Intelligens Fejlesztőeszkozok - 4. beadandó

Sándor Burian

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1 feladat

Intervallum felező módszerrel [1]

$$5x - 4 = sin(tanh(-3x + 2)); [-10, 10] intervallumon \tag{1}$$

$$\begin{gathered}
f(x) = 5x - 4 - sin(tanh(-3x + 2)) \\
f(-10) = -50 - 4 - sin(tanh(32)) = -54 - 0,017452406 = -54,017452406 \\
f(10) = 50 - 4 - sin(tanh(-28)) = 46 - (-0,017452406) = 46,017452406 \\
\\
\Rightarrow f(a)f(b) < 0 \Rightarrow intervallumfelezes : \frac{a+b}{2} \Rightarrow \frac{10 + (-10)}{2} = 0
\end{gathered} \tag{2}$$

$$\begin{gathered}
f(0) = -4 - sin(tanh(2)) = -4 - 0,016824661 = -4,016824661 \\
\Rightarrow f(-10)f(0) > 0 \Rightarrow [0, 10] intervallumotfelezzuk \Rightarrow [0, +5]
\end{gathered} \tag{3}$$

[1] https://www.uni-miskolc.hu/ matgt/pdf/nummod/NumMod.pdf (elérés 2022-10-01) alapján

$$f(5) = 25 - 4 - sin(tanh(-13)) = 21,017452406$$
$$f(0)f(5) < 0 \Rightarrow ujintervallum[0, 2.5]$$
$$f(2.5) = 12.5 - 4 - sin = tanh(-5.5)) = 8,517451824$$
$$\Rightarrow f(0)f(2.5) < 0 ujintervallum : [0, 1.25]$$
$$f(1.25) = 6.25 - 4 - sin(tanh(-1.75)) = 2,266429363$$
$$\Rightarrow f(0)f(1.25) < 0 \Rightarrow ujintervallum[0, 0.625]$$
$$f(0.625) = 3.125 - 4 - sin = tanh(0.125)) = -0,877170368$$
$$\Rightarrow f(0.625)f(10) < 0 \Rightarrow ujintervallum[0.625, 10]felezese[0.625, 5.3125]$$
$$f(5.3125) = 26.5625 - 4 - sin(tanh(-13.9375)) = 22,017452406$$
$$\Rightarrow \frac{0.625 + 5.3125}{2} = 2.96875 \Rightarrow [0.625, 2.96875]$$
$$f(2.96875) = 14.84375 - 4 - sin(tanh(-6.90625)) = 10.861202371$$
$$\Rightarrow ujintervallum[0.625, 1.796875]$$
$$f(1.796875) = 8.984375 - 4 - sin(tanh(-3.3903625)) = 5.001787843$$
$$\Rightarrow ujintervallum[0.625, 0.91796875]$$
$$f(0.91796875) == 4.58984375 - 4 - sin(tanh(-0.314453125)) = -0.137263922$$
$$\Rightarrow ujintervallum[0.625, 0.771484375]$$
$$f(0.771484375) = 3.857421875 - 4 - sin(tanh(-0.314453125)) = -0.137263922$$

$$\Rightarrow f(0.625)f(771484375) > 0 \quad (4)$$

Mivel a két függvényérték szorzata nem negatív, ezért leállunk. A függvény zéruspontja a [0.625, 771484375]. f(0.625) = -0,877170368 és f(0.771484375) = -0.137263922 küzül az utóbbi van közelebb a 0-hoz, így látható, hogy 0.771484375 pontról van szó.

2 feladat

$$x + 3 = e^{sin(x+3)} \quad (5)$$

Intervallum felező módszerrel [-10,10] között.

$$
\begin{gathered}
f(x) = x + 3 - e^{sin(x+3)} \\
f(-10) = -7 - e^{0.121869343} = -7.885264027 \\
f(10) = 13 - e^{0.224951054} = 11.747738578 \\
\Rightarrow f(-10)f(10) < 0 \Rightarrow \frac{10+10}{2} = 0 \\
\\
f(0) = 3 - e^{0.052335956} = 1.94627031 \\
\Rightarrow [-10, 0] \rightarrow [-10, -5] \\
f(-5) = -2 - e^{0.034899497} = 2.965702467 \\
\Rightarrow ujintervalum[-10, -7.5] \\
f(-7.5) = -4.5 - e^{0.078459096} = -5.424539877 \\
\Rightarrow f(-10)f(-7.5) > 0 \Rightarrow intervallumvaltas[-7.5, -5] \rightarrow -6.25 \\
f(-6.25) = -3.25 - e^{0.108866875} = -4.146849803 \\
\Rightarrow [-6.25, -5] intervallumfelezese - 5.625 \\
f(-5.625) = -2.625 - e^{0.09801714} = -2.906633364 \\
\Rightarrow [-5.625, -5] fele - 5.3125 \\
f(-5.3125) = -2.3125 - e^{0.040349782} = -3.272953431 \\
\Rightarrow [-5.3125, -5] fele - 5.15625 \\
f(-5.15625) = -2.15625 - e^{-0.037624779} = -3.119324239 \\
\Rightarrow [-5.15625, -5] fele - 5.078125 \\
f(-5.078125) = -2.078125 - e^{-0.036262172} = -3.042512425 \\
\Rightarrow [-5.078125, -5] fele - 5.0390625
\end{gathered} \tag{6}
$$

Tehát egyértelműen tart *-5*-be a végtelenben.

3 feladat

$$6x + 3 = tanh(tan(cos(-4x^2 - 3))) \tag{7}$$

[-10,10] tartományon, Húr módszerrel

Húr engyelete:

$$y - f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$$
$$x = \frac{af(b) - bf(a)}{f(b) - f(a)} \tag{8}$$

Legyen c (c, 0) pont az OX tengely metszéspontja a húron.

Ekkor:

$$f(-10) = -60 + 3 - tanh(tan(cos(-403))) = -57.01276453$$
$$f(10) = 60 + 3 - tanh(tan(cos(-403))) = 62.98723547 \tag{9}$$

A(-10, -57.01276453)) és B(10, 62.98723547) húr és OX tengely metszéspontját keressük.

$$c = \frac{-10 * 62.98723547 - 10 * (-57.01276453)}{62.98723547 + 57.01276453} = 0.497872578$$
$$f(-0.497872578) = -6\ddot{0}.497872578 + 3 - 0.017410957 = -0.004646425 \tag{10}$$

Tehát A'(-0.497872578, -0.004646425) és B pontokat összekötő húr OX metszéspontját:

$$c' = \frac{0.497872578 * 62.987235457 - 10(-0.004646425)}{62.987235457 + 0.004646425} = -0.497098231$$
$$f(-0.497098231) = -6 * 0.497098231 + 3 - 0,017411023 = -0.000000409 \tag{11}$$

4 feladat

$$x + 2 = x^3 \tag{12}$$

[-10,10] tartományon, Newton-Raphson módszerrel

$$f(x) = x + 2 - x^3$$
$$f'(x) = 1 - 3x^2$$
$$f''(x) = -6x \tag{13}$$

$$
\begin{gathered}
f(-10) = -8 + 1000 = 992 \\
f'(-10) = 1 - 300 = -299 \\
f"(-10) = 60 \\
f(10) = 12 - 1000 = -988 \\
x1 = \frac{-10 - 992}{-299} = -10 + 3.317725753 = -6.682274247 \\
f(-6.682274247) = 293,699908494 \\
x2 = -6.682274247 - \frac{f(-6.682274247)}{f'(-6.682274247)} = -4,473312793 \\
f(-4,473312793) = 87,04003517 \\
x3 = -4,473312793 - \frac{f(-4,473312793)}{f'(-4,473312793)} = -2,998847224 \\
f(-2,998847224) = 25,970039783 \\
x4 = -2,998847224 - \frac{f(-2,998847224)}{f'(-2,998847224)} = -1,999201901 \\
f(-1,999201901) = 7,991224732 \\
x5 = -1,999201901 - \frac{f(-1,999201901)}{f'(-1,999201901)} = -1,272093993 \\
f(-1,272093993) = 2,786437926 \\
x6 = -1,272093993 - \frac{f(-1,272093993)}{f'(-1,272093993)} = -0,549220602 \\
f(-0,549220602) = 1,616448096 \\
x7 = -0,549220602 - \frac{f(-0,549220602)}{f'(-0,549220602)} = -1,397781052 \\
f(-1,397781052) = 3,333192202
\end{gathered} \tag{14}
$$

Az előző pontosabb, így leállási feltétel teljesül f(-0,549220602)=1,616448096

5 feladat

$$f(x) = sin(x - 5) \tag{15}$$

[-10,10] tartományon, Fixpont iterációval

$f(x) = 0$

$$sin(x-5) = 0$$
$$sin(x)cos(5) - cos(x)sin(5) = 0 \tag{16}$$

jelöljük:

$y = sin(x)$

$$sin^2(x) + cos^2(x) = 1 \Rightarrow cos^2(x) = 1 - y^2 \tag{17}$$

$sin(x)cos(5) = cos(x)sin(5)$

$$y^2cos^2(5) = (1-y^2)sin^2(5)$$
$$y^2cos^2(5) + y^2sin^2(5) = sin^2(5) \leftarrow sin^a + cos^2(a) = 1$$
$$y^2(cos^2(5) + sin^2(5)) = sin^2(5)$$
$$y^2 = sin^2(5) \leftarrow y = sin(x)$$
$$sin(x) = sin(5) \Rightarrow x = 5 \tag{18}$$

a függvény x+g(x) alakban:

$$sin(x-5) = x - 5 - \frac{(x-5)^3}{3!} + \frac{(x-5)^5}{5!} - \frac{(x-5)^7}{7!} + ...x_0 = 5.5$$
$$x_1 = 0.5 - \frac{0.125}{6} + \frac{0.03125}{120} - \frac{0.0078125}{5040} = 0.479425533$$
$$x_2 = 0.479425533 - 5 - \frac{(0.479425533-5)^3}{6} + \frac{(0.479425533-5)^5}{120} - \frac{0.479425533-5)^7}{5040} = 2.798730405$$
$$\Rightarrow x_2 - 5 = -2.201269595$$
$$x_3 = 2,798730405 - 5 - \frac{-2,201269595^3}{6} + \frac{-2,201269595^5}{120} - \frac{-2,201269595^7}{5040} = -0,804547209$$
$$\Rightarrow x_3 - 5 = -5,804547209 \tag{19}$$

6 feladat

$$f(x) = cos(x-6) = 0 \tag{20}$$

$$
\begin{aligned}
&cos(x)cos(6) + sin(x)sin(6) = 0 \\
&\qquad cos(x)cos(6) = -sin(x)sin(6) = 0 \\
&\qquad cos^2(x)cos^2(6) = sin^2(x)sin^2(6) \\
&\qquad (1 - sin^2(x))cos^2(6) = sin^2(x)sin^2(6) \\
&\qquad sin^2(x)(sin^2(x)cos^2(6) = 1) \\
&\qquad sin^2(x) = 1 \\
&\qquad x = \pm 90 \qquad (21)
\end{aligned}
$$

$$
\begin{cases} 5x - 4 = sin(tanh(-3x + 2)) \end{cases} \qquad (22)
$$