Intelligens Fejlesztőeszkozok - 4. beadandó

Sándor Burian

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1 feladat

Intervallum felező módszerrel

$$5x - 4 = sin(tanh(-3x + 2)); [-10, 10] intervallumon \tag{1}$$

$$f(x) = 5x - 4 - sin(tanh(-3x + 2))$$
$$a_1 = -10$$
$$b_1 = 10$$
$$f(a_1) = f(-10) = -50-4-sin(tanh(32)) = -54-0,017452406 = -54,017452406$$
$$f(b_1) = f(10) = 50-4-sin(tanh(-28)) = 46-(-0,017452406) = 46,017452406$$

$$\Rightarrow f(a)f(b) < 0 \Rightarrow intervallumfelezes : \frac{a+b}{2} \Rightarrow \frac{10+(-10)}{2} = 0 \tag{2}$$

$$
\begin{gathered}
f(0) = -4 - sin(tanh(2)) = -4 - 0,016824661 = -4,016824661 \\
a_2 = (a_1 + b_1)/2 = 0; f(a_2) < 0 b_2 = b_1 = 10 \\
f(b_2) > 0 \\
f(\frac{a_2 + b_2}{2}) = f(5) = 25 - 4 - sin(tanh(-13)) = 21,017452406 \\
b_3 = 5 \\
f(b_3) > 0 \\
a_3 = a_2 = 0 \\
f(a_3) < 0 \\
f(0)f(5) < 0 \\
f(2.5) = 12.5 - 4 - sin = tanh(-5.5)) = 8,517451824 \\
... \\
f(\frac{a_2 3 + b_2 3}{2}) = f(0,798685552) = 0,000000094 \\
\Rightarrow x = 0,798685552
\end{gathered} \tag{3}
$$

Julia kódként:

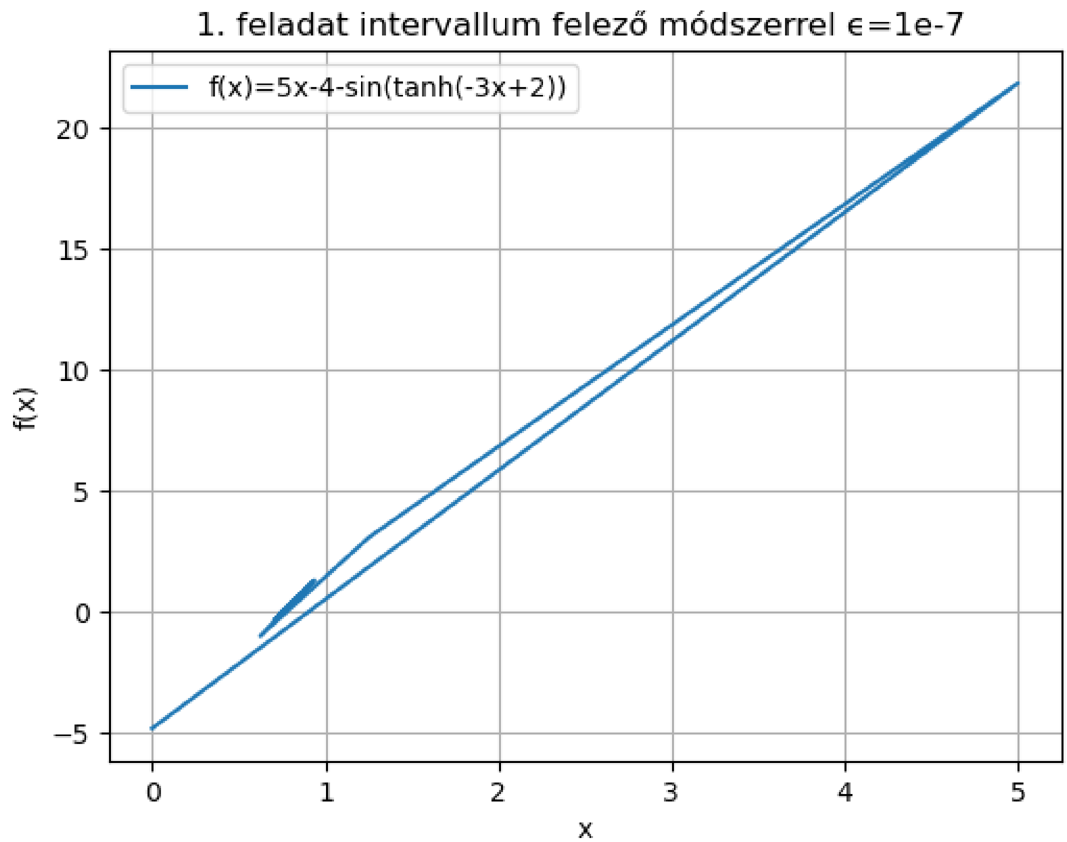
```
f(x)=5*x-4-sin(tanh(-3*x+2))
a=-10
b=10
=1e-7 #(10^(-7))

while true
    global a,b
    c=(a+b)/2
    println("x= ",c)
    println("f(x)= ",f(c))
    println()
    if sign(f(c))==sign(f(a))
        a=c
    end
    if sign(f(c))==sign(f(b))
        b=c
    end
    if abs(f(c))<
        break
    end
end
```

Logok:

```
x= 0.0
f(x)= -4.821494815516438

x= 5.0
f(x)= 21.841470984802374

x= 2.5
f(x)= 9.341452936704993

x= 1.25
f(x)= 3.0583686122902405

x= 0.625
f(x)= -0.9990327572022525

x= 0.9375
f(x)= 1.3092437102511458

x= 0.78125
f(x)= 0.2310697309851627

x= 0.703125
f(x)= -0.37564942969465204

x= 0.7421875
f(x)= -0.06813638549908974

x= 0.76171875
f(x)= 0.08270992936515631

x= 0.751953125
f(x)= 0.007575155275291817

x= 0.7470703125
f(x)= -0.030211616196670926

x= 0.74951171875
f(x)= -0.011300580891388218

x= 0.750732421875
f(x)= -0.001858251535921368

x= 0.7513427734375
f(x)= 0.0028595732281952724
```

```
x= 0.75103759765625
f(x)= 0.0005009404338206236

x= 0.750885009765625
f(x)= -0.0006785857483954938

x= 0.7509613037109375
f(x)= -8.880519482826199e-5

x= 0.7509994506835938
f(x)= 0.0002060719865843441

x= 0.7509803771972656
f(x)= 5.863448746595834e-5

x= 0.7509708404541016
f(x)= -1.5085080807220042e-5

x= 0.7509756088256836
f(x)= 2.1774771550742145e-5

x= 0.7509732246398926
f(x)= 3.3448624267573557e-6

x= 0.7509720325469971
f(x)= -5.8701049265030836e-6

x= 0.7509726285934448
f(x)= -1.2626201839338602e-6

x= 0.7509729266166687
f(x)= 1.0411213878791514e-6

x= 0.7509727776050568
f(x)= -1.107493314278507e-7

x= 0.7509728521108627
f(x)= 4.6518604487899573e-7

x= 0.7509728148579597
f(x)= 1.7721836090278664e-7

x= 0.7509727962315083
f(x)= 3.323451580605763e-8
```

Ábrázolva:



2 feladat

$$x + 3 = e^{sin(x+3)} \tag{4}$$

Intervallum felező módszerrel [-10,10] között.
Matematikai levezetés hasonlatos.
Julia kód:

```
f(x)=x+3-^(sin(x+3))
a=-10
b=10
=1e-7 #(10^(-7))

while true
    global a,b
    c=(a+b)/2
    println("x= ",c)
```

```
    println("f(x)= ",f(c))
    println()
    if sign(f(c))==sign(f(a))
        a=c
    end
    if sign(f(c))==sign(f(b))
        b=c
    end
    if abs(f(c))<
        break
    end
end
```

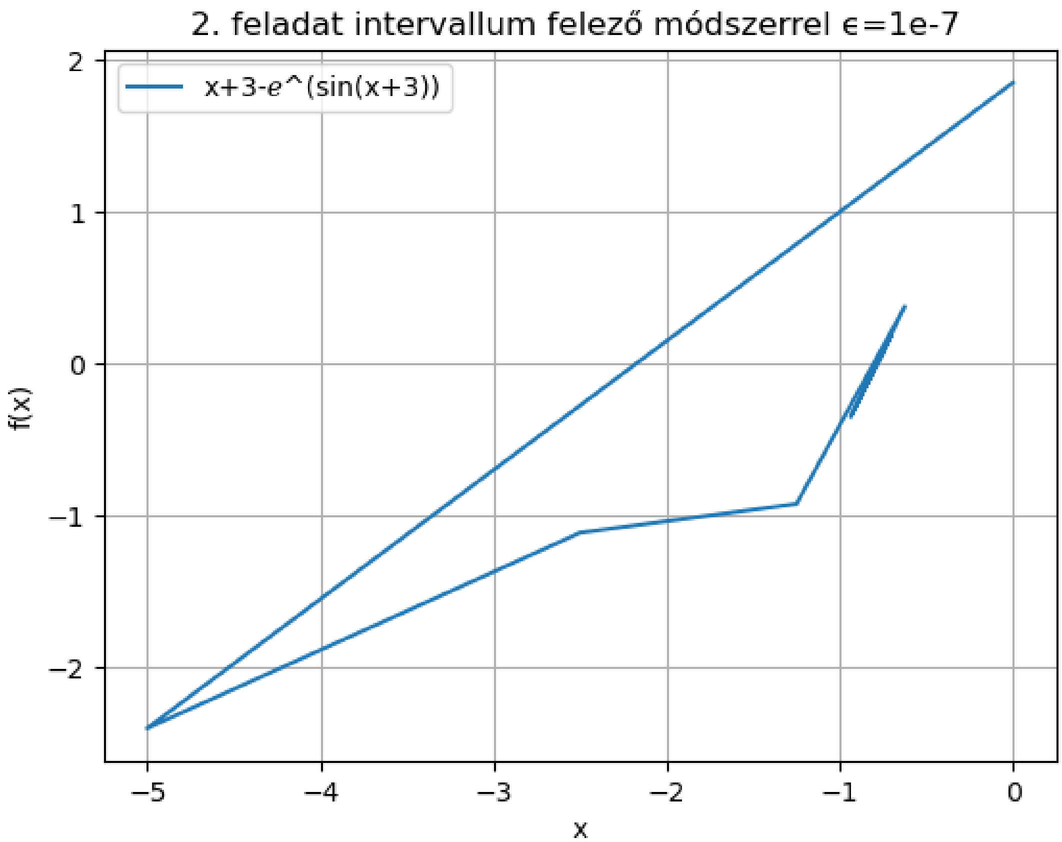
Logok:

```
x= 0.0
f(x)= 1.848437163485465

x= -5.0
f(x)= -2.402807126123528

x= -2.5
f(x)= -1.1151462964420837

x= -1.25
f(x)= -0.9250978172453692

x= -0.625
f(x)= 0.3739240078773549

x= -0.9375
f(x)= -0.35209068903115126

x= -0.78125
f(x)= -0.0008356627438894648

x= -0.703125
f(x)= 0.18456173681073818

x= -0.7421875
f(x)= 0.0912498148899128

x= -0.76171875
f(x)= 0.04503881542828969

x= -0.771484375
```

```
f(x)= 0.022057630510200266

x= -0.7763671875
f(x)= 0.010599761786235007

x= -0.77880859375
f(x)= 0.004879214512186092

x= -0.780029296875
f(x)= 0.0020210634445905207

x= -0.7806396484375
f(x)= 0.0005925217794442439

x= -0.78094482421875
f(x)= -0.00012161518258224646

x= -0.780792236328125
f(x)= 0.00023544213054549346

x= -0.7808685302734375
f(x)= 5.691068110946773e-5

x= -0.7809066772460938
f(x)= -3.23529490668939e-5

x= -0.7808876037597656
f(x)= 1.2278691452927148e-5

x= -0.7808971405029297
f(x)= -1.0037172450960696e-5

x= -0.7808923721313477
f(x)= 1.1207485899333847e-6

x= -0.7808947563171387
f(x)= -4.458214657887538e-6

x= -0.7808935642242432
f(x)= -1.668733716098103e-6

x= -0.7808929681777954
f(x)= -2.739927333905712e-7

x= -0.7808926701545715
f(x)= 4.233778856388426e-7
```

```
x= -0.7808928191661835
f(x)= 7.469256546599468e-8
```

Ábrázolva:



3 feladat

$$6x + 3 = tanh(tan(cos(-4x^2 - 3))) \tag{5}$$

[-10,10] tartományon, Húr módszerrel
Húr engyelete:

$a_0 = -10$

$$\begin{gathered} x_0 = 10 \\ f(-10) = -60 + 3 - tanh(tan(cos(-403))) = -57.01276453 \\ f(10) = 60 + 3 - tanh(tan(cos(-403))) = 62.98723547 \end{gathered} \tag{6}$$

innen

$$x_{n+1} = a - \frac{x_n - a}{f(x_n) - f(a)} \cdot f(a)$$
$$f(x_n)\epsilon \Rightarrow leall \Rightarrow x_n \quad (7)$$

Julia kód:

```
f(x)=6*x+3-tanh(tan(cos(-4*x^2-3)))
a=-10
x=10
=1e-7

while true
    global x
    x=a-f(a)*(x-a)/(f(x)-f(a))
    println("x= ",x)
    println("f(x)= ",f(x))
    println()
    if abs(f(x))<
        break
    end
end
```

logok

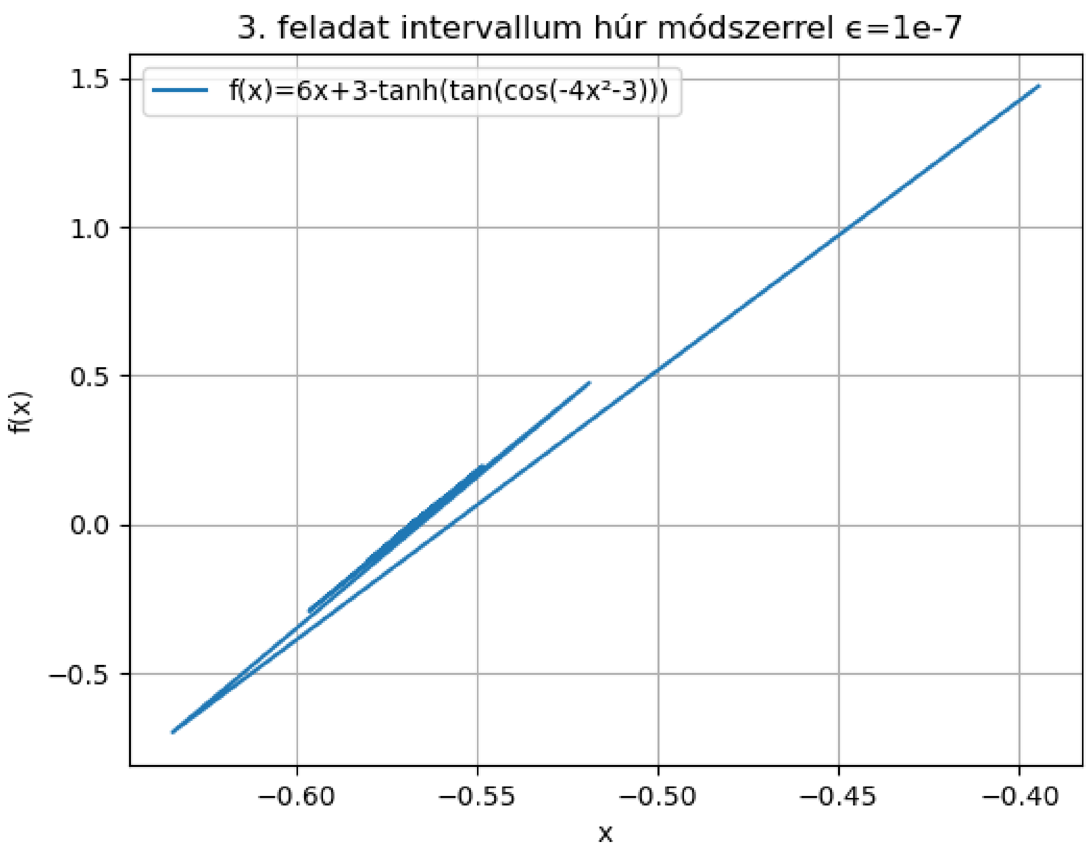
```
x= -0.3946673776396814
f(x)= 1.473222596433957

x= -0.6340843608639908
f(x)= -0.7005580211701712

x= -0.518834024842656
f(x)= 0.475196224620007

x= -0.5963702836862499
f(x)= -0.29263090201165215

x= -0.5483789490399076
f(x)= 0.19549112038083016

x= -0.5803310204365939
f(x)= -0.1252049477592092

x= -0.5598223266568301
f(x)= 0.08271525237683225
```

```
x= -0.5733517486254289
f(x)= -0.05364104222537219

x= -0.564569711701985
f(x)= 0.035233678627016984

x= -0.5703345962277488
f(x)= -0.022957362273294757

x= -0.5665768482960285
f(x)= 0.015039215196470612

x= -0.5690378817376001
f(x)= -0.009817990684844624

x= -0.5674309813764502
f(x)= 0.006424128784621008

x= -0.5684822946895061
f(x)= -0.004197205014267735

x= -0.5677953690318347
f(x)= 0.002744922160006602

x= -0.568244588846385
f(x)= -0.001794004529417248

x= -0.5679509821710642
f(x)= 0.0011730000376027339

x= -0.5681429513807323
f(x)= -0.0007667509786616344

x= -0.5680174658453687
f(x)= 0.0005012887629938789

x= -0.5680995054428433
f(x)= -0.0003276959548840219

x= -0.5680458752817863
f(x)= 0.0002142334239423893

x= -0.5680809362282222
f(x)= -0.00014004957144803099

x= -0.5680580159837305
```

```
f(x)= 9.15567689105945e-5

x= -0.5680729999652794
f(x)= -5.98535484045426e-5

x= -0.5680632044536296
f(x)= 3.912869655703366e-5

x= -0.568069608173257
f(x)= -2.557978649297965e-5

x= -0.5680654218377033
f(x)= 1.6722493205334477e-5

x= -0.5680681586059535
f(x)= -1.0932096461913066e-5

x= -0.5680663694815777
f(x)= 7.146723589424031e-6

x= -0.5680675390994843
f(x)= -4.672074870593068e-6

x= -0.5680667744773746
f(x)= 3.054309705152747e-6

x= -0.5680672743393593
f(x)= -1.9967148497945786e-6

x= -0.5680669475611424
f(x)= 1.3053267006735148e-6

x= -0.5680671611882016
f(x)= -8.533403115240645e-7

x= -0.5680670215322934
f(x)= 5.57860228067586e-7

x= -0.5680671128305423
f(x)= -3.6469388708937345e-7

x= -0.5680670531454961
f(x)= 2.3841391838530512e-7

x= -0.5680670921638242
f(x)= -1.5586001317346998e-7
```

```
x= -0.5680670666560967
f(x)= 1.0189147403583121e-7

x= -0.5680670833314441
f(x)= -6.66102302204763e-8
```

árbrázolva:



4 feladat

$$x + 2 = x^3 \tag{8}$$

[-10,10] tartományon, Newton-Raphson módszerrel

$$x_{n+1} = \frac{f(x_n)}{f'(x_n)} \tag{9}$$

ahol jelen esetben

$$
\begin{gathered}
f(x) = x + 2 - x^3 \\
f'(x) = 1 - 3x^2 \\
f"(x) = -6x \\
\\
f(-10) = -8 + 1000 = 992 \\
f'(-10) = 1 - 300 = -299 \\
f"(-10) = 60 \\
f(10) = 12 - 1000 = -988
\end{gathered}
\tag{10}
$$

alapján

$$
\begin{gathered}
sign(f(-10)) = 1 \\
sign(f'(-10)) = -1 \\
sign(f"(-10)) = 1 \\
sign(f(10)) = -1
\end{gathered}
\tag{11}
$$

Mivel *f'(-10)* és *f(-10)* valamint *f"(-10)* és *f(10)* előjelei megegyeznek ezért helyes intervallumon vagyunk, a művelet elvégezhető.

Julia kód:

```
f(x)=x+2-x^3
df(x)=1-3*x^2

x=-10
=1e-7

while true
    global x
    x=x-f(x)/df(x)
    println("x= ",x)
    println("f(x)= ",f(x))
    println()
    if abs(f(x))<
        break
    end
end
```

Logok:

```
x= -6.682274247491639
f(x)= 293.6999085590051

x= -4.473312792869828
f(x)= 87.04003516198537

x= -2.9988472242547655
f(x)= 25.970039789119255

x= -1.999201900879969
f(x)= 7.991224730944533

x= -1.2720939925431056
f(x)= 2.7864379244631436

x= -0.5492206014655907
f(x)= 1.6164480962034022

x= -17.551900926196666
f(x)= 5391.648634395187

x= -11.711775501850084
f(x)= 1596.741938527522

x= -7.821998670703618
f(x)= 472.75653358358846

x= -5.232275467424355
f(x)= 140.01019468202122

x= -3.506526815777292
f(x)= 41.608781234980135

x= -2.3470941558915683
f(x)= 12.58269777701162

x= -1.5366954902088426
f(x)= 4.092107996851008

x= -0.864126976223906
f(x)= 1.7811299712985802

x= 0.572098718429206
f(x)= 2.3848525564336365
```

```
x= -131.12099781689727
f(x)= 2.2541969650863465e6

x= -87.4156545880499
f(x)= 667901.0175286231

x= -58.27955805586903
f(x)= 197890.64076094175

x= -38.8566558177144
f(x)= 58630.4649589283

x= -25.909715855150342
f(x)= 17369.62909815448

x= -17.280731383904506
f(x)= 5145.154818537878

x= -11.53112653863139
f(x)= 1523.7267835392222

x= -7.701711255066577
f(x)= 451.13573733453586

x= -5.152188160809331
f(x)= 133.61286731020152

x= -3.4530383049803026
f(x)= 39.71917254207688

x= -2.3107118145696806
f(x)= 12.027077638259073

x= -1.5098765713570435
f(x)= 3.9322302087075345

x= -0.8364551305833876
f(x)= 1.7487767118602822

x= 0.7548296498241522
f(x)= 2.3247520206776104

x= 4.032343923946572
f(x)= -59.5327518144143
```

```
x= 2.7863516954778125
f(x)= -16.84620236002299

x= 2.030620667549976
f(x)= -4.342481805448995

x= 1.6487049095196127
f(x)= -0.8328507392434155

x= 1.5322985411023986
f(x)= -0.06544468594788322

x= 1.5214701702523892
f(x)= -0.0005377329652014318

x= 1.5213797130880249
f(x)= -3.734754283613029e-8
```

Mivel az utolsó kapott érték az intervallumon belül van így elfogadjuk.

5 feladat

$$f(x) = sin(x-5) \tag{12}$$

[-10,10] tartományon, Fixpont iterációval kiindulópont $x_0 = 5,5$

A Fixpont iteráció lényege, hogy *f(x)*-ből kifejezzük x-et x = *g(x)* formában, ahol jobb oldalba behelyettesítünk minden lépésnél.

$$\begin{gathered} f(x) = 0 \\ sin(x-5) = 0 \end{gathered} \tag{13}$$

A feladatban szereplő *sin* függvényt hatványsorként lehet kifejteni *x+g(x)* alakban, ahogy alább látható:

$$\begin{gathered} sin(x-5) = x - 5 - \frac{(x-5)^3}{3!} + \frac{(x-5)^5}{5!} - \frac{(x-5)^7}{7!} + ...x_0 = 5.5 \\ x_1 = g(x_0) = 5 + 0.5 - \frac{0.125}{6} + \frac{0.03125}{120} - \frac{0.0078125}{5040} = 5,020574467 \\ x_2 = g(x_1) = 5,000001452 \\ x_3 = g(x_2) = 5 \\ x_4 = g(x_3) = 5 \end{gathered} \tag{14}$$

Tehát ez az eredmény, próba:

$$f(5) = sin(5 - 5) = 0 \tag{15}$$

Tehát az eredmény helyes.

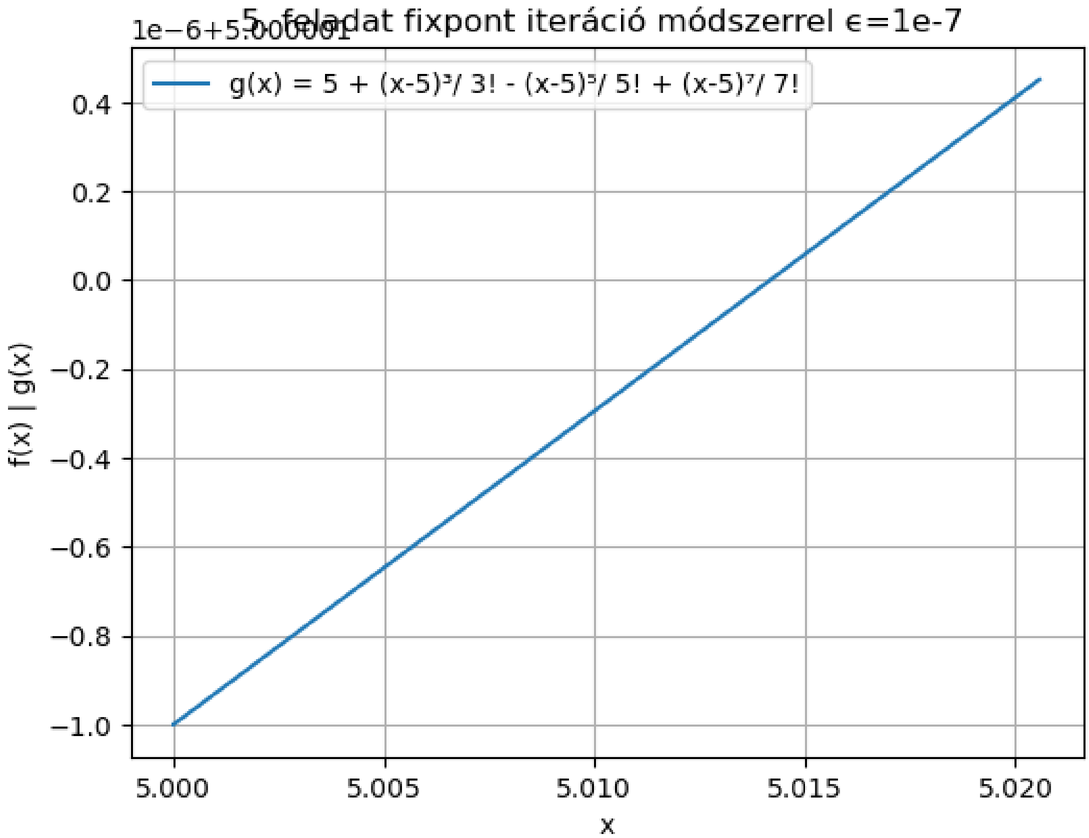
Julia kód:

```
#f(x)=sin(x-5)
#x = 5 + (x-5)^3/ 3! - (x-5)^5/ 5! + (x-5)^7/ 7!
#x=g(x)

g(x)=5+((x-5)^3)/factorial(3) - ((x-5)^5)/factorial(5) + ((x-5)^7)/factorial(7)
x=5.5
=1e-7

while true
    global x
    x=g(x)
    println("x= ",x)
    println("g(x)= ",g(x))
    println("f(x)= ",sin(x-5))
    if abs(x-g(x))<
        break
    end
end
```

logok:

```
x= 5.020574466765873
g(x)= 5.000001451527681
f(x)= 0.0205730152381915
x= 5.000001451527681
g(x)= 5.0
f(x)= 1.4515276811616805e-6
x= 5.0
g(x)= 5.0
f(x)= 0.0
```

Ábrázolva:



6 feladat

$$f(x) = cos(x - 6) = 0 \tag{16}$$

hasonlóan, kiindulópont $x_0 = 2$;

Julia kód:

```
#f(x)=cos(x-5)
#x = 1 - (x-6)2/ 2! + (x-6)4/ 4! - (x-6)6/ 6! + (x-6)8/ 8!
#x=g(x)

g(x)= 1- ((x-6)^2)/ factorial(2) + ((x-6)^4)/ factorial(4) - ((x-6)^6)/ factorial(6) + ((x-6
x=2
=1e-7

while true
    global x
```

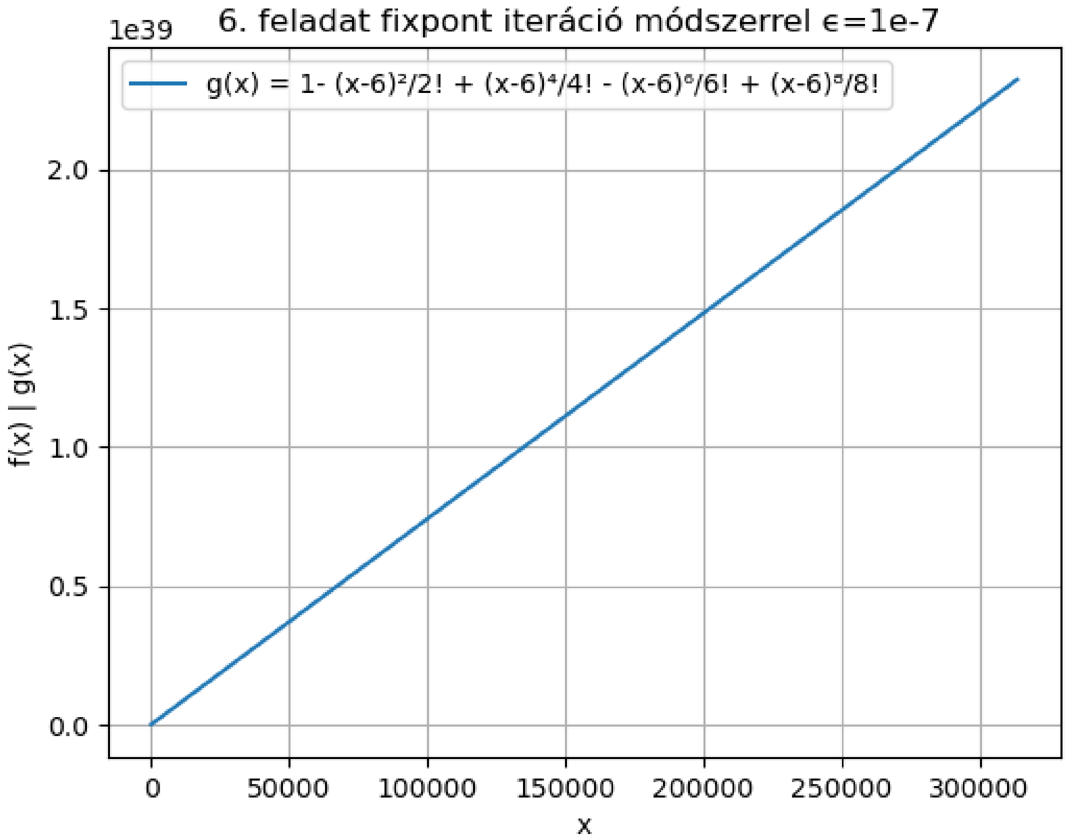
```
    x=g(x)
    println("x= ",x)
    println("g(x)= ",g(x))
    println("f(x)= ",cos(x-5))
    if abs(x-g(x))<
        break
    end
end
```

logok:

```
x= -0.39682539682539764
g(x)= 24.680488397364314
f(x)= 0.6322364612197151
x= 24.680488397364314
g(x)= 313658.34769635653
f(x)= 0.6741873636005243
x= 313658.34769635653
g(x)= 2.3230995305336858e39
f(x)= -0.9926587770008191
x= 2.3230995305336858e39
g(x)= Inf
f(x)= -0.9171030127173074
x= Inf
g(x)= NaN
```

Az eredmény végtelen, azaz értelmezhetetlen, mivel a fixpont iteráció nem minden periodikus függvény esetén alkalmazható.

Ábrázolása:


6. feladat fixpont iteráció módszerrel ϵ=1e-7
1e39
g(x) = 1- (x-6)²/2! + (x-6)⁴/4! - (x-6)⁶/6! + (x-6)⁸/8!
2.0
1.5
1.0
0.5
0.0
f(x) | g(x)
0
50000
100000
150000
200000
250000
300000
x