12. Reflection and Refraction

Wave fronts

Spherical waves are solutions to Maxwell's equations

Molecules scatter spherical waves

Spherical waves can add up to plane waves

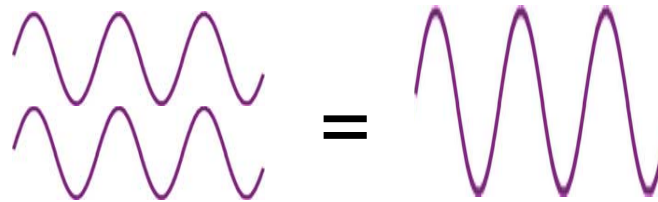Reflection and refraction from surfaces
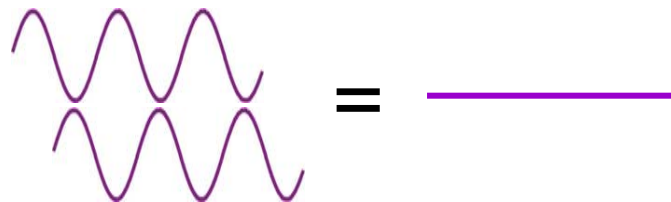
Snell's Law

Mirages and prisms

Constructive vs. destructive interference; Coherent vs. incoherent interference

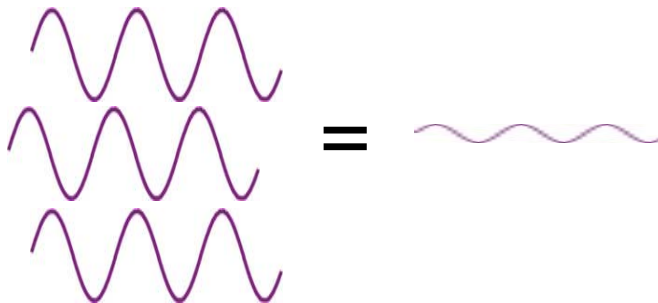Waves that combine **in phase** add up to a higher irradiance.

Constructive interference (coherent)

Waves that combine **180° out of phase** cancel out and yield zero irradiance.

Destructive interference (coherent)

Waves that combine with **lots of different phases** nearly cancel out and yield low irradiance.
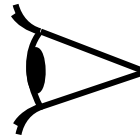
Random addition (incoherent)

Often, constructive interference will occur in only one direction in space.

Why can't we see a light beam?

Unless the light beam is propagating right into your eye or is scattered into it, you won't see it. This is true for any light beam.
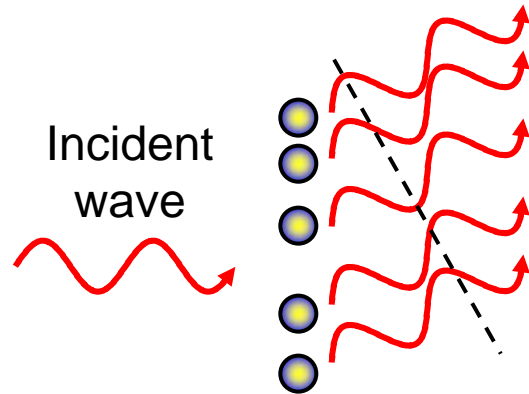
This eye sees no light

This eye is blinded…

To photograph light beams, you need something for the beam to scatter off…
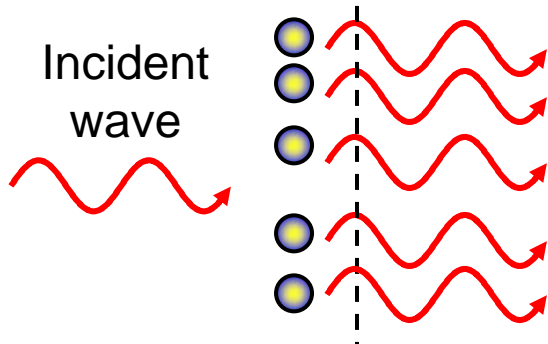
Incoherent interference

Incident wave

Incoherent scattering produces a weak, but non-zero intensity. This is because all of the wavefronts have random phases relative to one another.

Because the scattered radiation is relatively weak, the most dramatic effects are often related to the frequency-dependence of the scattering.

Light from the sun

Air
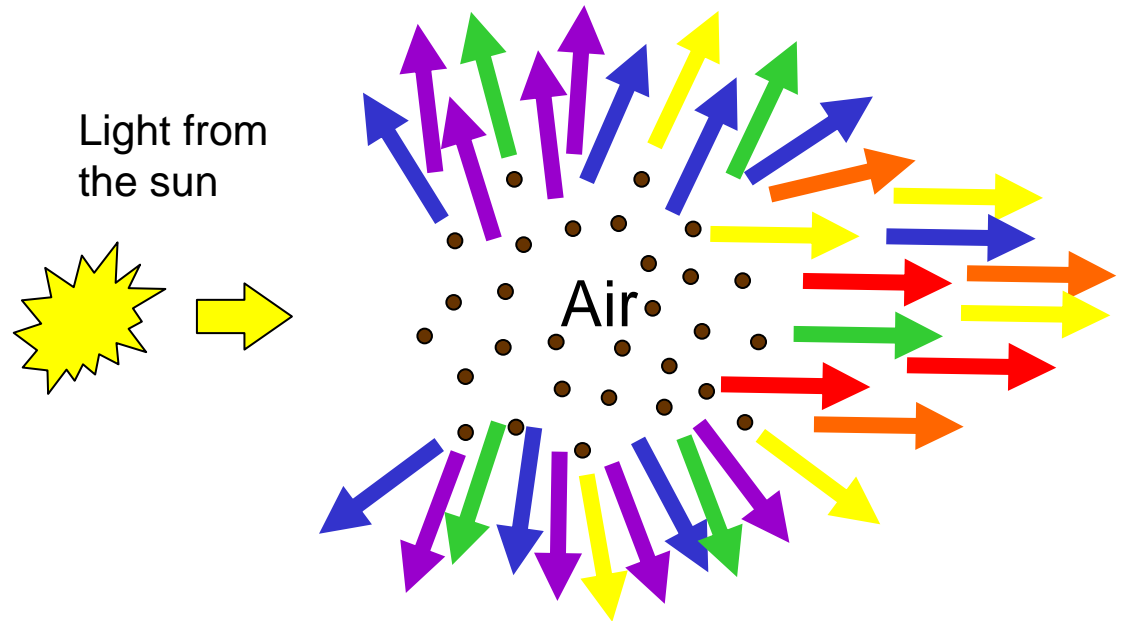
Coherent interference

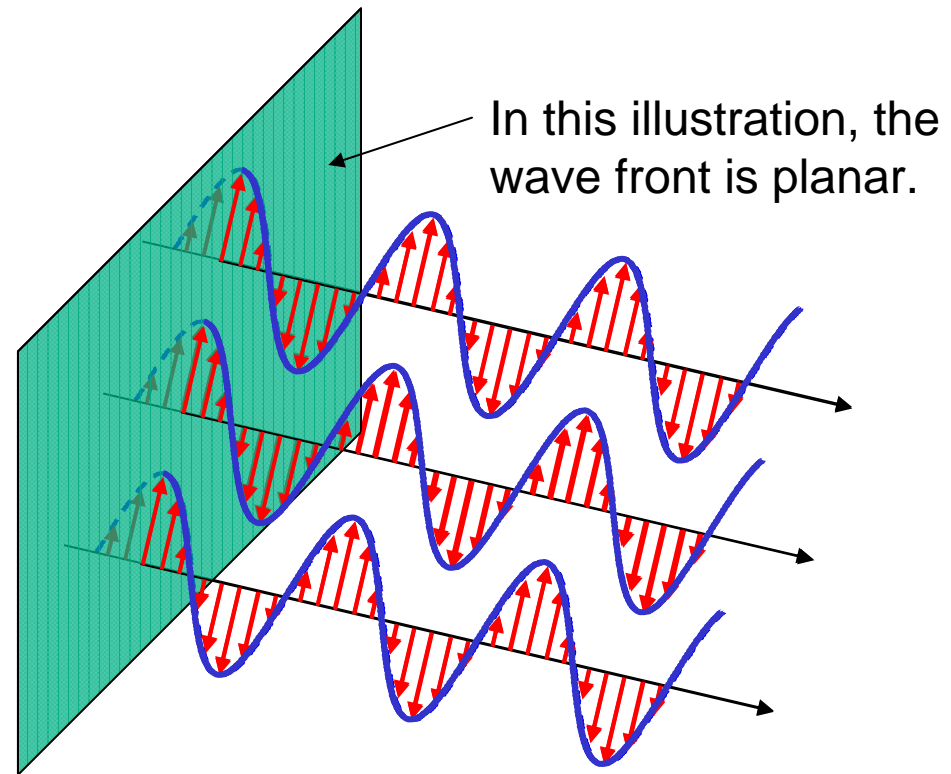Incident wave

Coherent scattering produces a wave with higher intensity, since it involves constructive interference. All of the scattered wavelets are in phase.
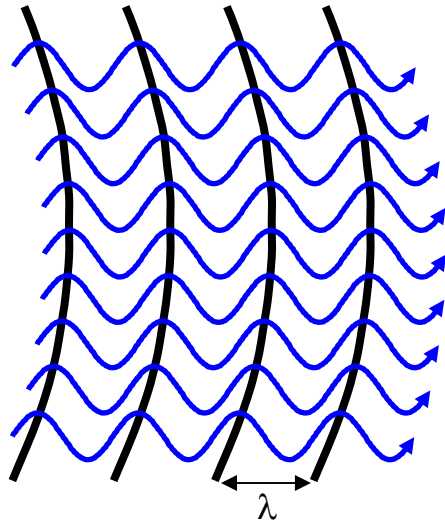
It is often possible to define a surface in space, connecting all the points at which each wavelet reaches a maximum.

This surface is known as a "wave front".

In this illustration, the wave front is planar.

Wave-fronts

At a given time, a wave's "wave-fronts" are the surfaces where the wave has its maxima: in other words, a **surface of constant phase**

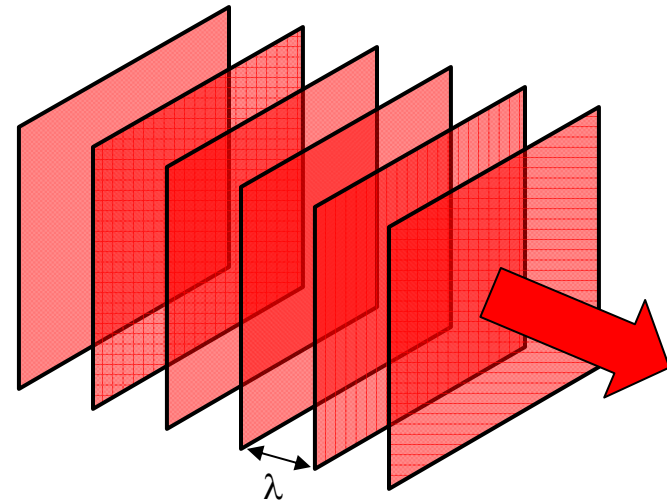Wave-fronts are helpful for drawing pictures of the shape of a wave.

Wave-fronts are sometimes planar.

A wave's wave-fronts are equally spaced, one wavelength apart.

They're perpendicular to the propagation direction, and they propagate at the speed of the wave.

Plane wave: a wave with a planar wave front

By now we are very familiar with this solution to the wave equation:

$$\vec{E}\left(\vec{r},t\right) = \vec{E}_0 e^{j\left(\vec{k}\cdot\vec{r}-\omega t+\theta\right)}$$
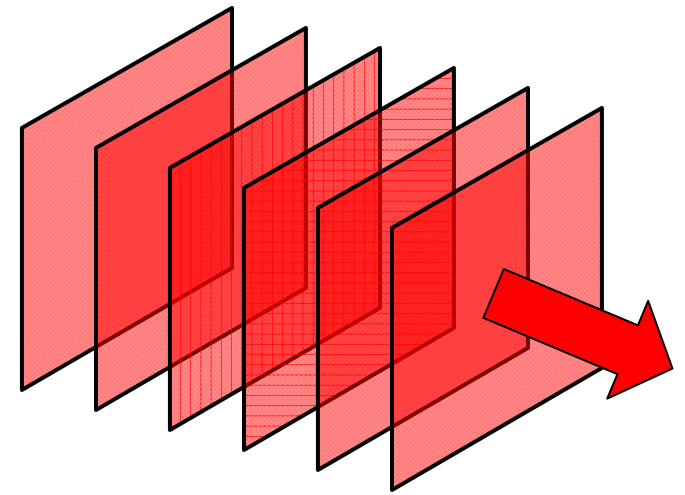
To find out what the wavefronts of this wave look like, we set the phase equal to a constant. For example, a wave traveling in the z direction:

$$kz - \omega t + \theta = \text{constant}$$
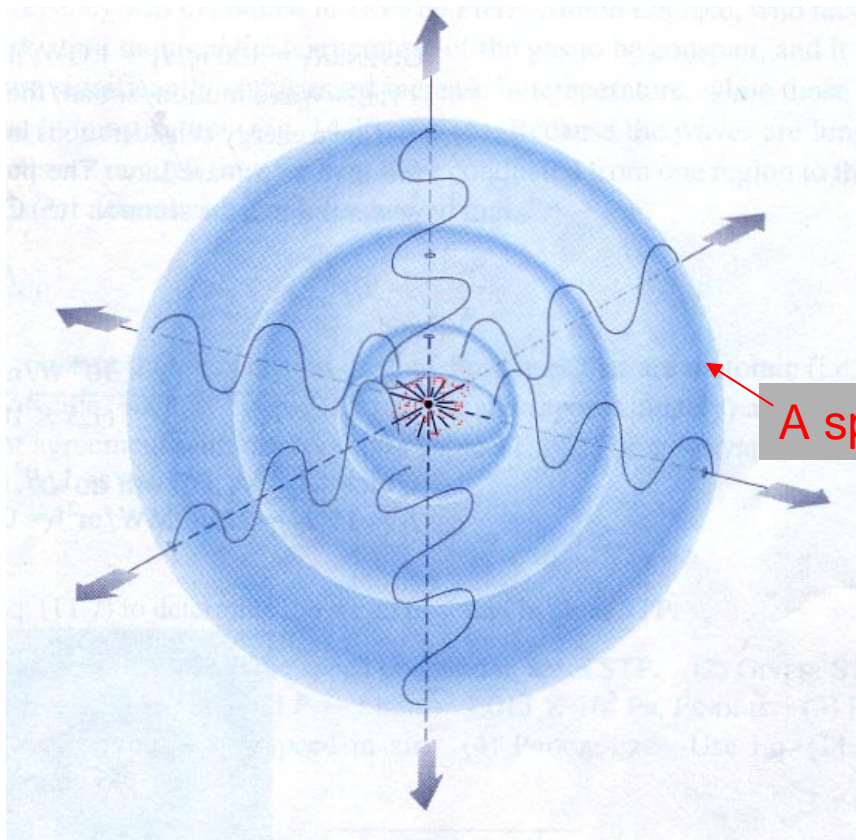
At a particular instant of time, t = constant, this translates to: z = constant. That is the definition of a plane perpendicular to the z axis.

$\vec{E}_0 e^{j\left(\vec{k}\cdot\vec{r}-\omega t+\theta\right)}$ is a plane wave.

defined by the mathematical condition: $\vec{k}\cdot\vec{r} = \text{constant}$
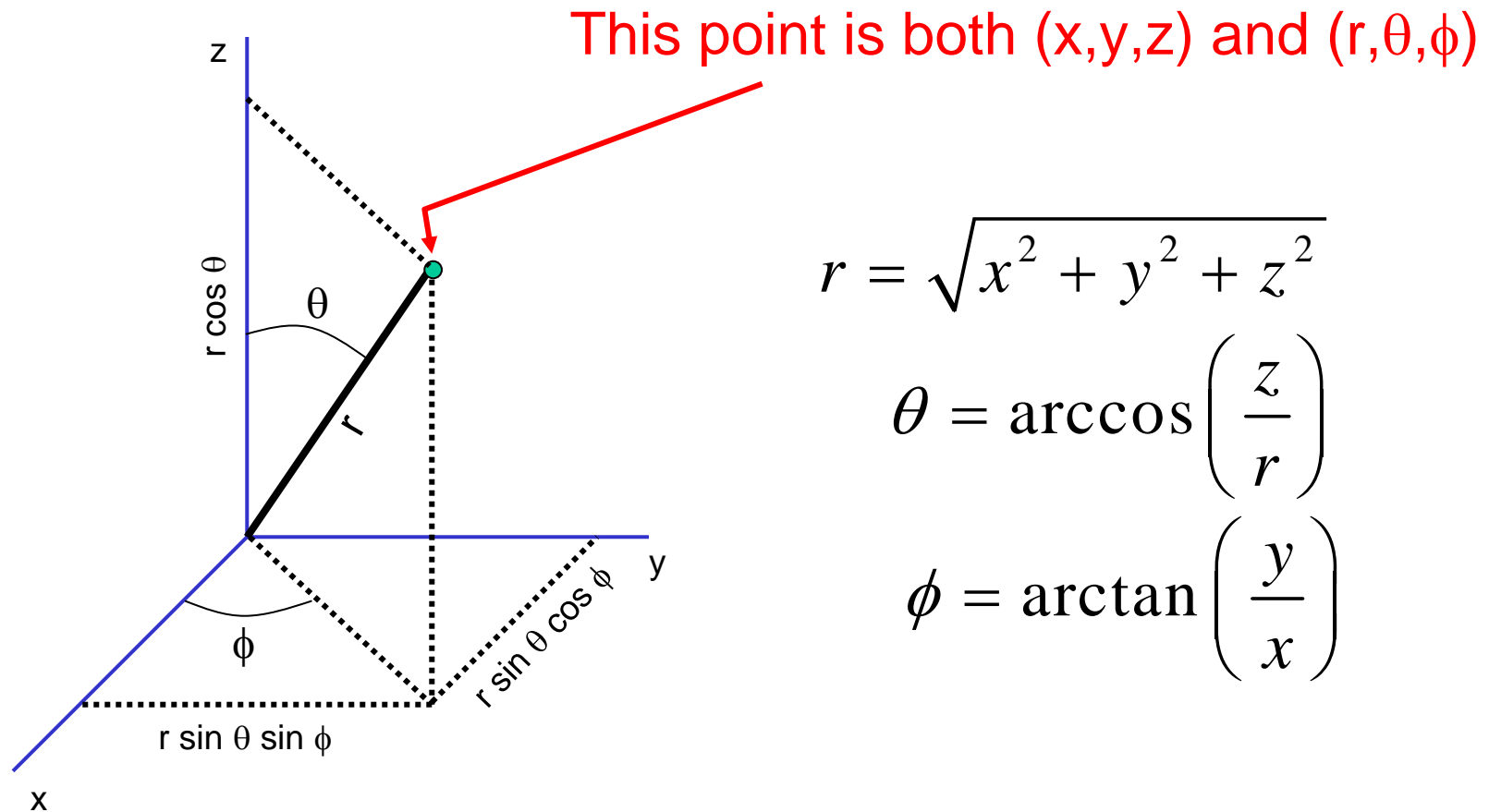
Wave fronts don't have to be planar

Consider a single molecule or point particle, emitting radiation at frequency ω in all directions. This produces a *spherical wave*.

A spherical wave has spherical wave-fronts.

Note: this does not mean that the intensity is the same in every direction. For a spherical wave, the phase is uniform on a sphere, but the amplitude need not be the same everywhere on the sphere.

Unlike a plane wave, whose amplitude remains constant as it propagates, a spherical wave weakens. The irradiance in any direction drops in proportion to $1/r^2$, where r is the distance from the emission point.

To deal with spherical waves, we use spherical coordinates

This point is both (x,y,z) and (r,θ,φ)

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \arccos\left(\frac{z}{r}\right)$$

$$\phi = \arctan\left(\frac{y}{x}\right)$$

Wave equation in spherical coordinates

$$\frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

This is really just three independent wave equations, one each for the x-, y-, and z- components of E.

1st derivative:

$$\frac{\partial \vec{E}}{\partial x} = \frac{\partial \vec{E}}{\partial r} \cdot \frac{\partial r}{\partial x} = \frac{\partial \vec{E}}{\partial r} \cdot \frac{\partial}{\partial x}\sqrt{x^2 + y^2 + z^2} \longrightarrow \frac{x}{r}$$

2nd derivative:

$$\frac{\partial^2 \vec{E}}{\partial x^2} = \frac{\partial \vec{E}}{\partial r} \cdot \frac{1}{r} + \frac{\partial^2 \vec{E}}{\partial r^2} \cdot \frac{x^2}{r^2} - \frac{\partial \vec{E}}{\partial r}\frac{\partial r}{\partial x} \cdot \frac{x^2}{r^3} = \frac{\partial^2 \vec{E}}{\partial r^2} \cdot \frac{x^2}{r^2} + \frac{\partial \vec{E}}{\partial r}\left(\frac{r^2 - x^2}{r^3}\right)$$

and similar expressions for the y and z derivatives. Put them all together to find the wave equation in spherical coordinates:

$$\frac{\partial^2 \vec{E}}{\partial r^2} + \frac{2}{r}\frac{\partial \vec{E}}{\partial r} = \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

Spherical wave solutions to the wave equation

This wave equation can be recast in the following form:

$$\mu\varepsilon\frac{\partial^2}{\partial t^2}\left(r\vec{E}\right) = \frac{\partial^2}{\partial r^2}\left(r\vec{E}\right)$$

which is exactly the same form as the 1D wave equation with which we are very familiar. Thus, if the product rE has the same form as the known solutions to the 1D equation:

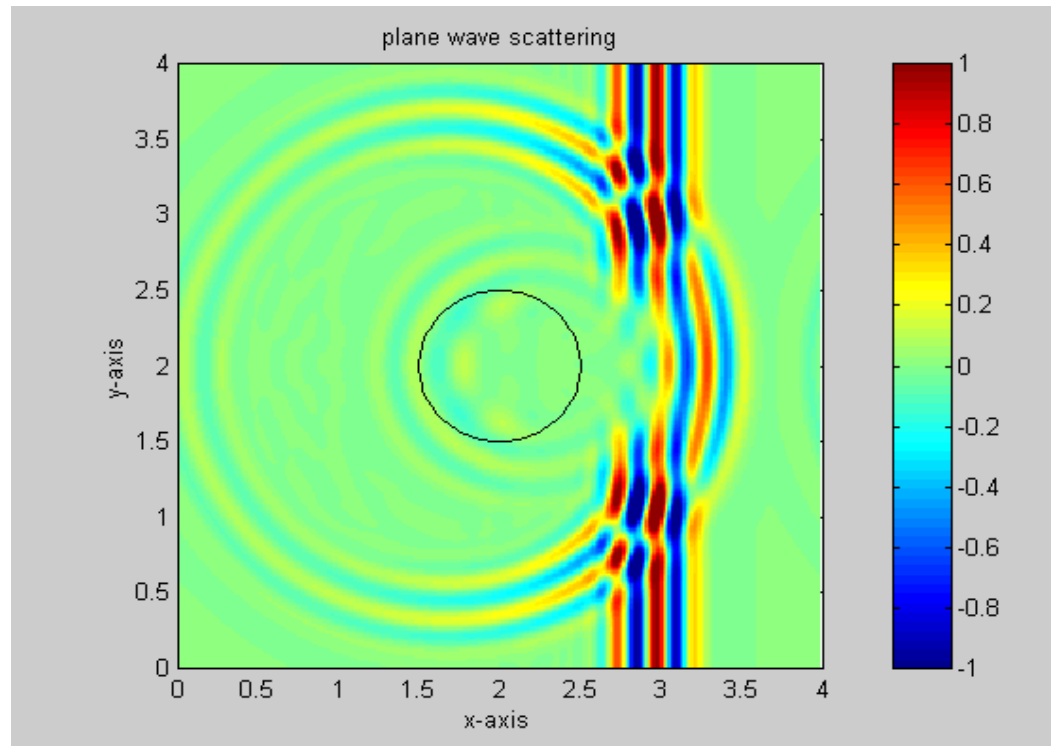$$r\vec{E}\left(r,t\right) = \vec{E}_0 e^{j\left(kr-\omega t+\theta\right)}$$

then we can immediately determine the solution to the 3D equation:

$$\vec{E}\left(r,t\right) = \frac{\vec{E}_0}{r}e^{j\left(kr-\omega t+\theta\right)}$$

Surfaces of constant phase are spheres!

Irradiance: $I \propto \left|\vec{E}\right|^2 \propto \frac{1}{r^2}$

Decreases with increasing distance from the emission point

A plane wave impinging on a molecule or particle scatters into a spherical wave.


plane wave scattering

Huygens' Principle says that waves propagate as if each point on a wave-front emits a spherical wave (whether or not there's a molecule or particle involved).

Scattering from an individual molecule or particle is weak, but many such scatterings can add up--especially if one can arrange it so that interference is constructive.

Most wave fronts are complicated

Plane waves and spherical waves are important examples. But most wavefronts have much more complicated shapes.

• A source of spherical waves (upper left corner)
• A region of lower refractive index (blue area across the middle)
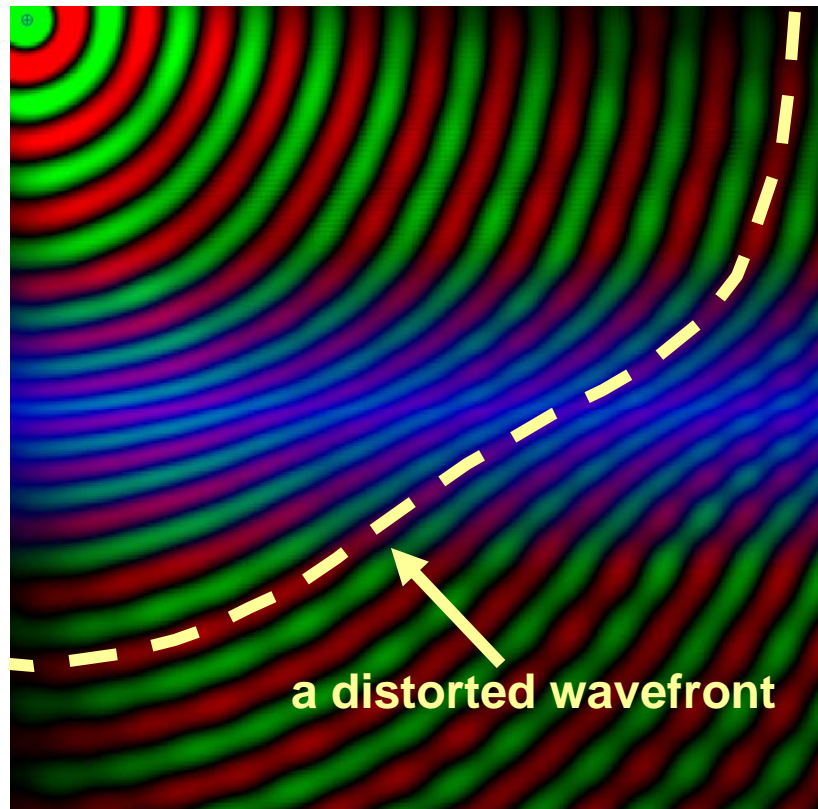
Light reflected from a non-smooth object has a curved and distorted wavefront in the shape of the object.

a distorted wavefront

nose-shaped wavefront
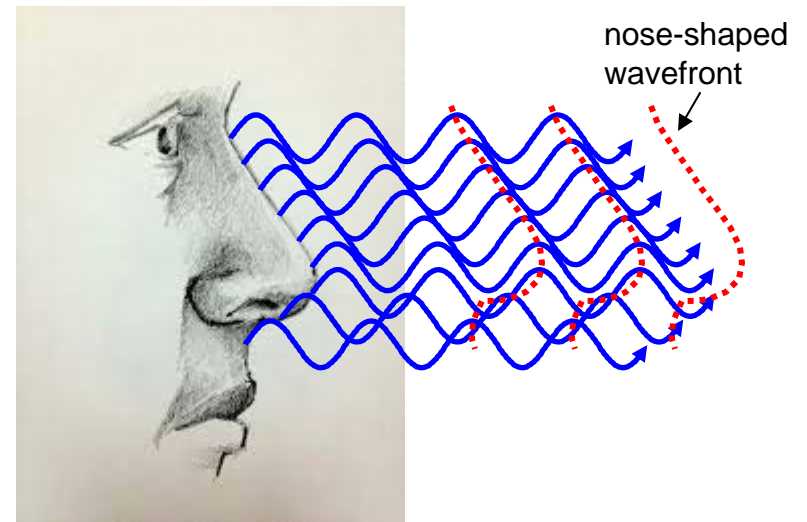
Any arbitrarily complicated wavefront can be written as a sum of plane waves or of spherical waves.

Spherical waves can combine to form plane waves.

A plane wave impinging on a surface will produce a reflected plane wave because all the spherical wavelets interfere constructively along a flat surface.
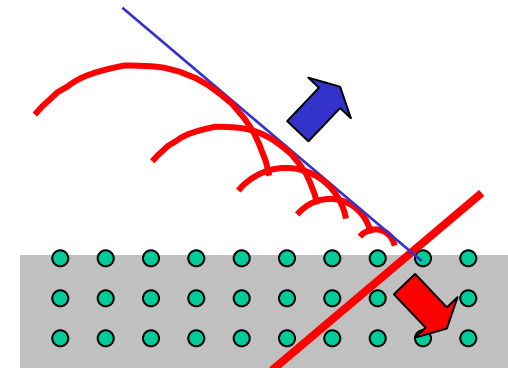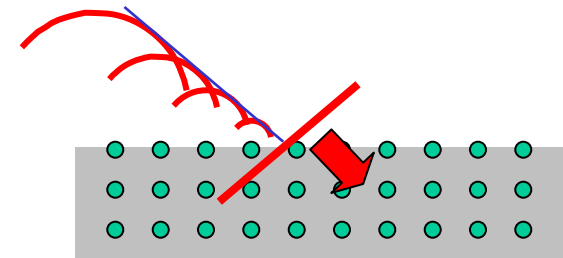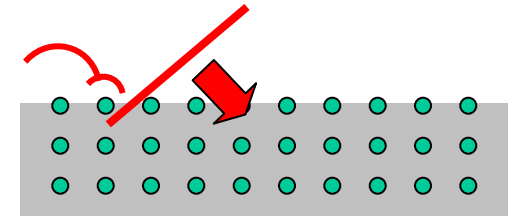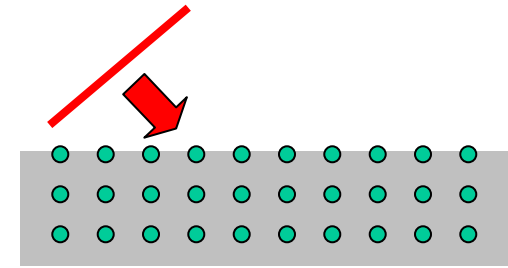
A note on the way we define angles

When considering the problem of a light beam approaching a surface, we always define the angles relative to the surface normal, *not* to the plane of the surface.
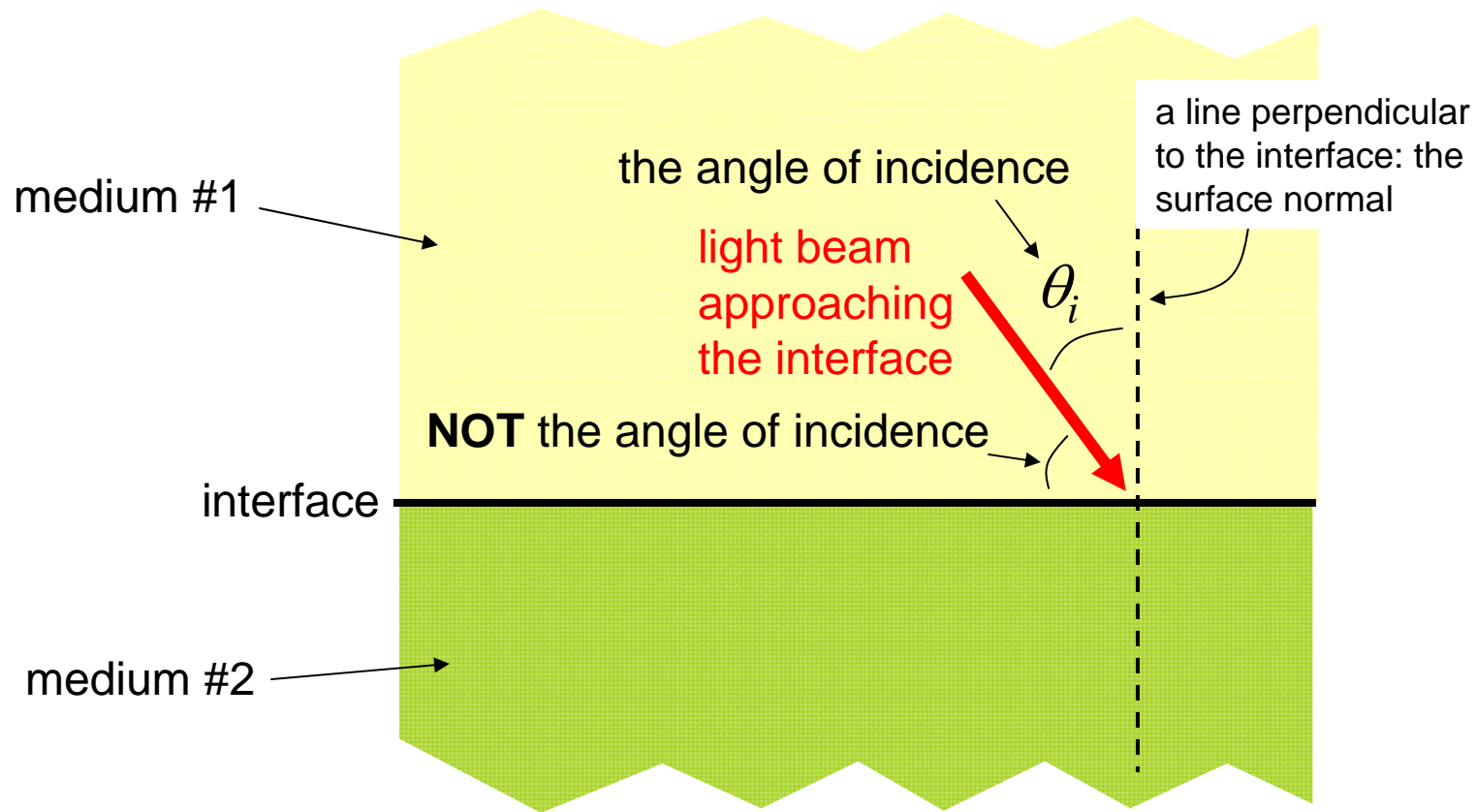
What happens to light when it encounters a surface?

It is scattered by the surface molecules. But a beam can remain a beam if there is a direction for which constructive interference occurs.

incident

reflected

refracted

θ_i θ_r

mirror
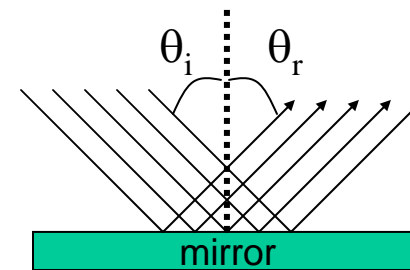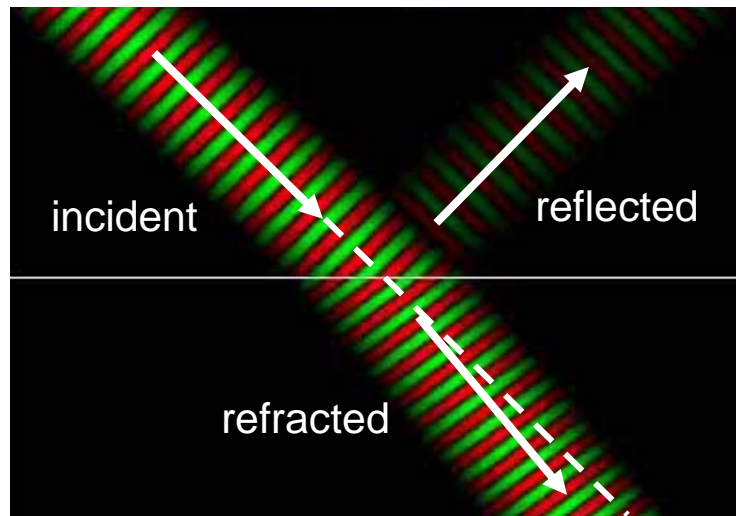
Constructive interference occurs for a reflected beam if the angle of incidence = the angle of reflection.
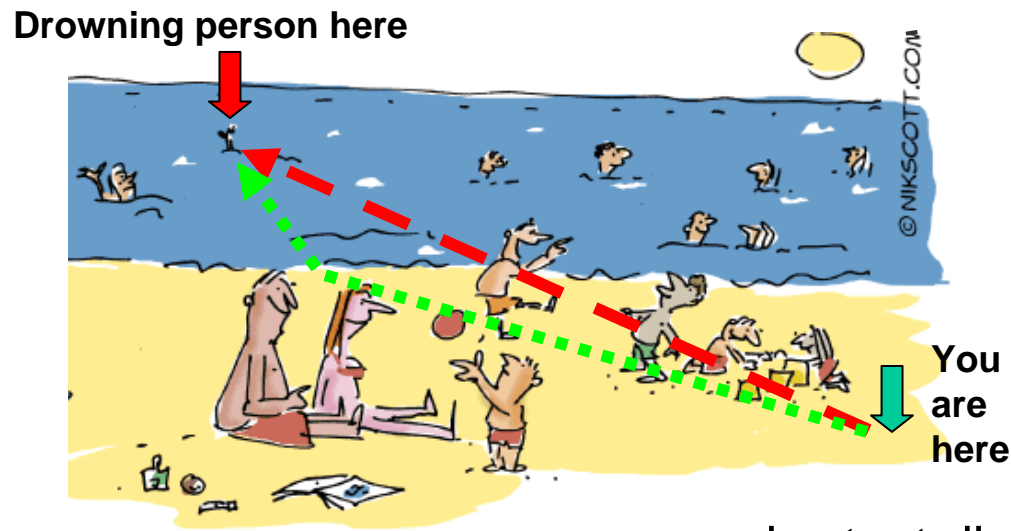
For a transmitted beam, the result is a bit more complicated. The angle of the beam changes. This is known as 'refraction'.

Snell's Law

Snell's Law was first described by Ibn Sahl in 984 AD.

Unaware of Sahl, Snell derived it in 1621. Descartes worked it out at about the same time. In France, it is known as the Snell-Decartes Law.
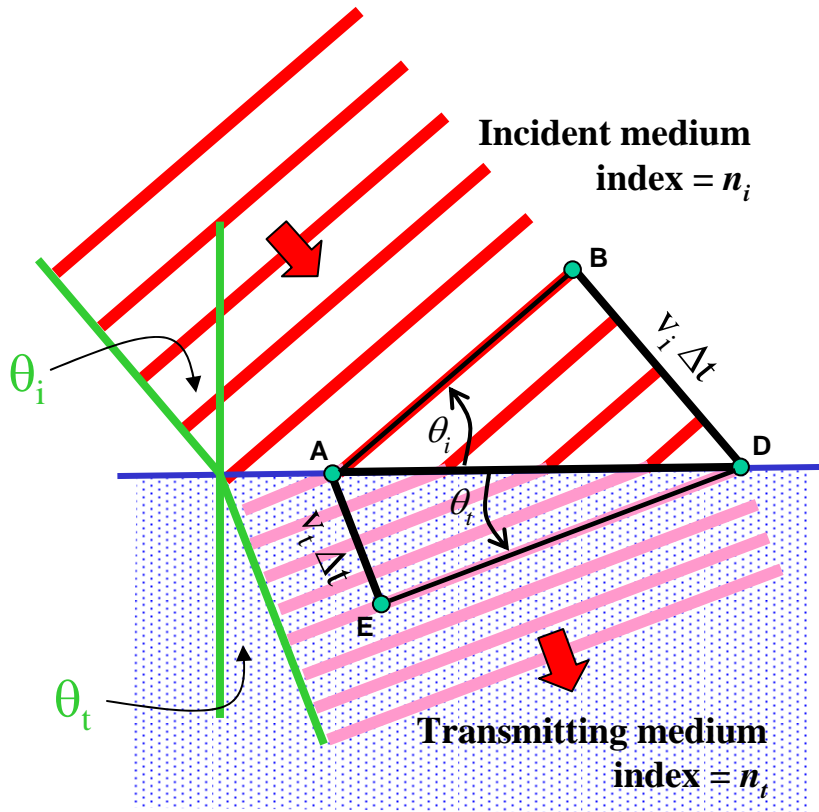
One interesting description comes from Feynman…

Willebrord Snel van Royen

shortest distance ≠ shortest time!

One proof of Snell's Law

The time it takes for a wave to propagate a certain number of wavelengths is independent of the medium, because both the speed and the wavelength change by the same factor.

Incident medium index = n_i

Transmitting medium index = n_t

$AD = BD/\sin(\theta_i)$ and $AD = AE/\sin(\theta_t)$

So: $BD/\sin(\theta_i) = AE/\sin(\theta_t)$
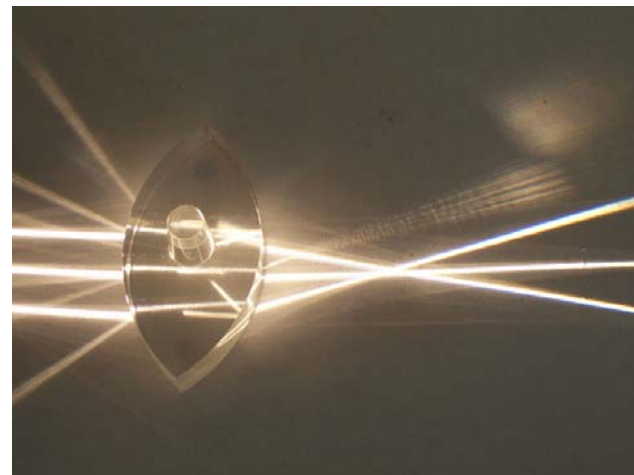
But: $BD = v_i\, \Delta t = (c_0/n_i)\, \Delta t$

same Δt!

& $AE = v_t\, \Delta t = (c_0/n_t)\, \Delta t$

So: $(c_0/n_i)\, \Delta t\, /\, \sin(\theta_i)$
$= (c_0/n_t)\, \Delta t\, /\, \sin(\theta_t)$

Or: $\boxed{n_i \sin(\theta_i) = n_t \sin(\theta_t)}$

Snell's Law java applet: http://www.ps.missouri.edu/rickspage/refract/refraction.html

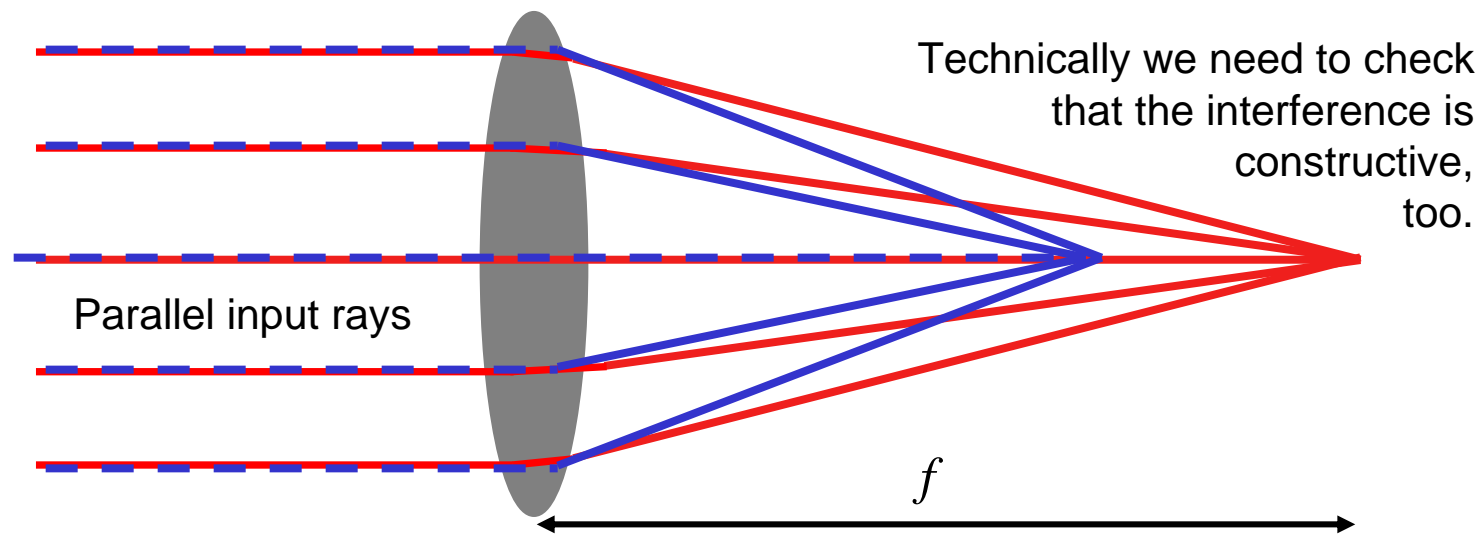# Snell's Law explains many everyday effects

Snell's Law explains why things look bent when they stick out of a pool of water
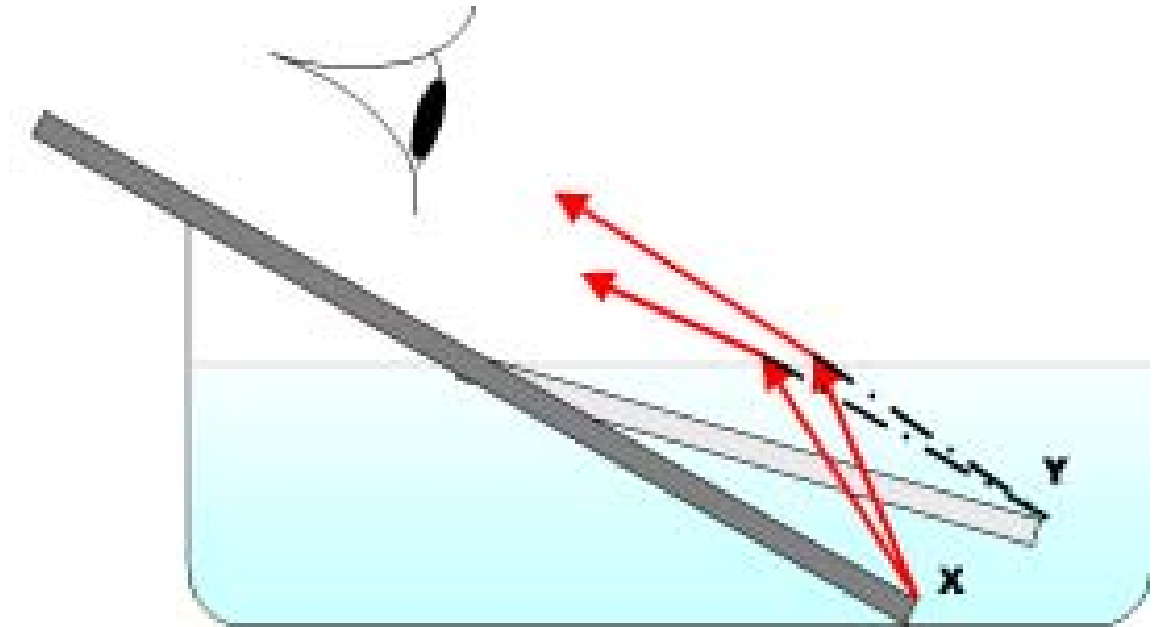
Refraction makes lenses possible

Because refraction bends rays toward the thickest part of a lens,
parallel rays incident on a convex lens converge.

Technically we need to check
that the interference is
constructive,
too.

Parallel input rays

f

The distance at which parallel rays converge is the lens focal length, f.

Dispersion means that the focal length of a lens can depend on
wavelength, $f(\lambda)$. This is called 'chromatic abberation'. Lens designers
go to great lengths to compensate for dispersion.

Snell's Law explains mirages

cool air

hot air

apparent reflecting surface
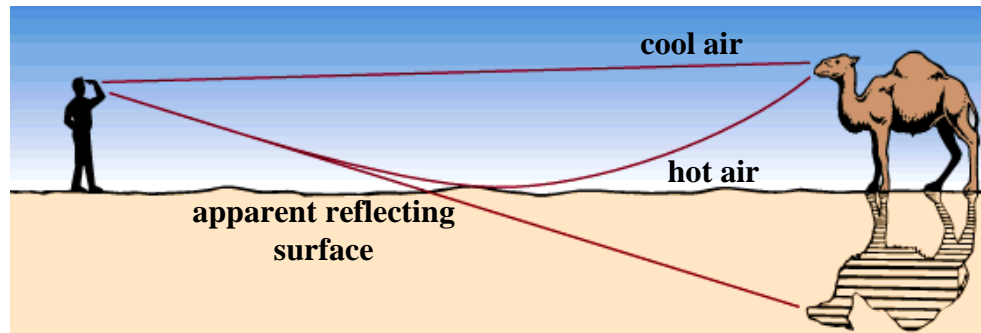
Snell's law helps us to understand the origin of mirages…

which can be very dramatic!

A mirage appears off the shore of Penglai City in eastern China's Shandong Province on Sunday, May 7, 2005.

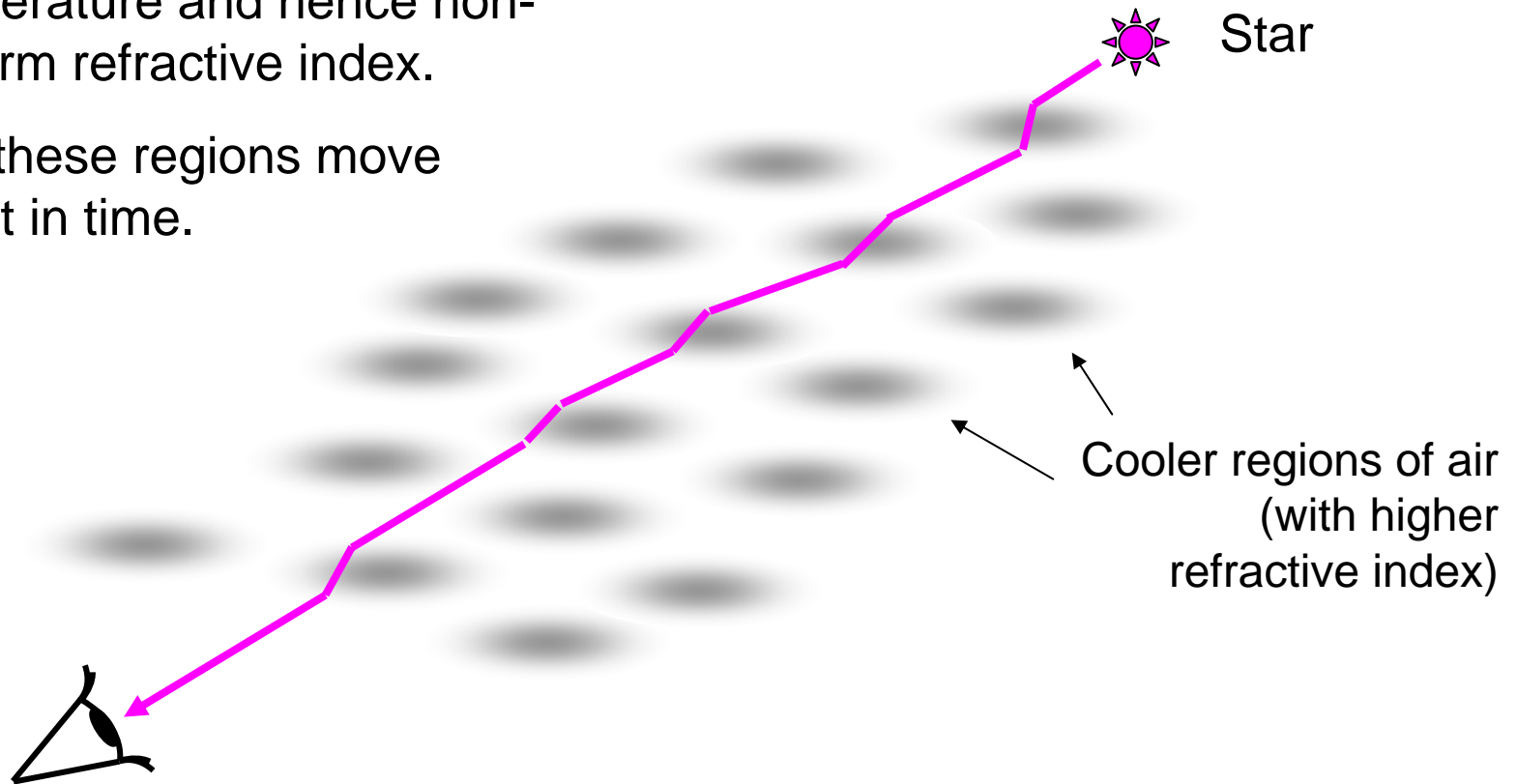The refractive index varies with density (and hence temperature)

Snell's Law explains why the sun flattens when it sets

Snell's Law explains why stars twinkle

The atmosphere has non-uniform temperature and hence non-uniform refractive index.

And these regions move about in time.

Star

Cooler regions of air (with higher refractive index)

As the air masses move about, the amount of light reaching our eyes from the star varies.

Prisms disperse light.

Because the refractive index depends on wavelength,
the refraction angle also depends on wavelength.

Because n generally
decreases with wave-
length ($dn/d\lambda < 0$), smaller
wavelengths experience
greater refraction angles.

Assuming $n_{air} = 1$:

$$\sin(\theta_t) = n(\lambda) \sin(\theta_i)$$

Differentiating implicitly
with respect to λ:

$$\cos(\theta_t) \frac{d\theta_t}{d\lambda} = \frac{dn}{d\lambda} \sin(\theta_i)$$

We obtain the "prism dispersion:"

$$\frac{d\theta_t}{d\lambda} = \frac{dn}{d\lambda} \frac{\sin(\theta_i)}{\cos(\theta_t)}$$
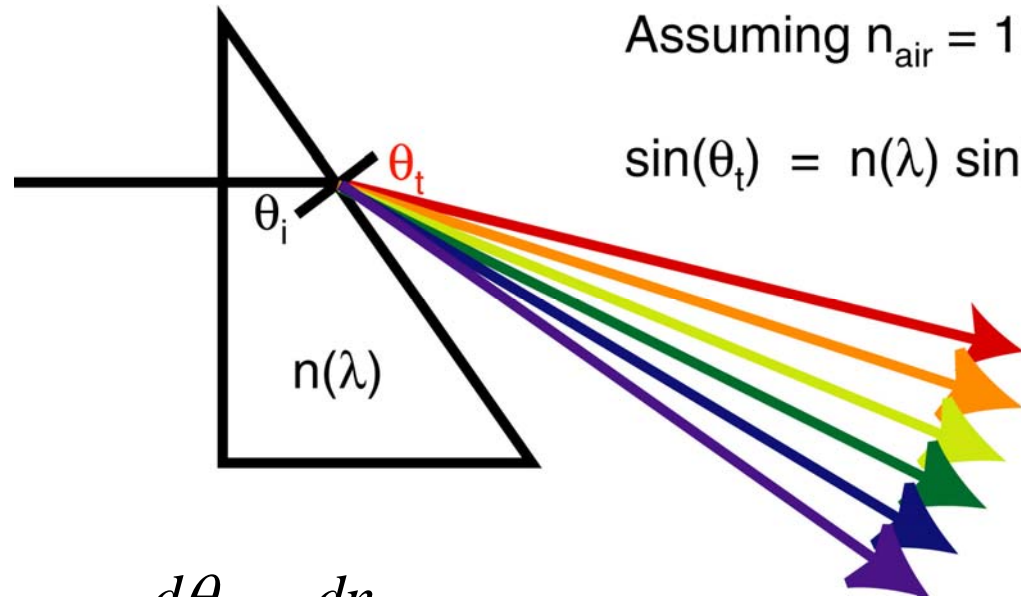
If the prism were
made from a
material for which
$dn/d\lambda = 0$, then that
prism would make
no rainbow!