Exercises

11.25 The following data pertain to the growth of a colony of bacteria in a culture medium:

Days since inoculation x	Count y
3	115,000
6	147,000
9	239,000
12	356,000
15	579,000
18	864,000

(a) Plot log y_i versus x_i to verify that it is reasonable to fit an exponential curve.

(b) Fit an exponential curve to the given data.

(c) Use the result obtained in part (b) to estimate the bacteria count at the end of 20 days.

11.26 The following data pertain to water pressure at various depths below sea level:

Depth (m) x	Pressure (psi) y
10	140
50	74
150	218
1,600	2,400
2,110	3,060
3,580	5,150
4,800	7,000

(a) Fit an exponential curve.

(b) Use the result obtained in part (a) to estimate the mean pressure at a depth of 1,000 m.

11.27 With reference to the preceding exercise, change the equation obtained in part (a) to the form $\widehat{y} = a \cdot e^{-cx}$, and use the result to rework part (b).

11.28 Refer to Example 10. Two new observations are available.

Rate of discharge(A)	Capacity(Ah)	ln(Capacity)
5	149.4	5.0066
18	108.2	6.5840

Add these observations to the data set in Example 10 and rework the example.

11.29 Fit a **Gompertz curve** of the form

$$y = e^{e^{\alpha x + \beta}}$$

to the data of Exercise 11.26.

11.30 Plot the curve obtained in the preceding exercise and the one obtained in Exercise 11.26 on one diagram and compare the fit of these two curves.

11.31 The number of inches which a newly built structure is settling into the ground is given by

$$y = 3 - 3\,e^{-\alpha x}$$

where x is its age in months.

x	2	4	6	12	18	24
y	1.07	1.88	2.26	2.78	2.97	2.99

Use the method of least squares to estimate α. [*Hint*: Note that the relationship between $\ln(3 - y)$ and x is linear.]

11.32 The following data pertain to the amount of hydrogen present, y, in parts per million in core drillings made at 1-foot intervals along the length of a vacuum-cast ingot, x, core location in feet from base:

x	1	2	3	4	5	6	7	8	9	10
y	1.28	1.53	1.03	0.81	0.74	0.65	0.87	0.81	1.10	1.03

(a) Draw a scatter plot to check whether it is reasonable to fit a parabola to the given data.

(b) Fit a parabola by the method of least squares.

(c) Use the equation obtained in part (b) to estimate the amount of hydrogen present at $x = 7.5$.

11.33 When fitting a polynomial to a set of paired data, we usually begin by fitting a straight line and using the method on page 339 to test the null hypothesis $\beta_1 = 0$. Then we fit a second-degree polynomial and test whether it is worthwhile to carry the quadratic term by comparing $\widehat{\sigma}_1^2$, the **residual variance** after fitting the straight line, with $\widehat{\sigma}_2^2$, the residual variance after fitting the second-degree polynomial. Each of these residual variances is given by the formula

$$\frac{\sum (y - \widehat{y})^2}{\text{degrees of freedom}} = \frac{\text{SSE}}{\nu}$$

with $\widehat{y}$ determined, respectively, from the equation of the line and the equation of the second-degree polynomial. The decision whether to carry the quadratic term is based on the statistic

$$F = \frac{\text{SSE}_1 - \text{SSE}_2}{\widehat{\sigma}_2^2} = \frac{\nu_1\,\widehat{\sigma}_1^2 - \nu_2\,\widehat{\sigma}_2^2}{\widehat{\sigma}_2^2}$$