

# F05 - Sortering och sökning

## 5DV149 Datastrukturer och algoritmer

### Kapitel 15

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  - ▶ s. 59, Kap 15.

# Sortering

# Varför sortering?

- ▶ För att snabba upp andra algoritmer!
  - ▶ Sökning går fortare
- ▶ Nödvändigt för stora datamängder



# Sortering kontra Sorterad datatyp

- ▶ Sortering förändrar **ordningen** mellan objekten i en struktur efter en **sorteringsordning** (fallande, ökande)
  - ▶ Gränsytan är **oförändrad**
  - ▶ Exempel: Vi **sorterar** en Lista
- ▶ Sorterad **datatyp**:
  - ▶ De strukturförändrande operationerna i gränsytan (Insert, Remove) **upprätthåller** en **sorteringsordning** mellan de lagrade elementen
  - ▶ Exempel: Gränsytan för Sorterad lista behöver modifieras jämfört med Lista

# Saker att beakta

- ▶ Absolut komplexitet:
  - ▶ Totala komplexiteten i alla implementationssteg
    - ▶ Tabell (konstruerad som) Lista (implementerad som) dubbellänkad lista
- ▶ Passar en viss typ av sortering för en viss typ av **konstruktion**?
  - ▶ Fält-baserad lista
  - ▶ Länkad lista
- ▶ Ska man
  1. sortera och sedan söka, eller
  2. behålla listan osorterad och göra linjär sökning?
- ▶ Sortering är ofta  $O(n \log n)$ 
  - ▶ Sökning i sorterad mängd är ofta  $O(\log n)$
- ▶ Linjärsökning är  $O(n)$

# Stabilitet

- ▶ Den inbördes relationen mellan två objekt med samma **sorteringsnyckel** bibehålls vid sortering:
  - ▶ Lista sorterad efter efternamn:  
(Alm, Lars), (Bok, Bo), (Ek, Eva), (Gran, Anna), (Löv, Eva)
  - ▶ Lista stabilt omsorterad efter andra elementvärdet:  
(Gran, Anna), (Bok, Bo), (Ek, Eva), (Löv, Eva), (Alm, Lars)
- ▶ Ett till exempel på stabil sortering (index för att förtydliga):

Före:	1	0 <sub>1</sub>	5	2	3 <sub>1</sub>	0 <sub>2</sub>	4	3 <sub>2</sub>	7
Efter:	0 <sub>1</sub>	0 <sub>2</sub>	1	2	3 <sub>1</sub>	3 <sub>2</sub>	4	5	7
- ▶ Alla sorteringsalgoritmer går inte att göra stabila
- ▶ Mer om detta senare

# Grundprinciper

- ▶ Urvalssortering
  - ▶ Välj ut det in-element som är på tur att sättas in
  - ▶ Sätt in det först/sist
- ▶ Instickssortering
  - ▶ Välj ett godtyckligt in-element och sätt in det på rätt plats
- ▶ Utbytessortering
  - ▶ Byt plats på objekt som ligger fel inbördes
- ▶ Samsortering
  - ▶ Sammanslagning av redan sorterade strukturer
- ▶ Nyckelsortering:
  - ▶ Kräver mer information/kunskap om nyckelmängden
  - ▶ T.ex. sortera 200 tentor i kodordning där koden har max 3 siffror



# Inplace

- ▶ En del sorteringsalgoritmer finns i två versioner
  - ▶ En version som bygger en "ny" sorterad kopia av indata
    - ▶ Behöver typiskt  $O(n)$  minne
    - ▶ Oftast enklare algoritm
  - ▶ En version som jobbar direkt i indata, oftast ett fält
    - ▶ Kallas för **in-place**-versioner
    - ▶ Behöver endast  $O(1)$  minne
    - ▶ Ibland komplexa algoritmer

# Sorteringsalgoritmer

- ▶ Idag:
  - ▶ Urvalssortering (*Selection Sort*)
  - ▶ Instickssortering (*Insertion Sort*)
  - ▶ Bubbelsortering (*Bubble Sort*)
  - ▶ Mergesort
  - ▶ Quicksort
  - ▶ Facksortering (*Bucket sort*)
- ▶ Senare:
  - ▶ Heapsort
  - ▶ (Radix Exchange Sort)

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# *Selection sort* — urvalssortering

## Selection sort av lista

- ▶ Algoritmen i grova drag:
  - ▶ Välj ut det **bästa** elementet i **in-listan**
  - ▶ Stoppa in elementet **sist** i **ut-listan**

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output:

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output:



## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output:

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output:

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
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Output:

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
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Output:

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
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Output:

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output:

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
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Output:

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
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Output:



## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output:

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output:

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output:

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output:

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output:

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0
---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0
---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0
---



## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0
---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0
---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0
---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0
---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0
---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0
---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0
---

## Selection sort — exempel

Input: 

7
---

3
---

15
----

0
---

2
---

7
---

6
---

5
---

19
----

9
---

Output: 

0
---



## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0
---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0
---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0
---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2
---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2
---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2
---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2
---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2
---	---



## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2
---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2
---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2
---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2
---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2
---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2
---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2
---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3
---	---	---



## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3
---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3
---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3
---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3
---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3
---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3
---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3
---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3
---	---	---



## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3
---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3
---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3
---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5
---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5
---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5
---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5
---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5
---	---	---	---



## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5
---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5
---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5
---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5
---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5
---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6
---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6
---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6
---	---	---	---	---



## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6
---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6
---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6
---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6
---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6
---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7
---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7
---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7
---	---	---	---	---	---



## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7
---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7
---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7
---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7
---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7
---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7
---	---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7
---	---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7
---	---	---	---	---	---	---



## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7
---	---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7
---	---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7
---	---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7
---	---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7	9
---	---	---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7	9
---	---	---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7	9
---	---	---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7	9
---	---	---	---	---	---	---	---



## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7	9
---	---	---	---	---	---	---	---

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7	9	15
---	---	---	---	---	---	---	---	----

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7	9	15
---	---	---	---	---	---	---	---	----

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7	9	15
---	---	---	---	---	---	---	---	----

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7	9	15
---	---	---	---	---	---	---	---	----

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7	9	15	19
---	---	---	---	---	---	---	---	----	----

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7	9	15	19
---	---	---	---	---	---	---	---	----	----

► Komplexitet:

## Selection sort — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7	9	15	19
---	---	---	---	---	---	---	---	----	----

► Komplexitet:

►  $O(n^2)$



# Inplace Selection sort av fält

- ▶ Jobba i fältet
- ▶ Betrakta fältet som två delar:
  - ▶ en **sorterad** del i början
  - ▶ en **osorterad** del i slutet
  - ▶ Den sorterade delen växer med ett element för varje varv av huvudalgoritmen
- ▶ Algoritmen i grova drag:
  - ▶ Initialt är alla element osorterade
  - ▶ Leta upp det **bästa** elementet bland de osorterade elementet
  - ▶ Byt plats på det **bästa** och det **första** osorterade elementet
  - ▶ Öka den sorterade delen av fältet med ett element

## *Inplace Selection sort* — exempel

Indata


8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

## Inplace Selection sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1



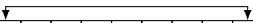
8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

## Inplace Selection sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1



0	3	9	4	7	5	6	2	8	1
---	---	---	---	---	---	---	---	---	---

## Inplace Selection sort — exempel

Indata

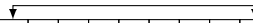
8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

0	3	9	4	7	5	6	2	8	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

0	3	9	4	7	5	6	2	8	1
---	---	---	---	---	---	---	---	---	---



## Inplace Selection sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

0	3	9	4	7	5	6	2	8	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

0	1	9	4	7	5	6	2	8	3
---	---	---	---	---	---	---	---	---	---



## Inplace Selection sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

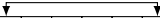
0	3	9	4	7	5	6	2	8	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

0	1	9	4	7	5	6	2	8	3
---	---	---	---	---	---	---	---	---	---

Iteration 3

0	1	9	4	7	5	6	2	8	3
---	---	---	---	---	---	---	---	---	---



## Inplace Selection sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

0	3	9	4	7	5	6	2	8	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

0	1	9	4	7	5	6	2	8	3
---	---	---	---	---	---	---	---	---	---

Iteration 3

0	1	2	4	7	5	6	9	8	3
---	---	---	---	---	---	---	---	---	---





## Inplace Selection sort — exempel

Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
8	3	9	4	7	5	6	2	0	1		
Iteration 1	<table><tr><td>0</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>1</td></tr></table>	0	3	9	4	7	5	6	2	8	1
0	3	9	4	7	5	6	2	8	1		
Iteration 2	<table><tr><td>0</td><td>1</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>3</td></tr></table>	0	1	9	4	7	5	6	2	8	3
0	1	9	4	7	5	6	2	8	3		
Iteration 3	<table><tr><td>0</td><td>1</td><td>2</td><td>4</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>3</td></tr></table>	0	1	2	4	7	5	6	9	8	3
0	1	2	4	7	5	6	9	8	3		
Iteration 4	<table><tr><td>0</td><td>1</td><td>2</td><td>4</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>3</td></tr></table>	0	1	2	4	7	5	6	9	8	3
0	1	2	4	7	5	6	9	8	3		

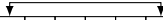


## Inplace Selection sort — exempel

Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
8	3	9	4	7	5	6	2	0	1		
Iteration 1	<table><tr><td>0</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>1</td></tr></table>	0	3	9	4	7	5	6	2	8	1
0	3	9	4	7	5	6	2	8	1		
Iteration 2	<table><tr><td>0</td><td>1</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>3</td></tr></table>	0	1	9	4	7	5	6	2	8	3
0	1	9	4	7	5	6	2	8	3		
Iteration 3	<table><tr><td>0</td><td>1</td><td>2</td><td>4</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>3</td></tr></table>	0	1	2	4	7	5	6	9	8	3
0	1	2	4	7	5	6	9	8	3		
Iteration 4	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>4</td></tr></table>	0	1	2	3	7	5	6	9	8	4
0	1	2	3	7	5	6	9	8	4		

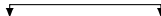


## Inplace Selection sort — exempel

Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
8	3	9	4	7	5	6	2	0	1		
Iteration 1	<table><tr><td>0</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>1</td></tr></table>	0	3	9	4	7	5	6	2	8	1
0	3	9	4	7	5	6	2	8	1		
Iteration 2	<table><tr><td>0</td><td>1</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>3</td></tr></table>	0	1	9	4	7	5	6	2	8	3
0	1	9	4	7	5	6	2	8	3		
Iteration 3	<table><tr><td>0</td><td>1</td><td>2</td><td>4</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>3</td></tr></table>	0	1	2	4	7	5	6	9	8	3
0	1	2	4	7	5	6	9	8	3		
Iteration 4	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>4</td></tr></table>	0	1	2	3	7	5	6	9	8	4
0	1	2	3	7	5	6	9	8	4		
Iteration 5	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>4</td></tr></table> <div></div>	0	1	2	3	7	5	6	9	8	4
0	1	2	3	7	5	6	9	8	4		

## Inplace Selection sort — exempel

Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
8	3	9	4	7	5	6	2	0	1		
Iteration 1	<table><tr><td>0</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>1</td></tr></table>	0	3	9	4	7	5	6	2	8	1
0	3	9	4	7	5	6	2	8	1		
Iteration 2	<table><tr><td>0</td><td>1</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>3</td></tr></table>	0	1	9	4	7	5	6	2	8	3
0	1	9	4	7	5	6	2	8	3		
Iteration 3	<table><tr><td>0</td><td>1</td><td>2</td><td>4</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>3</td></tr></table>	0	1	2	4	7	5	6	9	8	3
0	1	2	4	7	5	6	9	8	3		
Iteration 4	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>4</td></tr></table>	0	1	2	3	7	5	6	9	8	4
0	1	2	3	7	5	6	9	8	4		
Iteration 5	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
0	1	2	3	4	5	6	9	8	7		



## Inplace Selection sort — exempel

Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
8	3	9	4	7	5	6	2	0	1		
Iteration 1	<table><tr><td>0</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>1</td></tr></table>	0	3	9	4	7	5	6	2	8	1
0	3	9	4	7	5	6	2	8	1		
Iteration 2	<table><tr><td>0</td><td>1</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>3</td></tr></table>	0	1	9	4	7	5	6	2	8	3
0	1	9	4	7	5	6	2	8	3		
Iteration 3	<table><tr><td>0</td><td>1</td><td>2</td><td>4</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>3</td></tr></table>	0	1	2	4	7	5	6	9	8	3
0	1	2	4	7	5	6	9	8	3		
Iteration 4	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>4</td></tr></table>	0	1	2	3	7	5	6	9	8	4
0	1	2	3	7	5	6	9	8	4		
Iteration 5	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
0	1	2	3	4	5	6	9	8	7		
Iteration 6	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
0	1	2	3	4	5	6	9	8	7		



## Inplace Selection sort — exempel

Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
8	3	9	4	7	5	6	2	0	1		
Iteration 1	<table><tr><td>0</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>1</td></tr></table>	0	3	9	4	7	5	6	2	8	1
0	3	9	4	7	5	6	2	8	1		
Iteration 2	<table><tr><td>0</td><td>1</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>3</td></tr></table>	0	1	9	4	7	5	6	2	8	3
0	1	9	4	7	5	6	2	8	3		
Iteration 3	<table><tr><td>0</td><td>1</td><td>2</td><td>4</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>3</td></tr></table>	0	1	2	4	7	5	6	9	8	3
0	1	2	4	7	5	6	9	8	3		
Iteration 4	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>4</td></tr></table>	0	1	2	3	7	5	6	9	8	4
0	1	2	3	7	5	6	9	8	4		
Iteration 5	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
0	1	2	3	4	5	6	9	8	7		
Iteration 6	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
0	1	2	3	4	5	6	9	8	7		



## Inplace Selection sort — exempel

Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
8	3	9	4	7	5	6	2	0	1		
Iteration 1	<table><tr><td>0</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>1</td></tr></table>	0	3	9	4	7	5	6	2	8	1
0	3	9	4	7	5	6	2	8	1		
Iteration 2	<table><tr><td>0</td><td>1</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>3</td></tr></table>	0	1	9	4	7	5	6	2	8	3
0	1	9	4	7	5	6	2	8	3		
Iteration 3	<table><tr><td>0</td><td>1</td><td>2</td><td>4</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>3</td></tr></table>	0	1	2	4	7	5	6	9	8	3
0	1	2	4	7	5	6	9	8	3		
Iteration 4	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>4</td></tr></table>	0	1	2	3	7	5	6	9	8	4
0	1	2	3	7	5	6	9	8	4		
Iteration 5	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
0	1	2	3	4	5	6	9	8	7		
Iteration 6	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
0	1	2	3	4	5	6	9	8	7		
Iteration 7	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
0	1	2	3	4	5	6	9	8	7		




## Inplace Selection sort — exempel

Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
8	3	9	4	7	5	6	2	0	1		
Iteration 1	<table><tr><td>0</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>1</td></tr></table>	0	3	9	4	7	5	6	2	8	1
0	3	9	4	7	5	6	2	8	1		
Iteration 2	<table><tr><td>0</td><td>1</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>3</td></tr></table>	0	1	9	4	7	5	6	2	8	3
0	1	9	4	7	5	6	2	8	3		
Iteration 3	<table><tr><td>0</td><td>1</td><td>2</td><td>4</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>3</td></tr></table>	0	1	2	4	7	5	6	9	8	3
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Iteration 4	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>4</td></tr></table>	0	1	2	3	7	5	6	9	8	4
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Iteration 5	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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Iteration 6	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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Iteration 7	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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## Inplace Selection sort — exempel

Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
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Iteration 2	<table><tr><td>0</td><td>1</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>3</td></tr></table>	0	1	9	4	7	5	6	2	8	3
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Iteration 4	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>4</td></tr></table>	0	1	2	3	7	5	6	9	8	4
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Iteration 5	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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Iteration 7	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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Iteration 8	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table> <div></div>	0	1	2	3	4	5	6	9	8	7
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## Inplace Selection sort — exempel

Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
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Iteration 1	<table><tr><td>0</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>1</td></tr></table>	0	3	9	4	7	5	6	2	8	1
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Iteration 2	<table><tr><td>0</td><td>1</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>3</td></tr></table>	0	1	9	4	7	5	6	2	8	3
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Iteration 3	<table><tr><td>0</td><td>1</td><td>2</td><td>4</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>3</td></tr></table>	0	1	2	4	7	5	6	9	8	3
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Iteration 4	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>4</td></tr></table>	0	1	2	3	7	5	6	9	8	4
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Iteration 5	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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Iteration 6	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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Iteration 7	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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## Inplace Selection sort — exempel

Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
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Iteration 1	<table><tr><td>0</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>1</td></tr></table>	0	3	9	4	7	5	6	2	8	1
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Iteration 3	<table><tr><td>0</td><td>1</td><td>2</td><td>4</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>3</td></tr></table>	0	1	2	4	7	5	6	9	8	3
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Iteration 4	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>4</td></tr></table>	0	1	2	3	7	5	6	9	8	4
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Iteration 5	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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Iteration 6	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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Iteration 7	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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Iteration 8	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr></table>	0	1	2	3	4	5	6	7	8	9
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Iteration 9	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr></table>	0	1	2	3	4	5	6	7	8	9
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## Inplace Selection sort — exempel

Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
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Iteration 9	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr></table>	0	1	2	3	4	5	6	7	8	9
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## Inplace Selection sort — exempel

Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
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Iteration 1	<table><tr><td>0</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>1</td></tr></table>	0	3	9	4	7	5	6	2	8	1
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Iteration 2	<table><tr><td>0</td><td>1</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>3</td></tr></table>	0	1	9	4	7	5	6	2	8	3
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Iteration 5	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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Iteration 6	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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Iteration 7	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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Iteration 8	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr></table>	0	1	2	3	4	5	6	7	8	9
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Iteration 9	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr></table>	0	1	2	3	4	5	6	7	8	9
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## Inplace Selection sort — exempel

Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
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Iteration 1	<table><tr><td>0</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>1</td></tr></table>	0	3	9	4	7	5	6	2	8	1
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Iteration 5	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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Iteration 6	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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Iteration 7	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
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Iteration 9	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr></table>	0	1	2	3	4	5	6	7	8	9
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► Stabil?

## Inplace Selection sort — exempel

Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
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Iteration 2	<table><tr><td>0</td><td>1</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>8</td><td>3</td></tr></table>	0	1	9	4	7	5	6	2	8	3
0	1	9	4	7	5	6	2	8	3		
Iteration 3	<table><tr><td>0</td><td>1</td><td>2</td><td>4</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>3</td></tr></table>	0	1	2	4	7	5	6	9	8	3
0	1	2	4	7	5	6	9	8	3		
Iteration 4	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>7</td><td>5</td><td>6</td><td>9</td><td>8</td><td>4</td></tr></table>	0	1	2	3	7	5	6	9	8	4
0	1	2	3	7	5	6	9	8	4		
Iteration 5	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
0	1	2	3	4	5	6	9	8	7		
Iteration 6	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
0	1	2	3	4	5	6	9	8	7		
Iteration 7	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>9</td><td>8</td><td>7</td></tr></table>	0	1	2	3	4	5	6	9	8	7
0	1	2	3	4	5	6	9	8	7		
Iteration 8	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr></table>	0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9		
Iteration 9	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr></table>	0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9		
Utdata	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr></table>	0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9		

► Stabil?

► Nej!

# Selection sort — algorithm

## ► Algorithm:

```
Algorithm selection_sort(a: Array, n: Int)
  // i indicates first unsorted element in a

  for i ← 0 to n - 2 do

    // find the smallest value among the unsorted
    ix ← i
    // start with the ith element as the smallest
    for j ← i + 1 to n - 1 do
      if a[j] < a[ix] then
        ix ← j

    // swap the ith element for the smallest
    t ← a[j]
    a[j] ← a[ix]
    a[ix] ← t

  return a
```



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# *Insertion sort* — instickssortering

## Insertion sort av lista

- ▶ Algoritmen i grova drag:
  - ▶ Plocka ut **första** elementet från **in-listan**
  - ▶ **Leta upp** platsen i **ut-listan** där elementet ska skjutas in

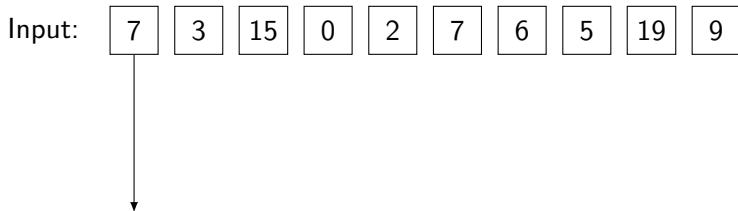
## Insertion sort av lista — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output:

## Insertion sort av lista — exempel



Output:

## Insertion sort av lista — exempel

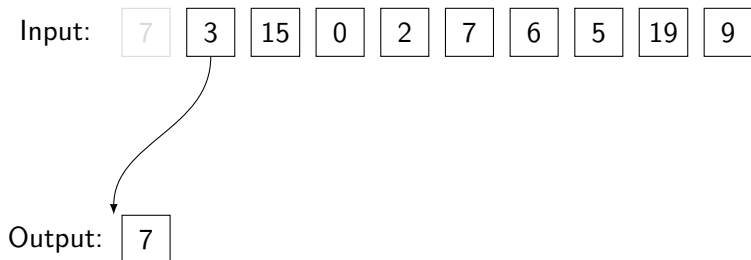
Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

7
---

## Insertion sort av lista — exempel



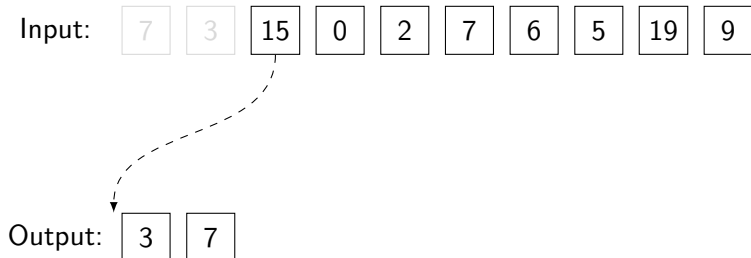


## Insertion sort av lista — exempel

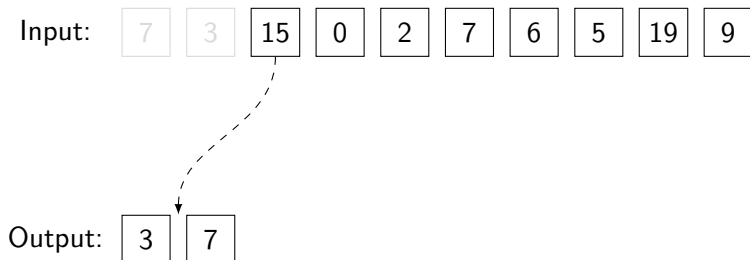
Input:

Output:

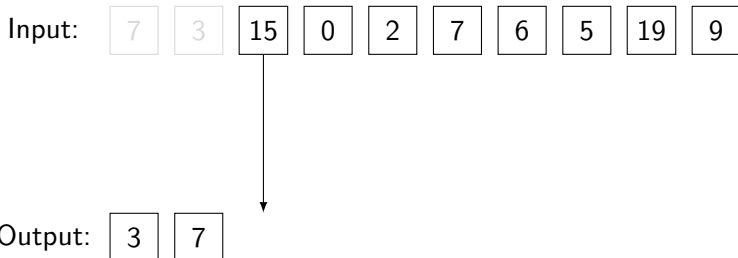
## Insertion sort av lista — exempel



## Insertion sort av lista — exempel



## Insertion sort av lista — exempel



## Insertion sort av lista — exempel

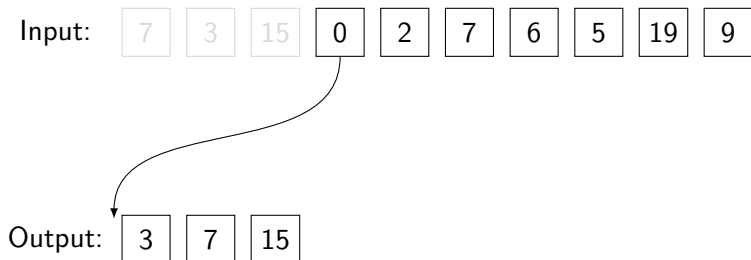
Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

3	7	15
---	---	----

## Insertion sort av lista — exempel



## Insertion sort av lista — exempel

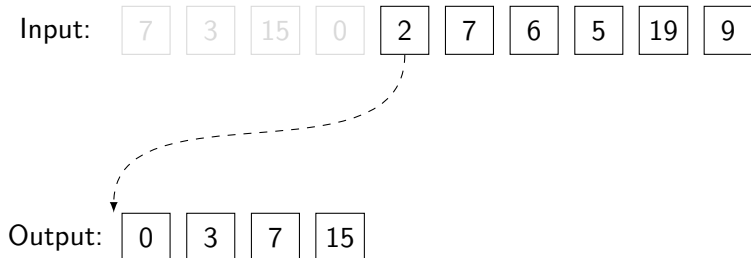
Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

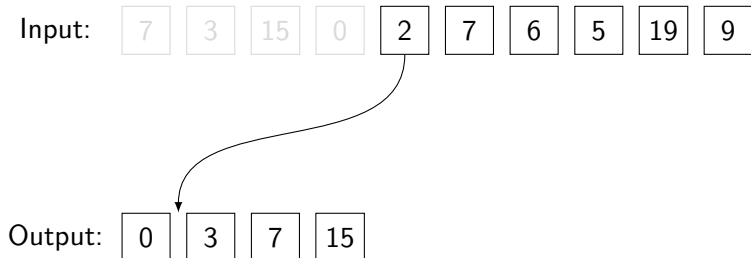
0	3	7	15
---	---	---	----

## Insertion sort av lista — exempel





## Insertion sort av lista — exempel



## Insertion sort av lista — exempel

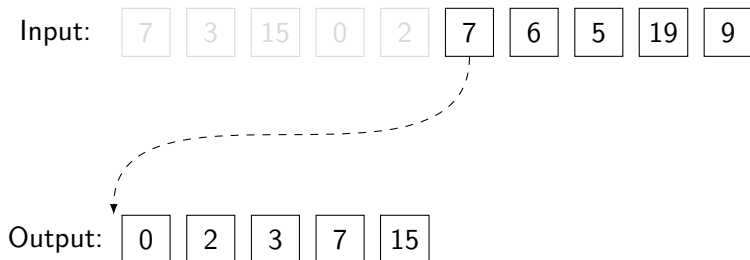
Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

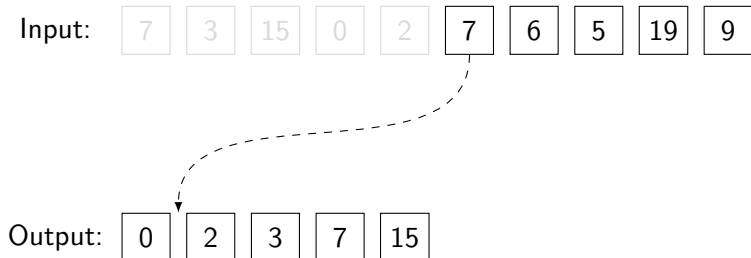
Output: 

0	2	3	7	15
---	---	---	---	----

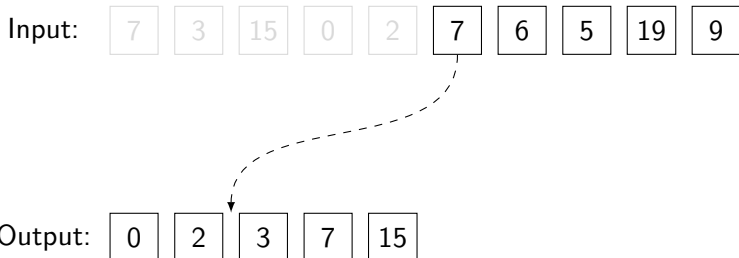
## Insertion sort av lista — exempel



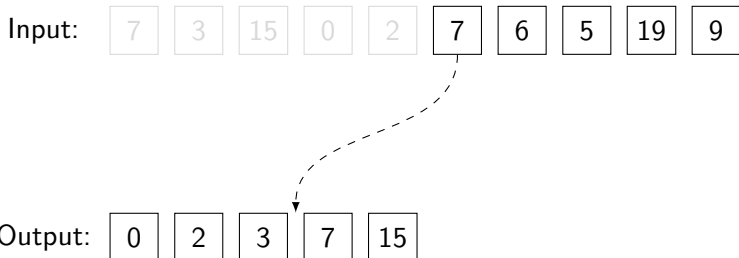
## Insertion sort av lista — exempel



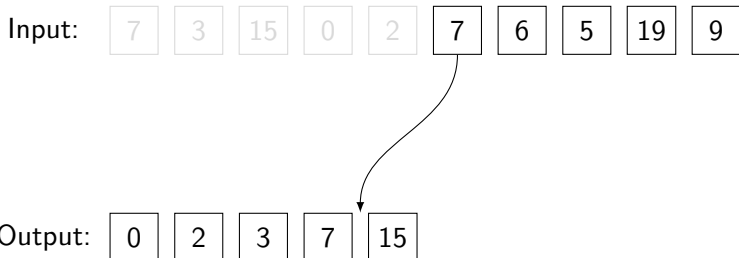
## Insertion sort av lista — exempel



## Insertion sort av lista — exempel



## Insertion sort av lista — exempel



## Insertion sort av lista — exempel

Input: 

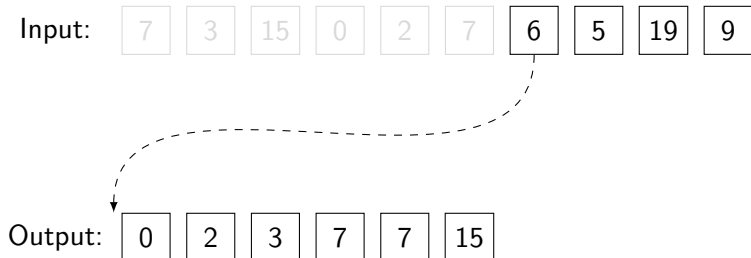
7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

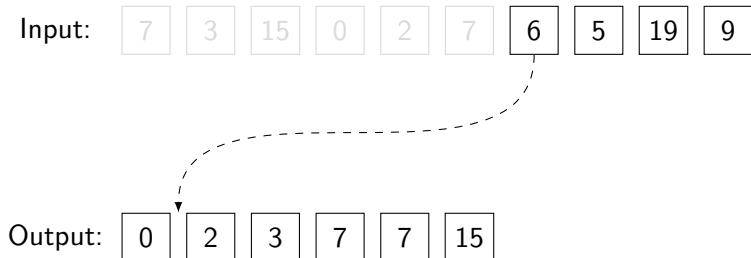
0	2	3	7	7	15
---	---	---	---	---	----



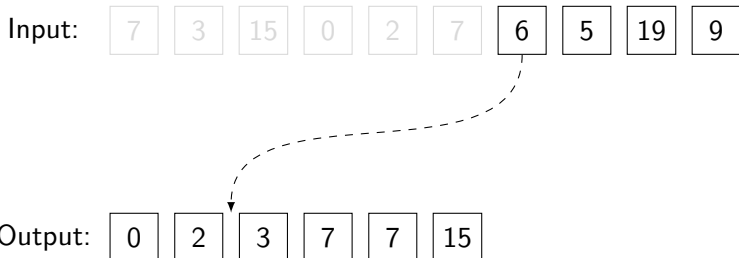
## Insertion sort av lista — exempel



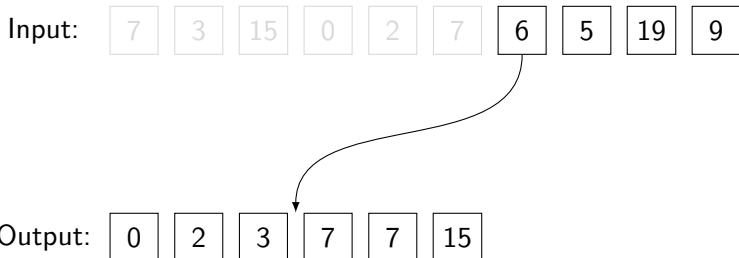
## Insertion sort av lista — exempel



## Insertion sort av lista — exempel



## Insertion sort av lista — exempel



## Insertion sort av lista — exempel

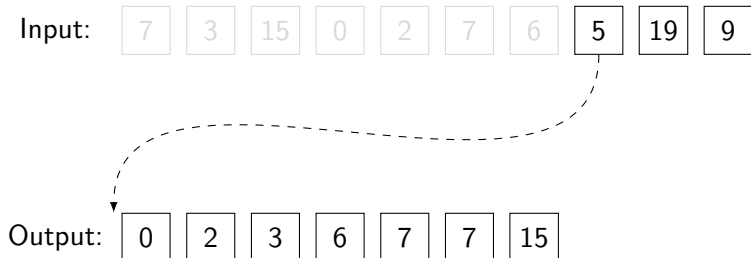
Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

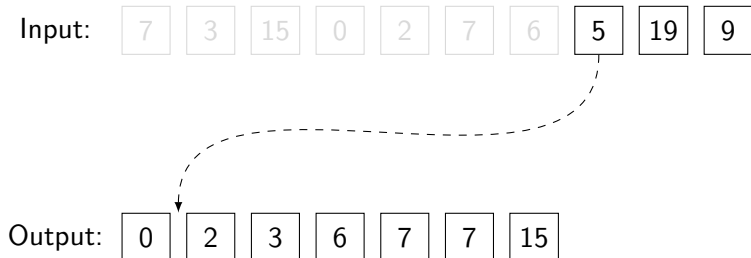
Output: 

0	2	3	6	7	7	15
---	---	---	---	---	---	----

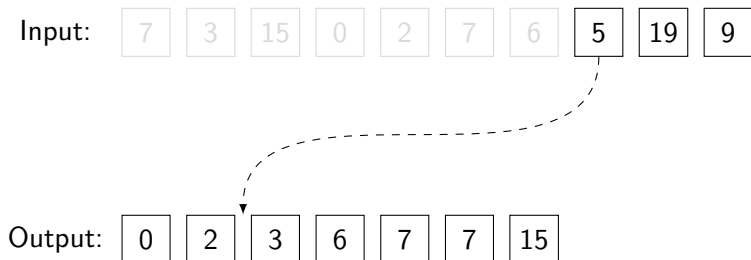
## Insertion sort av lista — exempel



## Insertion sort av lista — exempel

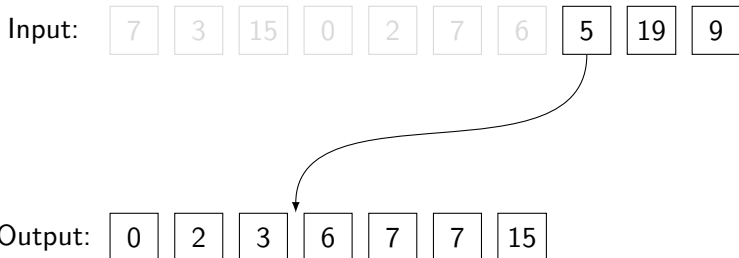


## Insertion sort av lista — exempel





## Insertion sort av lista — exempel



## Insertion sort av lista — exempel

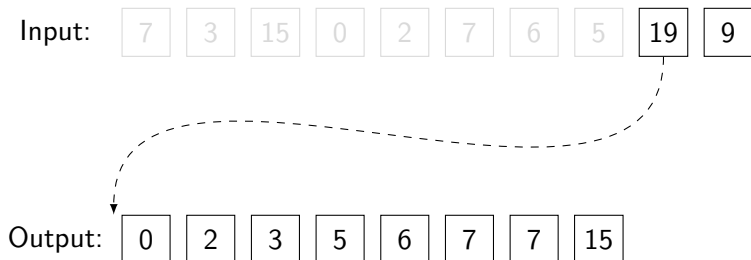
Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

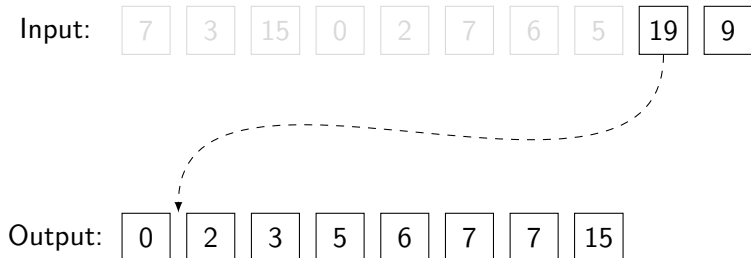
Output: 

0	2	3	5	6	7	7	15
---	---	---	---	---	---	---	----

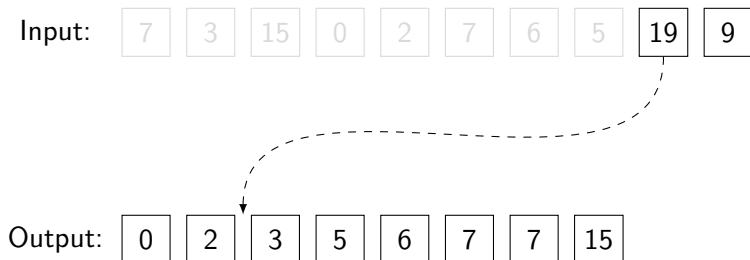
## Insertion sort av lista — exempel



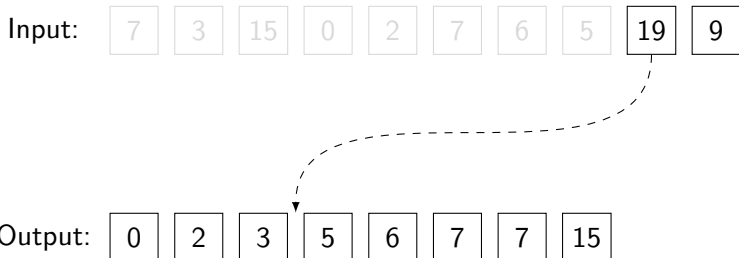
## Insertion sort av lista — exempel



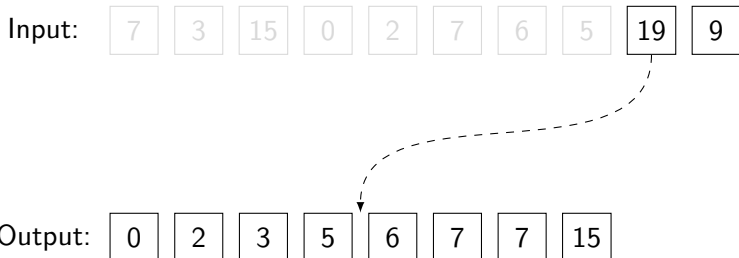
## Insertion sort av lista — exempel



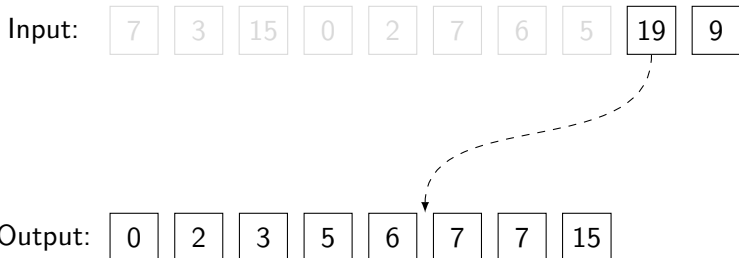
## Insertion sort av lista — exempel



## Insertion sort av lista — exempel

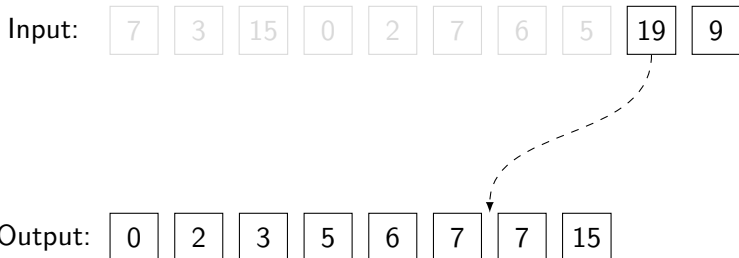


## Insertion sort av lista — exempel

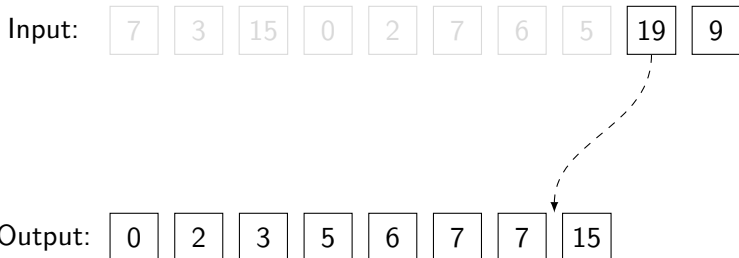




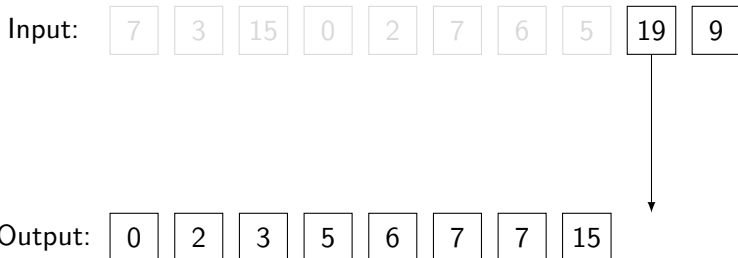
## Insertion sort av lista — exempel



## Insertion sort av lista — exempel



## Insertion sort av lista — exempel



## Insertion sort av lista — exempel

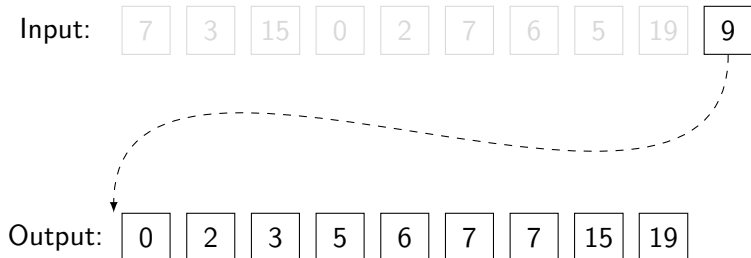
Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

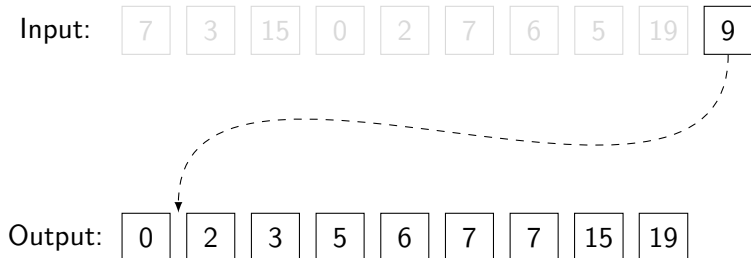
Output: 

0	2	3	5	6	7	7	15	19
---	---	---	---	---	---	---	----	----

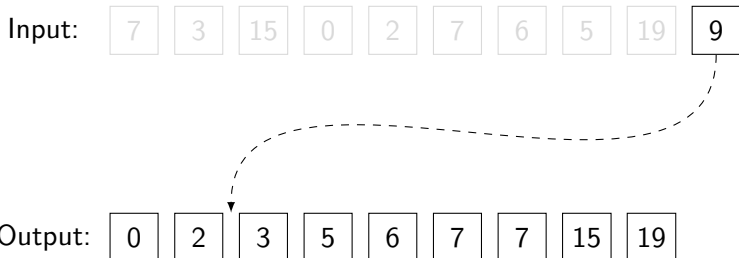
## Insertion sort av lista — exempel



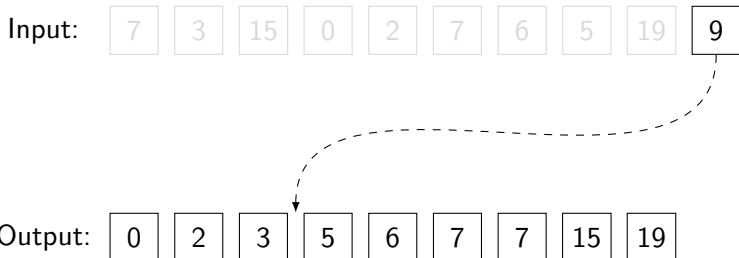
## Insertion sort av lista — exempel



## Insertion sort av lista — exempel

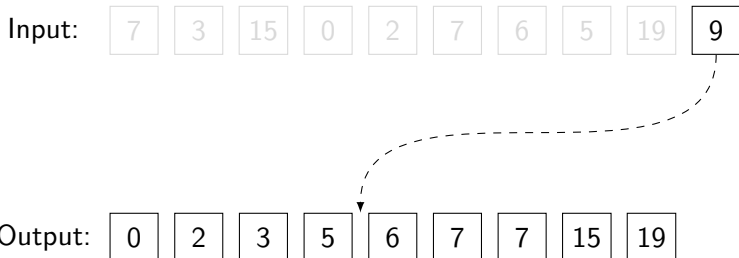


## Insertion sort av lista — exempel

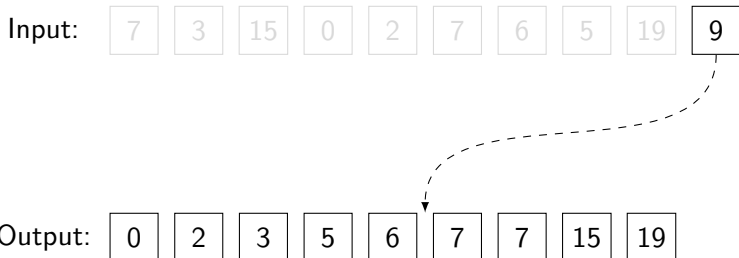




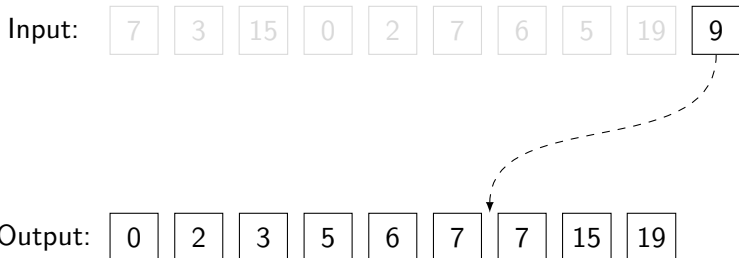
## Insertion sort av lista — exempel



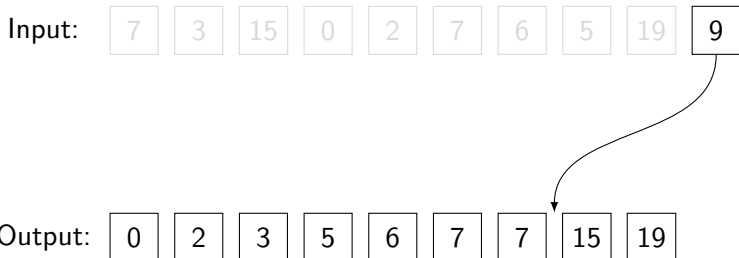
## Insertion sort av lista — exempel



## Insertion sort av lista — exempel



## Insertion sort av lista — exempel



## Insertion sort av lista — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7	9	15	19
---	---	---	---	---	---	---	---	----	----

## Insertion sort av lista — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7	9	15	19
---	---	---	---	---	---	---	---	----	----

► Komplexitet:

## Insertion sort av lista — exempel

Input: 

7	3	15	0	2	7	6	5	19	9
---	---	----	---	---	---	---	---	----	---

Output: 

0	2	3	5	6	7	7	9	15	19
---	---	---	---	---	---	---	---	----	----

► Komplexitet:

►  $O(n^2)$

## Inplace Insertion sort av fält

- ▶ Jobba i fältet
- ▶ Betrakta fältet som två delar:
  - ▶ en **sorterad** del i början
  - ▶ en **osorterad** del i slutet
  - ▶ Den sorterade delen växer med ett element för varje varv av huvudalgoritmen
- ▶ Algoritmen i grova drag:
  - ▶ Börja med **ett element** (ett element är **sorterat**)
  - ▶ Ta det **första** osorterade elementet och **sortera in** på rätt plats bland de sorterade elementen



## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0

8	3	9	4	7	5	6	2	0	1

## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---



## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0


8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---



## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 3

3	4	8	9	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---



## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1


3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 3

3	4	8	9	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---



## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 3

3	4	8	9	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 4

3	4	7	8	9	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

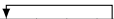
3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 3

3	4	8	9	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 4

3	4	7	8	9	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---



## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 3

3	4	8	9	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 4

3	4	7	8	9	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 5

3	4	5	7	8	9	6	2	0	1
---	---	---	---	---	---	---	---	---	---

## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 3

3	4	8	9	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 4

3	4	7	8	9	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 5

3	4	5	7	8	9	6	2	0	1
---	---	---	---	---	---	---	---	---	---



## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 3

3	4	8	9	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 4

3	4	7	8	9	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 5

3	4	5	7	8	9	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 6

3	4	5	6	7	8	9	2	0	1
---	---	---	---	---	---	---	---	---	---

## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 3

3	4	8	9	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 4

3	4	7	8	9	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 5

3	4	5	7	8	9	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 6

3	4	5	6	7	8	9	2	0	1
---	---	---	---	---	---	---	---	---	---

## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 3

3	4	8	9	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 4

3	4	7	8	9	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 5

3	4	5	7	8	9	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 6

3	4	5	6	7	8	9	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 7

2	3	4	5	6	7	8	9	0	1
---	---	---	---	---	---	---	---	---	---



## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 3

3	4	8	9	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 4

3	4	7	8	9	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 5

3	4	5	7	8	9	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 6

3	4	5	6	7	8	9	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 7

2	3	4	5	6	7	8	9	0	1
---	---	---	---	---	---	---	---	---	---

## Insertion sort — exempel

Indata

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 0

8	3	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 1

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 2

3	8	9	4	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 3

3	4	8	9	7	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 4

3	4	7	8	9	5	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 5

3	4	5	7	8	9	6	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 6

3	4	5	6	7	8	9	2	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 7

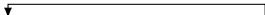
2	3	4	5	6	7	8	9	0	1
---	---	---	---	---	---	---	---	---	---

Iteration 8

0	2	3	4	5	6	7	8	9	1
---	---	---	---	---	---	---	---	---	---

## Insertion sort — exempel

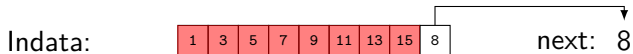
Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
8	3	9	4	7	5	6	2	0	1		
Iteration 0	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
8	3	9	4	7	5	6	2	0	1		
Iteration 1	<table><tr><td>3</td><td>8</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	3	8	9	4	7	5	6	2	0	1
3	8	9	4	7	5	6	2	0	1		
Iteration 2	<table><tr><td>3</td><td>8</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	3	8	9	4	7	5	6	2	0	1
3	8	9	4	7	5	6	2	0	1		
Iteration 3	<table><tr><td>3</td><td>4</td><td>8</td><td>9</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	3	4	8	9	7	5	6	2	0	1
3	4	8	9	7	5	6	2	0	1		
Iteration 4	<table><tr><td>3</td><td>4</td><td>7</td><td>8</td><td>9</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	3	4	7	8	9	5	6	2	0	1
3	4	7	8	9	5	6	2	0	1		
Iteration 5	<table><tr><td>3</td><td>4</td><td>5</td><td>7</td><td>8</td><td>9</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	3	4	5	7	8	9	6	2	0	1
3	4	5	7	8	9	6	2	0	1		
Iteration 6	<table><tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>2</td><td>0</td><td>1</td></tr></table>	3	4	5	6	7	8	9	2	0	1
3	4	5	6	7	8	9	2	0	1		
Iteration 7	<table><tr><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>0</td><td>1</td></tr></table>	2	3	4	5	6	7	8	9	0	1
2	3	4	5	6	7	8	9	0	1		
Iteration 8	<table><tr><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>1</td></tr></table>	0	2	3	4	5	6	7	8	9	1
0	2	3	4	5	6	7	8	9	1		



## Insertion sort — exempel

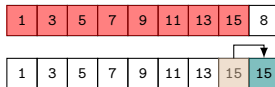
Indata	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
8	3	9	4	7	5	6	2	0	1		
Iteration 0	<table><tr><td>8</td><td>3</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	8	3	9	4	7	5	6	2	0	1
8	3	9	4	7	5	6	2	0	1		
Iteration 1	<table><tr><td>3</td><td>8</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	3	8	9	4	7	5	6	2	0	1
3	8	9	4	7	5	6	2	0	1		
Iteration 2	<table><tr><td>3</td><td>8</td><td>9</td><td>4</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	3	8	9	4	7	5	6	2	0	1
3	8	9	4	7	5	6	2	0	1		
Iteration 3	<table><tr><td>3</td><td>4</td><td>8</td><td>9</td><td>7</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	3	4	8	9	7	5	6	2	0	1
3	4	8	9	7	5	6	2	0	1		
Iteration 4	<table><tr><td>3</td><td>4</td><td>7</td><td>8</td><td>9</td><td>5</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	3	4	7	8	9	5	6	2	0	1
3	4	7	8	9	5	6	2	0	1		
Iteration 5	<table><tr><td>3</td><td>4</td><td>5</td><td>7</td><td>8</td><td>9</td><td>6</td><td>2</td><td>0</td><td>1</td></tr></table>	3	4	5	7	8	9	6	2	0	1
3	4	5	7	8	9	6	2	0	1		
Iteration 6	<table><tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>2</td><td>0</td><td>1</td></tr></table>	3	4	5	6	7	8	9	2	0	1
3	4	5	6	7	8	9	2	0	1		
Iteration 7	<table><tr><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>0</td><td>1</td></tr></table>	2	3	4	5	6	7	8	9	0	1
2	3	4	5	6	7	8	9	0	1		
Iteration 8	<table><tr><td>0</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>1</td></tr></table>	0	2	3	4	5	6	7	8	9	1
0	2	3	4	5	6	7	8	9	1		
Utdata	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr></table>	0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9		

## Insertion sort — Sidospår: insättning



## Insertion sort — Sidospår: insättning

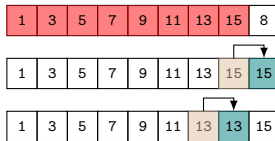
Indata:



next: 8

## Insertion sort — Sidospår: insättning

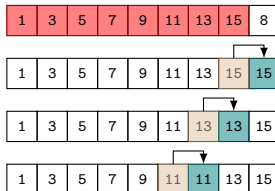
Indata:



next: 8

## Insertion sort — Sidospår: insättning

Indata:

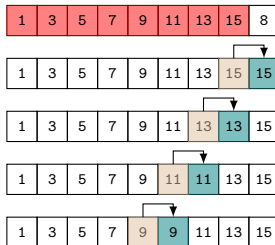


next: 8



## Insertion sort — Sidospår: insättning

Indata:



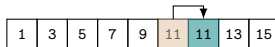
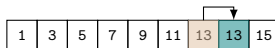
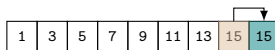
next: 8

## Insertion sort — Sidospår: insättning

Indata:



next: 8



Utdata:



# Insertion sort — algorithm

## ► Algorithm:

```
Algorithm insertion_sort(a: Array, n: Int)
  // i indicates first unsorted element in a

  for i ← 1 to n - 1 do

    // new value to insert in sorted part of a
    next ← a[i]

    // start with last sorted element
    j ← i - 1

    // as long as new element is smaller and
    // we're inside the array
    while j >= 0 and next < a[j] do

      // shift element right
      a[j + 1] ← a[j]

      // continue to the left
      j ← j - 1

    // insert new value in its sorted place
    a[j+1] ← next

  return a
```

# Blank

# *Bubble sort* — bubbelsortering

# Bubble Sort

- ▶ Algoritmen i grova drag:
  - ▶ Upprepa följande tills **ingen förändring** sker:
    - ▶ Jämför alla elementen **ett par i taget**
      - ▶ Börja med element 0 och 1, därefter 1 och 2, osv
    - ▶ Om elementen är i **fel ordning**, **byt plats** på dem

# Bubble Sort — algorithm

## ► Algorithm:

```
Algorithm bubble_sort(a: Array, n: Int)
do
  // so far no swap has taken place
  swapped ← false

  // for each adjacent pair in a...
  for j ← 0 to n - 2 do

    // if the elements are in the wrong order...
    if a[j] > a[j + 1] then

      // ...swap the elements
      tmp ← a[j]
      a[j] ← a[j + 1]
      a[j + 1] ← tmp

      // remember that a swap has taken place
      swapped ← true

while swapped = true

return a
```

## Bubble Sort exempel

a      

8	3	9	4	7
---	---	---	---	---

j      

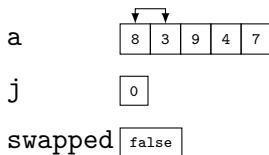
x
---

swapped 

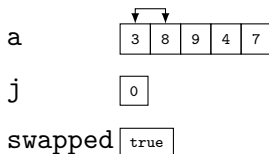
x
---



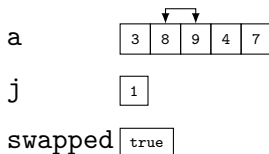
## Bubble Sort exempel



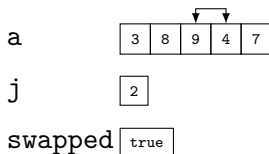
## Bubble Sort exempel



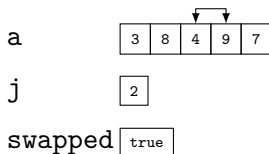
## Bubble Sort exempel



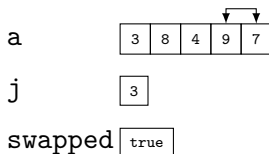
## Bubble Sort exempel



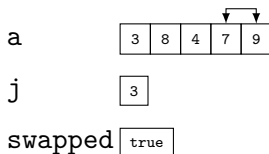
## Bubble Sort exempel



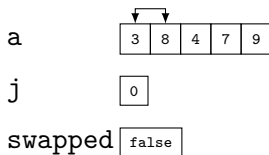
## Bubble Sort exempel



## Bubble Sort exempel

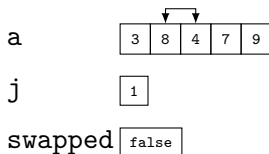


## Bubble Sort exempel

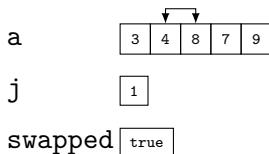




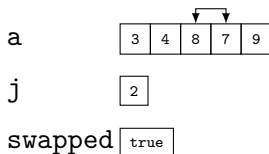
## Bubble Sort exempel



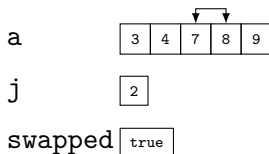
## Bubble Sort exempel



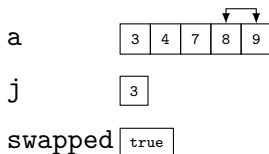
## Bubble Sort exempel



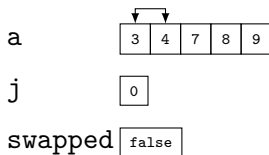
## Bubble Sort exempel



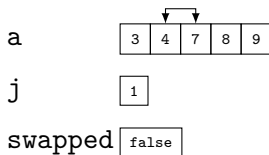
## Bubble Sort exempel



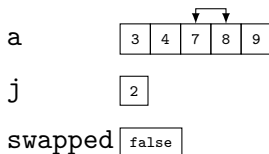
## Bubble Sort exempel



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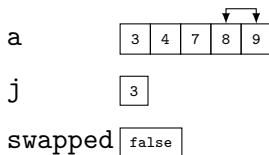


## Bubble Sort exempel





## Bubble Sort exempel



## Bubble Sort exempel

a      

3	4	7	8	9
---	---	---	---	---

j      

3
---

swapped 

false
-------

# Bubble Sort — stabilitet, komplexitet

## ► Algoritmen:

```
Algorithm bubble_sort(a: Array, n: Int)
do
  // so far no swap has taken place
  swapped ← false

  // for each adjacent pair in a...
  for j ← 0 to n - 2 do

    // if the elements are in the wrong order...
    if a[j] > a[j + 1] then

      // ...swap the elements
      tmp ← a[j]
      a[j] ← a[j + 1]
      a[j + 1] ← tmp

      // remember that a swap has taken place
      swapped ← true

while swapped = true

return a
```

## ► Stabil sortering?

## ► Tidskomplexitet för sortering av fält?

### ► Bästa-falls?

### ► Värsta-falls?

## ► Går att få samma komplexitet för sortering av lista då vi alltid refererar till ett element och dess efterföljare

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```

- Stabil sortering?
  - Ja!
- Tidskomplexitet för sortering av fält?
  - Bästa-falls?
  - Värsta-falls?
- Går att få samma komplexitet för sortering av lista då vi alltid refererar till ett element och dess efterföljare

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## ► Stabil sortering?

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## ► Tidskomplexitet för sortering av fält?

► Bästa-falls?

►  $O(n)$  (inga utbyten)

► Värsta-falls?

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while swapped = true

return a
```

## ► Stabil sortering?

► Ja!

## ► Tidskomplexitet för sortering av fält?

► Bästa-falls?

►  $O(n)$  (inga utbyten)

► Värsta-falls?

►  $O(n^2)$

## ► Går att få samma komplexitet för sortering av lista då vi alltid refererar till ett element och dess efterföljare

# Divide-and-Conquer

- ▶ Rekursiv algoritmprincip:
  - ▶ Grundidén är att dela upp problemet i mindre och mindre problem
  - ▶ Lös problemen för basfallet
  - ▶ Slå ihop till en totallösning
- ▶ *Mergesort* och *Quicksort* är av denna typ
- ▶ Komplexitet:  $O(n \log n)$

# Blank



# Blank

# *Merge sort* — samsortering

# Merge Sort

- ▶ Algoritmen i grova drag
  - ▶ Om sekvensen har **ett** element
    - ▶ Returnera sekvensen (den är redan **sorterad**)
  - ▶ annars
    - ▶ **Dela** sekvensen i **två** ungefär lika stora **delsekvenser**
    - ▶ **Sortera** delsekvenserna **rekursivt**
    - ▶ **Slå samman** delsekvenserna (*Merge*)
    - ▶ Returnera den **sammanslagna sekvensen**

# Merge

- ▶ *Merge Sort* använder en delalgorithm — *Merge*
- ▶ Algorithm för att slå samman två **redan sorterade** sekvenser:
  - ▶ Så länge **bägge sekvenserna har element**:
    - ▶ Jämför **första** (=minsta) elementet i vardera sekvensen
    - ▶ Flytta det **minsta av de två elementen** till utsekvensen
  - ▶ Flytta över alla element som **finns kvar** i sekvenserna

## Merge — exempel

A

1	3	5	7	9	11	13	15
---	---	---	---	---	----	----	----

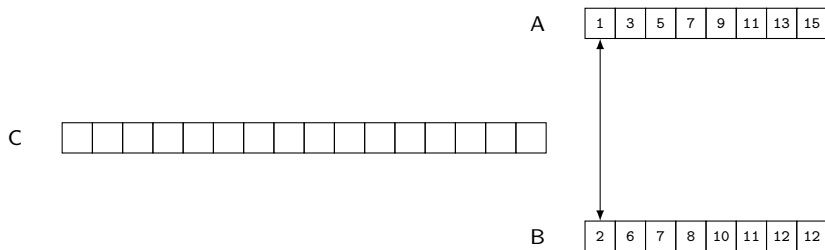
C

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

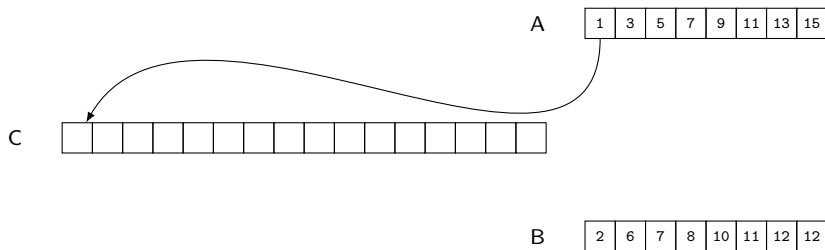
B

2	6	7	8	10	11	12	12
---	---	---	---	----	----	----	----

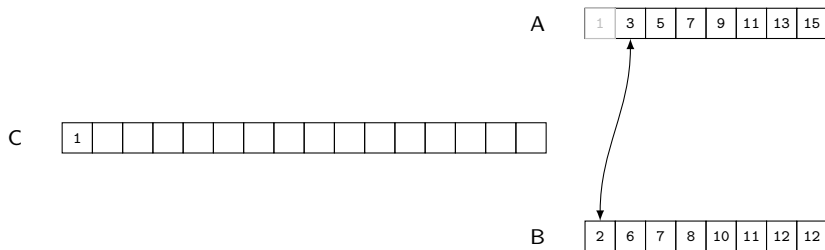
## Merge — exempel



## Merge — exempel

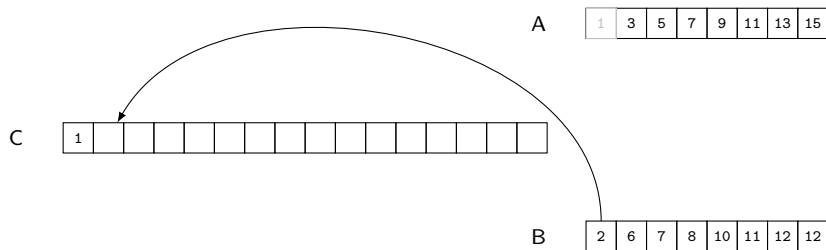


## Merge — exempel

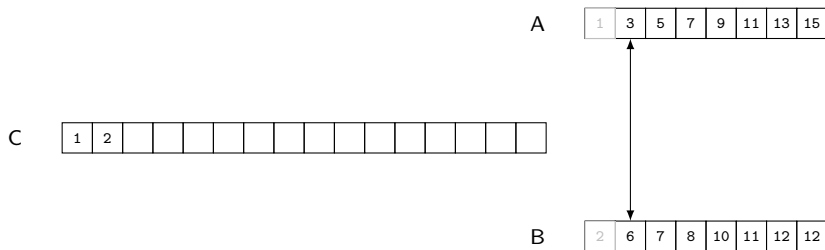




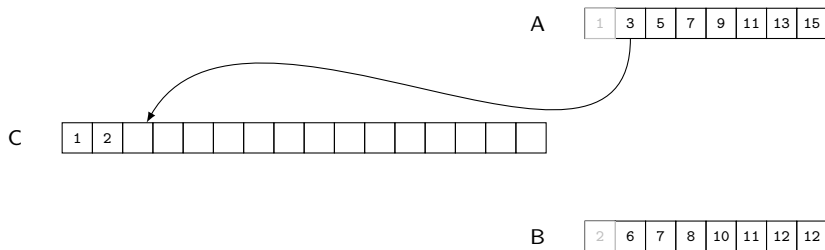
## Merge — exempel



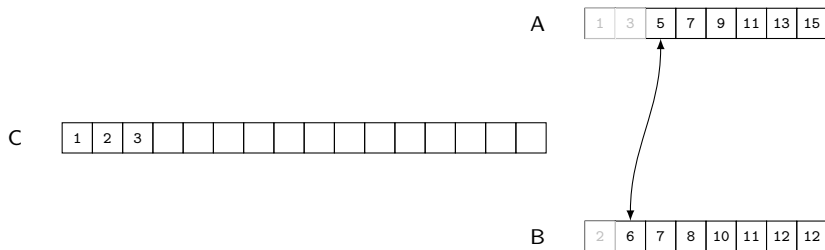
## Merge — exempel



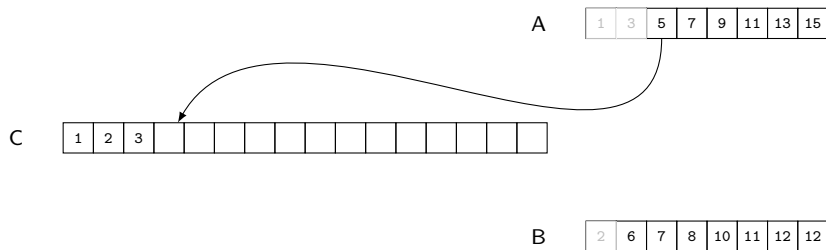
## Merge — exempel



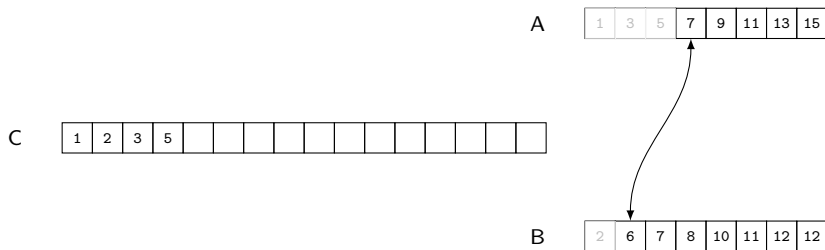
## Merge — exempel



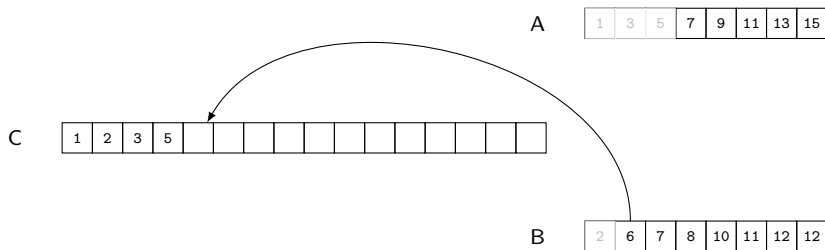
## Merge — exempel



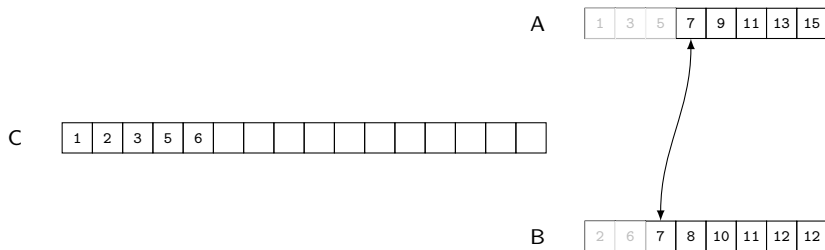
## Merge — exempel



## Merge — exempel

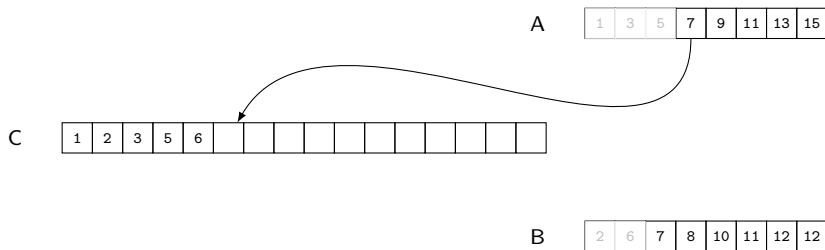


## Merge — exempel

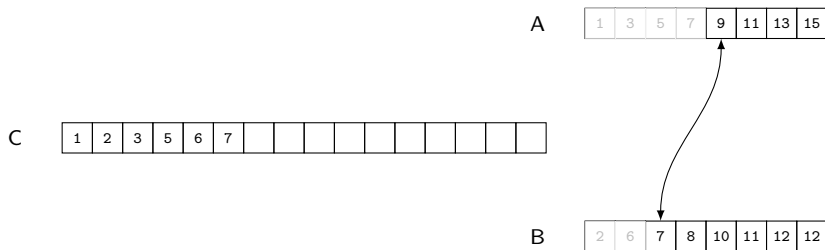




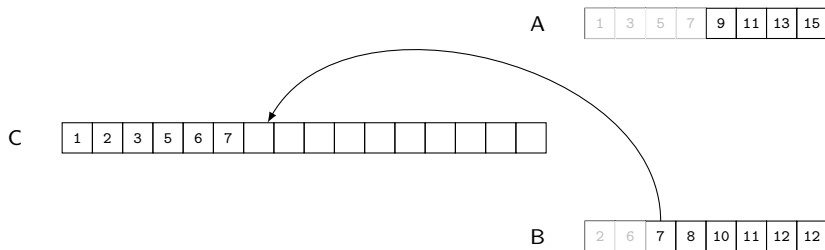
## Merge — exempel



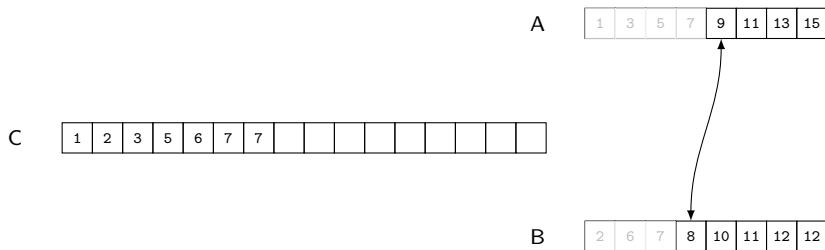
## Merge — exempel



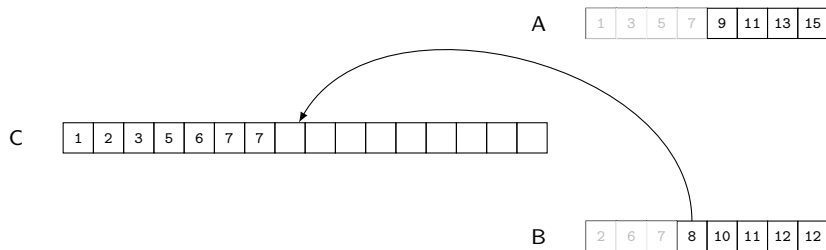
## Merge — exempel



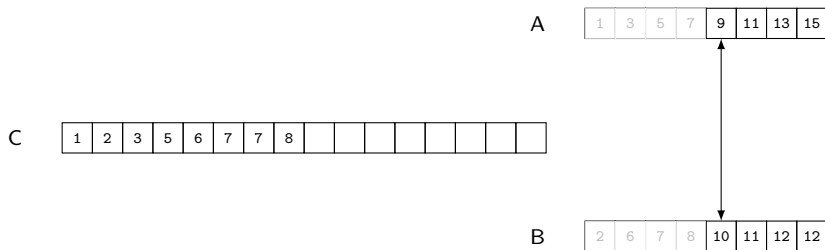
## Merge — exempel



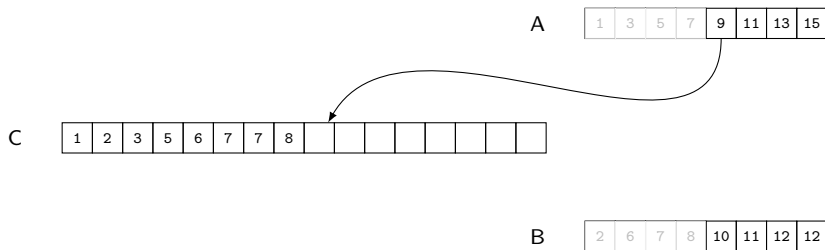
## Merge — exempel



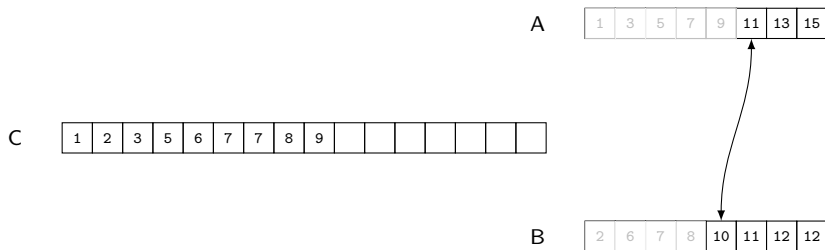
## Merge — exempel



## Merge — exempel

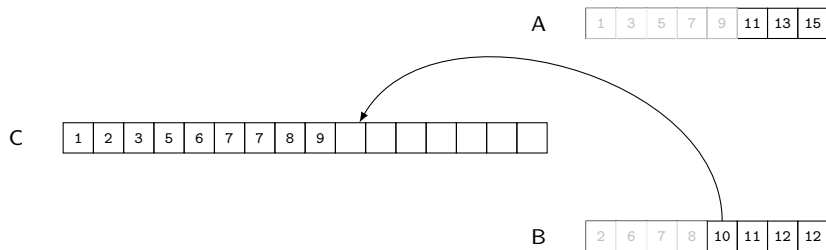


## Merge — exempel

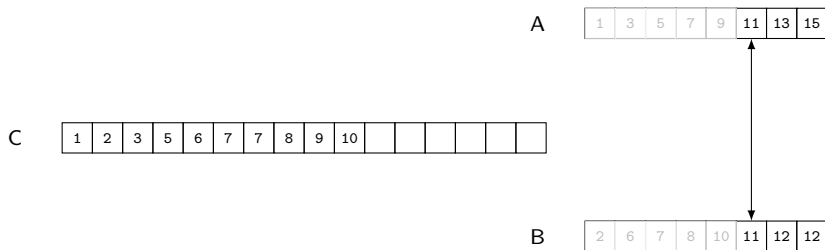




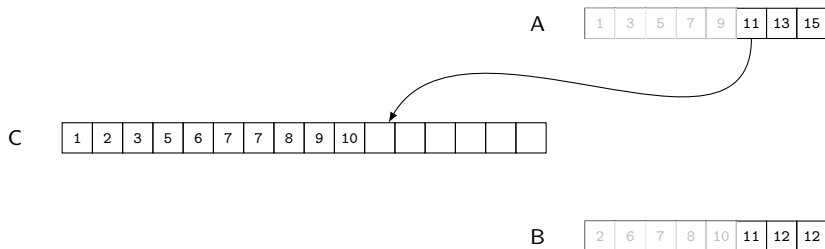
## Merge — exempel



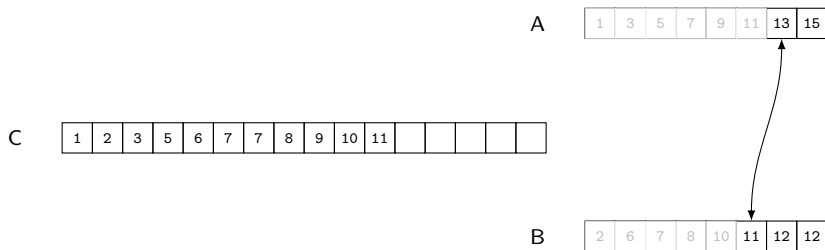
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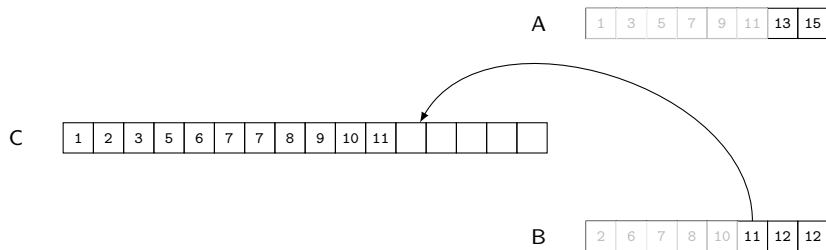
## Merge — exempel



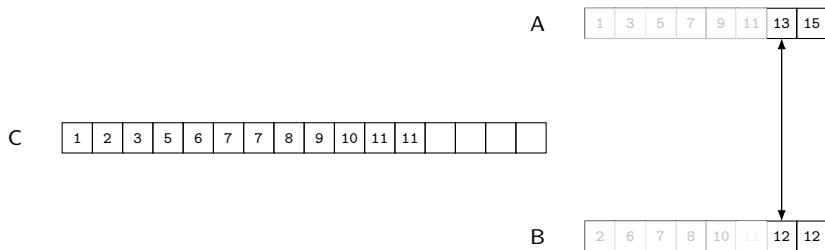
## Merge — exempel



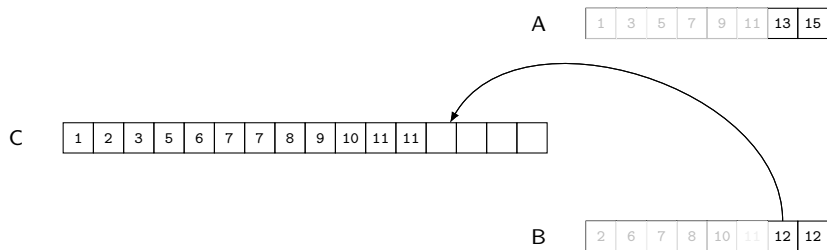
## Merge — exempel



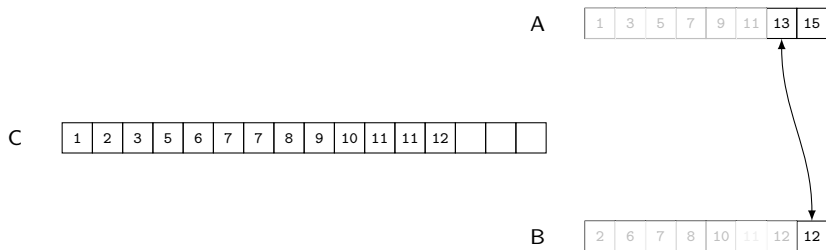
## Merge — exempel



## Merge — exempel

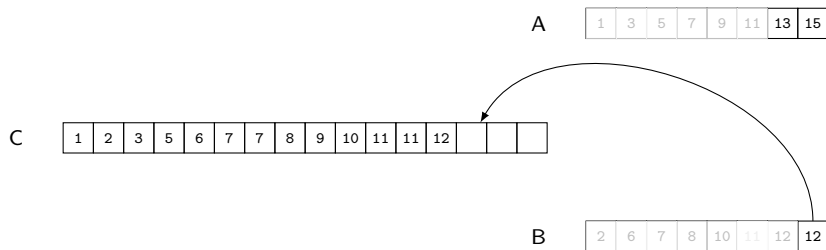


## Merge — exempel

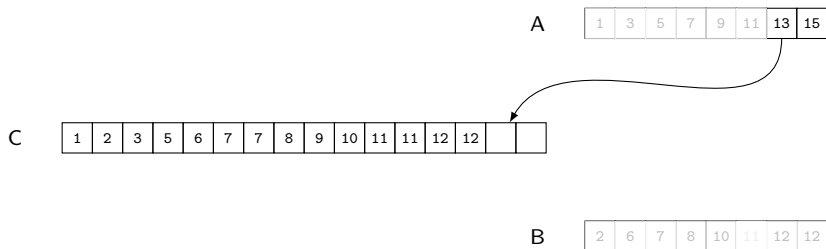




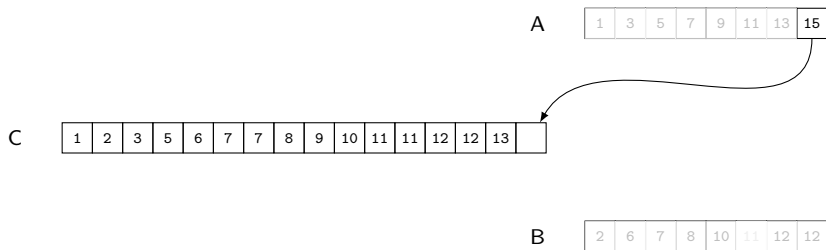
## Merge — exempel



## Merge — exempel



## Merge — exempel



## Merge — exempel

A

1	3	5	7	9	11	13	15
---	---	---	---	---	----	----	----

C

1	2	3	5	6	7	7	8	9	10	11	11	12	12	13	15
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----

B

2	6	7	8	10	11	12	12
---	---	---	---	----	----	----	----

# Merge

## ► Algorithm für Merge:

```
Algorithm merge(A, B: Array, na, nb: Int)
  C ← create_array(na + nb)

  ia ← 0 // Where to read from in A
  ib ← 0 // Where to read from in B
  ic ← 0 // Where to write to in C

  // While there are elements in both A and B...
  while ia < na and ib < nb do
    if A[ia] <= B[ib] then // Smallest in A...
      C[ic] ← A[ia] // ...copy from A
      ia ← ia + 1 // ...advance in A
    else // Smallest in B...
      C[ic] ← B[ib] // ...copy from B
      ib ← ib + 1 // ...advance in B

    ic ← ic + 1 // Advance in C

  // While there are elements in A...
  while ia < na do
    C[ic] ← A[ia] // ...copy from A
    ia ← ia + 1 // ...advance in A and C
    ic ← ic + 1

  // While there are elements in B...
  while ib < nb do
    C[ic] ← B[ib] // ...copy from B
    ib ← ib + 1 // ...advance in B and C
    ic ← ic + 1

  return C
```

# Merge Sort — algorithm

## ► Algorithm for Merge Sort:

```
Algorithm merge_sort(a: Array, n: Int)
  if n < 2 then
    // Already sorted
    return a

  // Split a in two parts
  (left, right) ← split(a, n/2)

  // Lengths of left and right parts, respectively
  nl ← floor(n/2)
  nr ← n - nl

  // Sort left half recursively
  left ← merge_sort(left, nl)

  // Sort right half recursively
  right ← merge_sort(right, nr)

  // Merge sorted arrays
  a ← merge(left, right, nl, nr)

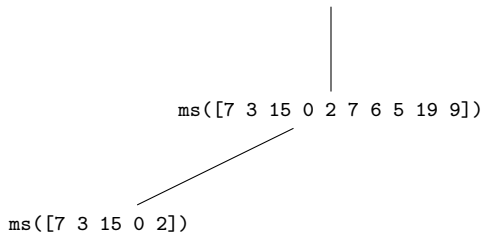
  return a
```

## Merge sort, anropsträd

ms([7 3 15 0 2 7 6 5 19 9])

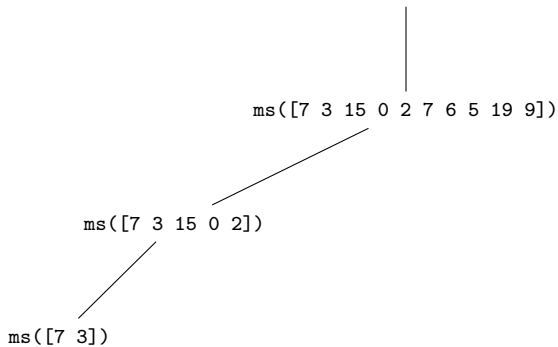


## Merge sort, anropsträd

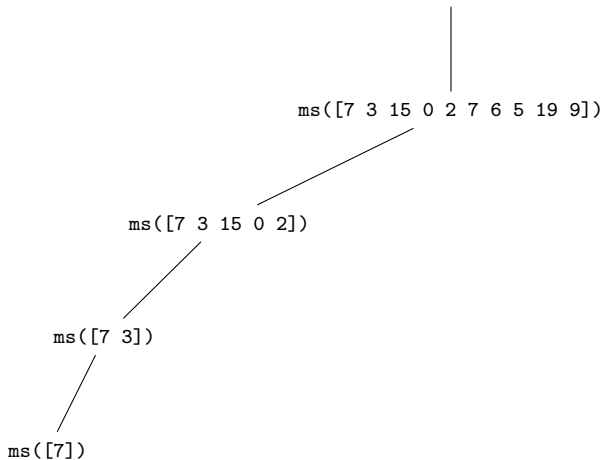




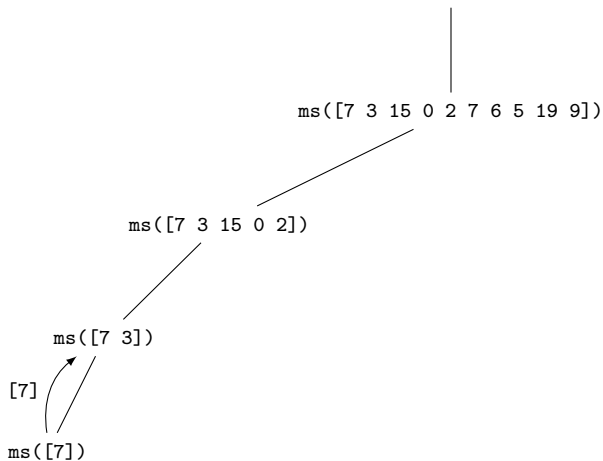
# Merge sort, anropsträd



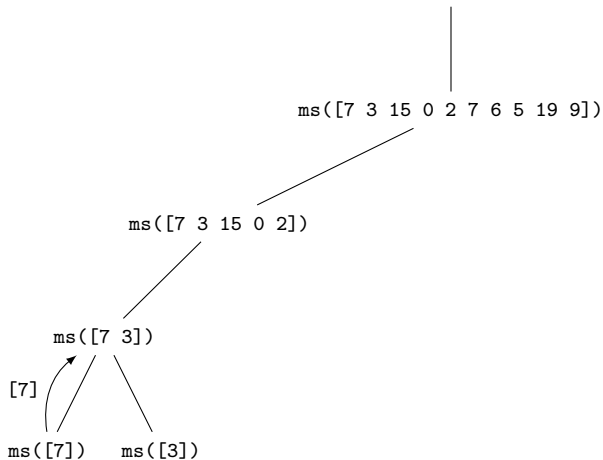
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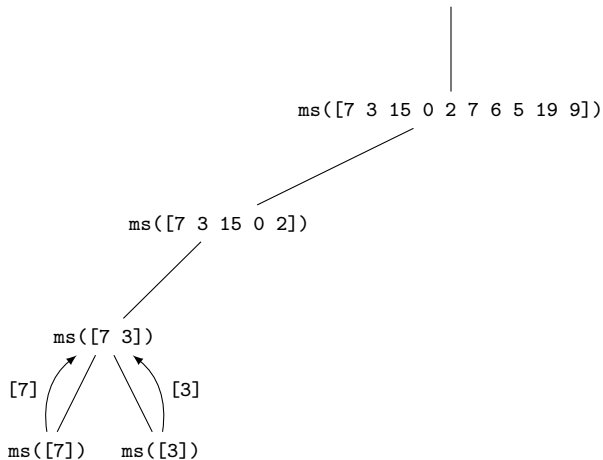
# Merge sort, anropsträd



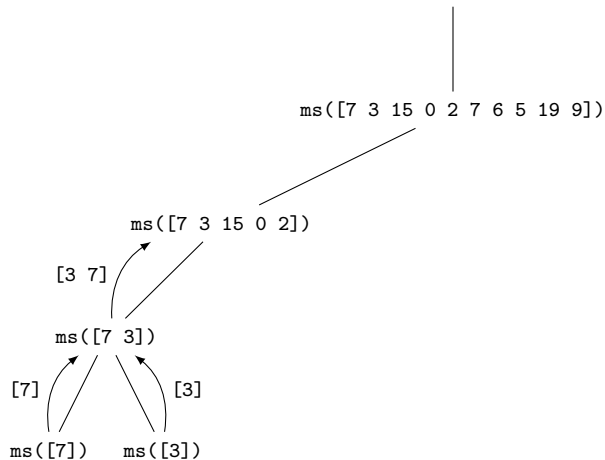
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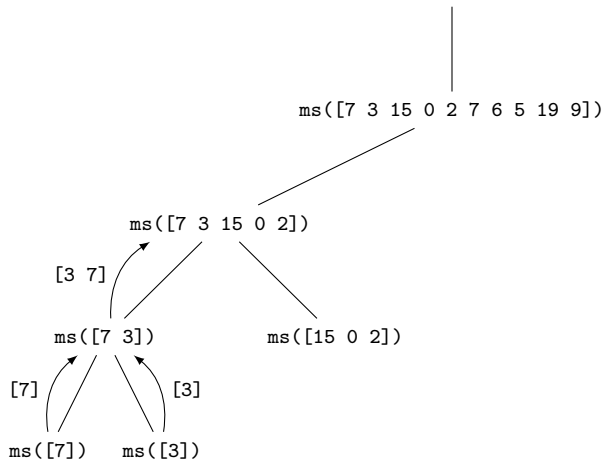
# Merge sort, anropsträd



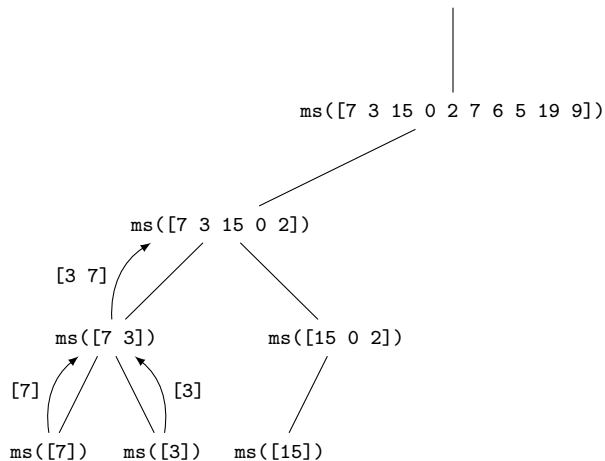
# Merge sort, anropsträd



# Merge sort, anropsträd

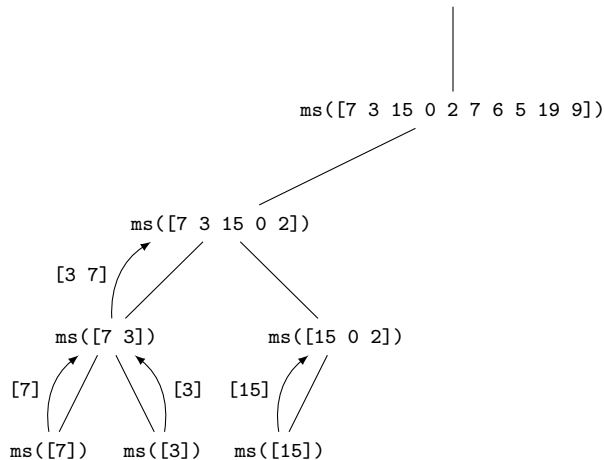


# Merge sort, anropsträd

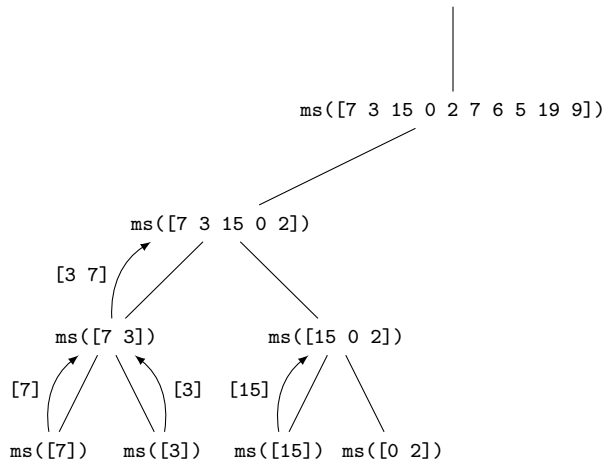




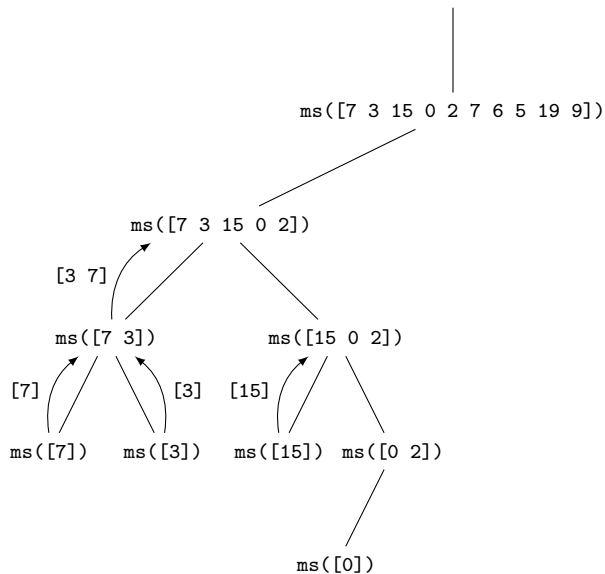
# Merge sort, anropsträd



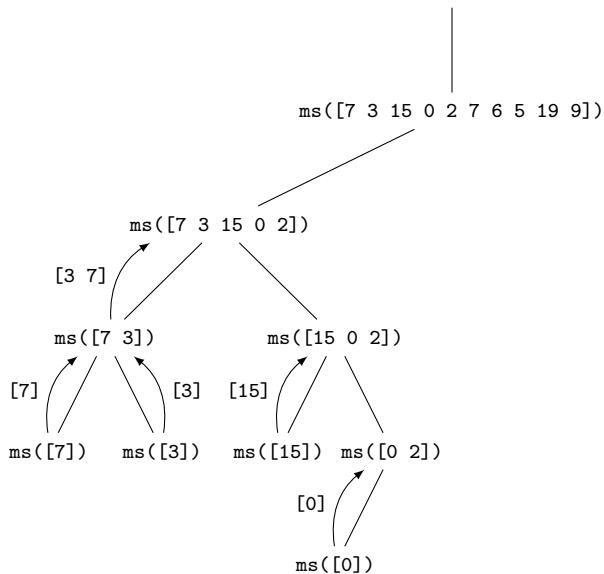
# Merge sort, anropsträd



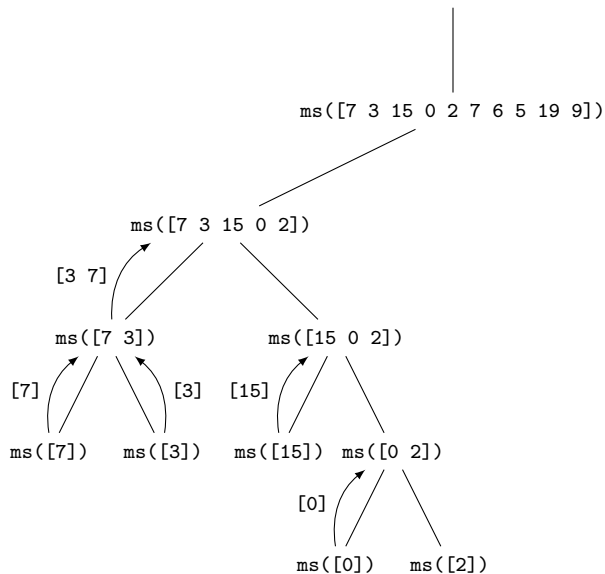
# Merge sort, anropsträd



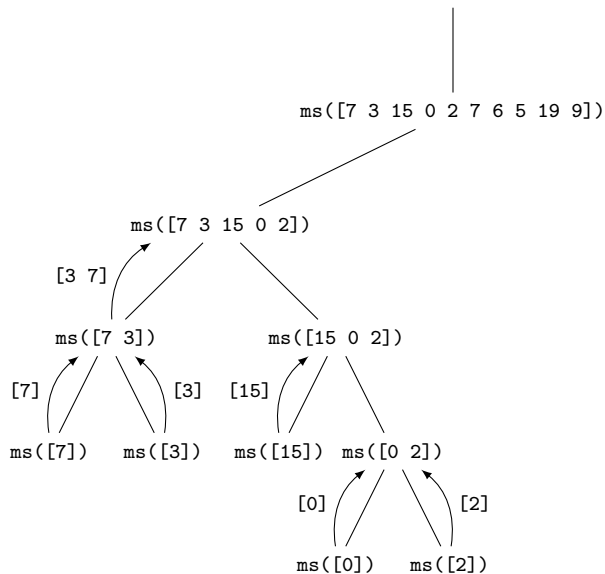
# Merge sort, anropsträd



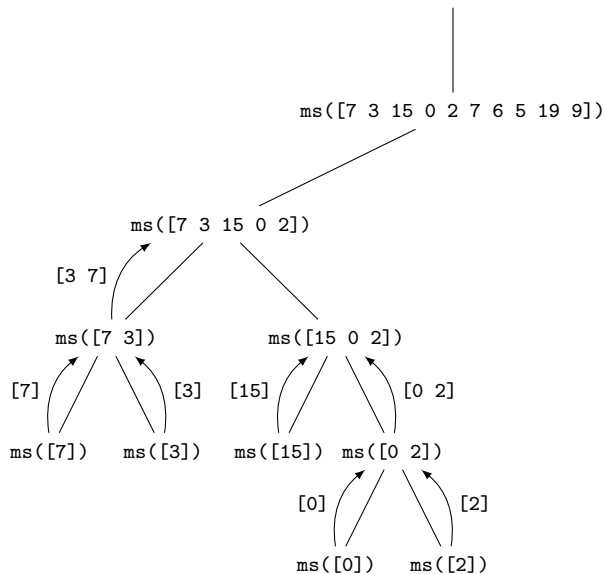
# Merge sort, anropsträd



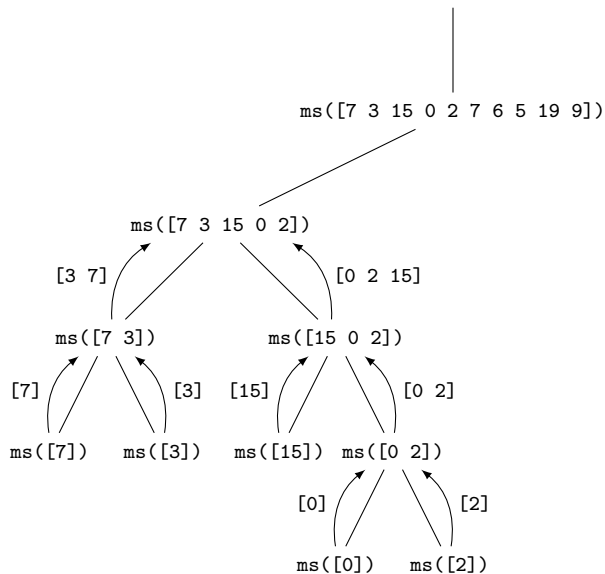
# Merge sort, anropsträd



# Merge sort, anropsträd

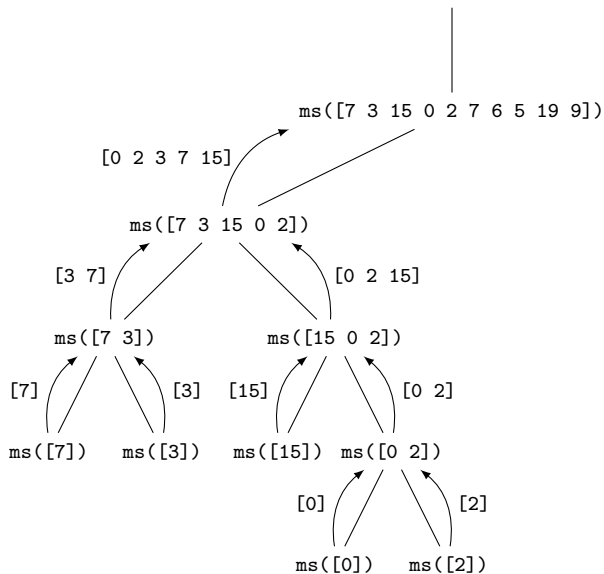


# Merge sort, anropsträd

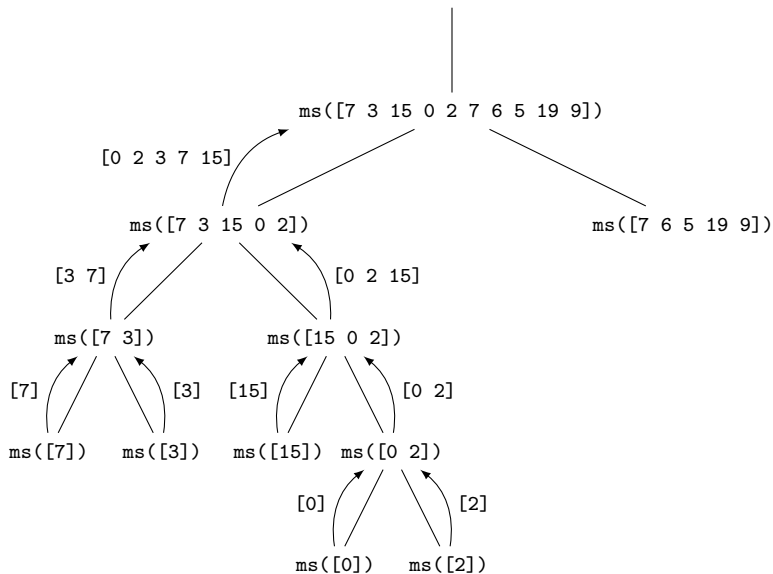




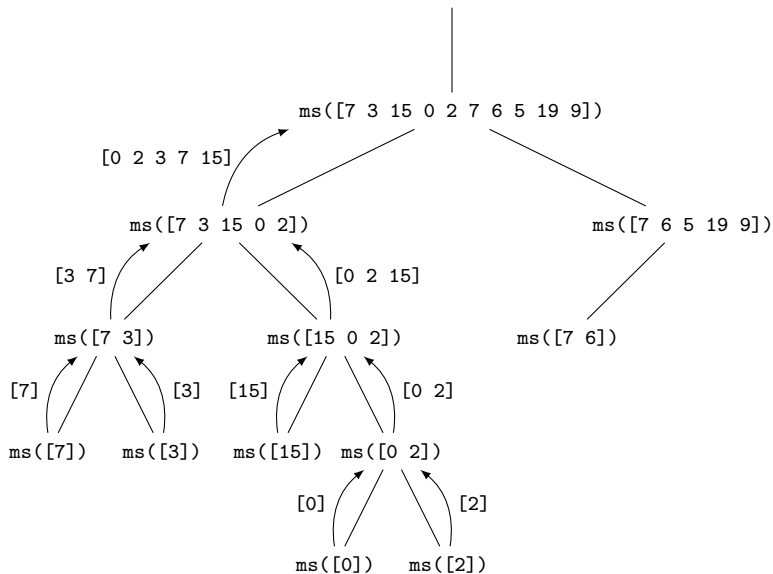
# Merge sort, anropsträd



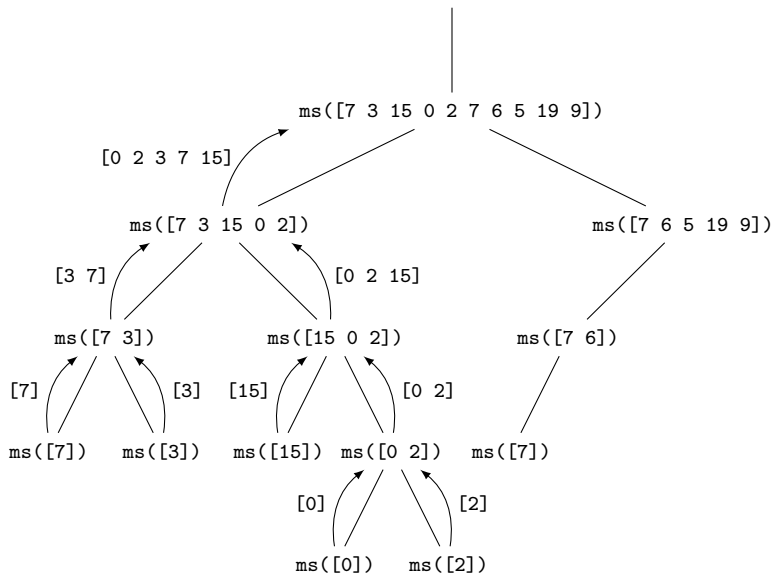
# Merge sort, anropsträd



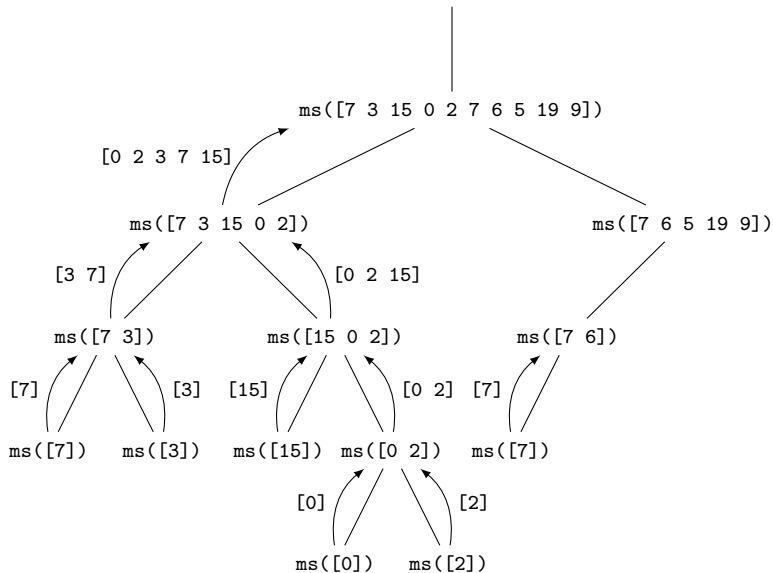
# Merge sort, anropsträd



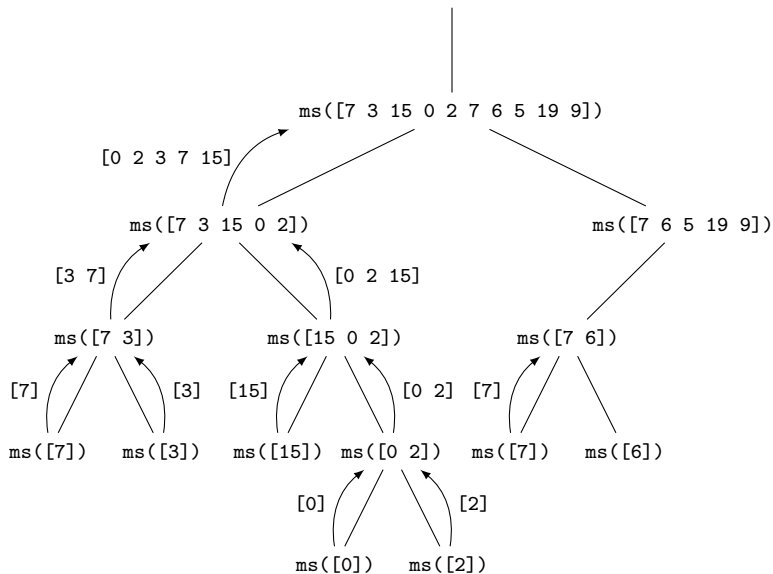
# Merge sort, anropsträd



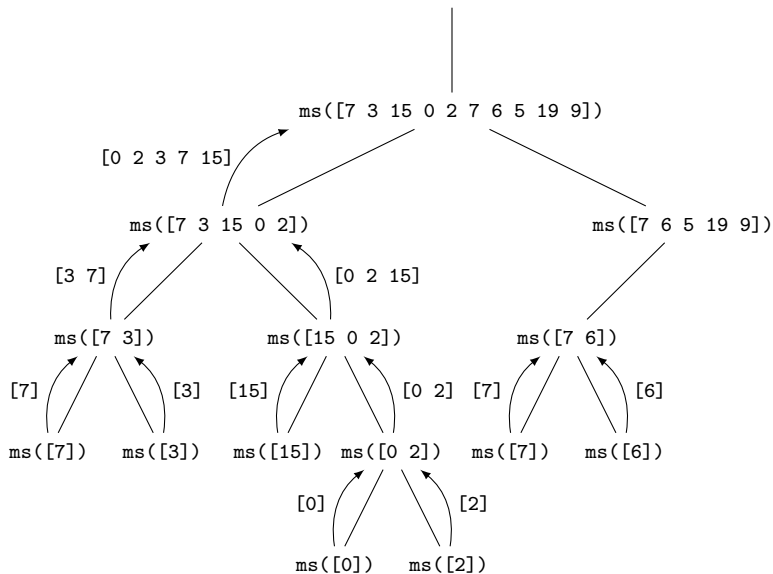
# Merge sort, anropsträd



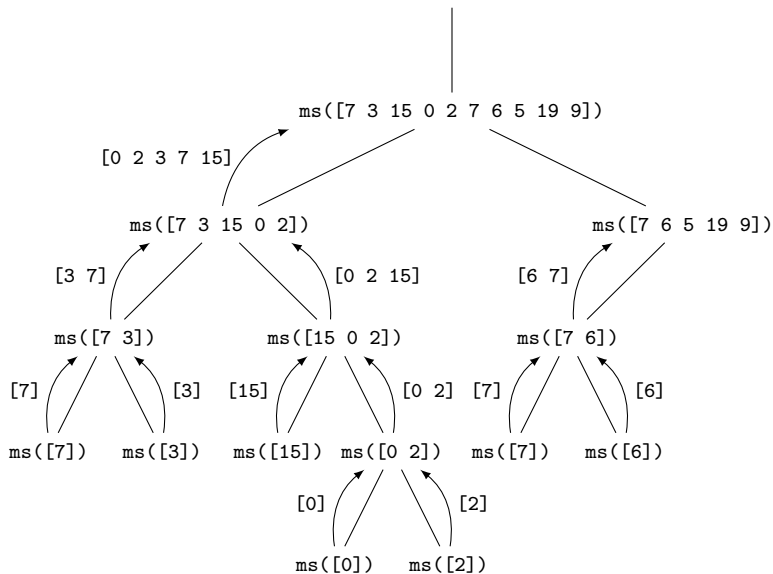
# Merge sort, anropsträd



# Merge sort, anropsträd

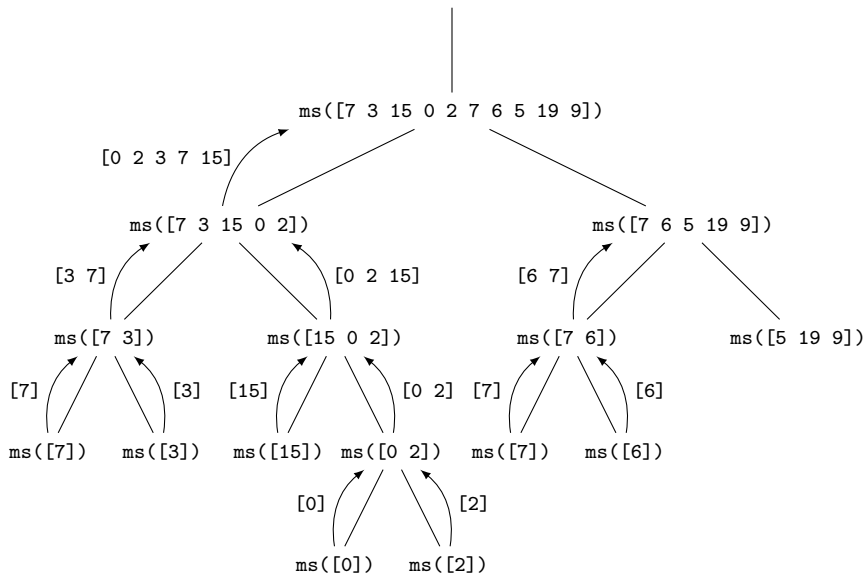


# Merge sort, anropsträd

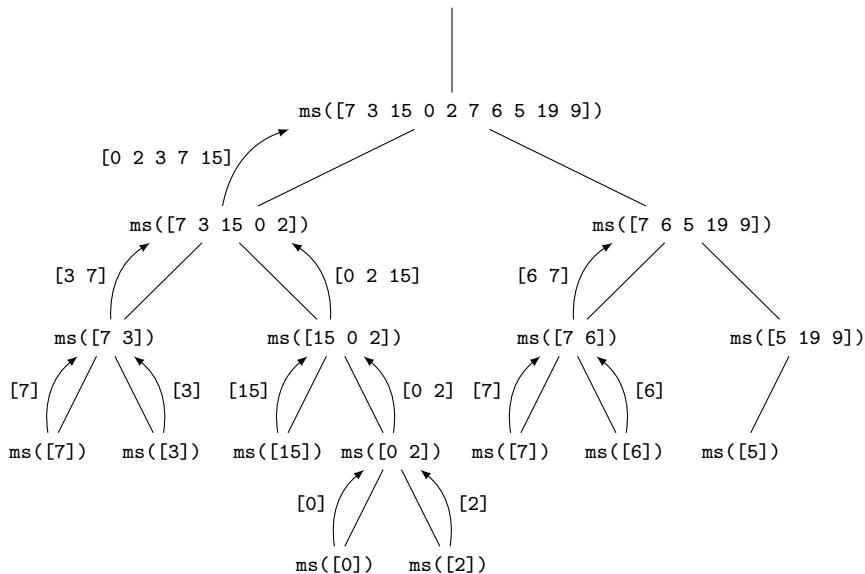




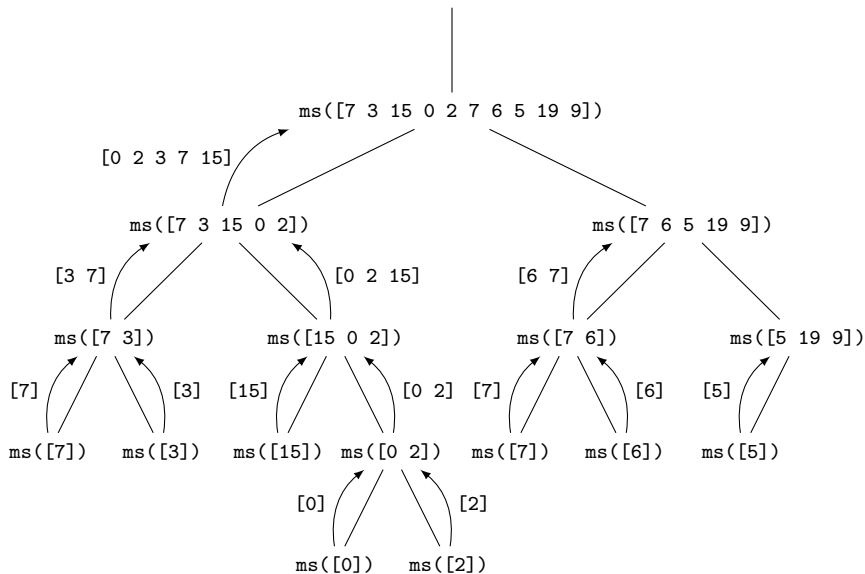
# Merge sort, anropsträd



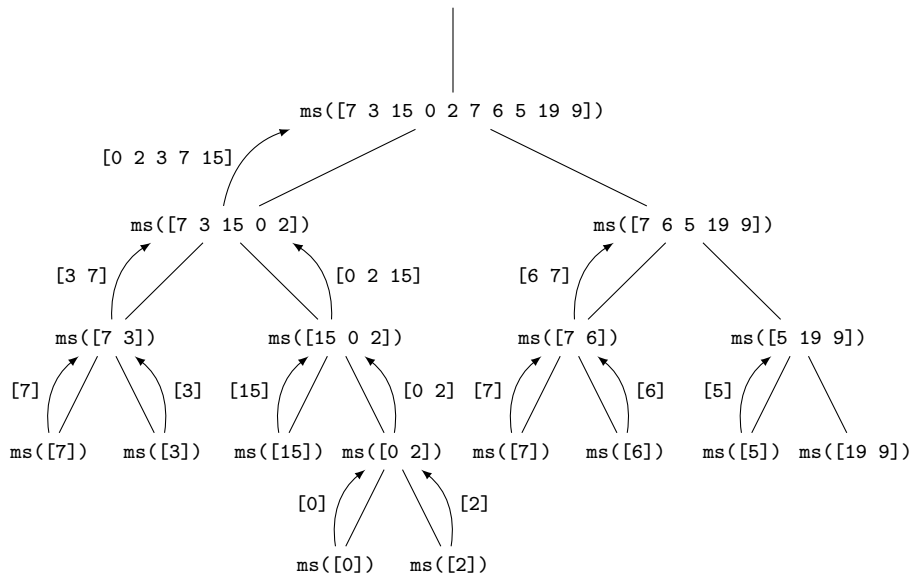
# Merge sort, anropsträd



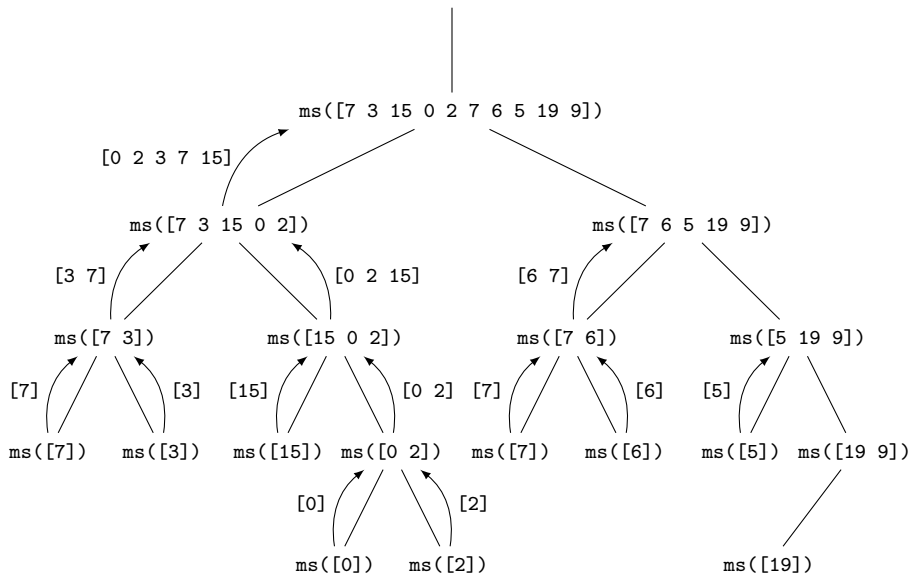
# Merge sort, anropsträd



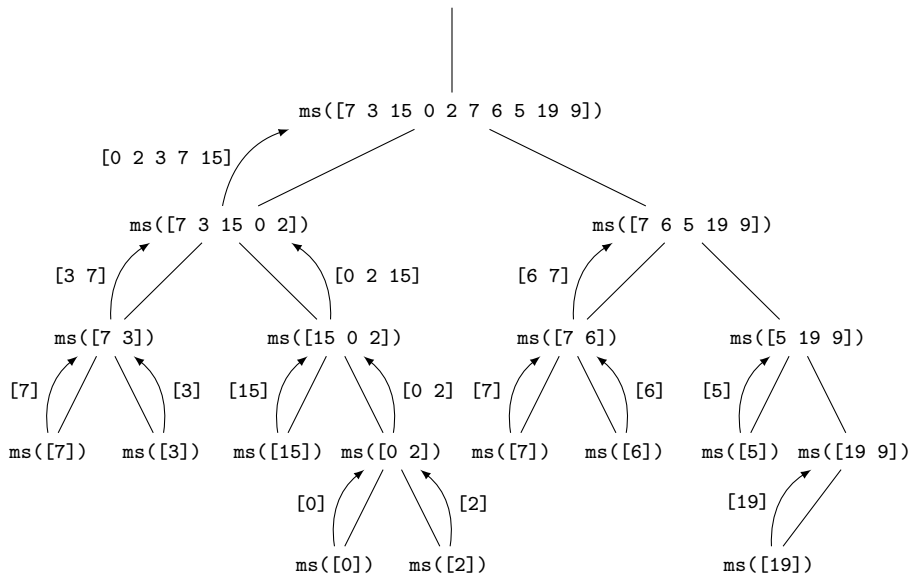
# Merge sort, anropsträd



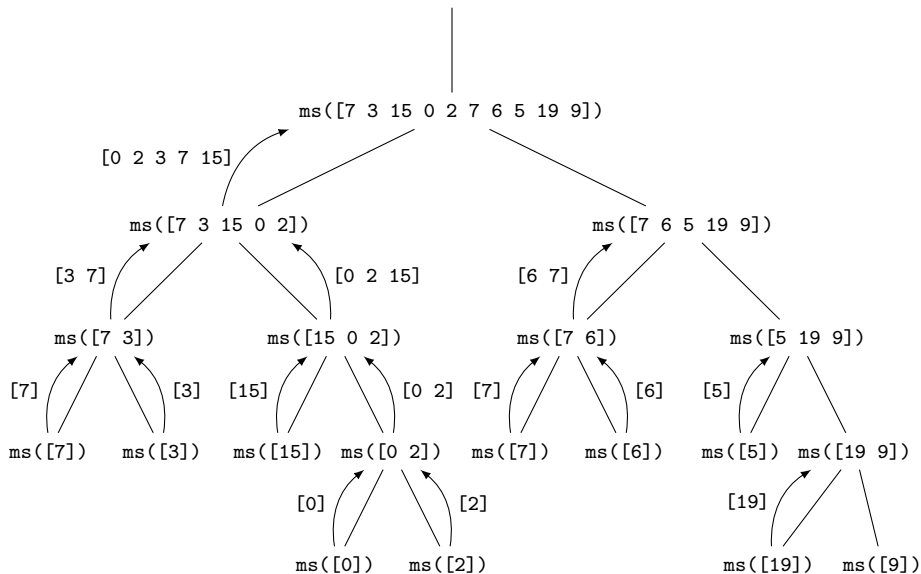
# Merge sort, anropsträd



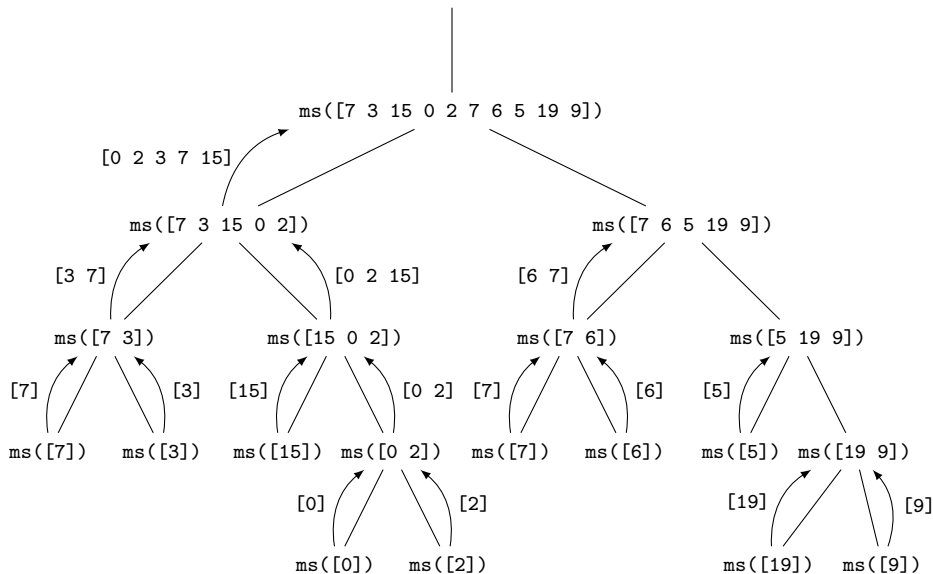
# Merge sort, anropsträd



# Merge sort, anropsträd

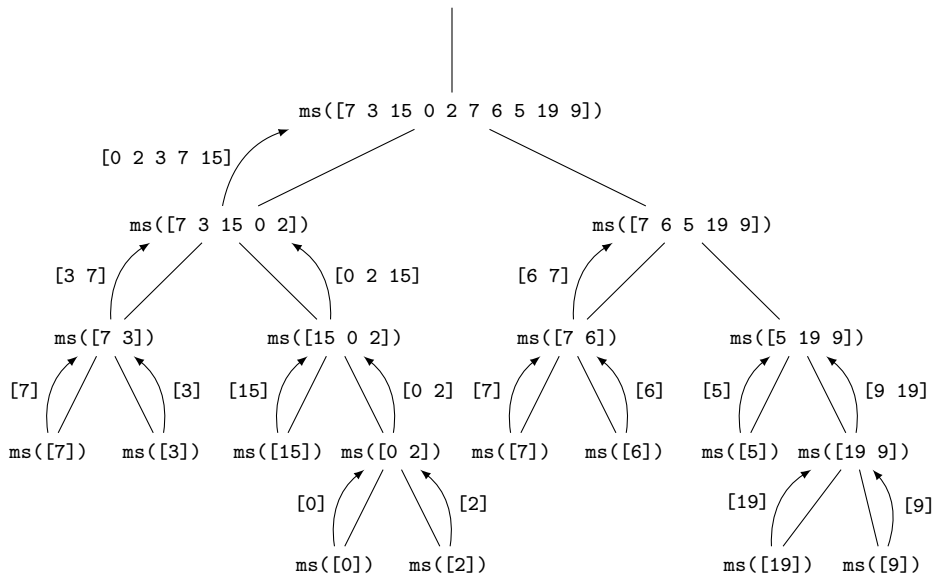


# Merge sort, anropsträd

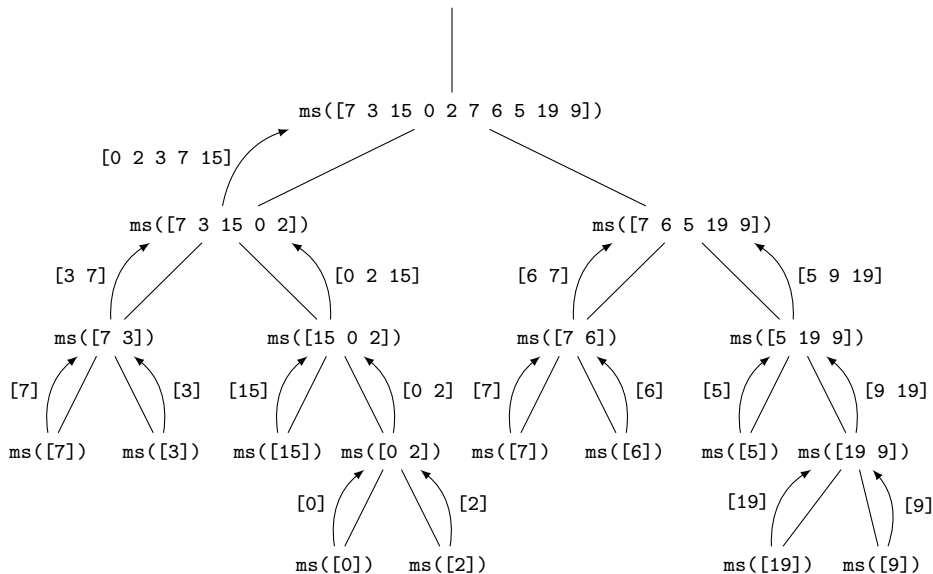




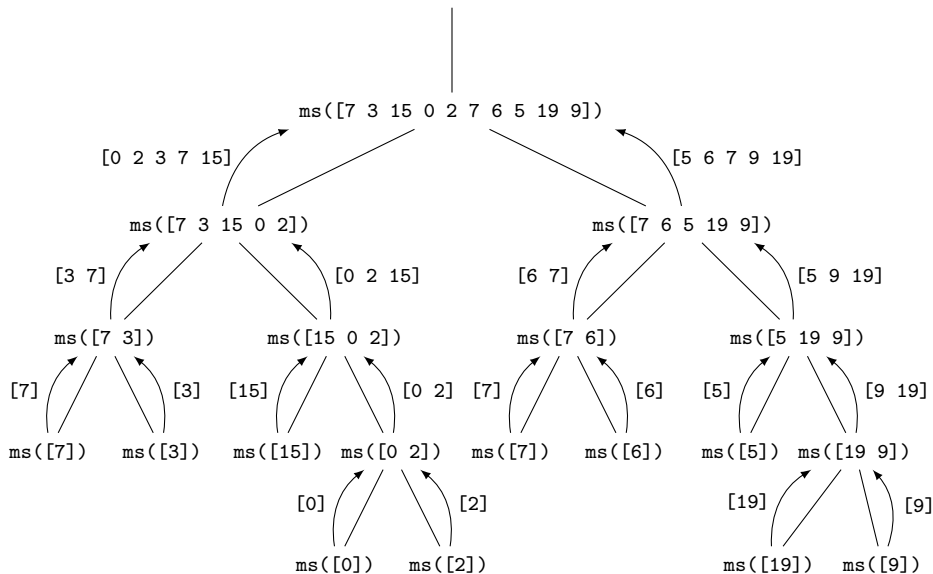
# Merge sort, anropsträd



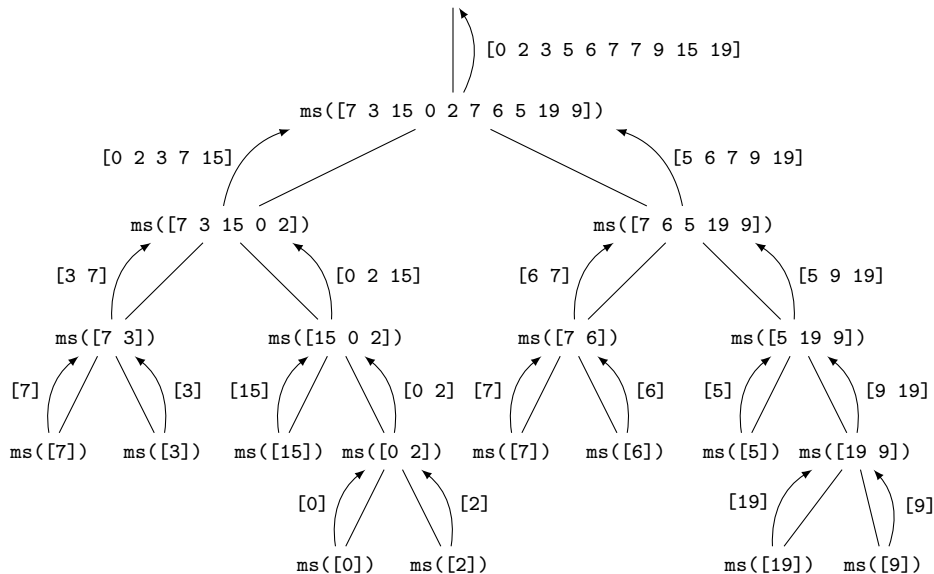
# Merge sort, anropsträd



# Merge sort, anropsträd



# Merge sort, anropsträd



## Merge sort, stabil?

- ▶ Är Merge sort **stabil**?

## Merge sort, stabil?

- ▶ Är *Merge sort* **stabil**?
  - ▶ Ja, om *Merge sort* lägger den första halvan i left
    - ▶ Lika element i A sorteras före B av *Merge*

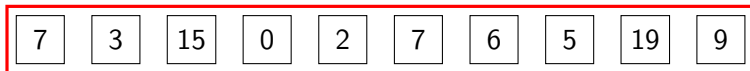
# *Quicksort*

# Quicksort

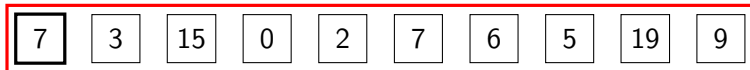
- ▶ Algoritm:
  - ▶ Välj ut ett pivåelement
  - ▶ Dela upp listan i tre delar: Less, Equal, Greater
  - ▶ Sortera Less och Greater rekursivt
  - ▶ Slå ihop Less+Equal+Greater



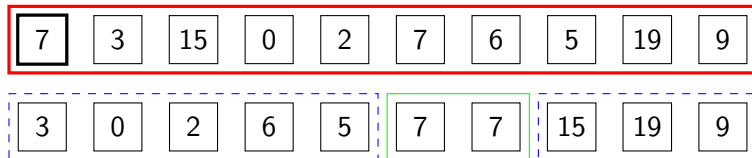
## Quicksort — Exempel



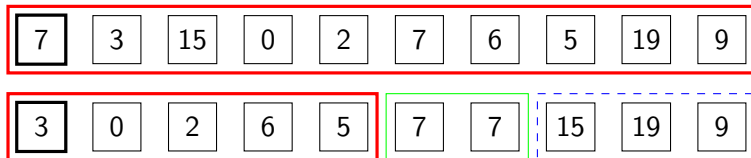
## Quicksort — Exempel



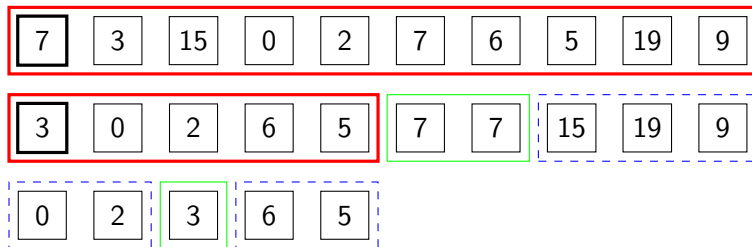
## Quicksort — Exempel



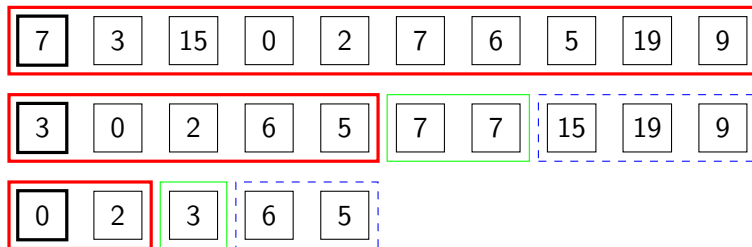
## Quicksort — Exempel



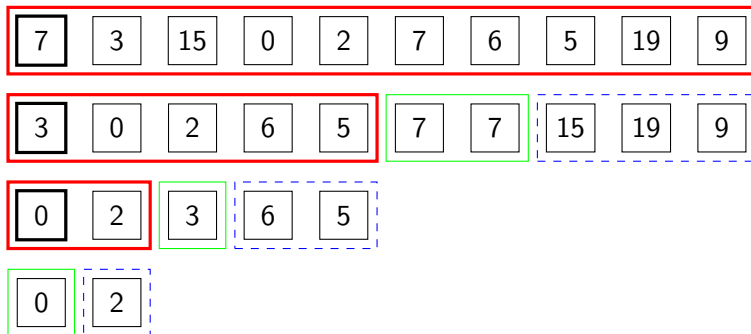
## Quicksort — Exempel



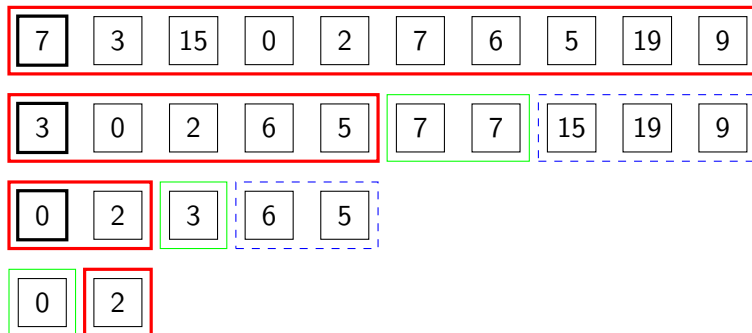
## Quicksort — Exempel



# Quicksort — Exempel

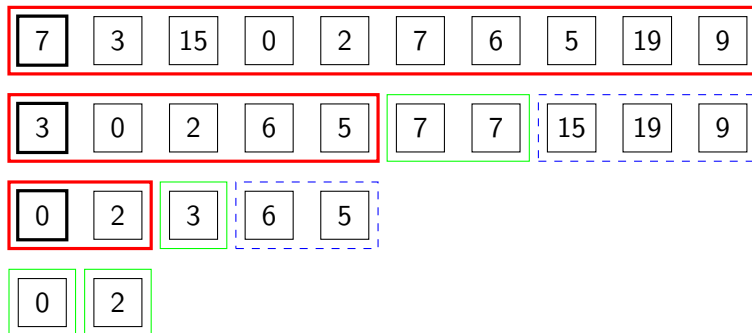


# Quicksort — Exempel

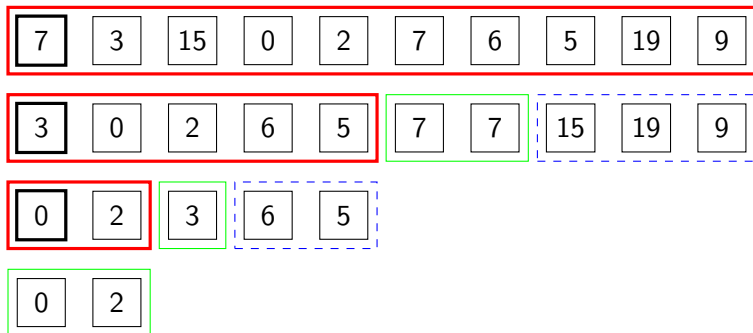




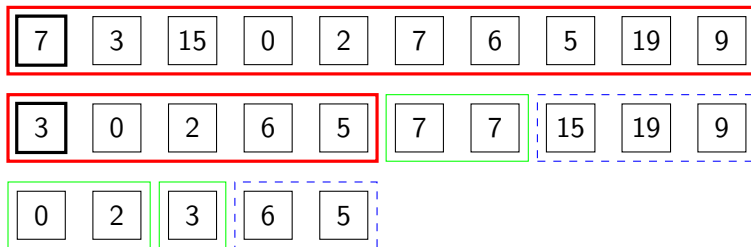
# Quicksort — Exempel



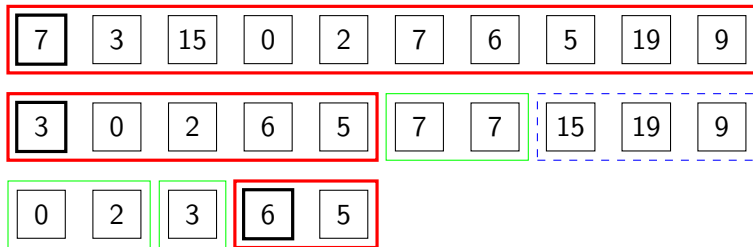
# Quicksort — Exempel



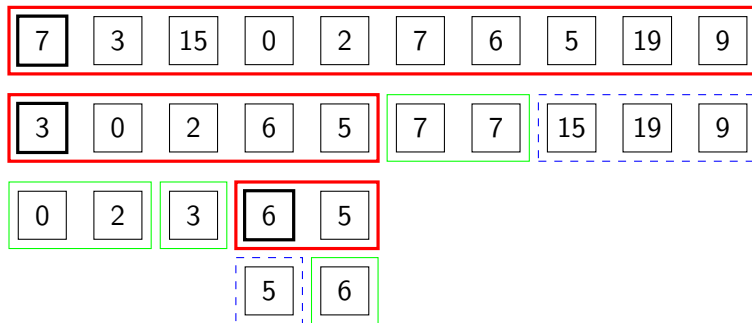
## Quicksort — Exempel



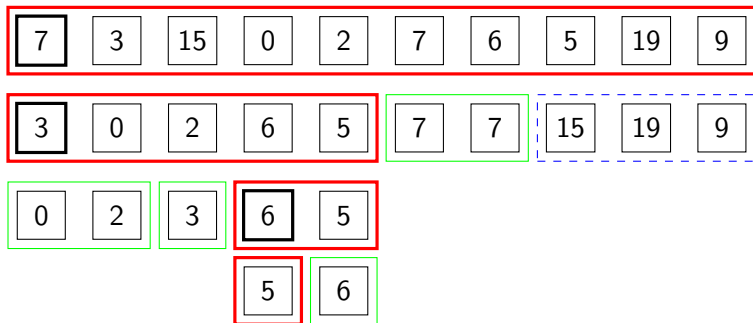
## Quicksort — Exempel



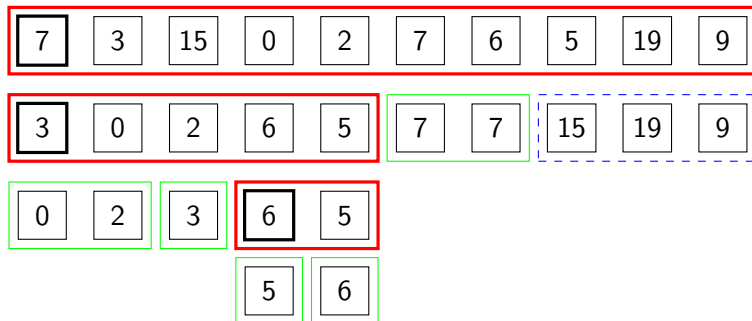
# Quicksort — Exempel



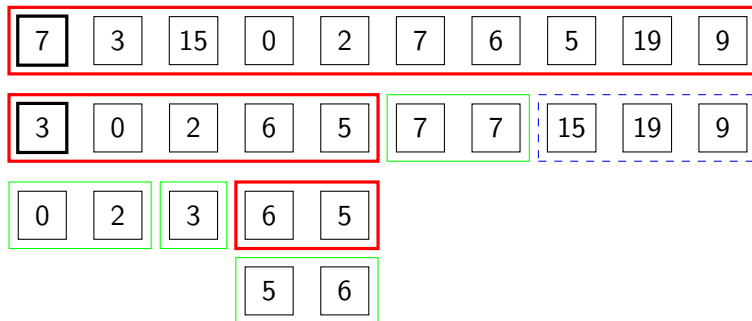
# Quicksort — Exempel



# Quicksort — Exempel

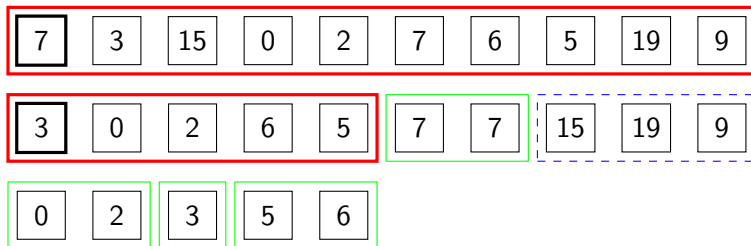


# Quicksort — Exempel

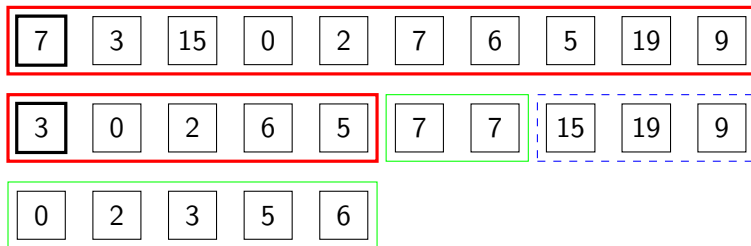




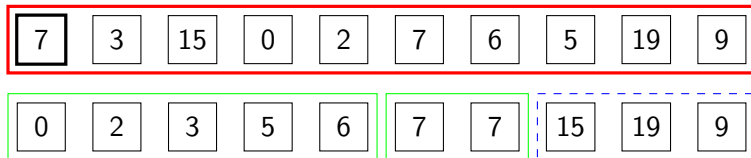
## Quicksort — Exempel



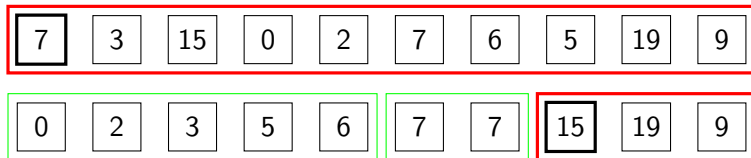
## Quicksort — Exempel



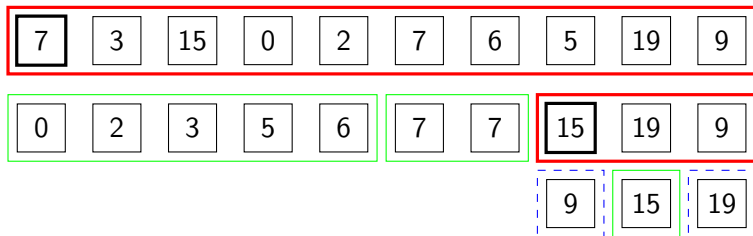
## Quicksort — Exempel



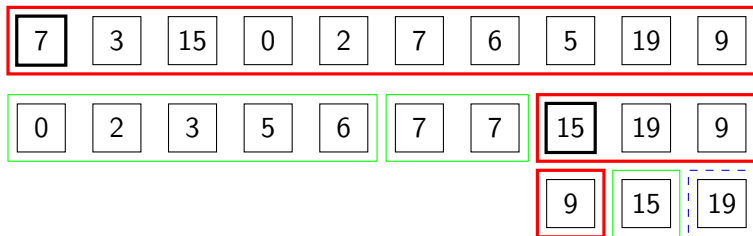
## Quicksort — Exempel



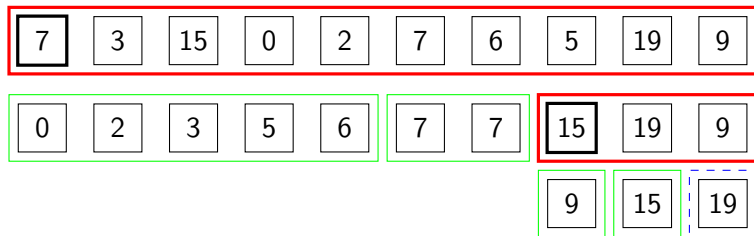
# Quicksort — Exempel



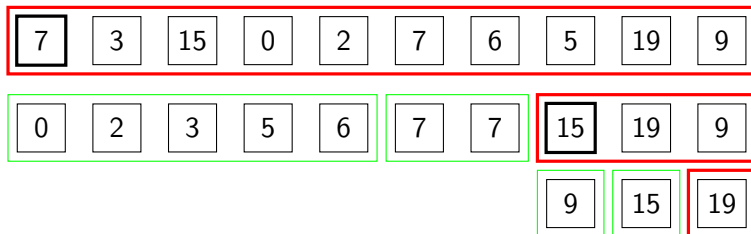
# Quicksort — Exempel



# Quicksort — Exempel

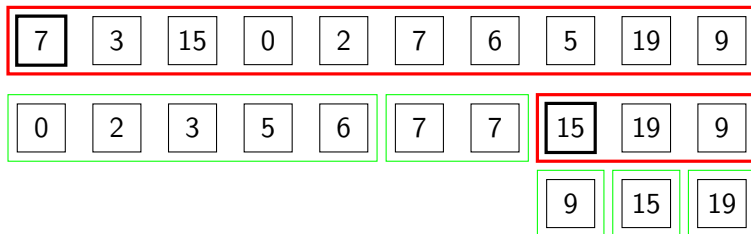


# Quicksort — Exempel

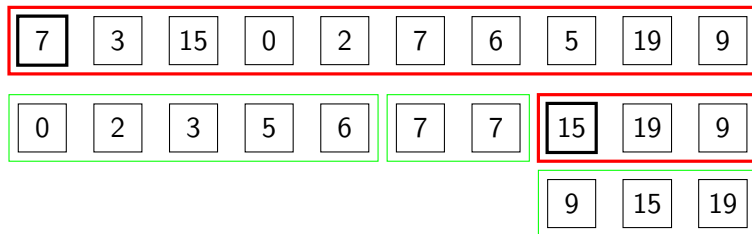




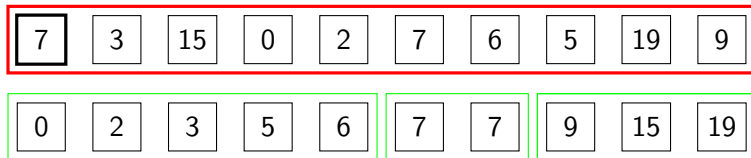
# Quicksort — Exempel



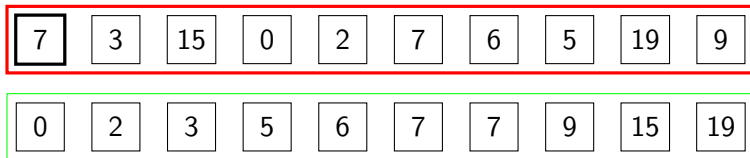
# Quicksort — Exempel



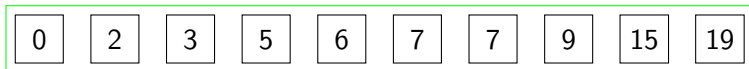
## Quicksort — Exempel



## Quicksort — Exempel



## Quicksort — Exempel



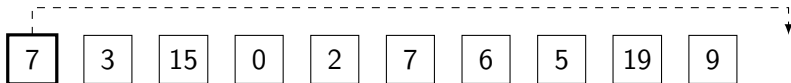
# Inplace Quicksort

- ▶ **Algorithm:**
  - ▶ Välj ett pivåelement (PE)
  - ▶ Traversera parallellt från båda hållen i S:
    - ▶ Gå framåt från början av S tills man hittar ett element som är  $\geq$  PE
    - ▶ Gå bakåt från slutet av S tills man hittar ett element som är  $<$  PE
    - ▶ Byt plats på dessa två element.
  - Upprepa till traverseringarna möts.
  - ▶ Stoppa in PE på rätt plats.
  - ▶ Rekursivt anrop för Less och Greater.

## *Inplace* Quicksort — exempel

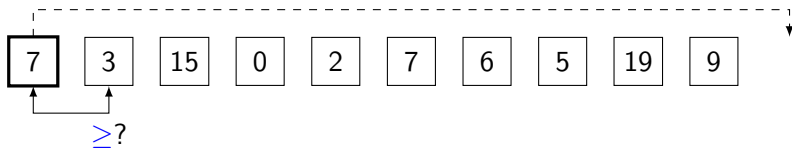


## Inplace Quicksort — exempel

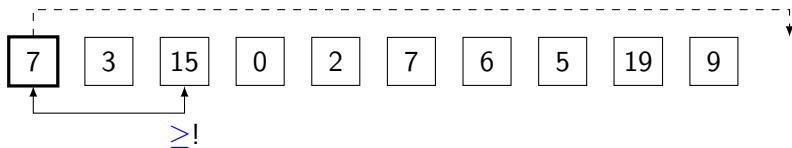




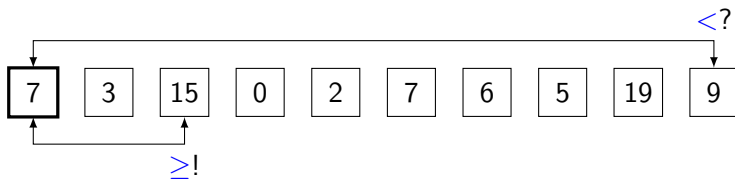
## Inplace Quicksort — exempel



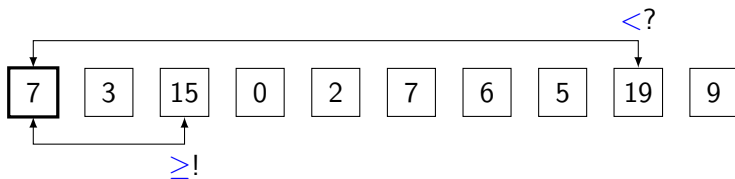
## Inplace Quicksort — exempel



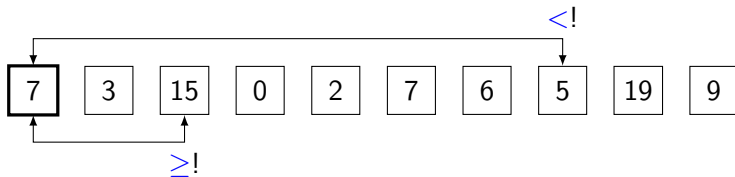
## Inplace Quicksort — exempel



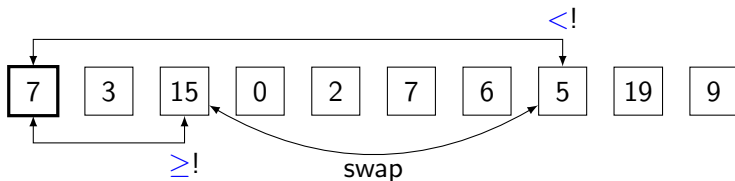
## Inplace Quicksort — exempel



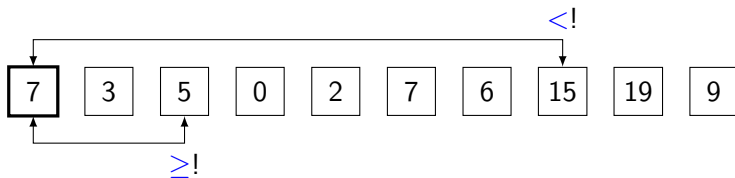
## Inplace Quicksort — exempel



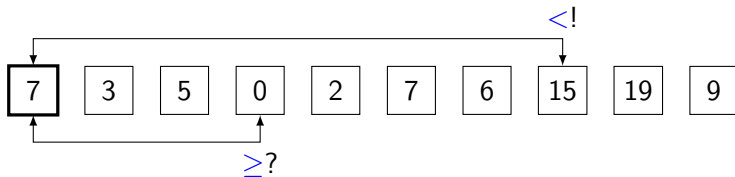
## Inplace Quicksort — exempel



## Inplace Quicksort — exempel

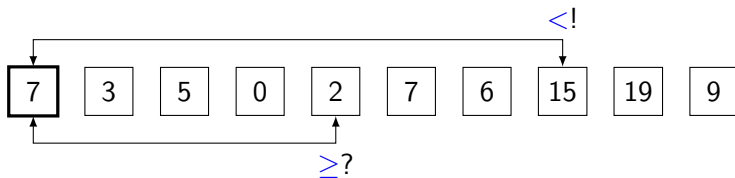


## Inplace Quicksort — exempel

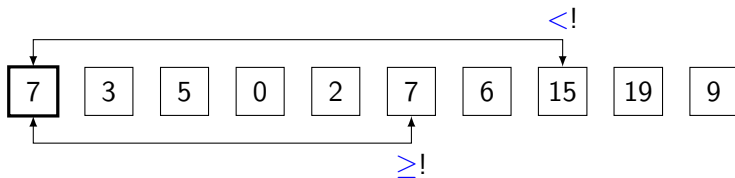




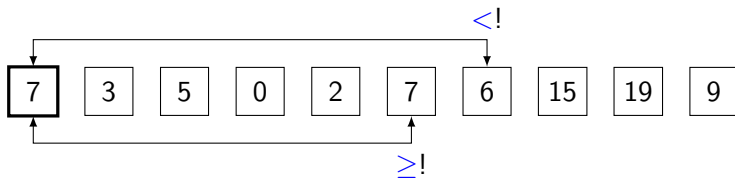
## Inplace Quicksort — exempel



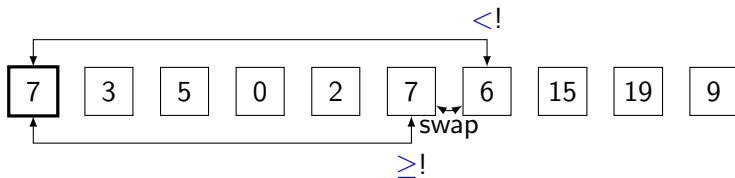
## Inplace Quicksort — exempel



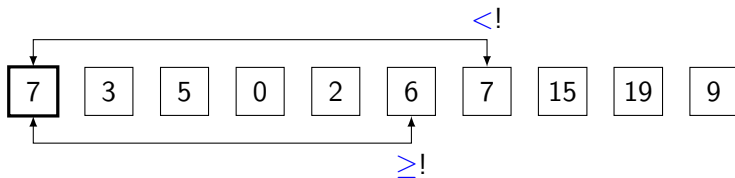
## Inplace Quicksort — exempel



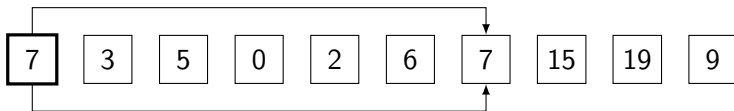
## Inplace Quicksort — exempel



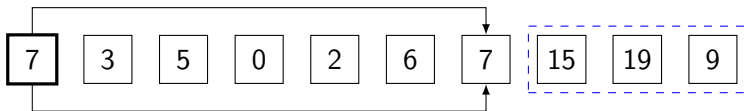
## Inplace Quicksort — exempel



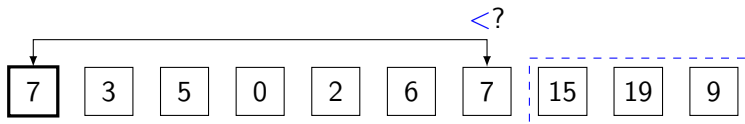
## Inplace Quicksort — exempel



## Inplace Quicksort — exempel

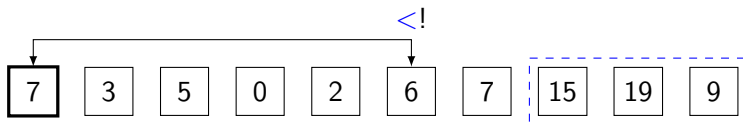


## Inplace Quicksort — exempel

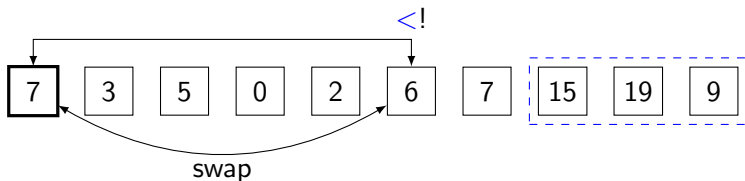




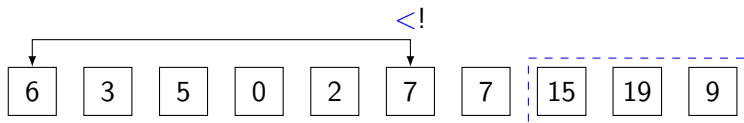
## Inplace Quicksort — exempel



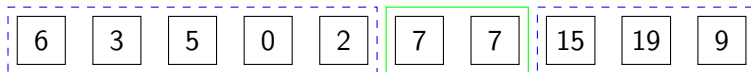
## Inplace Quicksort — exempel



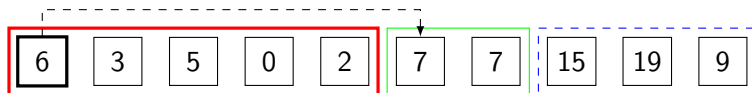
## Inplace Quicksort — exempel



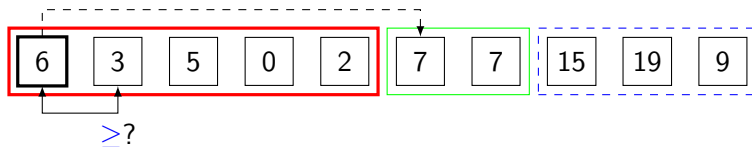
## *Inplace* Quicksort — exempel



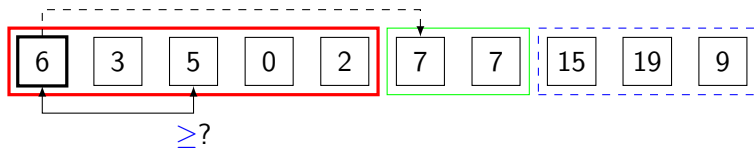
## Inplace Quicksort — exempel



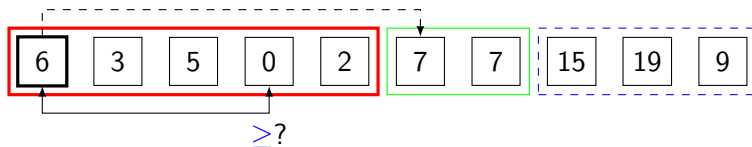
## Inplace Quicksort — exempel



## Inplace Quicksort — exempel

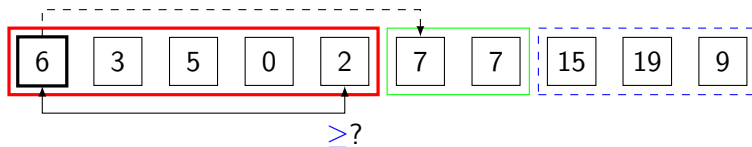


## Inplace Quicksort — exempel

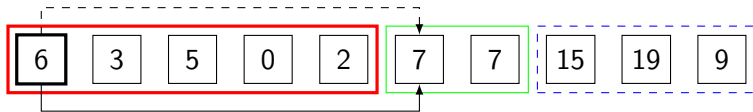




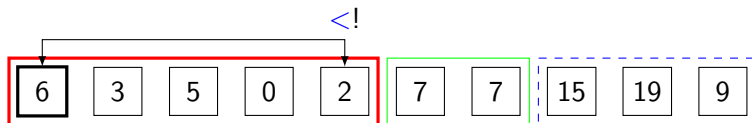
## Inplace Quicksort — exempel



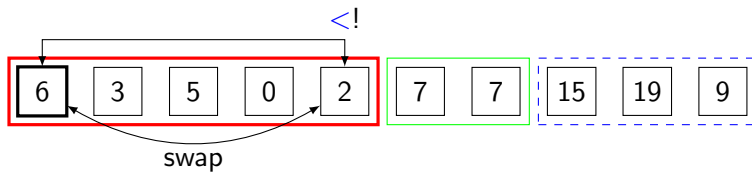
## Inplace Quicksort — exempel



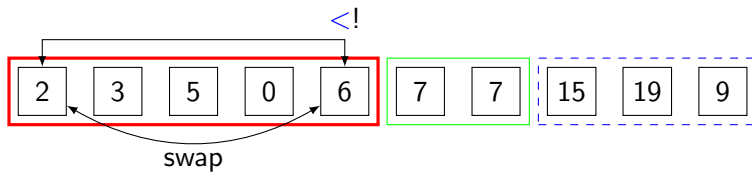
## Inplace Quicksort — exempel



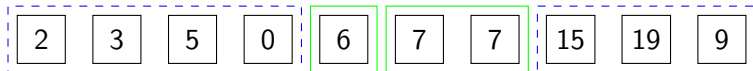
## Inplace Quicksort — exempel



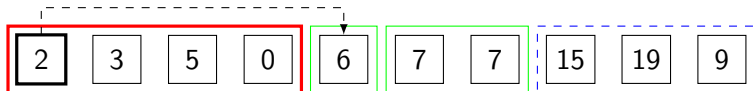
## Inplace Quicksort — exempel



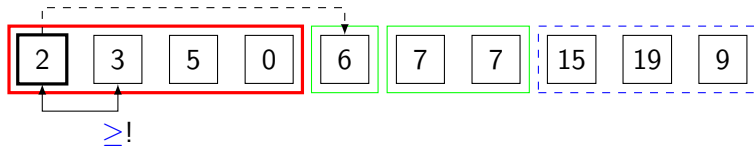
## *Inplace* Quicksort — exempel



## Inplace Quicksort — exempel

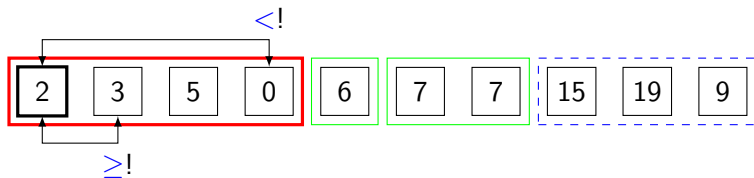


## Inplace Quicksort — exempel

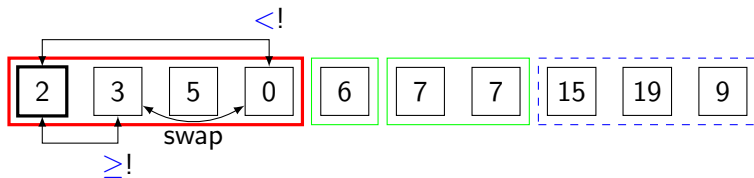




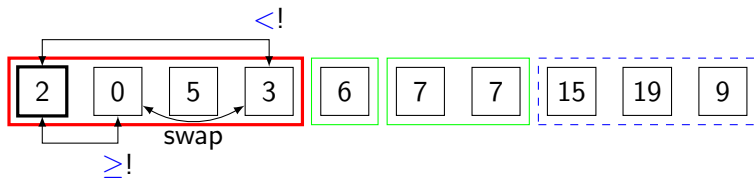
## Inplace Quicksort — exempel



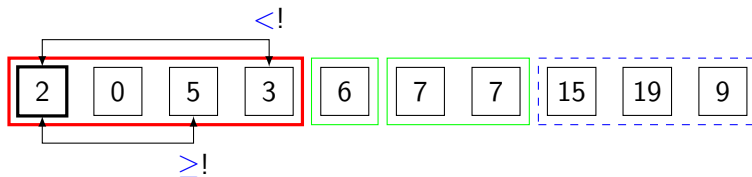
## Inplace Quicksort — exempel



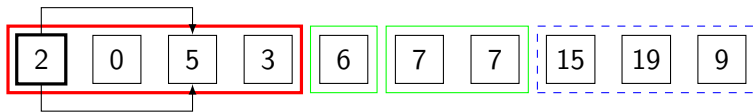
## Inplace Quicksort — exempel



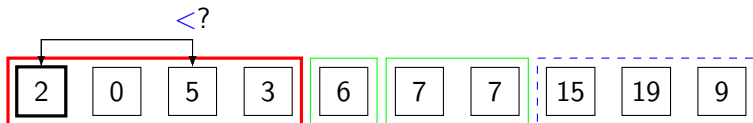
## Inplace Quicksort — exempel



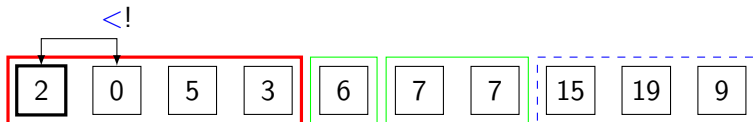
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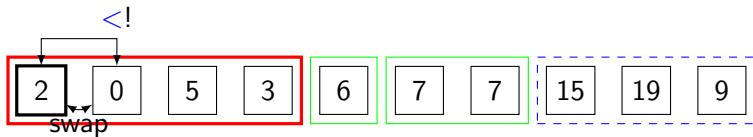
## Inplace Quicksort — exempel



## Inplace Quicksort — exempel

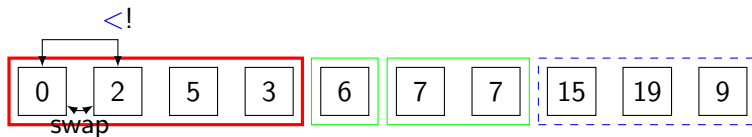


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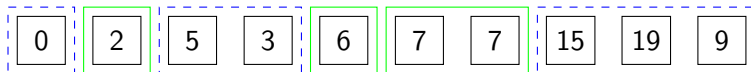




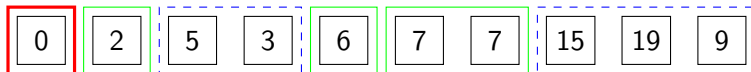
## Inplace Quicksort — exempel



## Inplace Quicksort — exempel



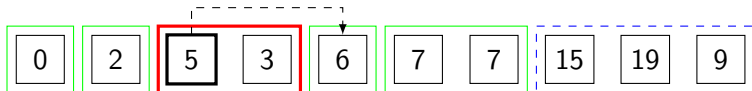
## Inplace Quicksort — exempel



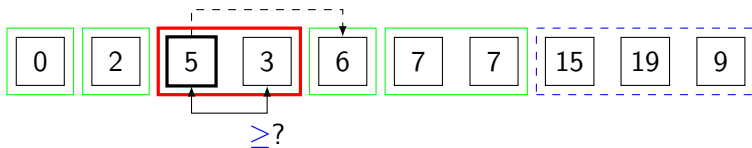
## *Inplace* Quicksort — exempel



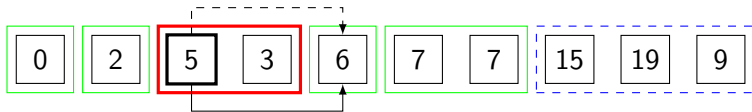
## Inplace Quicksort — exempel



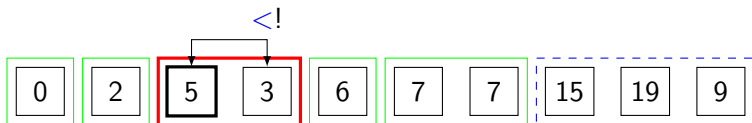
## Inplace Quicksort — exempel



## Inplace Quicksort — exempel

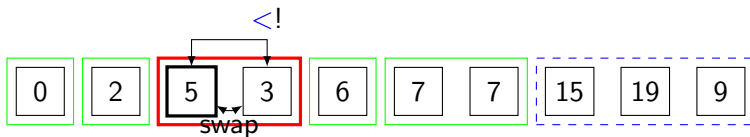


## Inplace Quicksort — exempel

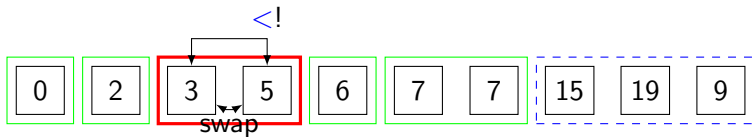




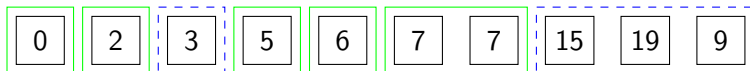
## Inplace Quicksort — exempel



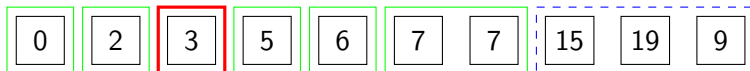
## Inplace Quicksort — exempel



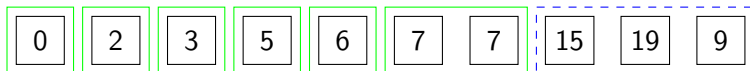
## Inplace Quicksort — exempel



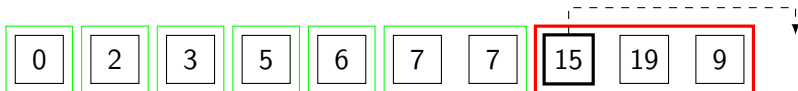
## Inplace Quicksort — exempel



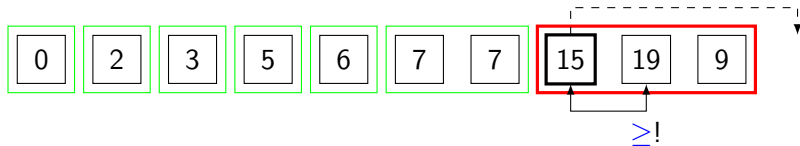
## *Inplace* Quicksort — exempel



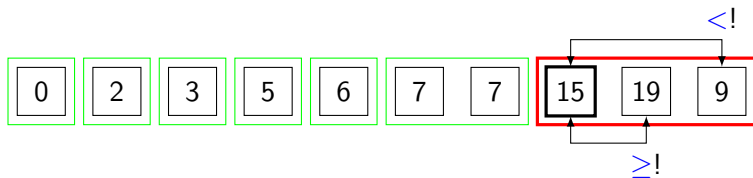
## Inplace Quicksort — exempel



## Inplace Quicksort — exempel

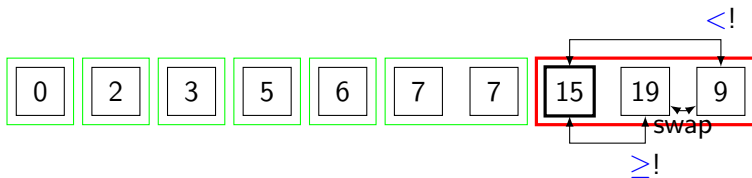


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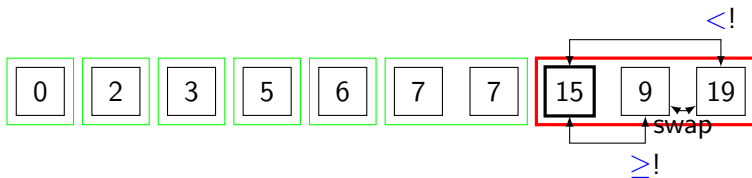




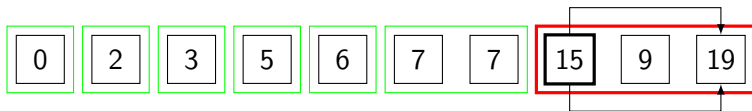
## Inplace Quicksort — exempel



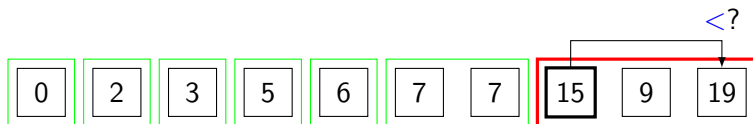
## Inplace Quicksort — exempel



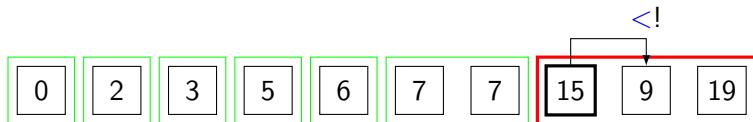
## Inplace Quicksort — exempel



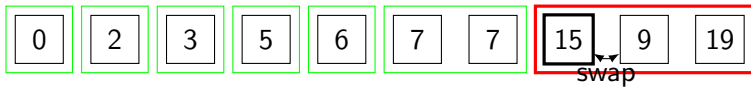
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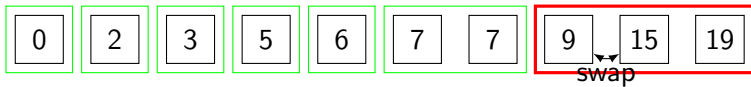
## Inplace Quicksort — exempel



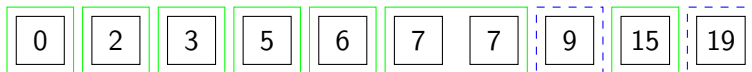
## Inplace Quicksort — exempel



## Inplace Quicksort — exempel

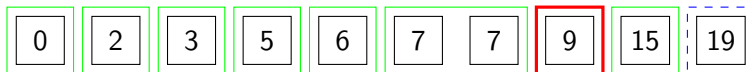


## *Inplace* Quicksort — exempel

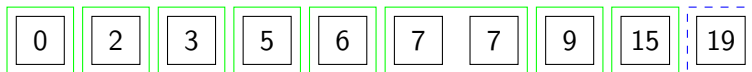




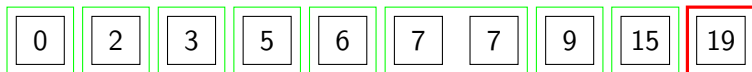
## *Inplace* Quicksort — exempel



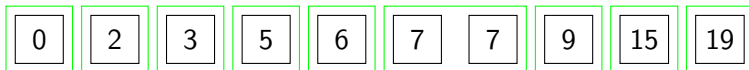
## *Inplace* Quicksort — exempel



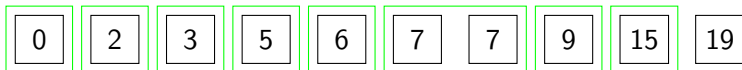
## *Inplace* Quicksort — exempel



## *Inplace* Quicksort — exempel

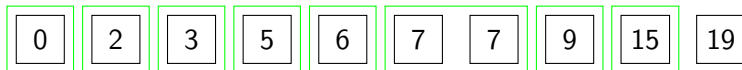


## Inplace Quicksort — exempel



► Stabil?

## Inplace Quicksort — exempel



- Stabil?
  - Nej!

# Quicksort — val av pivåelement

- ▶ Komplexiteten är  $O(n \log n)$  i bästa fallet
- ▶ Valet av pivåelement är **kritiskt**:
  - ▶ Vill ha ett pivåelement som har ett mitten-värde
  - ▶ Vid sned fördelning får man i praktiken insticks-/urvalssortering med  $O(n^2)$
- ▶ Alternativ för att få en enkel tilldelning:
  - ▶ Välj första/sista, slumpmässigt
  - ▶ Medel/median mellan några stycken
    - ▶ Median mellan första/mitten/sista
  - ▶ Det största av de två första som skiljer sig åt

# Hur snabbt kan man sortera?

- ▶ Det snabbaste vi kan sortera med jämförelsebaserad algoritmer är  $O(n \log n)$
- ▶ Den enda information vi använder är resultatet av en jämförelse av två nyckelvärden
  - ▶  $a < b$  ?
- ▶ Nyckelsortering använder mer information om nyckeltypen
  - ▶ Kan göras snabbare än  $O(n \log n)$



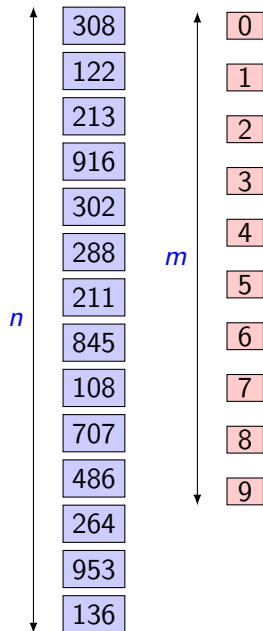
# Hur sortera snabbare än $O(n \log n)$ ?

- ▶ Vi måste **veta mer** om **nycklarna** vi vill sortera efter:
  - ▶ Det måste gå att **avbilda nycklarna på heltal**  $V$
  - ▶ Vi behöver känna till ett **minsta** och **största** värde
    - ▶  $V_{\min} \dots V_{\max}$  för  $V$
- ▶ Komplexiteten blir  $O(n + m)$  där  $n$  är antalet element och  $m$  är  $V_{\max} - V_{\min}$

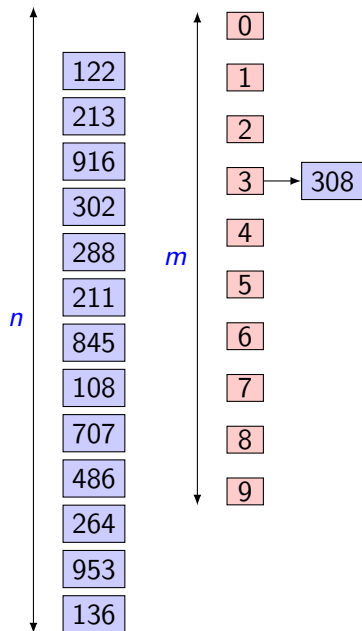
# Facksortering (*Bucket sort*)

- ▶ Notera! På Wikipedia kallas denna algoritm *Pidgeonhole sort*
- 1. Skapa ett fack för varje nyckelvärde i intervallet  $V_{\min} \dots V_{\max}$
- 2. Gå igenom sekvensen  $S$ 
  - ▶ Lägg elementen i det fack dess nyckelvärde motsvarar
  - ▶ Komplexitet  $O(n)$
- 3. Länka samman facken till en sekvens i ordning  $V_{\min} \dots V_{\max}$ 
  - ▶ Komplexitet  $O(m)$
- ▶ Kan göras stabil

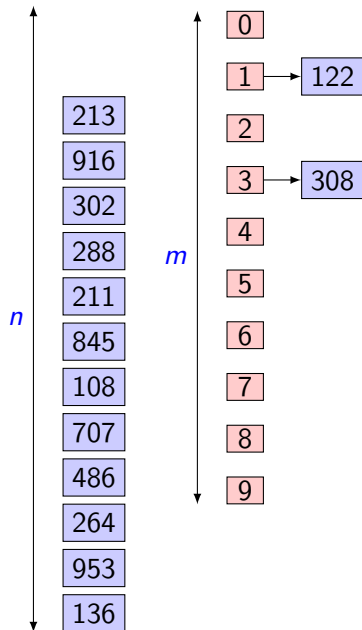
# Facksortering, nyckelvärdet är $v/100$



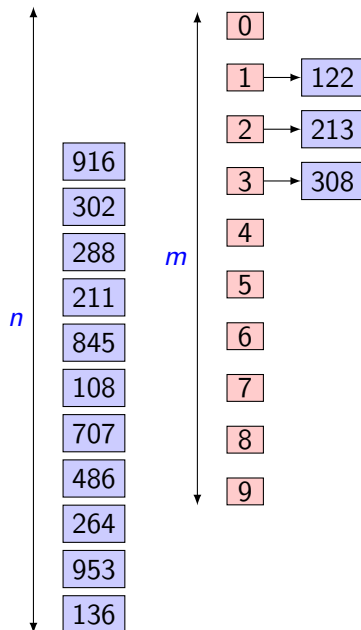
# Facksortering, nyckelvärdet är $v/100$



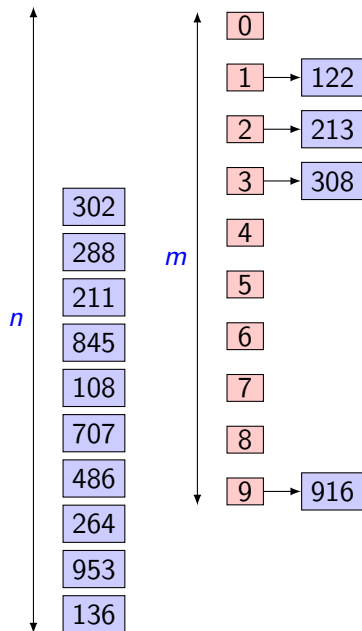
# Facksortering, nyckelvärdet är $v/100$



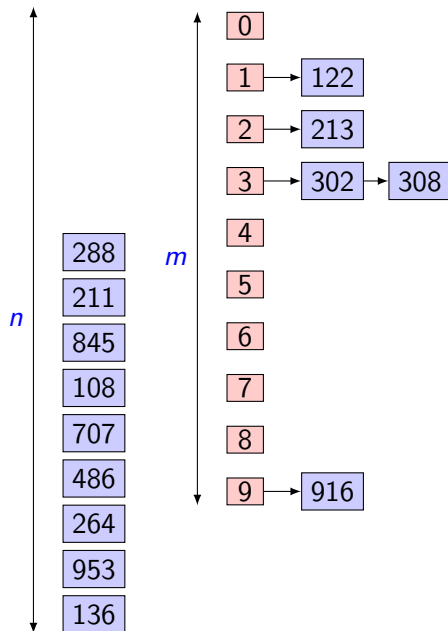
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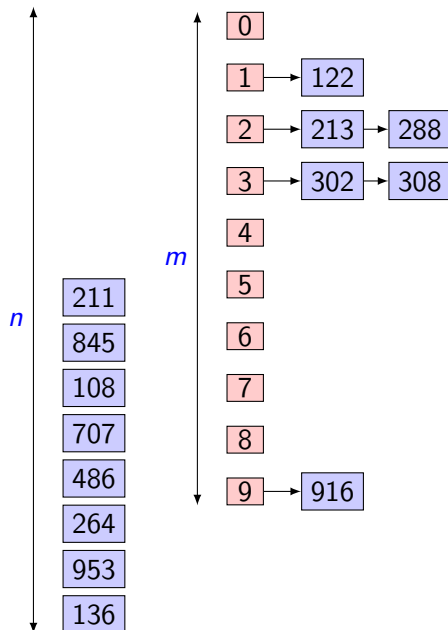


# Facksortering, nyckelvärdet är $v/100$

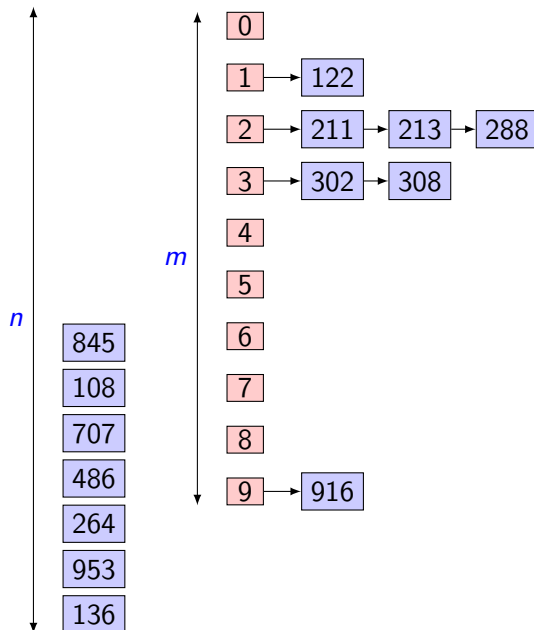




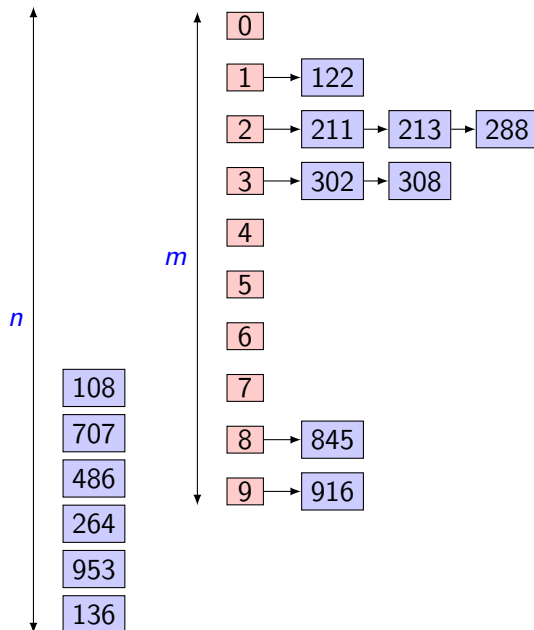
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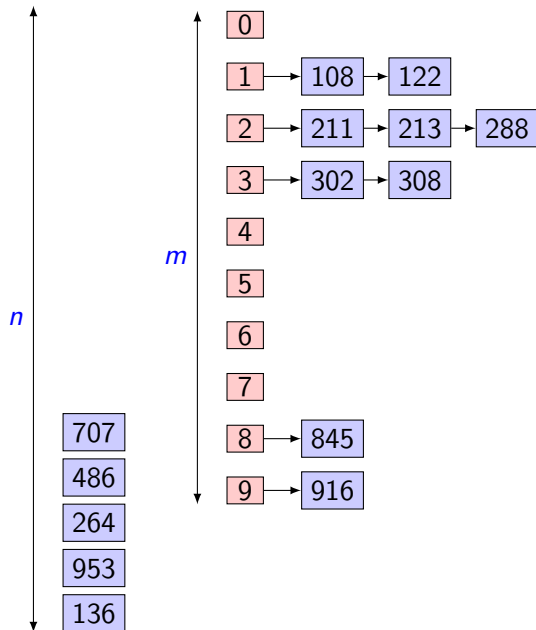
# Facksortering, nyckelvärdet är $v/100$



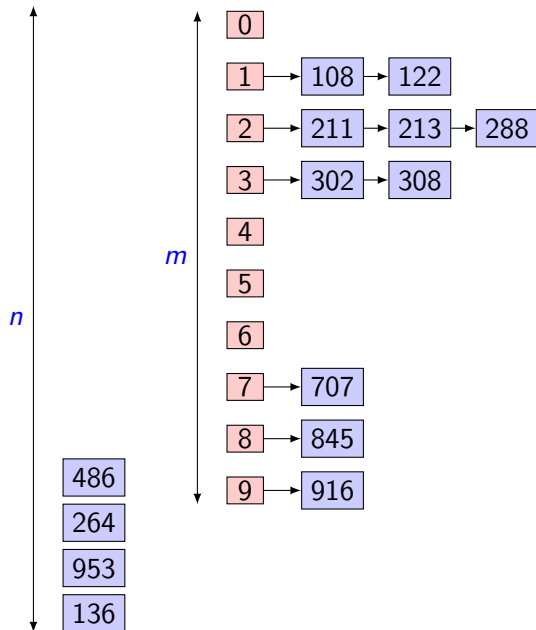
# Facksortering, nyckelvärdet är $v/100$



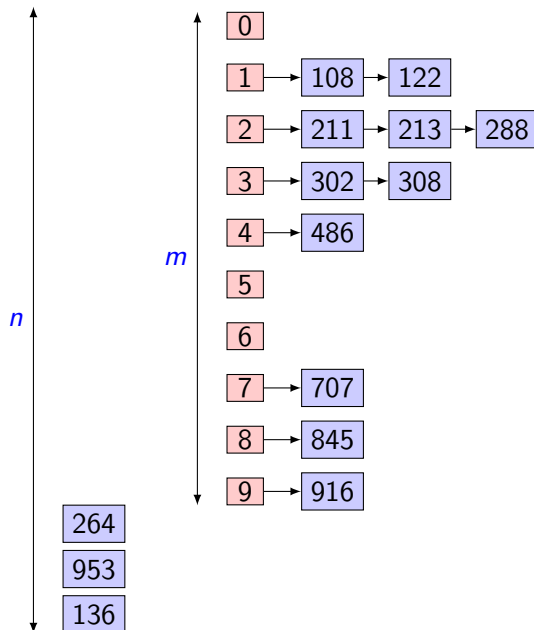
# Facksortering, nyckelvärdet är $v/100$



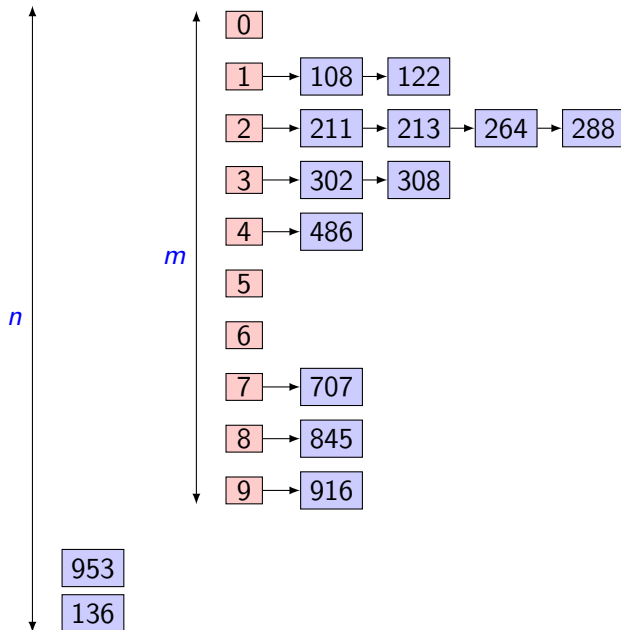
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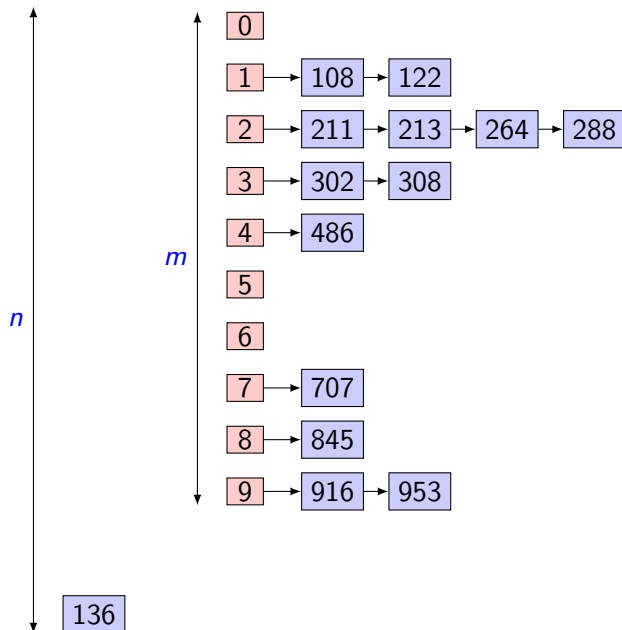
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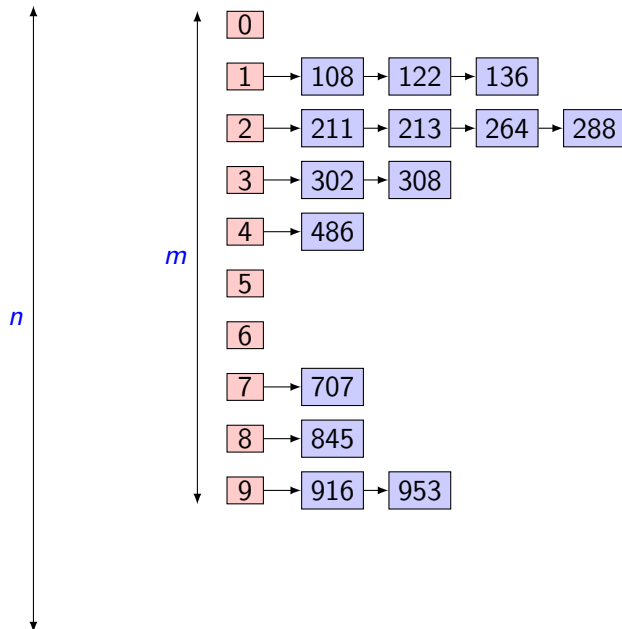


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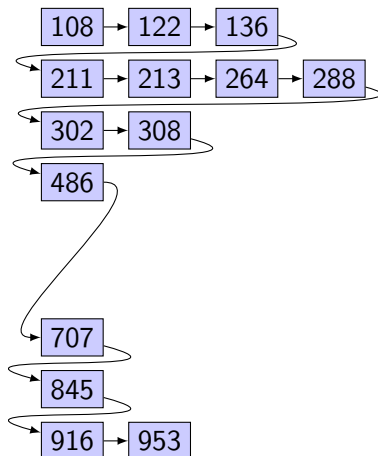




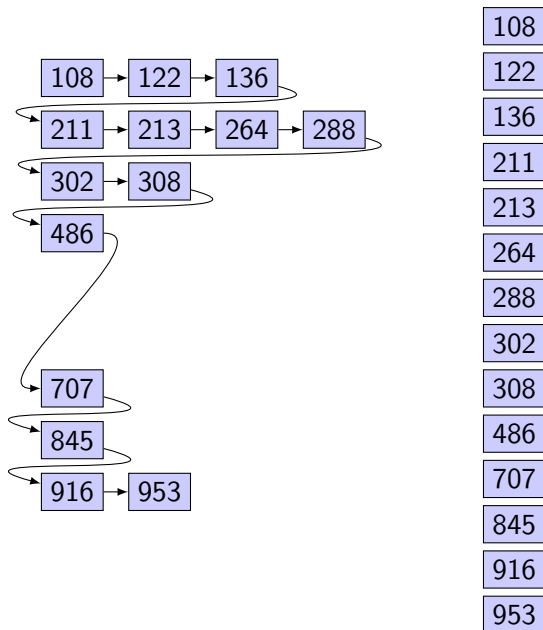
# Facksortering, nyckelvärdet är $v/100$



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# Sammanfattning

Algoritm	Tidskomplexitet		Stabil?	Minnesbehov
	bästa	värsta		
Insertion Sort	$O(n^2)$	$O(n^2)$	Ja	$O(1)$
Selection Sort	$O(n)$	$O(n^2)$	Nej <sup>1</sup>	$O(1)$
Bubble Sort	$O(n)$	$O(n^2)$	Ja	$O(1)$
Merge Sort	$O(n \log n)$	$O(n \log n)$	Ja	$O(n)$
Quicksort	$O(n \log n)$	$O(n^2)$	Nej	$O(\log n)$ <sup>2</sup>
Heapsort	$O(n \log n)$	$O(n \log n)$	Nej	$O(1)$
Bucket sort	$O(n + m)$	$O(n + m)$	Ja	$O(m)$

► Fundera på:

- Hur hanterar algoritmen en redan sorterad lista?
- Hur hanterar algoritmen en motsatt sorterad lista?

---

<sup>1</sup>Ja om  $O(n)$  extra minne.

<sup>2</sup> $O(n)$  i värsta-fallet

# Länkar

- ▶ Wikipedia — Sorting algorithms
  - ▶ [https://en.wikipedia.org/wiki/Sorting\\_algorithm](https://en.wikipedia.org/wiki/Sorting_algorithm)
- ▶ Animering av sorteringsalgorithmer
  - ▶ <http://www.sorting-algorithms.com>
- ▶ Dansat *merge sort*
  - ▶ [https://www.youtube.com/watch?v=XaqR3G\\_NVoo](https://www.youtube.com/watch?v=XaqR3G_NVoo)

# Sökning

# Vad betyder "lika med"?

- ▶ Vid sökning och sortering definierar man ofta en extern **likhetsfunktion** (*match function*) som avgör om två elementvärden *a* och *b* är lika
- ▶ Det går att låta funktionen ta två argument och returnera True of argumenten anses lika, annars False
- ▶ Om man i stället definierar en **jämförelsefunktion** (*compare function*) och begär att den ska returnera ett heltal
  - <0 om *a* kommer **före** *b* i sorteringsordningen
  - 0 om *a* och *b* är **lika** enligt sorteringsordningen
  - >0 om *a* kommer **efter** *b* i sorteringsordningenså blir algoritmerna ännu flexiblare på sorterade data
- ▶ En jämförelsefunktion gör det möjligt att använda samma söknings- och sorteringsalgoritmer på olika data

# Jämförelsefunktioner, exempel

```
Algorithm Compare-int(a, b: Int)
// Input: Two integers a and b.
// Output: An integer
//          < 0 if and only if a < b
//          = 0 if and only if a = b, and
//          > 0 if and only if a > b
return a - b
```

```
Algorithm Compare-strings(a, b: String)
// Input: Two strings a and b.
// Output: An integer
//          < 0 if and only if a comes before b
//          = 0 if and only if a is equal to b
//          > 0 if and only if a comes after b
if Isless-than(a,b) then
    return -1
else if Isequal(a,b) then
    return 0
else
    return +1
```



# Jämförelsefunktioner, exempel

```
Algorithm Compare-record(a, b: Record)
// Input: Two records a and b with fields lastname, firstname,
//        and age
// Output: An integer <0, =0, or >0 to indicate whether a is
//         considered to be before, equal to, or after b,
//         respectively. The ordering is decided by last name,
//         then first name. In case of a tie, a younger age is
//         given precedence.
cmp ← Compare-strings(a.lastname, b.lastname)
if cmp = 0 then
    cmp ← Compare-strings(a.firstname, b.firstname)
if cmp = 0 then
    cmp ← Compare-int(a.age, b.age)
return cmp
```

# Linjär sökning

- ▶ Starta från början och sök tills elementet hittats eller sekvensen är slut
- ▶ Komplexitet:
  - ▶ Om elementet finns: I medel, gå igenom halva listan  $O(n)$
  - ▶ Om elementet saknas: Gå igenom hela listan  $O(n)$
- ▶ Om listan är sorterad:
  - ▶ Om elementet finns: I medel, gå igenom halva listan  $O(n)$
  - ▶ Om elementet saknas: I medel, gå igenom halva lista  $O(n)$

# Algorithm linjär sökning, osorterad lista (jämför seek)

```
Algorithm Linsearch(l: List, v: Value, Value-isequal: Function)
// Input: An unsorted list, a search value, and a equality
//         function. The Value-isequal function should accept
//         two element values and return True if the values
//         are considered equal.
// Output: (True, pos), where pos is the position for the first
//         match, or (False, None) if no match is found.
p ← First(l)
while not Isend(p, l) do
    if Value-isequal(v, Inspect(p, l)) then
        return (True, p)
    p ← Next(p, l)
return (False, None)
```

# Algorithm linjär sökning, sorterad lista

```
Algorithm Linsearch-sorted(l: List, v: Value, Compare: Function)
// Input: A sorted list, a search value, and a compare function.
// Output: (True, pos), where pos is the position for the first
//          match, or (False, None) if no match is found.
p ← First(l)
while not Isend(l) do
    // Compare
    c ← Compare(v, Inspect(p, l))
    if c = 0 then
        // Found it
        return (True, p)
    else if c > 0 then
        // Past where it could be, give up
        return (False, None)
    else
        // Still before where it could be, continue
        p ← Next(p, l)
return (False, None)
```

# Binär sökning

- ▶ Om sekvensen har index (t.ex. i ett fält) kan man söka **binärt**
- ▶ Successiv **halvering** av sökintervallet
- ▶ Sök efter elementet med värde  $v$ :
  - ▶ Jämför med elementet  $x[m]$  närmast mitten av intervallet
    - ▶ Om likhet — klart!
    - ▶ Om  $v$  kommer **före**  $x[m]$  i sorteringsordningen, fortsätt sökningen rekursivt i delintervallet till **vänster** om  $m$
    - ▶ Om  $v$  kommer **efter**  $x[m]$  i sorteringsordningen, fortsätt sökningen rekursivt i delintervallet till **höger** om  $m$
- ▶ Vi får värsta-falls och medelkomplexitet  $O(\log n)$

# Algorithm iterativ binär sökning

```
Algorithm Binssearch-iter(a: Array, v: Value, Compare: Function)
left ← Low(a)
right ← High(a)
while left ≤ right do
    // Check in the middle
    mid ← (left + right)/2
    // Compare
    c ← Compare(v, Inspect-value(a, mid))
    if c = 0 then
        // Found it
        return (True, mid)
    else if c < 0 then
        // Look left
        right ← mid - 1
    else
        // Look right
        left ← mid + 1
return (False, None)
```

# Algorithm rekursiv binär sökning

```
Algorithm Binsearch-rec(a: Array, v: Value, Compare: Function,  
                        left, right: Index)  
  
  if right < left then  
    return (False, None)  
  // Check in the middle  
  mid ← (left + right)/2  
  // Compare  
  c ← Compare(v, Inspect-value(a, mid))  
  if c = 0 then  
    // Found it  
    return (True, mid)  
  else if c < 0 then  
    // Look left  
    return Binsearch-rec(a, v, Compare, left, mid-1)  
  else  
    // Look right  
    return Binsearch-rec(a, v, Compare, mid+1, right)  
  
Algorithm binsearch-main(a: Array, v: Value, Compare: Function)  
  // Call the recursive function to do the work.  
  return Binsearch-rec(a, v, Compare, Low(a), High(a))
```

## Binär sökning, exempel

T

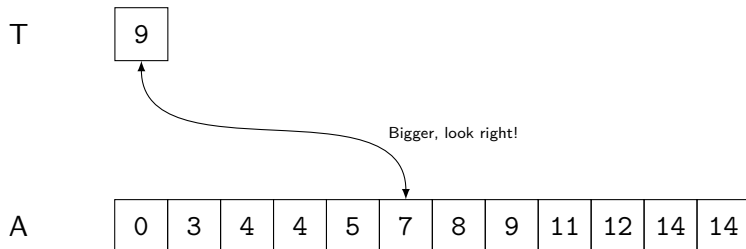
9
---

A

0	3	4	4	5	7	8	9	11	12	14	14
---	---	---	---	---	---	---	---	----	----	----	----



## Binär sökning, exempel



## Binär sökning, exempel

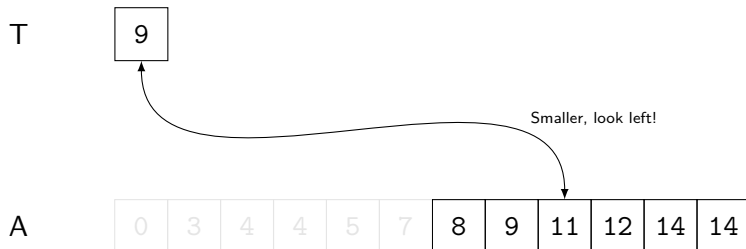
T

9
---

A

0	3	4	4	5	7	8	9	11	12	14	14
---	---	---	---	---	---	---	---	----	----	----	----

# Binär sökning, exempel



## Binär sökning, exempel

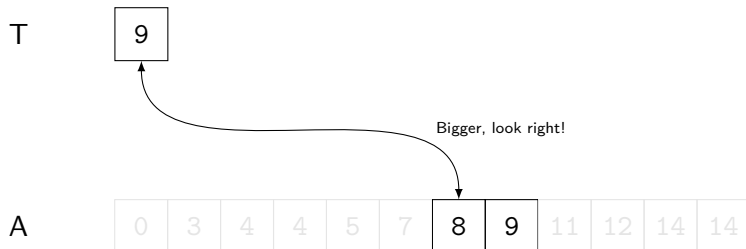
T

9
---

A

0	3	4	4	5	7	8	9	11	12	14	14
---	---	---	---	---	---	---	---	----	----	----	----

# Binär sökning, exempel



## Binär sökning, exempel

T

9
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A

0
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3
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4
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8
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9
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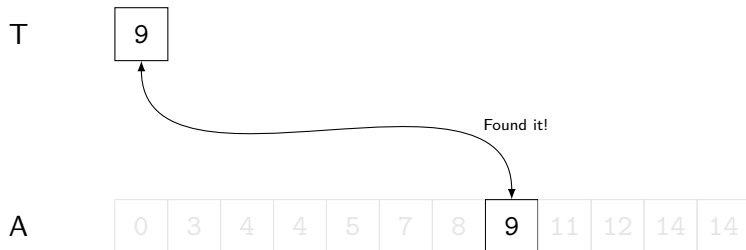
11
----

12
----

14
----

14
----

## Binär sökning, exempel



## Exempel, sorterat fält

- ▶ Sök efter elementvärdet 13:

- ▶ Linjär sökning: 8 jämförelser innan träff.

1	2	4	4	6	7	9	13	14	19
?	?	?	?	?	?	?	!		

- ▶ Binär sökning: 2 jämförelser innan träff.

1	2	4	4	6	7	9	13	14	19
[				?					]
					[		!		]

- ▶ Sök efter elementvärdet 10:

- ▶ Linjär sökning: 8 jämförelser innan man ger upp.

1	2	4	4	6	7	9	13	14	19
?	?	?	?	?	?	?	?		

- ▶ Binär sökning: 4 jämförelser innan man ger upp.

1	2	4	4	6	7	9	13	14	19
[				?					]
					[		?		]
					[?				]
						[?]			