Intelligens Fejlesztőeszkozok - 2. órai jegyzet

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Szeptember 2022

1 feladat

$$\begin{cases} y' - \frac{2xy}{x^2+1} = x^3 + x \\ y(0) = 1 \end{cases} \tag{1}$$

I.

$$\begin{gathered} y' - \frac{2xy}{x^2+1} = 0 \\ \frac{dy}{dx} = \frac{2xy}{x^2+1} \\ \int \frac{1}{4} dy = \int \frac{2x}{x^2+1} \\ ln|y| = ln|x^2+1| + ln|c| \\ yn = c|x^2+1 \end{gathered} \tag{2}$$

II.

$$\begin{gathered} yp = k(x)(x^2+1) \\ yp = k'(x)(x^2+1) + k(x)2x \\ k'(x)(x^2+1) - k(x)2x - \frac{2xkx(x^2+1}{x^2+1} = x(x^2+1) \end{gathered} \tag{3}$$

$$
\begin{gathered}
k(x) = x \\
\int k'(x)dx = \int xdx \\
k(x) = \frac{x^2}{2} \\
yp = \frac{x^2}{2}(x^2+1) = \frac{x^4+x^2}{2}
\end{gathered}
\tag{4}
$$

III.

$$
\begin{gathered}
y = Yn + yp = (x^2+1) + \frac{x^4+x^2}{2} \\
1 = c \\
y_{imp} = (x^2+1) + \frac{x^4+x^2}{2}
\end{gathered}
\tag{5}
$$

2 feladat

$$
y" - 4y' + 4y = 12x - 4 \tag{6}
$$

I.

$$
\begin{gathered}
y" - 4y' + 4y = 0 \\
\lambda^2 - 4\lambda + 4 = 0 \\
\lambda_{1,2} = \frac{4 \pm \sqrt{16-16}}{2} = 2 \\
yh = c_1 e^{2x} + c_2 e^{2x}
\end{gathered}
\tag{7}
$$

II.

$$yp = Ax + B$$
$$y'p = Ax$$
$$y"p = 0$$

$$-4Ax + 4B = 12x - 4$$
$$A = 3$$
$$b = 2$$
$$\Rightarrow yp = 3x + 2 \quad (8)$$

III.

$$y = c_1 e^{2x} + c_2 x e^{2x} + 3x + 2$$
$$y' = c_2 e^{2x} + c_2 e^{2x} + 2c_2 x e^{2x} + 3$$
$$3 = c_1 + 2$$
$$8 = 2 + c_2 + 3 \Rightarrow c_2 = 3 \quad (9)$$

$$y_{imp} = e^{2x} + 3x e^{2x} + 3x + 2 \quad (10)$$

3 feladat

$$\begin{cases} y' = f(x, y) \\ y(x_0 = y_0 \end{cases} \quad (11)$$

3.1 Euleres megoldása

$$\begin{cases} x_{n+1} = x_n + h \\ y_{n+1} - y_n = hAn \\ y_{n+1} = y_n + hAn \\ An = f(x_n, y_n) \end{cases} \quad (12)$$

$$\begin{cases} y' = x^2 - y^2 \\ y_0 = 1 \\ h = 0,1 \end{cases} \quad (13)$$

2 lépés:

n	x_n	y_n	An	hAn
0	0	1	-1	-0,1
1	0,1	0,9	-0,8	-0,08
2	0,2	0,82		

3.2 Runge-Kutta

$$\begin{cases} x_{n+1} = x_n + h \\ \rightarrow An - f(An, yn) \\ \rightarrow \hat{y_{n+1}} = y_n + hAn \\ \rightarrow B_n = f(y_{n+1}, \hat{Y}_{n+1}) \\ \rightarrow y_{n+1} = y_n + h(\frac{A_n + B_n}{2}) \end{cases} \tag{14}$$

3.3 RKM

$$\frac{A_n + 2B_n + 2C_n + B_n}{6} \tag{15}$$