Testing of Hypothesis:

A statistical hypothesis is a statement about the parameter of one or more populations. ~~Testing~~

Testing a hypothesis is a process for deciding wether to accept or reject of hypothesis.

There are two types of hypothesis:

1. Null Hypothesis: A hypothesis of No difference is called Null hypothesis. It is denoted by H_0

   Null hypothesis $H_0: \mu = \mu_0$

2. Alternative Hypothesis: A hypothesis which is complementary to the null hypothesis is called Alternative hypothesis.

   Alternative Hypothesis is denoted by H_1.

   $H_1: \mu \neq \mu_0$ (or)

   $H_1: \mu > \mu_0$ (or)

   $H_1: \mu < \mu_0$

Testing of Hypothesis [Type-I & Type-II error]

When we are testing Null hypothesis (H_0) against Alternative Hypothesis (H_1) There are four possibilities

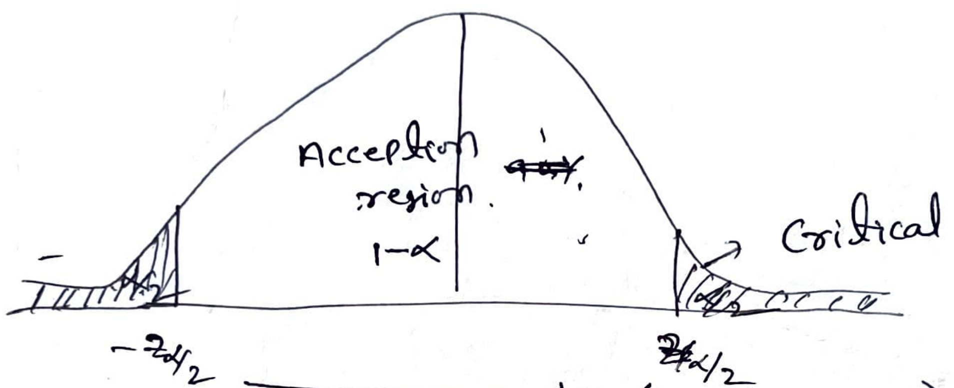
1. H_0 accepted when H_0 is true [correct]
2. H_0 rejected when H_0 is true [Type-I error]
3. H_0 accepted when H_0 is false [Type-II error]
4. H_0 rejected when H_0 is false [correct]

One - tailed and Two - tailed Tests :-

To test the statistical hypothesis to a testing of hypothesis problem, the critical region is more stretched towards the left and right tailed is known as two tailed tests.

If the critical region is more stretched towards the left or Right is known as one tailed test or single - tailed test or one sided.

Two - tailed test.

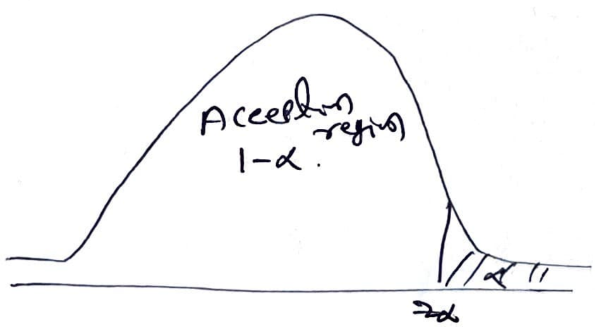


$H_0 : \mu = \mu_0$

$H_1 : \mu \neq \mu_0$

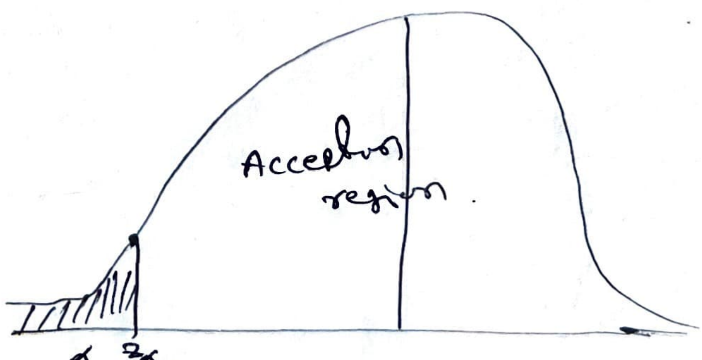
$\alpha\% + (1-\alpha)\%$
$= 100$

one - tailed test.



$H_0 = \mu = \mu_0$

$H_1 : \mu > \mu_0$



$H_0 : \mu = \mu_0$

$H_1 : \mu < \mu_0$

Critical values of Z at given α
(Level of significance).

test	$\alpha = 1\%$	2%	5%	10%
Two tailed	$Z_\alpha = 2.58$	$Z_\alpha = 2.33$	$Z_\alpha = 1.96$	Z_α 1.65 (1.645)
one tailed	$Z_\alpha = \pm 2.33$	$Z_\alpha : 1.96$	$Z_\alpha = 1.65$	$Z_\alpha : 1.28$.

Procedure for Testing of Hypothesis:

various steps involved in testing of hypothesis are given below.

step: 1: set up statistical hypothesis

set up a null hypothesis and Alternative hypothesis from the given hypothesis testing problem.

step: 2: List the Sample data:

List out the complete Sample data which is available from the hypothesis testing problem.

step 3: Test statistic:

use an appropriate test statistic to test the statistical hypothesis.

e.g: $z = \frac{t - E(t)}{S.E(t)} \sim N(0,1)$.

step 4: Calculation:

using the sample data, compute the test statistic and compare with its table (or) critical value of the test statistic at the given level of significance.

step 5: conclusion:

calculated value of z is less than or equal table value.

Then Ho is accepted. ($z_{cal} \leq z$ table value)

Ho is rejected ($z_{cal} > z$ table value).

Test for mean:

This large sample ~~that~~ test is used to test the hypothetical population mean μ (or) the random sample is drawn from the same population or not.

1. Set up ~~the statistical~~ H_0 and H_1.

   set up the statistical $H_0 : \mu = \mu_0$ (Population mean)

   $H_1 : \mu \neq \mu_0$ (Two tailed

2. Sample data: Now the sample data is given by

   $n, \ \bar{x} = \frac{1}{n} \Sigma x_i, \ \sigma^2, \quad \alpha = 1\%$ (or) 2% (or) 5% (or) 10%

3. Test statistic: $z = \dfrac{(\bar{x} - \mu)}{\frac{\sigma}{\sqrt{n}}}$ where $\frac{\sigma}{\sqrt{n}} = S.E$

4. Calculation: using the sample data, find calculated value of z and compare with table value of z ~~and~~ at given Level of significance α.

5. Conclusion: If $z_{cal} \leq z_{table}$ value H_0 is accepted. otherwise reject H_0.

1. From a class of a college, a sample of 40 students selected at random with the sample mean marks 26 and S.D of marks 2.5, Test the hypothesis that the average marks of the students in this college is 24. at 5% Level of significance.

Solution: To test the given hypothesis by testing of hypothesis procedure, we have

i. set up H_0 and H_1.

$H_0 : \mu = 24$. (Average marks of the students in the college is 24).

$H_1 : \mu \neq 24$. (Two tailed test).

2. Sample data:

The sample data is given by.

$n = 40$, $\bar{x} = 26$, $\sigma = 2.5$, $\alpha = 5\% = 0.05$

Z_{tab} value $= 1.96$.

3. Test the statistic:

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} =$$

4. Calculation: $Z = \left(\frac{26-24}{2.5}\right)\sqrt{40} = 5.0596$.

Z_{cal} value : 5.06

Z tab value : 1.96.

Conclusion:

$z_{cal} > z_{tab}$

$5.06 > 1.96$

H_0 is rejected.

we conclude that The average of marks of the student in this college is not 24 at 5% Level of Significance.

Note: Confidence limits for mean μ.

If the population mean μ is unknown then it is estimated by interval estimators.

ie $\mu \in \bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$$\mu \in \left(\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}\right).$$

Ex ①

$n = 40$, $\bar{x} = 26$, $\sigma = 2.5$ $\alpha = 5\%$.

$z_{\alpha/2} = 1.96$.

$$\mu \in \left(\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}\right)$$

$$\mu \in 26 \pm (1.96) \frac{2.5}{\sqrt{40}}.$$

2. A sample of 64 students have a mean weight of 70 kgs. Can this be regarded as a sample from a population with mean weight 56 kgs. and S.D 25 kgs.

Solution:

1. set up H_0 and H_1

$$H_0 : \mu = 56$$

$$H_1 : \mu \neq 56. \text{ (Two-tailed test)}$$

2. Sample data

$n = 64,\quad \bar{x} = 70 \quad \mu = 56 \quad \sigma = 25.$

Level of significance $\alpha = 5\%.$

$Z_\alpha = 1.96$

3. Test statistic.

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

4. Calculation:

$$Z_{cal} = \left(\frac{70-56}{25}\right)\sqrt{64}.$$

$$= (0.56)\,8$$

$$Z_{cal} = 4.48.$$

5. Conclusion

$Z_{cal} > Z$ tab value

$4.48 > 1.96.$

H_0 is rejected.

③ The mean life time of a sample of 100 light tubes produced by a company is found to be 1560 hrs with a population S.D of 90 hrs. Test the hypothesis that the mean life time of the tubes produced by the company is 1580 hrs.

Solution:

1. Set up H_0 and H_1.

Null Hypothesis $H_0 = \mu = 1580$ hrs.
(mean life time of the tubes produced by the company is 1580 hrs)

Alternative hypothesis $H_1 : \mu \neq 1580$.

2. Sample data

$n = 100$, $\bar{x} = 1560$.
$\sigma = 90$ hr.
$\alpha = 5\% = 0.05$
1.96.

3. Test statistic

$$z = \left(\frac{\bar{x}-\mu}{\sigma}\right)\sqrt{n}$$

4. calculation:

$$z = \left(\frac{1560-1580}{90}\right)\sqrt{100}$$
$$= \frac{-20}{90} \times 10 = 2.22.$$

5. conclusion: $z_{cal} > z_{tab}$
$2.22 > 1.96$.

H_0 is rejected. $\therefore \mu \neq 1580$.

4. A Sample of ~~9~~ 900 members has a mean 3.4 cms, and S.D 2.61 cms. Is the sample from a large population of mean 3.25 cm and S.D 2.61 cms. If the population is normal and its mean is unknown find the 95% fiducial limits of true mean.

solution:

1. Setup H_0 & H_1:

Null Hypothesis $H_0: \mu = 3.25$

Alternative Hypothesis $H_1: \mu \neq 3.25$.

2. Sample data

$n = 900, \quad \bar{x} = 3.4, \quad \sigma = 2.61.$

$\alpha = 0.05.$

$z_{tab} = 1.96.$

3. Test statistic $\quad z = \left(\frac{\bar{x} - \mu}{\sigma}\right)\sqrt{n}$

4. Calculation. $\quad = \left(\frac{3.4 - 3.25}{2.61}\right)\sqrt{900} = 1.724$

$z_{cal} < z_{tab}$

$1.724 < 1.96.$

5. conclusion: H_0 is Accepted.

Confidence limits.

$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}.$

$3.4 \pm 1.96 \times \frac{2.61}{\sqrt{900}}$

3.14 ± 0.1705

$\mu \in (3.2295, 3.57)$

(5) Mice with an average life span of 32 months will live upto 40 months when fed by a certain nutrious food. If 64 mice fed on this diet have an average life span of 38 months and S.D of 5.8 months, is there any reason to believe that average lifespan is less than 40 months. Use 0.01 level of significance. (8)

Solution.

N.H $H_0 : \mu = 40$ months (μ = Average life span of mice fed with nutritious food)

A.H. $H_1 : \mu < 40$.

(Left one tailed test)

$n = 64$, $\bar{x} = 38$, $\sigma = 5.8$.

$\alpha = 0.01$,

$Z_{tab} : -2.33$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$Z_{cal} = \left(\frac{38-40}{5.8}\right)\sqrt{64}.$$

$$= -2.76$$

$$-2.76 < -2.33$$

H_0 is ~~Accepted~~ Rejected.

Test for Difference of Two Means $(\mu_1 - \mu_2)$:

or

Test for Equality of Two means:

This large sample test can be used to test the significance difference between two hypothetical population means of two distinct and independent populations.

This test can also be used whether the samples are drawn from the same populations or not.

Set up statistical Hypothesis:

Null Hypothesis H_0 : $\mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

Alternative Hypothesis H_1 : $\mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$.

Sample data:

	I	II
Sample size	n_1	n_2
Sample mean	$\bar{x}$	$\bar{y}$
Sample variance	σ_1^2	σ_2^2

Level of significance α given.

Test statistic: $(n_1, n_2 > 30)$

$$z = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

where $SE(\bar{x} - \bar{y}) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

Calculation: using the sample data compute z test statistic and compare with its table value at a given los α.

Conclusion:

If $|z| \le z_{tab}$ Then H_0 is Accepted.

$|z| > z_{tab}$ H_0 is Rejected.

Confidence limits (Interval estimators).

$$\mu_1 - \mu_2 \in \bar{x} - \bar{y} \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.$$

$z_{\alpha/2}$ is table (critical) value of z at given α for two tailed test

Note ① If σ_1^2 and σ_2^2 are unknown then $\sigma_1^2 \approx s_1^2$, $\sigma_2^2 \approx s_2^2$

where s_1^2 and s_2^2 are sample variances.

$$z = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}.$$

② If $\sigma_1 = \sigma_2 = \sigma$ (say)

$$z = \frac{\bar{x} - \bar{y}}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

(10)

① From two colleges A and B, a sample of 40 and 50 student selected at random with their average marks 12 and 10, S.D of marks 2 and 3 respectively. Test the hypothesis that is There ~~is~~ any Significance difference between the marks of students from these two colleges A and B at 1% LOS.

Solution:

Statistical Hypothesis

$H_0 : \mu_1 - \mu_2 = 0$ (or) $\mu_1 = \mu_2$

H_1 $\mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$.

Sample data

College	A	B
	$n_1 = 40$	$n_2 = 50$
	$\bar{x} = 12$	$\bar{y} = 10$
	$\sigma_1^2 = 2^2 = 4$	$\sigma_2^2 = 3^2 = 9$

$\alpha = 1\% = 0.01$.

Then Z tab $= 2.58$.

Test statistic :

$$z = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}.$$

Calculation :-

$$z_{cal} = \frac{12-10}{\sqrt{\frac{4}{40}+\frac{9}{50}}} = 3.76$$

$z_{cal} > z_{tab}$.

Ho is rejected.

Conclusion: There is no evidence to accept Ho.

There is a significance difference between the average performance of the studentse from two colleges A ad B at 1% LOS.

confidence limits

$$\mu_1 - \mu_2 \in \bar{x} - \bar{y} \pm z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\in (12-10) \pm 2.58 \sqrt{\frac{4}{40} + \frac{9}{50}}.$$

$$\in 2 \pm 1.36.$$

(2) The means of two large samples of sizes 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of S.D 2.5 inches.

Test for Single proportion: [No mean, No S.D. with one mean]

1. Set up the statistical Hypothesis.

Null hypothesis $H_0: P = P_0$

Alternative Hypothesis $H_1: P \neq P_0$ (Two tail)

$P > P_0$ (or) $P < P_0$ (one tail).

2. Sample data: list out the complete sample data which is available from the hypothesis testing problem.

Level of significance.

3. Test statistic: use an appropriate test statistic to test the statistical hypothesis

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$P \rightarrow$ population proportion

$p \rightarrow$ sample proportion.

$p = \frac{x}{n}$

4. Canculation: using the sample data compute the test statistic compare with its table

5. Conclusion: Ho is accepted $|Z_{cal}| \leq Z$ table value.

Ho is rejected $|Z_{cal}| > Z$ table value.

(1) (13)

In a sample of 1000 people in Karnataka 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this state at 1% level of significance

Solution:

Set up the statistical Hypothesis

H_0: Population proportion is known.

H_0 < ~~P = P_0~~ P = population proportion of rice eaters $= \frac{1}{2} = 0.5$

$Q = 1 - 0.5 = 0.5$

$H_0 = P = 0.5$ (Both rice & wheat are equally popular)

H_1: $P \neq 0.5$

$n = 1000$, $x = 540$.

p = sample proportion of rice eaters $= \frac{540}{1000} = 5.40$

$$z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{5.4 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{1000}}} = 2.532.$$

The $z_{cal} = 2.532$

$z_{tab} - 2.58$

$z_{cal} < z_{tab}$.

$\therefore$ H_0 is Accepted.

both rice and wheat eaters are equally popular in the state. at 1% level of significance.

Test for Difference between two proportions: (No mean No S.D & 2 Sample)

Setup statistical Hypothesis:

Null Hypothesis $H_0: P_1 - P_2 = 0$ (or) $P_1 = P_2$

Alternative hypothesis $H_1: P_1 - P_2 \neq 0$ (or) $P_1 \neq P_2$ (two tail)

$P_1 > P_2$ (or) $P_1 < P_2$ (one tail)

Sample data: collect the sample data from the problem.

Level of significance.

Test statistic:

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} \; ; \quad Q = 1 - P$$

Calculation: Using the sample data compute Z test and compare with its table value at a given level of sig α.

Conclusion: If $|Z_{cal}| \leq Z$ table Then H_0 is accepted

$|Z| > Z$ table Then H_0 is rejected.

$P=0.5$ agai

solution: set up the statistical Hypothesis.

Null Hypothesis $H_0 : P=0.5$

Alternative Hypothesis $H_1 : P>0.5$

Sample data: $n=125 \quad x=68$

$$p=\frac{x}{n}=\frac{68}{125}=0.544.$$

Level of significance $\alpha=0.05$ (assumed)

Test statistic is $$z=\frac{p-P}{\sqrt{\frac{PQ}{n}}}$$

Calculation: $$z=\frac{0.544-0.5}{\sqrt{\frac{(0.5)(0.5)}{125}}}=0.9839$$

$|z_{cal}|=0.9839 \qquad z$ table value $=1.645$

conclusion: $|z_{cal}| < z$ table value

$0.9839 < 1.645$

H_0 is accepted at 5% level of significance.

① (16)

Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal are same, at 5% LOS.

Solution:

Null Hypothesis $H_0: P_1 = P_2 = P$ (Pr

Alternative Hypothesis $H_1: P_1 \neq P_2$.

Sample data $n_1 = 400$ $n_2 = 600$

$x = 200$ $y = 325$

Sample proportion of men $p_1 = \frac{200}{400} = 0.5$

Sample proportion of women $p_2 = \frac{325}{600} = 0.541$.

$\alpha = 0.05$ $z_{tab} = 1.96$.

Test statistic $z = \dfrac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$P = \dfrac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = 0.525$

$Q = 1 - P = 0.475$

$z = \dfrac{0.5 - 0.541}{\sqrt{(0.525)(0.475)\left(\frac{1}{400} + \frac{1}{600}\right)}} = -1.28$

$|z| = 1.25 < 1.96$
H_0 is Accepted. There is no differences of opinion between men and women as far as proposal of flyover is concerned