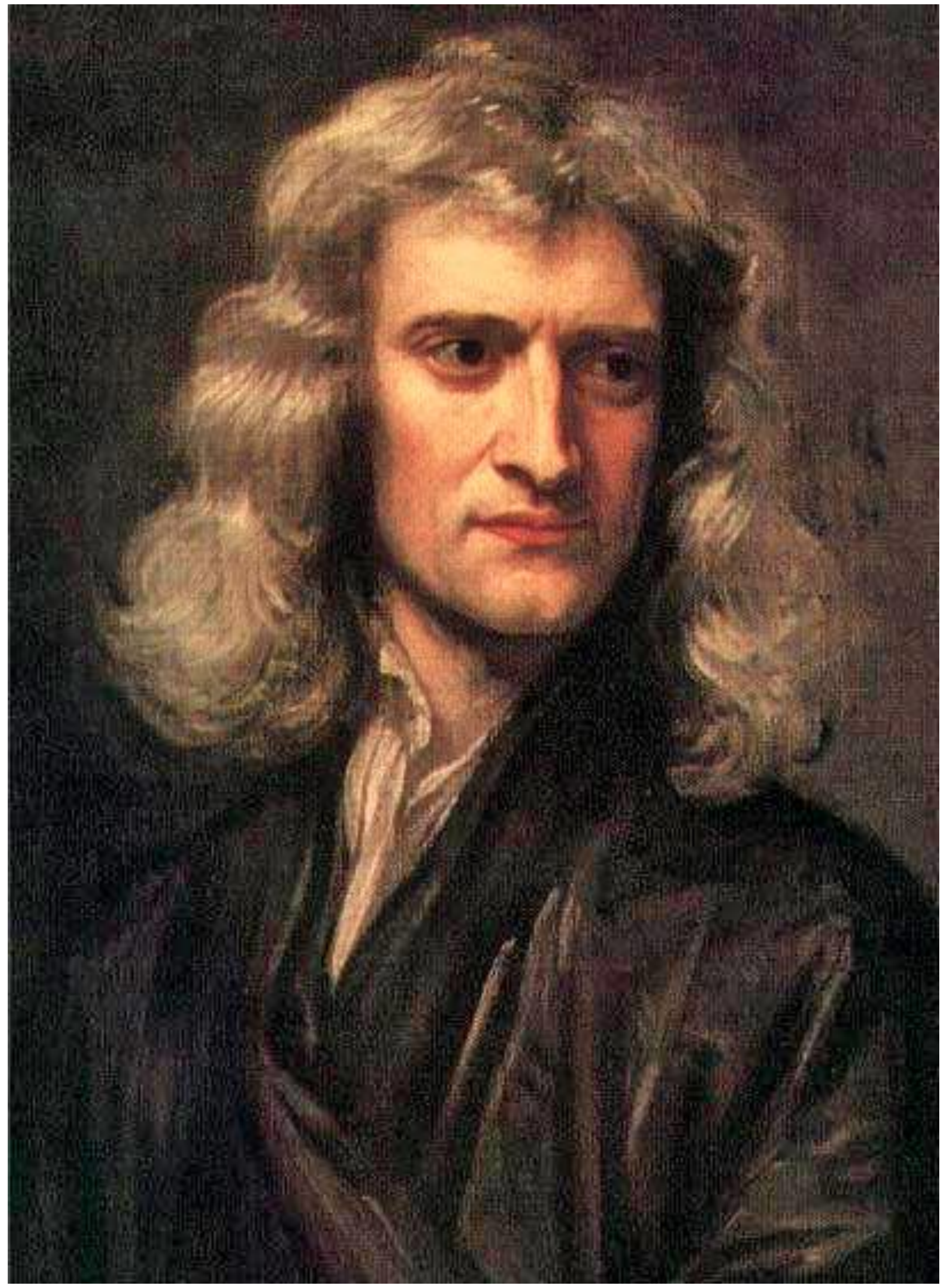
1.2.1 Newton’s laws and inertial frame



Space Mission Design and Operations

Prof. Claude Nicollier

Credits: PD, Wikipedia, Sir Godfrey Kneller

Newton's laws

1. In the absence of a force, a body either is at rest or moves in a straight line with constant speed.

2. A body experiencing a force $\vec{F}$ will be subject to an acceleration $\vec{a}$ such that $\vec{F} = m\vec{a}$, where m is the mass of the body.

3. Whenever a first body exerts a force $\vec{F}$ on a second body, the second body exerts a force $-\vec{F}$ on the first body. The two forces are of equal magnitude and opposite in direction.

Generalization of Newton's second law

The force is equal to the time derivative of momentum :

$$\boxed{\vec{F} = \frac{d\vec{p}}{dt}}$$

where

$$\boxed{\vec{p} = m\vec{v}}$$

This formulation is important in case m does not remain constant (rocket equation for instance, or any leaking system)

Concept of inertial frame

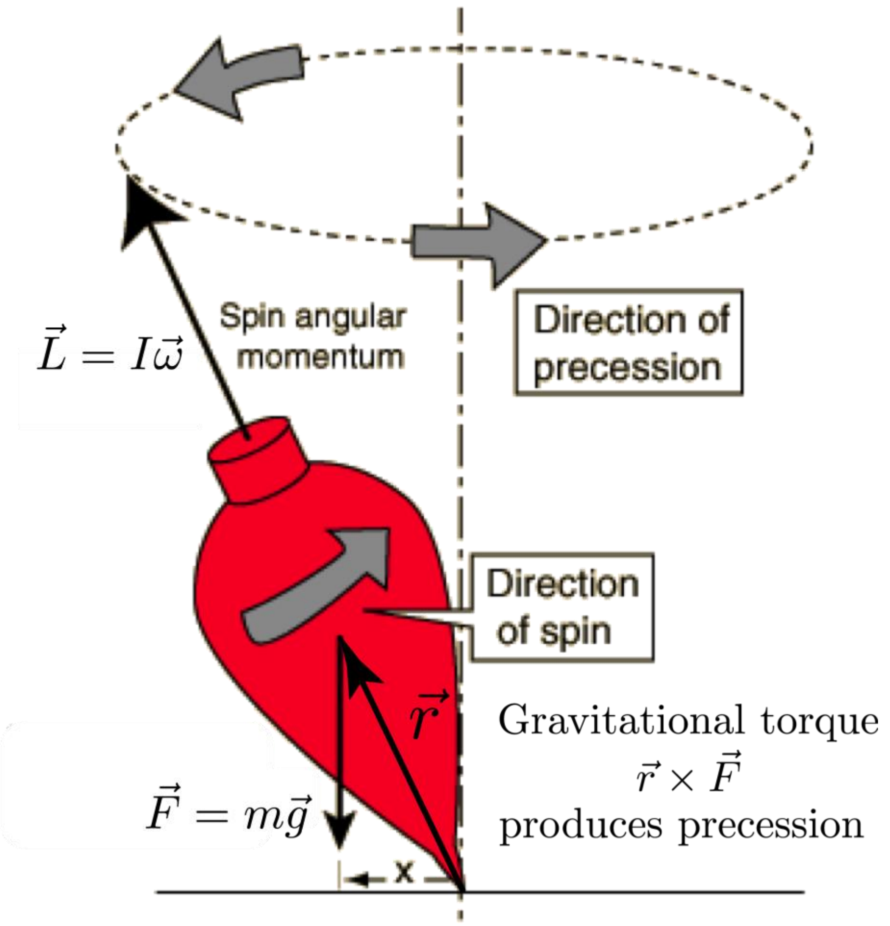
- **Inertial frame:**

  - It is a frame with respect to which the laws of Newton are valid.
  - An inertial frame has direction, direction of the axes which are towards distant stars, nodes moving on very slowly.
  - The center of the inertial frame, which is orthogonal coordinate system, will depend on the application.

- **Validity of Newton's laws:**

  - Motions of celestial bodies and man-made spacecraft.
  - In the vicinity of the Earth and in the solar system.
  - Speeds smaller than 10^{-3} c.
  - Lorentz factor is negligible.

$$\text{Lorentz factor} = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$



1.2.2 Laws of gravitation and rotating bodies

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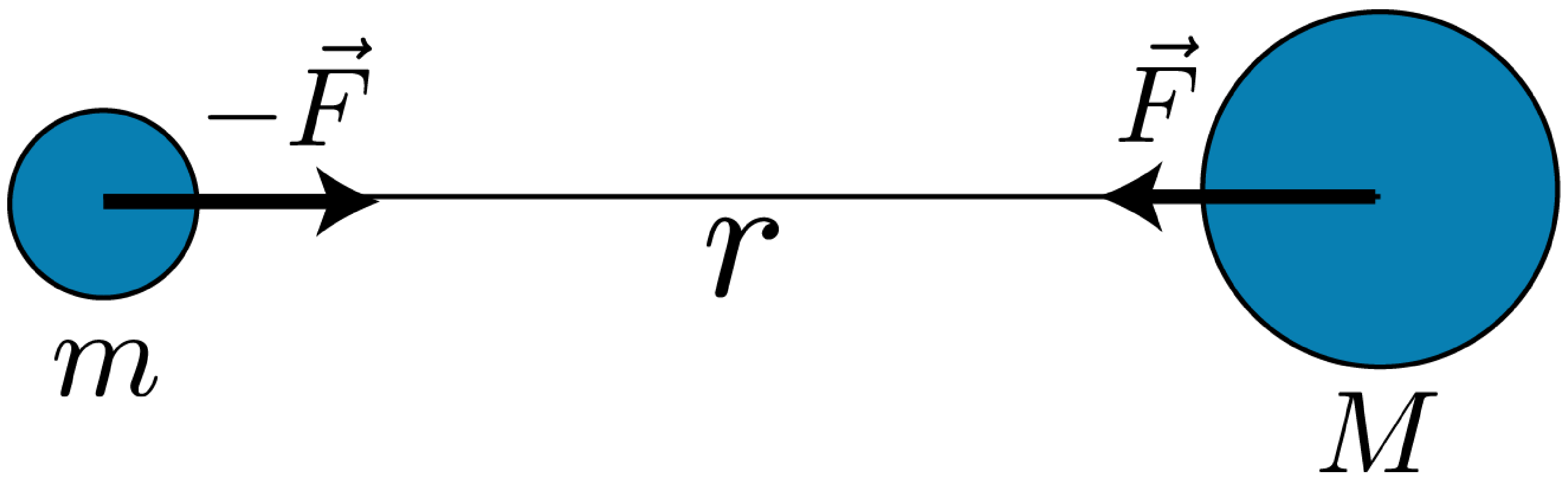
Credits: Adapted from Georgia State University Department of Physics and Astronomy, *Hyperphysics*, « Larmor precession »

Gravitation's law

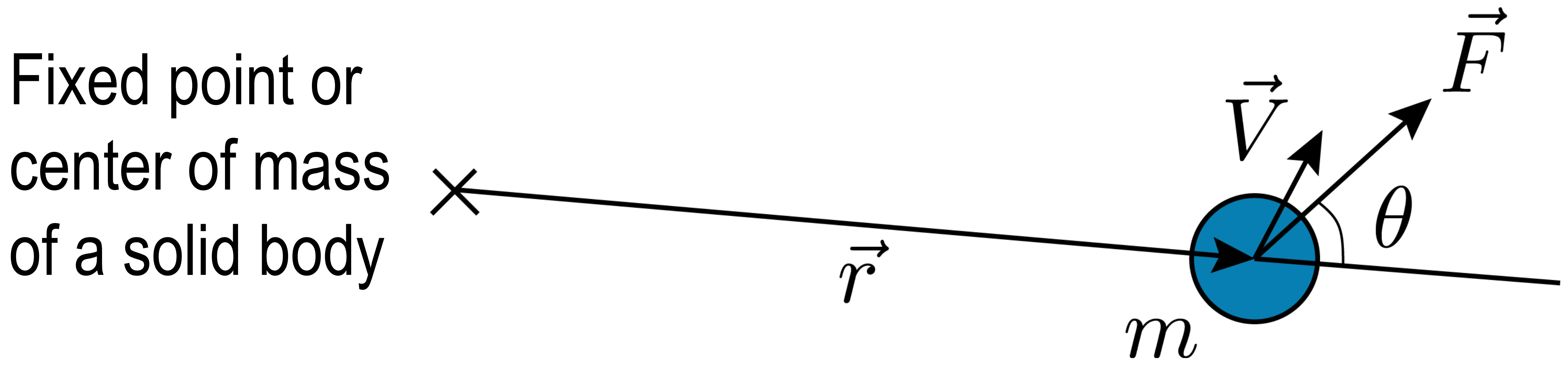


$$\boxed{F = G\frac{Mm}{r^2}} \qquad \boxed{\frac{F}{m} = \frac{\mu}{r^2}}$$

$$\mu = GM$$

$$G = 6.673 \times 10^{-11}\ \mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$$

Laws of mechanics for rotations



- Torque = moment of force: $\vec{T} = \vec{r} \times \vec{F}$
- Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$
- Newton's second law for rotations: $\boxed{\vec{T} = \dfrac{d\vec{L}}{dt}}$

Angular momentum and inertia

- $\vec{L}$ for a solid body, with a fixed rotation axis Δ:

$$\boxed{\vec{L} = I_{\Delta}\vec{\omega}}$$

With the moment of inertia I_{Δ} around the axis Δ.

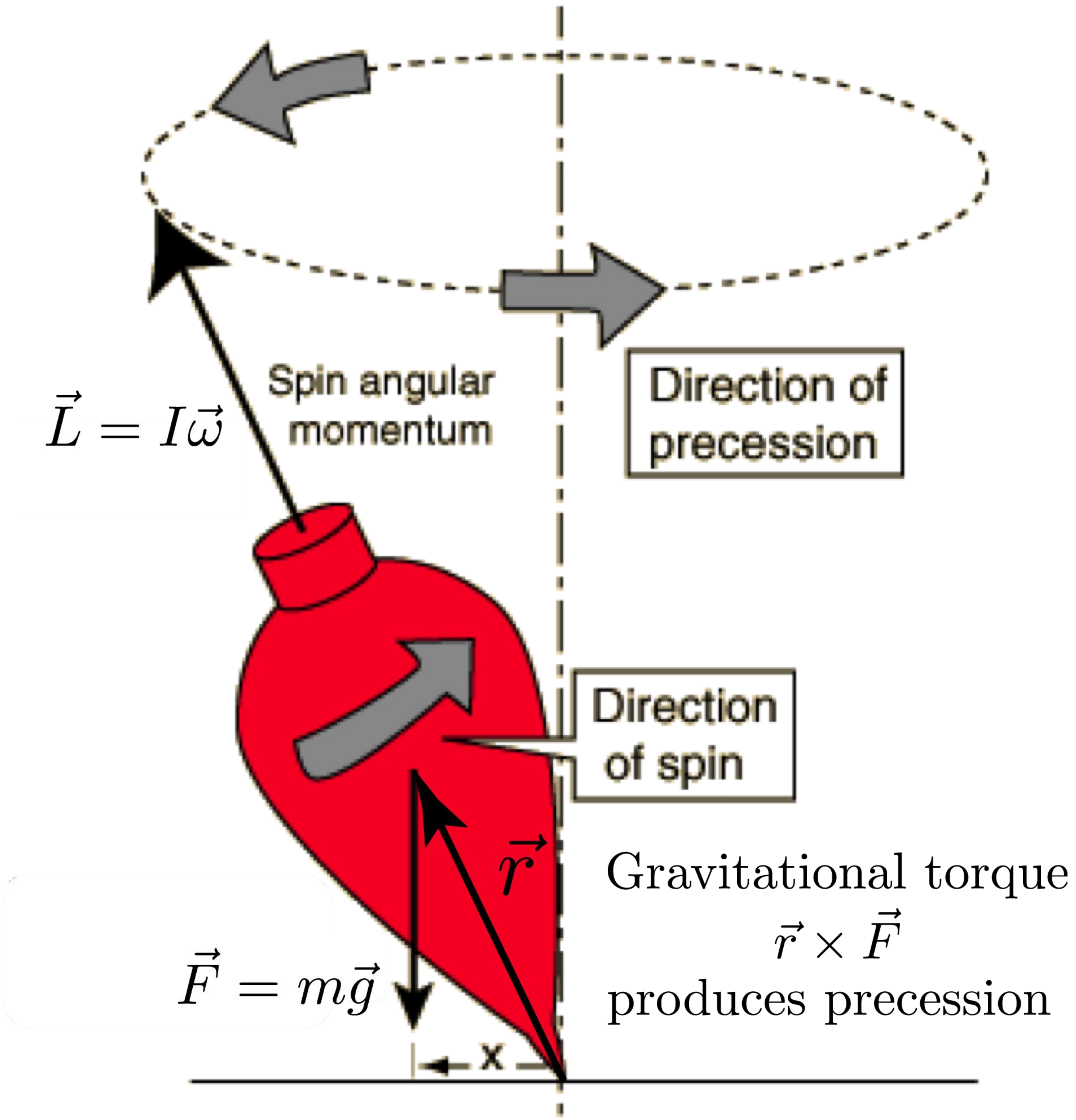
- Moment of inertia:

$$I_{\Delta} = \sum_i m_i r_i^2$$

$$I_{\Delta} = \iiint_V r^2 \rho\left(r\right) dV$$

- Where the r_i are distances of mass elements to the axis of rotation Δ.
- Where r is the distance of the mass element to the axis of rotation Δ.

Precession of a spinning top



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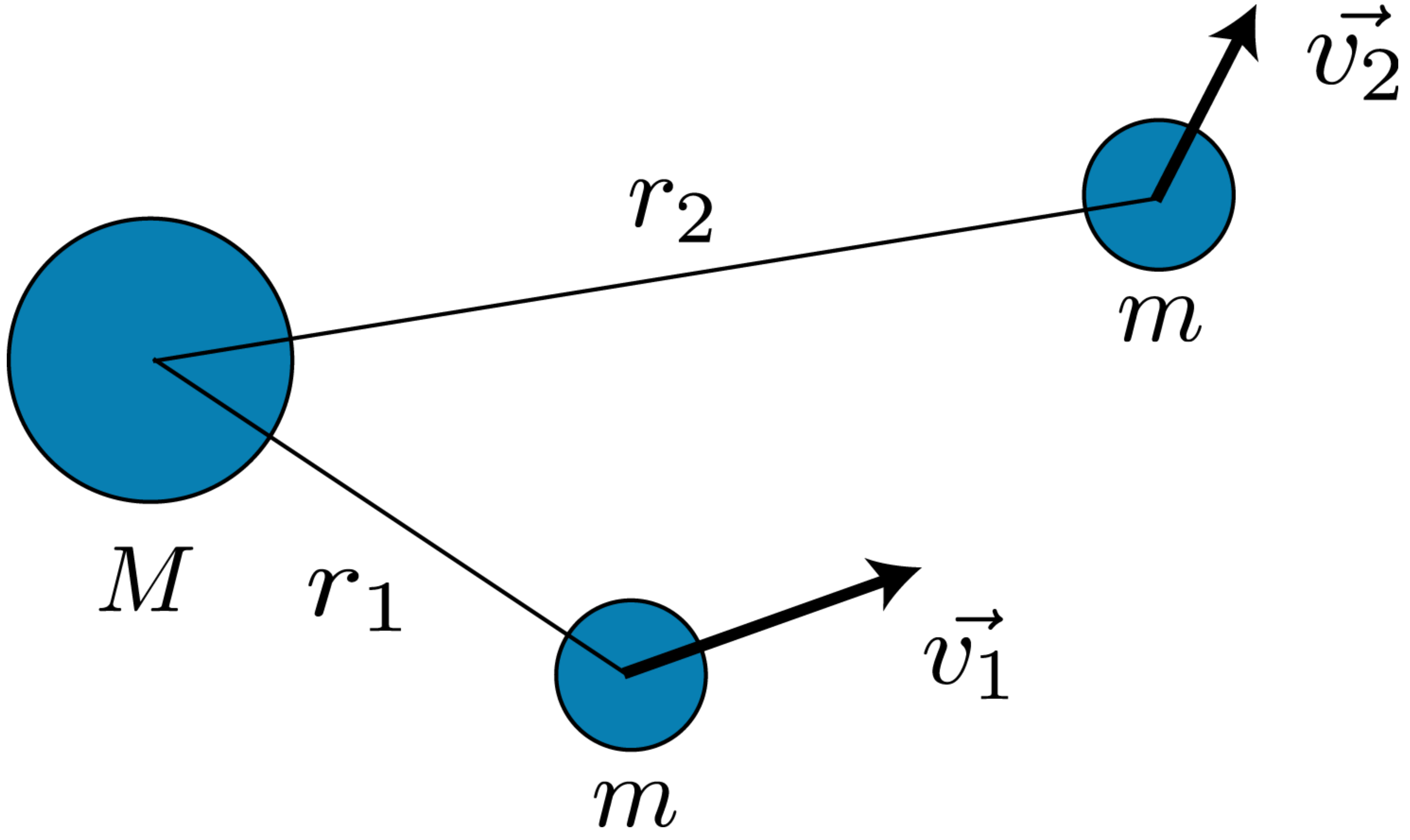

1.2.3 Conservation laws

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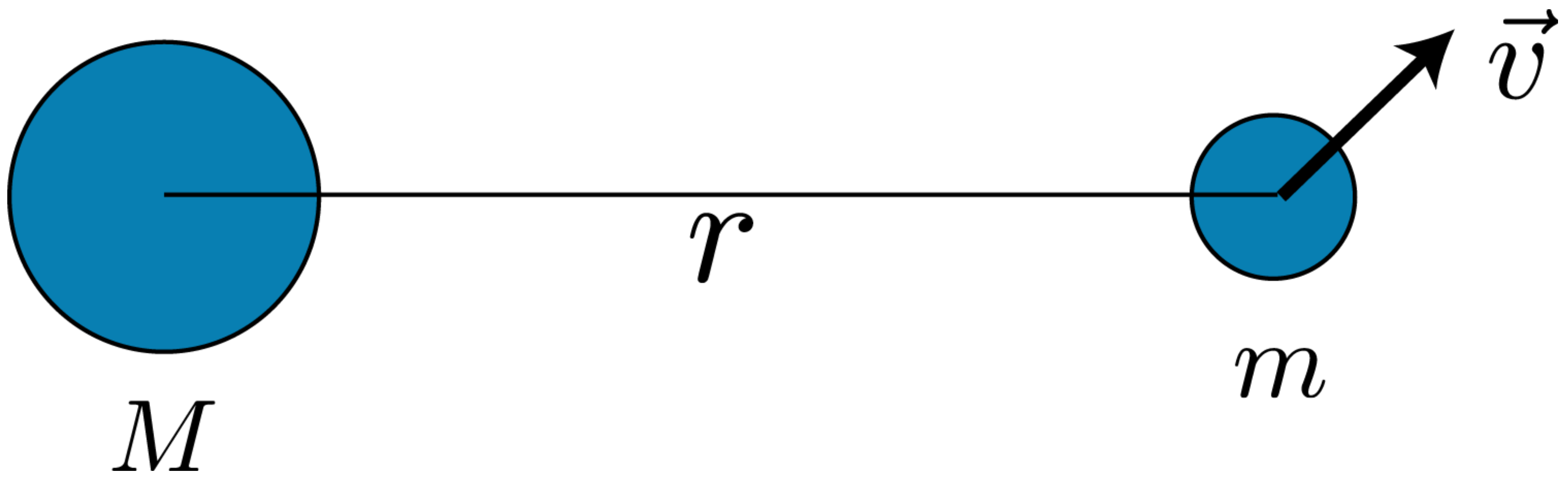
Potential energy of a spacecraft

- Potential energy of a spacecraft of mass m in the gravitational field of a much larger mass M:

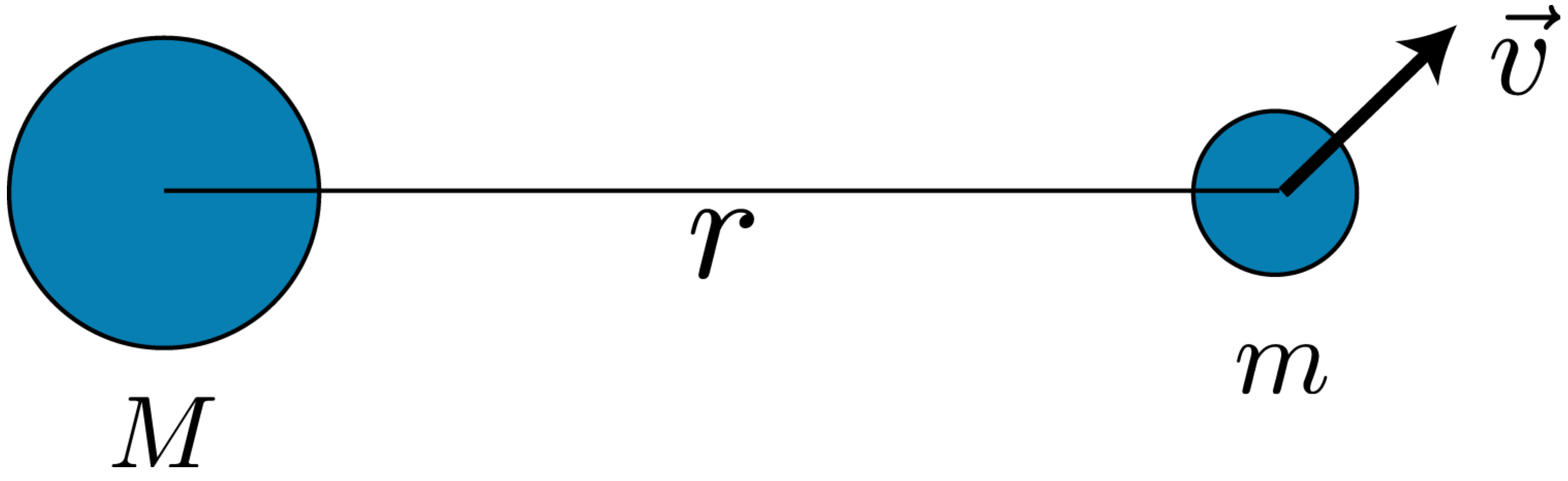


- Potential energy (*m* << *M* - the center of mass of the two bodies is at the center of the large mass M):

$$\boxed{E_{\text{pot}} = -GM\frac{m}{r}} \qquad \boxed{\frac{E_{\text{pot}}}{m} = -\frac{\mu}{r}}$$

Potential energy of a spacecraft

- Kinetic energy of a spacecraft of mass m in the gravitational field of a much larger mass M:



- Kinetic energy:

$$\boxed{E_{\text{kin}} = \frac{1}{2} m v^2}$$

$$\boxed{\frac{E_{\text{kin}}}{m} = \frac{1}{2} v^2}$$

Conservation's laws

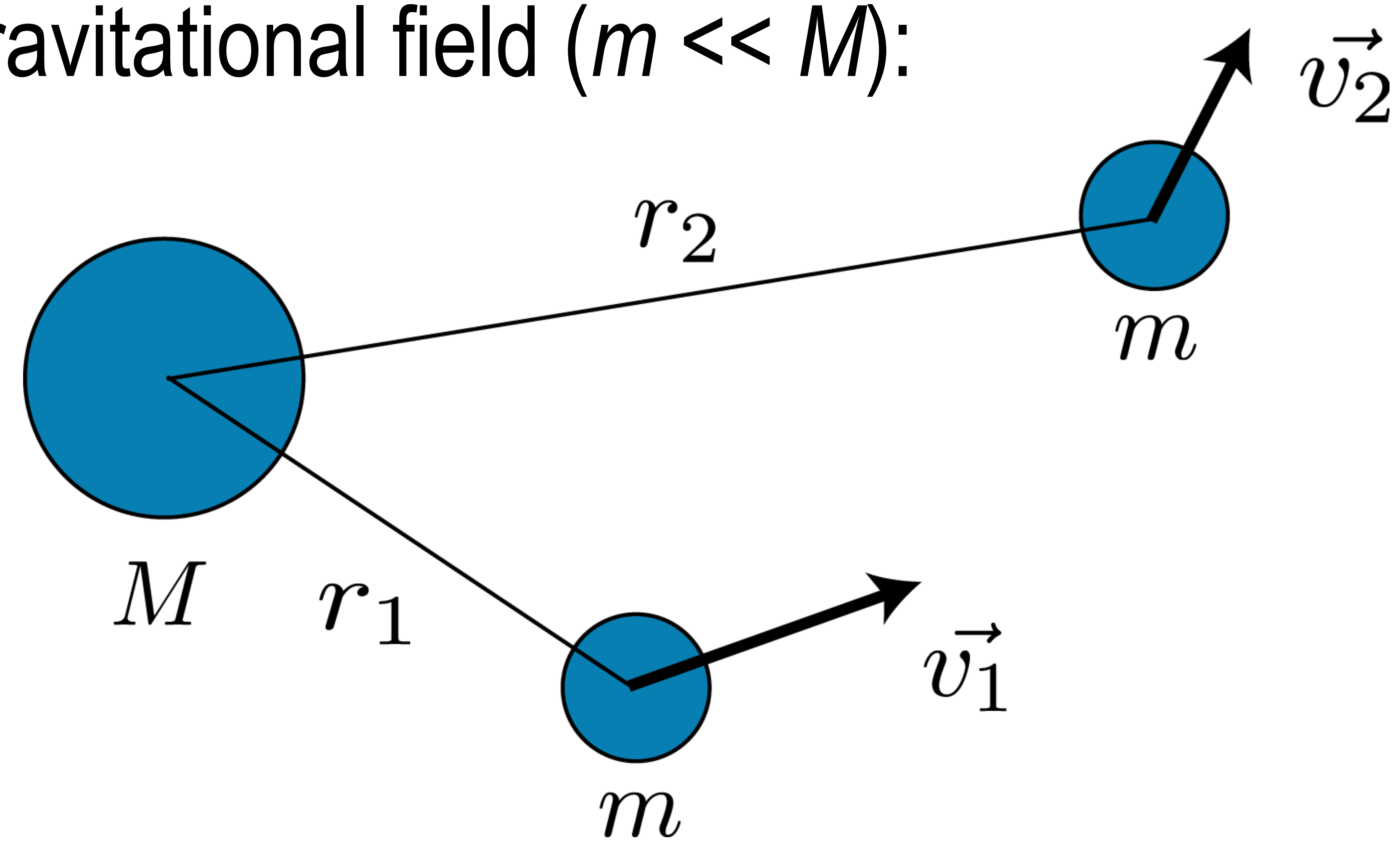
- **Conservation of momentum:**
  in an isolated system (absence of forces).

- **Conservation of angular momentum** for a rotation:
  in an isolated system (absence of torques).

- **Conservation of mechanical energy:**
  Potential and kinetic in an isolated system, in a conservative force field (gravitational force field is OK, in the absence of dissipative forces).

Conservation of mechanical energy

- Conservation of mechanical energy in a gravitational field (*m* << *M*):



$$\boxed{E_{\mathrm{pot1}} + E_{\mathrm{kin1}} = E_{\mathrm{pot2}} + E_{\mathrm{kin2}}}$$

- *E* is in joules.

Translation of important terms

Maths	English	Français	Deutsch	Español
$\vec{F}$	force	force	Kraft	fuerza
m	mass	masse	Masse	masa
$\vec{v}$	speed or velocity	vitesse	Geschwindigkeit	velocidad
$\vec{p} = m\vec{v}$	momentum	quantité de mouvement	Impuls	cantidad de movimiento
$\vec{L} = \vec{r} \times m\vec{v}$	angular momentum	moment cinétique	Drehimpuls	momento angular
$\vec{T}$	torque	moment de force	Moment	torque o momento
I_Δ	moment of inertia	moment d'inertie	Trägheitsmoment	momento de inercia