

Intelligens Fejlesztőeszközök - 4. beadandó

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1 feladat

Intervallum felező módszerrel

$$5x - 4 = \sin(\tanh(-3x + 2)); [-10, 10] \text{intervallumon} \quad (1)$$

$$f(x) = 5x - 4 - \sin(\tanh(-3x + 2))$$

$$a_1 = -10$$

$$b_1 = 10$$

$$f(a_1) = f(-10) = -50 - 4 - \sin(\tanh(32)) = -54,017452406 = -54,017452406$$

$$f(b_1) = f(10) = 50 - 4 - \sin(\tanh(-28)) = 46 - (-0,017452406) = 46,017452406$$

$$\Rightarrow f(a)f(b) < 0 \Rightarrow \text{intervallumfelezés} : \frac{a+b}{2} \Rightarrow \frac{10 + (-10)}{2} = 0 \quad (2)$$

$$\begin{aligned}
f(0) &= -4 - \sin(\tanh(2)) = -4 - 0,016824661 = -4,016824661 \\
a_2 &= (a_1 + b_1)/2 = 0; f(a_2) < 0, b_2 = b_1 = 10 \\
f(b_2) &> 0 \\
f\left(\frac{a_2 + b_2}{2}\right) &= f(5) = 25 - 4 - \sin(\tanh(-13)) = 21,017452406 \\
b_3 &= 5 \\
f(b_3) &> 0 \\
a_3 &= a_2 = 0 \\
f(a_3) &< 0 \\
f(0)f(5) &< 0 \\
f(2.5) &= 12.5 - 4 - \sin(\tanh(-5.5)) = 8,517451824 \\
&\dots \\
f\left(\frac{a_2 3 + b_2 3}{2}\right) &= f(0,798685552) = 0,000000094 \\
&\Rightarrow x = 0,798685552
\end{aligned}$$

(3)

Julia kódként:

```

f(x)=5*x-4-sin(tanh(-3*x+2))
a=-10
b=10
=1e-7 #(10^(-7))

while true
    global a,b
    c=(a+b)/2
    println("x= ",c)
    println("f(x)= ",f(c))
    println()
    if sign(f(c))==sign(f(a))
        a=c
    end
    if sign(f(c))==sign(f(b))
        b=c
    end
    if abs(f(c))<
        break
    end
end
end

```

Logok:

x= 0.0
f(x)= -4.821494815516438

x= 5.0
f(x)= 21.841470984802374

x= 2.5
f(x)= 9.341452936704993

x= 1.25
f(x)= 3.0583686122902405

x= 0.625
f(x)= -0.9990327572022525

x= 0.9375
f(x)= 1.3092437102511458

x= 0.78125
f(x)= 0.2310697309851627

x= 0.703125
f(x)= -0.37564942969465204

x= 0.7421875
f(x)= -0.06813638549908974

x= 0.76171875
f(x)= 0.08270992936515631

x= 0.751953125
f(x)= 0.007575155275291817

x= 0.7470703125
f(x)= -0.030211616196670926

x= 0.74951171875
f(x)= -0.011300580891388218

x= 0.750732421875
f(x)= -0.001858251535921368

x= 0.7513427734375
f(x)= 0.0028595732281952724

x= 0.75103759765625
f(x)= 0.0005009404338206236

x= 0.750885009765625
f(x)= -0.0006785857483954938

x= 0.7509613037109375
f(x)= -8.880519482826199e-5

x= 0.7509994506835938
f(x)= 0.0002060719865843441

x= 0.7509803771972656
f(x)= 5.863448746595834e-5

x= 0.7509708404541016
f(x)= -1.5085080807220042e-5

x= 0.7509756088256836
f(x)= 2.1774771550742145e-5

x= 0.7509732246398926
f(x)= 3.3448624267573557e-6

x= 0.7509720325469971
f(x)= -5.8701049265030836e-6

x= 0.7509726285934448
f(x)= -1.2626201839338602e-6

x= 0.7509729266166687
f(x)= 1.0411213878791514e-6

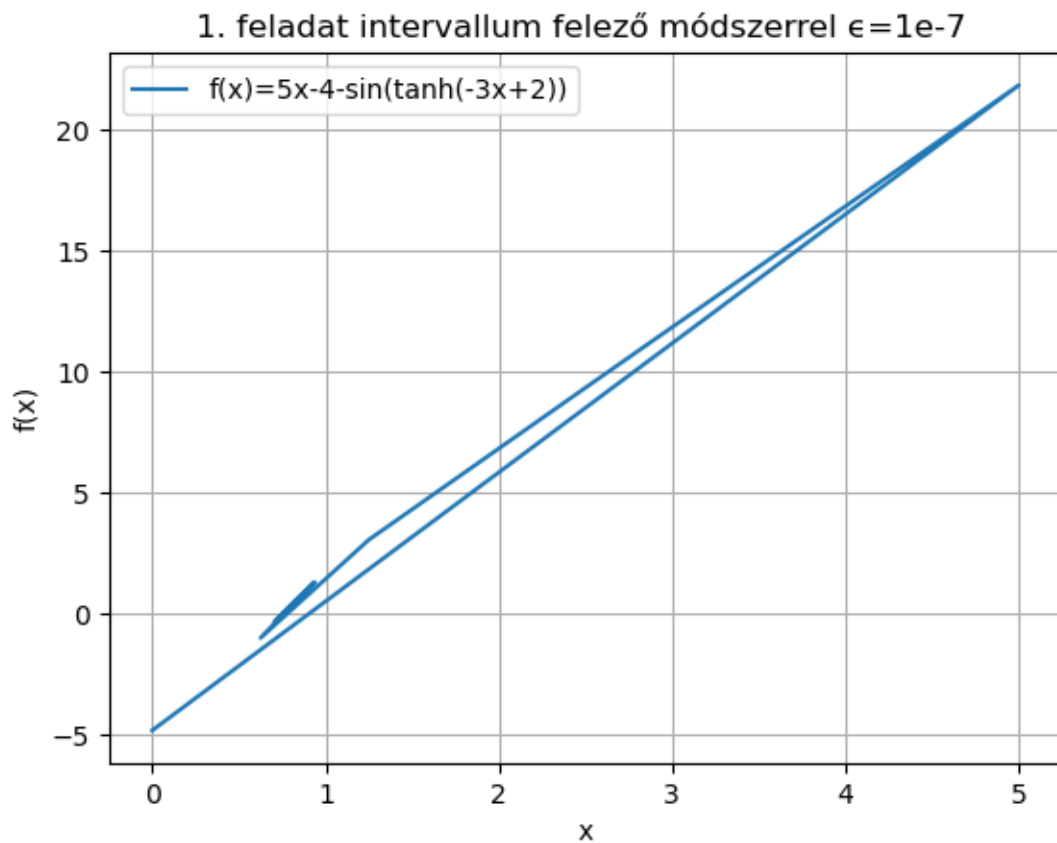
x= 0.7509727776050568
f(x)= -1.107493314278507e-7

x= 0.7509728521108627
f(x)= 4.6518604487899573e-7

x= 0.7509728148579597
f(x)= 1.7721836090278664e-7

x= 0.7509727962315083
f(x)= 3.323451580605763e-8

Ábrázolva:



2 feladat

$$x + 3 = e^{\sin(x+3)} \quad (4)$$

Intervallum felező módszerrel $[-10, 10]$ között.

Matematikai levezetés hasonló.

Julia kód:

```
f(x)=x+3-^(sin(x+3))
a=-10
b=10
=1e-7 #(10^(-7))

while true
    global a,b
    c=(a+b)/2
    println("x= ",c)
```

```

        println("f(x)= ",f(c))
        println()
        if sign(f(c))==sign(f(a))
            a=c
        end
        if sign(f(c))==sign(f(b))
            b=c
        end
        if abs(f(c))<
            break
        end
    end
end

```

Logok:

```

x= 0.0
f(x)= 1.848437163485465

x= -5.0
f(x)= -2.402807126123528

x= -2.5
f(x)= -1.1151462964420837

x= -1.25
f(x)= -0.9250978172453692

x= -0.625
f(x)= 0.3739240078773549

x= -0.9375
f(x)= -0.35209068903115126

x= -0.78125
f(x)= -0.0008356627438894648

x= -0.703125
f(x)= 0.18456173681073818

x= -0.7421875
f(x)= 0.0912498148899128

x= -0.76171875
f(x)= 0.04503881542828969

x= -0.771484375

```

$f(x) = 0.022057630510200266$

$x = -0.7763671875$
 $f(x) = 0.010599761786235007$

$x = -0.77880859375$
 $f(x) = 0.004879214512186092$

$x = -0.780029296875$
 $f(x) = 0.0020210634445905207$

$x = -0.7806396484375$
 $f(x) = 0.0005925217794442439$

$x = -0.78094482421875$
 $f(x) = -0.00012161518258224646$

$x = -0.780792236328125$
 $f(x) = 0.00023544213054549346$

$x = -0.7808685302734375$
 $f(x) = 5.691068110946773e-5$

$x = -0.7809066772460938$
 $f(x) = -3.23529490668939e-5$

$x = -0.7808876037597656$
 $f(x) = 1.2278691452927148e-5$

$x = -0.7808971405029297$
 $f(x) = -1.0037172450960696e-5$

$x = -0.7808923721313477$
 $f(x) = 1.1207485899333847e-6$

$x = -0.7808947563171387$
 $f(x) = -4.458214657887538e-6$

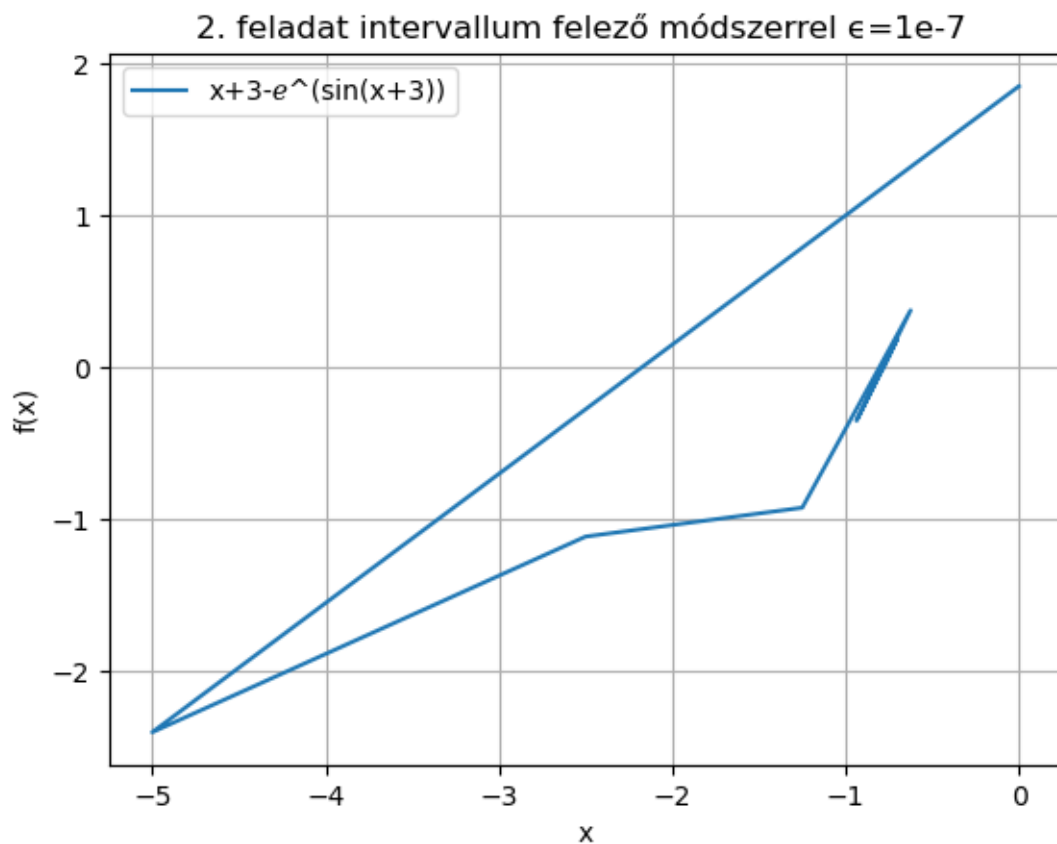
$x = -0.7808935642242432$
 $f(x) = -1.668733716098103e-6$

$x = -0.7808929681777954$
 $f(x) = -2.739927333905712e-7$

$x = -0.7808926701545715$
 $f(x) = 4.233778856388426e-7$

x= -0.7808928191661835
f(x)= 7.469256546599468e-8

Ábrázolva:



3 feladat

$$6x + 3 = \tanh(\tan(\cos(-4x^2 - 3))) \quad (5)$$

[-10,10] tartományon, Húr módszerrel

Húr egyenlete:

$$a_0 = -10$$

$$x_0 = 10$$

$$f(-10) = -60 + 3 - \tanh(\tan(\cos(-403))) = -57.01276453$$

$$f(10) = 60 + 3 - \tanh(\tan(\cos(-403))) = 62.98723547$$

(6)

innen

$$x_{n+1} = a - \frac{x_n - a}{f(x_n) - f(a)} f(a)$$

$$f(x_n)\epsilon \Rightarrow leall \Rightarrow x_n \quad (7)$$

Julia kód:

```
f(x)=6*x+3-tanh(tan(cos(-4*x^2-3)))  
a=-10  
x=10  
=1e-7
```

```
while true  
    global x  
    x=a-f(a)*(x-a)/(f(x)-f(a))  
    println("x= ",x)  
    println("f(x)= ",f(x))  
    println()  
    if abs(f(x))<  
        break  
    end  
end
```

logok

```
x= -0.3946673776396814  
f(x)= 1.473222596433957
```

```
x= -0.6340843608639908  
f(x)= -0.7005580211701712
```

```
x= -0.518834024842656  
f(x)= 0.475196224620007
```

```
x= -0.5963702836862499  
f(x)= -0.29263090201165215
```

```
x= -0.5483789490399076  
f(x)= 0.19549112038083016
```

```
x= -0.5803310204365939  
f(x)= -0.1252049477592092
```

```
x= -0.5598223266568301  
f(x)= 0.08271525237683225
```

x= -0.5733517486254289
f(x)= -0.05364104222537219

x= -0.564569711701985
f(x)= 0.035233678627016984

x= -0.5703345962277488
f(x)= -0.022957362273294757

x= -0.5665768482960285
f(x)= 0.015039215196470612

x= -0.5690378817376001
f(x)= -0.009817990684844624

x= -0.5674309813764502
f(x)= 0.006424128784621008

x= -0.5684822946895061
f(x)= -0.004197205014267735

x= -0.5677953690318347
f(x)= 0.002744922160006602

x= -0.568244588846385
f(x)= -0.001794004529417248

x= -0.5679509821710642
f(x)= 0.0011730000376027339

x= -0.5681429513807323
f(x)= -0.0007667509786616344

x= -0.5680174658453687
f(x)= 0.0005012887629938789

x= -0.5680995054428433
f(x)= -0.0003276959548840219

x= -0.5680458752817863
f(x)= 0.0002142334239423893

x= -0.5680809362282222
f(x)= -0.00014004957144803099

x= -0.5680580159837305

$$f(x) = 9.15567689105945e-5$$

$$x = -0.5680729999652794$$

$$f(x) = -5.98535484045426e-5$$

$$x = -0.5680632044536296$$

$$f(x) = 3.912869655703366e-5$$

$$x = -0.568069608173257$$

$$f(x) = -2.557978649297965e-5$$

$$x = -0.5680654218377033$$

$$f(x) = 1.6722493205334477e-5$$

$$x = -0.5680681586059535$$

$$f(x) = -1.0932096461913066e-5$$

$$x = -0.5680663694815777$$

$$f(x) = 7.146723589424031e-6$$

$$x = -0.5680675390994843$$

$$f(x) = -4.672074870593068e-6$$

$$x = -0.5680667744773746$$

$$f(x) = 3.054309705152747e-6$$

$$x = -0.5680672743393593$$

$$f(x) = -1.9967148497945786e-6$$

$$x = -0.5680669475611424$$

$$f(x) = 1.3053267006735148e-6$$

$$x = -0.5680671611882016$$

$$f(x) = -8.533403115240645e-7$$

$$x = -0.5680670215322934$$

$$f(x) = 5.57860228067586e-7$$

$$x = -0.5680671128305423$$

$$f(x) = -3.6469388708937345e-7$$

$$x = -0.5680670531454961$$

$$f(x) = 2.3841391838530512e-7$$

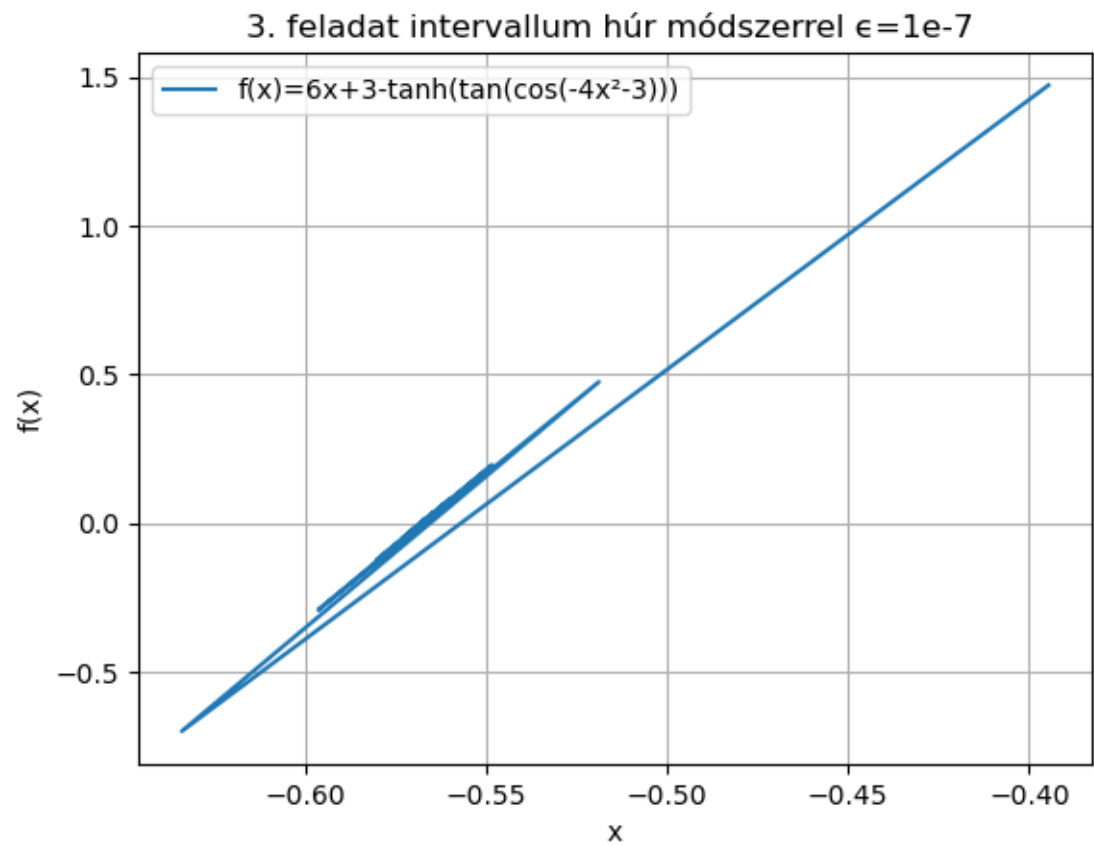
$$x = -0.5680670921638242$$

$$f(x) = -1.5586001317346998e-7$$

x= -0.5680670666560967
f(x)= 1.0189147403583121e-7

x= -0.5680670833314441
f(x)= -6.66102302204763e-8

ábrázolva:



4 feladat

$$x + 2 = x^3 \quad (8)$$

$[-10, 10]$ tartományon, Newton-Raphson módszerrel

$$x_{n+1} = \frac{f(x_n)}{f'(x_n)} \quad (9)$$

ahol jelen esetben

$$\begin{aligned}f(x) &= x + 2 - x^3 \\f'(x) &= 1 - 3x^2 \\f''(x) &= -6x\end{aligned}$$

$$f(-10) = -8 + 1000 = 992$$

$$f'(-10) = 1 - 300 = -299$$

$$f''(-10) = 60$$

$$f(10) = 12 - 1000 = -988$$

(10)

alapján

$$\begin{aligned}\text{sign}(f(-10)) &= 1 \\ \text{sign}(f'(-10)) &= -1 \\ \text{sign}(f''(-10)) &= 1 \\ \text{sign}(f(10)) &= -1\end{aligned}$$

(11)

Mivel $f'(-10)$ és $f(-10)$ valamint $f''(-10)$ és $f(10)$ előjelei megegyeznek ezért helyes intervallumon vagyunk, a művelet elvégezhető.

Julia kód:

```
f(x)=x+2-x^3
df(x)=1-3*x^2
```

```
x=-10
=1e-7
```

```
while true
    global x
    x=x-f(x)/df(x)
    println("x= ",x)
    println("f(x)= ",f(x))
    println()
    if abs(f(x))<
        break
    end
end
end
```

Logok:

x= -6.682274247491639
f(x)= 293.6999085590051

x= -4.473312792869828
f(x)= 87.04003516198537

x= -2.9988472242547655
f(x)= 25.970039789119255

x= -1.999201900879969
f(x)= 7.991224730944533

x= -1.2720939925431056
f(x)= 2.7864379244631436

x= -0.5492206014655907
f(x)= 1.6164480962034022

x= -17.551900926196666
f(x)= 5391.648634395187

x= -11.711775501850084
f(x)= 1596.741938527522

x= -7.821998670703618
f(x)= 472.75653358358846

x= -5.232275467424355
f(x)= 140.01019468202122

x= -3.506526815777292
f(x)= 41.608781234980135

x= -2.3470941558915683
f(x)= 12.58269777701162

x= -1.5366954902088426
f(x)= 4.092107996851008

x= -0.864126976223906
f(x)= 1.7811299712985802

x= 0.572098718429206
f(x)= 2.3848525564336365

x= -131.12099781689727
f(x)= 2.2541969650863465e6

x= -87.4156545880499
f(x)= 667901.0175286231

x= -58.27955805586903
f(x)= 197890.64076094175

x= -38.8566558177144
f(x)= 58630.4649589283

x= -25.909715855150342
f(x)= 17369.62909815448

x= -17.280731383904506
f(x)= 5145.154818537878

x= -11.53112653863139
f(x)= 1523.7267835392222

x= -7.701711255066577
f(x)= 451.13573733453586

x= -5.152188160809331
f(x)= 133.61286731020152

x= -3.4530383049803026
f(x)= 39.71917254207688

x= -2.3107118145696806
f(x)= 12.027077638259073

x= -1.5098765713570435
f(x)= 3.9322302087075345

x= -0.8364551305833876
f(x)= 1.7487767118602822

x= 0.7548296498241522
f(x)= 2.3247520206776104

x= 4.032343923946572
f(x)= -59.5327518144143

x= 2.7863516954778125
f(x)= -16.84620236002299

x= 2.030620667549976
f(x)= -4.342481805448995

x= 1.6487049095196127
f(x)= -0.8328507392434155

x= 1.5322985411023986
f(x)= -0.06544468594788322

x= 1.5214701702523892
f(x)= -0.0005377329652014318

x= 1.5213797130880249
f(x)= -3.734754283613029e-8

Mivel az utolsó kapott érték az intervallumon belül van így elfogadjuk.

5 feladat

$$f(x) = \sin(x - 5) \quad (12)$$

[-10,10] tartományon, Fixpont iterációval kiindulópont $x_0 = 5,5$

$$\begin{aligned} f(x) &= 0 \\ \sin(x - 5) &= 0 \end{aligned}$$

$$g(x) = \sin(x - 5) \quad (13)$$

$$x_0 = 5.5, x_1 = g(x_0) = \sin(0.5) = 0.479425538604203, x_2 = g(x_1) = \sin(0.479425538604203 - 5) = 0.981659930$$

Julia kód:

```
g(x)=sin(x-5) + x
x=5.5

=1e-7
```



```

while true
    global x
    x=g(x)
    println("x= ",x)
    println("g(x)= ",g(x))
    println("f(x)= ",sin(x-5))
    if abs(x-g(x))<
        break
    end
end
end

```

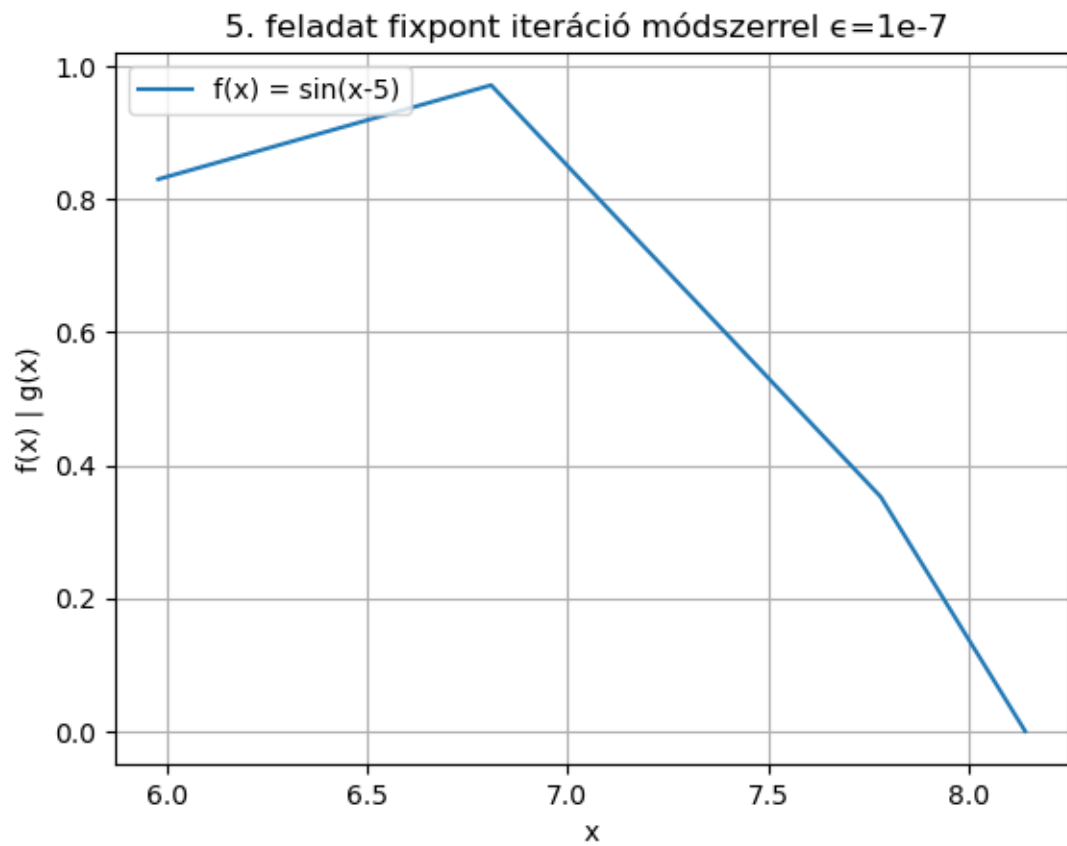
logok:

```

x= 5.979425538604203
g(x)= 6.809602784129557
f(x)= 0.830177245525354
x= 6.809602784129557
g(x)= 7.781223775283726
f(x)= 0.971620991154169
x= 7.781223775283726
g(x)= 8.13384321661877
f(x)= 0.35261944133504336
x= 8.13384321661877
g(x)= 8.141592576026204
f(x)= 0.007749359407434462
x= 8.141592576026204
g(x)= 8.141592653589793
f(x)= 7.756358945155285e-8

```

Ábrázolva:



6 feladat

$$f(x) = \cos(x - 6) = 0 \quad (15)$$

hasonlóan, kiindulópont $x_0 = 2$;

$$\begin{aligned} x_0 &= 2x_1 = g(x_0) = \cos(-4) = -0.653643620863612 \\ x_2 &= g(x_1) = \cos(-0.653643620863612 - 6) = 0.932161514392666 \\ x_3 &= g(x_2) = \cos(0.932161514392666 - 6) = 0.348011807099133 \end{aligned}$$

...

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Julia kód:

```
#fixpont iteráció
#f(x)=cos(x-5)
```

```

#x=g(x)

g(x)= cos(x-6)+x

x=2
=1e-7

while true
    global x
    x=g(x)
    println("x= ",x)
    println("g(x)= ",g(x))
    println("f(x)= ",cos(x-6))
    if abs(x-g(x))<
        break
    end
end
end

```

logok:

```

x= 1.3463563791363882
g(x)= 1.2876448023272073
f(x)= -0.05871157680918083
x= 1.2876448023272073
g(x)= 1.2876110196153163
f(x)= -3.37827118910034e-5
x= 1.2876110196153163
g(x)= 1.2876110196153099
f(x)= -6.400945957772979e-15

```

Ábrázolása:

