infty}^{\infty} |\sigma z| \cdot e^{-\frac{z^2}{2}} \sigma\, dz$

$(\because \frac{x-\mu}{\sigma} = z$

$x = \mu + z\sigma$

$$= \frac{\sigma}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} |z|\, e^{-z^2/2}\, dz$$

$$= \frac{2\sigma}{\sqrt{2\pi}} \int_0^{\infty} \cancel{y}\, z \cdot e^{-z^2/2} dz$$

$$= \frac{2\sigma}{\sqrt{2\pi}} \int_0^{\infty} e^{-t} \cdot \sqrt{2t} \cdot \frac{1}{\sqrt{2t}}\, dt \qquad (\because 0 < z < \infty \Rightarrow |z| = z)$$

$$= \frac{2\sigma}{\sqrt{2\pi}} \times \frac{\sqrt{2}}{\sqrt{2}} \int_0^{\infty} e^{-t} dt \quad (\because z^2/2 = t;$$

$$z = \sqrt{2t}.$$

$$dz = \frac{1}{\sqrt{2t}}\, dt.$$

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$$= \frac{\sqrt{2}\sigma}{\sqrt{2\pi}} \int_0^{\infty} e^{-t} \cdot t^{1-1}\, dt.$$

$$= \sigma \sqrt{\frac{2}{\pi}} \cdot 1$$

i.e $M.D = \sqrt{\frac{2}{\pi}} \cdot \sigma$

$$= \sqrt{\frac{2}{22/7}} \cdot \sigma$$

$$= \sqrt{\frac{14}{22}} \cdot \sigma$$

$$= \sqrt{\frac{7}{11}} \cdot \sigma$$

$$= (0.8)\, \sigma$$

$$= \frac{8}{10}\, \sigma$$

$$= \frac{4}{5}\, \sigma$$

$M.D = \frac{4}{5}\sigma$

$\therefore$

$$\boxed{M.D = \frac{4}{5}(S.D)}$$

Additive Property of Normal Distribution :- If $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$, then

$$\boxed{X + Y \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)}$$

Area Property of Normal Distribution :-

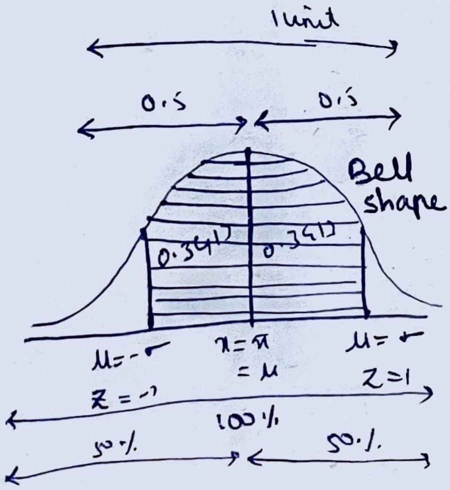
(i) The fundamental area property of normal distribution is used to find the probability of an event by converting a large range of variable X to very smaller range of standard normal variate Z.

(ii) This property can also be used to find the expected frequencies by fitting of N.D in area's method

Since $X \sim N(\mu, \sigma^2)$

Z for $\mu = 0$; $\sigma = 1$, its same as

$Z \sim N(0,1)$, the N.D is symmetric about mean



Here Mean = Median = Mode

From the figure $P(\mu - \sigma \leq x \leq \mu + \sigma)$ is same as i.e

$$P(\mu - \sigma \leq x \leq \mu + \sigma) = P(-\sigma \leq x - \mu \leq \sigma)$$

$$= P\left(-1 \leq \frac{x-\mu}{\sigma} \leq 1\right)$$

$$= P(-1 \leq Z \leq 1)$$

$$= 2 \times P(0 \leq Z \leq 1)$$

$$= 2 \times 0.3413$$

$$= 0.6826$$

$$= 68.26\%$$

Similarly:

$$P(\mu - 2\sigma \leq x \leq \mu + 2\sigma) = P(-2\sigma \leq x - \mu \leq 2\sigma)$$

$$= P\left(-2 \leq \frac{x-\mu}{\sigma} \leq 2\right)$$

$$= P(-2 \leq Z \leq 2)$$

$$= 2 \times P(0 \leq x \leq 2)$$

$$\Rightarrow 2 \times 0.4772$$

$$= 0.9544$$

$$= 95.44\%$$

$P(\mu - 3\sigma \le x \le \mu + 3\sigma)$

$= P(-3\sigma \le x - \mu \le 3\sigma)$

$= P\left(-3 \le \frac{x-\mu}{\sigma} \le 3\right)$

$= P(-3 \le z \le 3)$

$= 2 \times P(0 \le x \le 3)$

$= 0.99$

$= 99\%$

Finally:

$P(-3.99 \le z \le 3.99) = 2 \times P(0 \le z \le 3.99) = 2 \times 0.5 = 1$

Q Find the probability Area of z by a suitable diagram.

(i) $P(0 \le z \le 1)$

(ii) $P(-1 \le z \le 0)$

(iii) $P(0 \le z \le 1.25)$

(iv) $P(-1.25 \le z \le 1.25)$

* (v) $P(-1.25 \le z \le 1.45)$

(vi) $P(0 \le z \le 2)$

(vii) $P(|z| \le 2)$

(viii) $P(0 \le z \le 3)$

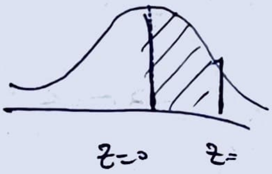
(ix) $P(-2.5 \le z \le 2.5)$

(x) $P(-3 \le z \le 0)$

(xi)

(i) $P(0 \le Z \le 1) = 0.3413$

$= 34.13\%$



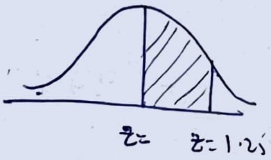
(ii) $P(-1 \le Z \le 0) = P(0 \le Z \le 1)$

$= 0.3413$

$= 34.13\%$

(iii) $P(0 \le Z \le 1.25) = 0.3944$

39.44%

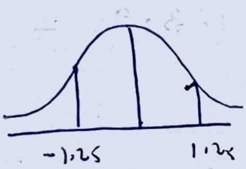


(iv) $P(-1.25 \le Z \le 1.25) = \cancel{0.394}$

2×0.3944

$= 0.7888$

$= 78.88\%$



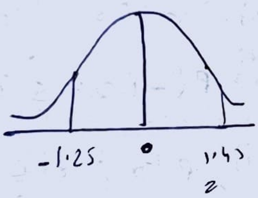
(v) $P(-1.25 \le Z \le 1.45)$

$= P(0 \le Z \le 1.25) + P(0 \le Z \le 1.45)$

$= 0.3944 + 0.4265$

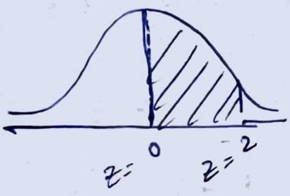
$=$

0.8209



(vi) $P(0 \le Z \le 2)$

$= 0.4772$ //



(vii) $P|z| \le 2$ $= P(-2 \le z \le 2)$

$= 2 \times P(0 \le u \le 2)$

$= 2 \times 0.4772$

$= \cancel{0.8548 \Rightarrow 85.48\%}$ 0.9544

$\Rightarrow 95.44\%$

(viii) $P(0 \le z \le 3)$

$= 0.4987$

$= 49.87$ //

(ix) $P(-2.5 \le z \le 2.5)$

$P(0 \le z \le 2.5)$

$\Rightarrow 2 \times P(0 \le z \le 2.5)$

$= 2 \times 0.4938$

$= 0.9876$

$= 98.76$ //

(x) $P(-3 \le x \le 0)$

$= P(0 \le z \le 3)$

$= 0.4987$ //.

(xi) $P(-3 \le z \le 2.5) = P(-3 \le z \le 0) + P(0 \le z \le 2.5)$

$= P(0 \le z \le 3) + P(0 \le z \le 2.5)$

$= 0.4987 + 0.4938$

$= 0.9925$

$= 99.25\%$

(xiii) $P(|Z| > 2)$

$= P(Z < -2) + P(Z > 2)$

$= P(Z > 2) + P(Z > 2)$

$= 2P(Z > 2)$

$= 2\left(0.5 - P(0 \leq Z < 2)\right)$

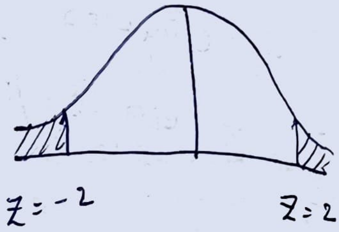
$= 1 - 2P(0 \leq Z < 2)$

$= 1 - 2(0.4772)$

$= 1 - 0.9544$

$= 0.0456$

$= 4.56\%$



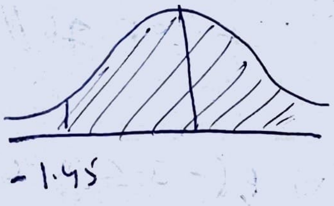
(xiv) $P(-1.45 \leq Z) = P(Z \geq 1.45)$

$= 0.5 + P(0 \leq Z \leq 1.45)$

$= 0.5 + 0.4265$

$= 0.9265$

$= 92.65\%$



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Q. A C.R.V X is normally distibuted with mean 12, S.D 4. Find the probability that

(i) $X \leq 20$

(ii) $X > 20$

(iii) $0 \leq X \leq 12$

(iv) $|X-12| > 4$. Also find x' if $P(x > x') = 0.24$

Sol:

Let $X \sim N(\mu, \sigma^2)$

Given $\mu = 12$

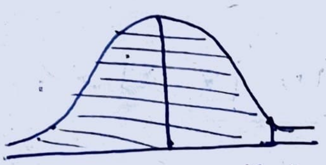
$\sigma = 4$

W.K.T $\quad Z = \dfrac{x-\mu}{\sigma} = \dfrac{X-12}{4}$

(i) At $X <= 20 \Rightarrow X = 20$

$Z = \dfrac{20-12}{4}$

$Z = 2$



$\therefore P(X \leq 20) = P(Z \leq 2)$

$= 0.5 + P(0 \leq Z \leq 2)$

$= 0.5 + 0.4772$

$= 0.9772 = 97.72\%$

(ii) $P(x > 20) = P(z > 2)$

$= 0.5 - P(0 \le z \le 2)$

$= 0.5 - 0.4772$

$= 0.0228$

$= 2.28\%$

(iii) $P(0 \le x \le 12)$

$z = \dfrac{x - \mu}{\sigma}$

At $x = 12$

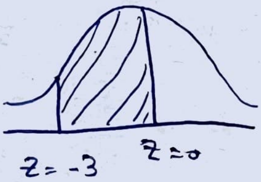
$z = \dfrac{12-12}{4} = 0$; $x = 0$; $z = \dfrac{-12}{4} = -3$

$\therefore P(0 \le x \le 12) = P(-3 \le z \le 0)$

$= P(0 \le z \le 3)$

$= 0.4987$

$= 49.87\%$



(iv) $|x - 12| > 4 = P(x-12 < -4) + P(x-12 > 4)$

$= P(x < 8) + P(x > 16)$

$= P(z < -1) + P(z > 1)$

$= P(z > 1) + P(z > 1)$

$= 2P(z > 1)$

$= 2 \times [0.5 - P(0 \le z < 1)]$

$= 1 - 2 \times P(0 < z < 1)$

$= 1 - 2 \times 0.3413$

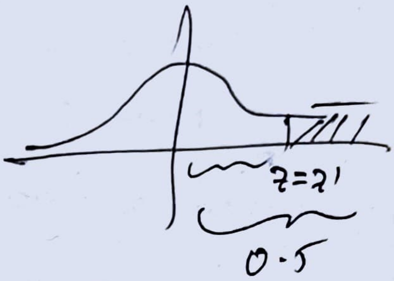
$= 1 - 0.6826$

$= 0.3174$

$= 31.74\%$

Further

at $x = x'$; $\frac{x'-12}{4} = z'$ (say)



$\therefore P(x > x') = 0.24$

$P(Z \geq z') = 0.24$

$0.5 - P(0 < z < z') = 0.24$

$P(0 < z < z') = 0.5 - 0.24$
$= 0.26$

Then from table $\boxed{z' = 0.71}$

$\therefore \frac{x'-12}{4} = 0.71$

$\Rightarrow x' = 14.84\%$

2. The marks in a class test is normally distributed with mean marks 30 and S.D with marks 5. Find probability that

(i) marks in b/w 26 and 40

(ii) marks more than 40

(iii) marks b/w 25 & 35. Also find expected number of students whose marks are in between 25 and 35. Then total number of students in a class is 100.

Sol:- given $\mu = 30$ and $\sigma = 5$

(i) b/w 26 and 40

$P(26 < z < 40)$

$$\Rightarrow \quad Z = \frac{x-\mu}{\sigma}$$

when $x = 26 \Rightarrow \dfrac{26-30}{5} = -\dfrac{4}{5} = -0.8$

Now $Z = -0.8$

when $x = 40$

$$Z = \frac{x-\mu}{\sigma} = \frac{40-30}{5} = \frac{10}{5} = 2$$

$\therefore \; P(-0.8 < Z < 2)$

$\Rightarrow \; P(0 < Z < 0.8) + P(0 \leq Z < 2)$

$\Rightarrow \; 0.2881 + 0.4772$

$\Rightarrow \; 0.7653 \Rightarrow 76.53\%$

(ii) ~~bto~~ more than 40 $\Rightarrow P(x > 40) = P(z > 2)$

$= 0.5 - P(0 < z < 2)$

$$Z = \frac{x-\mu}{\sigma} \Rightarrow \frac{40-30}{5} = \frac{10}{5} = 2.$$

$= 0.5 - 0.4772$

$P(0 < Z < 2) = 0.4772 \Rightarrow 47.72\%$ $= 0.0228$

(iii) $P(25 < x < 35)$

$\Rightarrow$ when $x = 25 \Rightarrow Z = \dfrac{25-30}{5} = \dfrac{-5}{5} = -1$

when $x = 35 \Rightarrow Z = \dfrac{35-30}{5} = \dfrac{5}{5} = 1$

Now $P(-1 < Z < 1) \Rightarrow P(0 < Z < 1) + P(0 < Z < 1)$

$= 2\,P(0 < Z < 1)$

$= 2 \times 0.3413$

$= 0.6826 \Rightarrow 68.26\%$

~~Now $\mu = 100$~~

~~$P(25 < x < 35)$~~

~~$\Rightarrow Z = \dfrac{25-100}{5} = \dfrac{-75}{5} = -15$~~

~~$Z = \dfrac{35-100}{5} =$~~

$\therefore$ There are 68 students. ~~pass~~

3) In a locality of a town the age of the children is normally distributed with avg age 8 and years and S.D is 4 years. Then find the percentage age of children is

(i) between 5 and 10 years

(ii) more than 5 years

(iii) between 10 and 12 years.

Sol :- From the above problem, we can assume the following :-

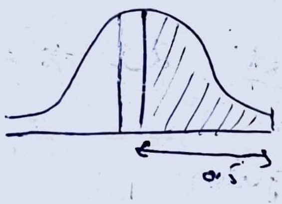
given:: $\sigma = 4$; $\mu = 8$

(i)

$P(5 < x < 10) \Rightarrow P(5 < z < 10) \Rightarrow$ $z = \frac{5-\mu}{4}$

$z = \frac{5-8}{4} = \frac{-3}{4} = -0.75$

when $x = 10 \Rightarrow z = \frac{10-8}{4} = \frac{2}{4} = 0.5$



$\therefore P(0.75 < z < 0.5)$

$\Rightarrow P(0 < z < 0.75) + P(0 < z < 0.5)$

$\Rightarrow 0.2734 + 0.1915 \Rightarrow 0.4649$ //

(ii) $P(x > 5)$

when we consider $x = 5$

$z = \frac{x-\mu}{\sigma} \Rightarrow \frac{5-8}{4} = \frac{-3}{4} = 0.75$

$\therefore P(z > 0.75) \Rightarrow 0.5 - P(0 < z < 0.75)$

$= 0.5 - 0.2734$

$= 0.2266$ //

(iii) $P(10 < x < 12)$

when $x = 10 \Rightarrow \quad z = \frac{10-8}{4} = \frac{2}{4} = \frac{1}{2} = 0.5$

when $x = 12 \Rightarrow \quad z = \frac{12-8}{4} = \frac{4}{4} = 1$

$P(0.5 < z < 1) \Rightarrow P(0 < z < 0.5) + P(0 < z < 1)$

$= 0.1915 + 0.3413$

$= 0.5328$ //

4. A C.R.V 'x' follows N.D, then exactly 7 percent items are under 35 and 89 % of items are under 63. Find mean and S.D of distribution.

Sol:- Let $X \sim N(\mu, \sigma^2)$

given

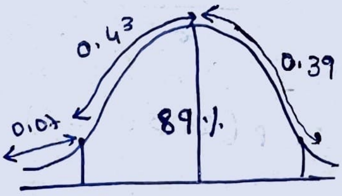
$P(X \leq 35) = 7\% = 0.07 \rightarrow (1)$

$P(X < 63) = 89\% = 0.89$

$P(X \geq 63) = 1 - 0.89 = 0.11 \rightarrow (2)$

W.K.T

$z = \frac{x - \mu}{\sigma}$



At $x = 35$

$\Rightarrow \frac{35 - \mu}{\sigma} = -z_1 \text{ (say)} \rightarrow (3)$

At $x = 63$

$$\frac{63-\mu}{\sigma} = -z_2 \text{ (say)} \rightarrow (4)$$

From figure, $P(-z_1 < z < 0) = 0.43$

$\Rightarrow P(0 < z < z_1) = 0.43$

$z_1 = 1.48$ ($\because$ from table)

Again from figure,

$P(0 < z < z_2) = 0.39$

$z_2 = 1.23$ (from table)

$\therefore$ (3) $\Rightarrow$ $35 - \mu = -1.48\sigma \rightarrow (5)$

(4) $\Rightarrow$ $63 - \mu = 1.23\sigma \rightarrow (6)$

6 - 5

$(6 - \mu = 1.23\sigma) - (35 - \mu = -1.48\sigma)$

$\Rightarrow 6 - \cancel{\mu} = 1.23\sigma$

$-35 + \cancel{\mu} = +1.48\sigma$

$-29 = 2.71\sigma$

$$\sigma = \frac{-29}{2.71}$$

$\sigma = 10.7$

σ in (5)

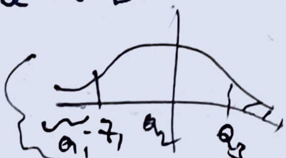
$35 - \mu = -1.48 \times 10.7$

$35 - \mu = -15.836$

$\mu = 50.836$ //

5. Find Quartile deviation of the N.D

Sol:- Hint



$$Q.D = \frac{Q_3 - Q_1}{2}$$

$P(X < Q_1) = 25\%$

& $P(X > Q_3) = 25\%$

Q_3 = 3rd (75%)

Q_1 = 1st (25%)

when $X = Q_1 \Rightarrow z = \frac{x-\mu}{\sigma} = \frac{Q_1-\mu}{\sigma} = -z_1 \rightarrow ①$ (say)

when $X = Q_3 \Rightarrow z = \frac{x-\mu}{\sigma} = \frac{Q_3-\mu}{\sigma} = z_1$ (say) ②

$P(X < Q_1) = 0.25$

$P(X \leq Q_3) = 0.75$

$P(X > Q_3) = 0.25$

if $P(X < Q) = 25\%$

$P(z < -z_1) = 0.25$

$P(z > z_1) = 0.25$

$0.5 - P(0 \leq z \leq z_1) = 0.25$

$P(0 \leq z \leq z_1) = 0.25$

$\boxed{Q.D = \frac{2}{3}\sigma}$

Now ② - ①

$z_1 = 0.67$ (By Table)

$\Rightarrow \frac{Q_3-\mu}{\sigma} - \frac{Q_1-\mu}{\sigma} = z_1 - (-z_1)$

$\frac{Q_3-Q_1}{\sigma} = 2z_1$

$\frac{Q_3-Q_1}{2} = \sigma z_1 \Rightarrow Q.D = 0.67\sigma$

$= \frac{2}{3}\sigma$

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Fitting of Normal Distribution :- (Areas Method)

Let $X \sim N(\mu, \sigma^2)$, then $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$-\infty < x < \infty$

$-\infty < \mu < \infty$

$\sigma^2 > 0$

and $Z \sim N(0,1)$

then $f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z^2)}$

where $z = \frac{x-\mu}{\sigma}$

here, parameters μ and σ^2 are unknown, which can be obtained by. $\mu = \frac{1}{N}\Sigma f x$ (or) $= A + \frac{h}{N}\Sigma f d$

$d = \frac{x-A}{h}$)

$\sigma^2 = \frac{1}{N}\Sigma f(x-\bar{x})^2$

(or) $\frac{h^2}{N}\left[\Sigma f d^2 - \frac{1}{N}(\Sigma f d)^2\right]$

$\star$ $Z \sim N(0,1)$

$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$

A - origin

h - scale

$\therefore$ $E.F = Nf(x) = N \times P(z_1 \le Z \le z_2)$.

$$Z_1 = \frac{X_1 - \mu}{\sigma} \; ; \; Z_2 = \frac{X_2 - \mu}{\sigma}$$

N = Total frequence = Σf.

Q. Fit a N.D to the following data:

Class :-	Frequency (f) :-
60 - 65	3
65 - 70	21
70 - 75	150
75 - 80	335
80 - 85	326
85 - 90	136
90 - 95	26
95 - 100	4

[Hint : $N = \Sigma f = 1000$

x: 62.5, 67.5, 72.5, 77.5, 82.5, 87.5, 92.5, 97.5

$$\mu = \frac{1}{N} \Sigma f_i x_i - \frac{79945}{1000} = 79.945$$

$$\sigma^2 = \frac{1}{N} = \frac{1}{N} \Sigma f(x - \bar{x})^2 \quad 5.444$$

Characteristics of Normal Distribution :-

(i) Pdf of Nd is $f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$-\infty < x < \infty$

$-\infty < \mu < \infty$

$\sigma > 0$

(ii) Std. normal dist'n

$$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \qquad -\infty < z < \infty$$

where $z = \frac{x-\mu}{\sigma} \sim N(0,1)$.

(iii) Mean = μ, variance = σ^2

(iv) Mean = Median = Mode

(v) Normal distribution is symmetric about its mean

(vi) N.D shape is bell shape.

(vii) M.D = 4/5 S.D

Q.D = 2/3 S.D

(viii) $X \sim N(\mu_1, \sigma_1^2)$

$Y \sim N(\mu_2, \sigma_2^2)$

$(X+Y) \sim N(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)$

(ix) Probability under the normal curve is unity.

(x) N.D is the limiting case of binomial, poisson.

(xi) N.D has the method of areas to obtain the expected or theoretical frequencies of given experimental data.

Q P.T for normal distribution, the quartile deviation, the mean deviation, standard deviation are approx 10:12:15

Sol, W.k. that Q.D of N.D $= \frac{2}{3}\sigma$

M.D of N.D $= \frac{4}{5}\sigma$

S.D of N.D $= \sigma$

Now Q.D : M.D : S.D $= \frac{2}{3}\sigma : \frac{4}{5}\sigma : \sigma$

$= \frac{2}{3} : \frac{4}{5} : 1$

$= 10 : 12 : 15$