

**Figure 11.9**

The 95% limits of prediction (solid) and confidence intervals (dotted) for aluminum sheets.

## Exercises

**11.1** A chemical engineer found that by adding different amounts of an additive to gasoline, she could reduce the amount of nitrous oxides (NOx) coming from an automobile engine. A specified amount will be added to a gallon of gas and the total amount of NOx in the exhaust collected. Initially, five runs with 1, 2, 3, 4, and 5 units of additive will be conducted.

- How would you randomize in this experiment?
- Suppose you properly calculate a point estimate of the mean value of NOx when the amount of additive is 8. What additional danger is there in using this estimate?

**11.2** A motorist found that the efficiency of her engine could be increased by adding lubricating oil to fuel. She experimented with different amounts of lubricating oil and the data are

Amount of lubricating oil (ml)	Efficiency (%)
0	60
25	70
50	75
75	81
100	84

- Obtain the least squares fit of a straight line to the amount of lubricating oil.
- Test whether or not the slope  $\beta = 0$ . Take  $\alpha = 0.05$  as your level of significance.
- Give a point estimate of the mean engine efficiency when the amount of lubricating oil is 450 ml.
- What additional danger is there when using your estimate in part (c)?
- How would you randomize this experiment?

**11.3** A textile company, wanting to know the effect of temperature on the tearing strength of a fiber, obtained the data shown in the following table.

Temperature (°C) <i>x</i>	Tearing strength (g) <i>y</i>
20	1,600
22	1,700
25	2,100
35	2,500
18	1,550
29	2,600
31	2,550
16	1,100
13	1,050
48	2,650

- Draw a scatter plot to verify that a straight line will provide a good fit to the data, draw a straight line by eye, and use it to predict the tearing strength one can expect when the temperature is 29°C.
- Fit a straight line to the given data by the method of least squares and use it to predict the tearing strength one can expect when the temperature is 29°C.

**11.4** In the accompanying table,  $x$  is the tensile force applied to a steel specimen in thousands of pounds, and  $y$  is the resulting elongation in thousandths of an inch:

$x$	1	2	3	4	5	6
$y$	14	33	40	63	76	85

- (a) Graph the data to verify that it is reasonable to assume that the regression of  $Y$  on  $x$  is linear.
- (b) Find the equation of the least squares line, and use it to predict the elongation when the tensile force is 3.5 thousand pounds.

**11.5** With reference to the preceding exercise,

- (a) construct a 95% confidence interval for  $\beta$ , the elongation per thousand pounds of tensile stress;
- (b) find 95% limits of prediction for the elongation of a specimen with  $x = 3.5$  thousand pounds.

**11.6** The following table shows how many days in December a sample of 6 students were present at their university and the number of lectures each attended on a given day.

Number of days present $x$	Number of lectures attended $y$
12	3
8	2
13	5
10	4
7	1
10	3

- (a) Find the equation of the least squares line which will enable us to predict  $y$  in terms of  $x$ .
- (b) Use the result in part (a) to estimate how many lectures someone who has been present for 15 days can be expected to attend each day.
- 11.7** With reference to the preceding exercise, test the null hypothesis  $\beta = 0.75$  against the alternative hypothesis  $\beta < 0.75$  at the 0.10 level of significance.
- 11.8** With reference to Exercise 11.6, find
- (a) a 90% confidence interval for the average number of classes attended each day by a student present for 15 days;
- (b) 90% limits of prediction for the number of classes attended each day by a student present for 15 days.
- 11.9** Scientists searching for higher performance flexible structures created a diode with organic and inorganic layers. It has excellent mechanical bending properties. Applying a bending strain to the diode actually leads to higher current density(mA/cm<sup>2</sup>). Metal curvature molds, each having a different radius, were used to apply strain(%). For one demonstration of the phenomena, the data are

(Courtesy of Jung-Hun Seo see Jung-Hun Seo et. al. (2013), A multifunction heterojunction formed between pentacene and a single-crystal silicon nanomembrane, *Advanced Functional Materials*, **23**(27), 3398–3403.)

Strain(%) $x$	Current density(mA/cm <sup>2</sup> ) $y$
0.00	3.47
0.25	3.57
0.49	3.68
0.64	3.73
0.80	3.86
1.08	3.99

- (a) Obtain the least squares line.
- (b) Predict the current density when the strain  $x = 0.50$ .

**11.10** With reference to Exercise 11.9, construct a 95% confidence interval for  $\alpha$ .

**11.11** With reference to Exercise 11.9, test the null hypothesis  $\beta = 0.40$  against the alternative hypothesis  $\beta > 0.40$  at the 0.05 level of significance.

**11.12** The level of pollution because of vehicular emissions in a city is not regulated. Measurements by the local government of the change in flow of vehicles and the change in the level of air pollution (both in percentages) on 12 days yielded the following results:

Change in flow of vehicles $x$	Change in level of air pollution $y$
28	22
36	26
15	15
19	18
24	21
18	17
25	21
40	31
63	52
12	8
16	17
21	20

- (a) Make a scatter plot to verify that it is reasonable to assume that the regression of  $y$  on  $x$  is linear.
- (b) Fit a straight line by the method of least squares.