Intelligens Fejlesztőeszkozok - 6. beadandó

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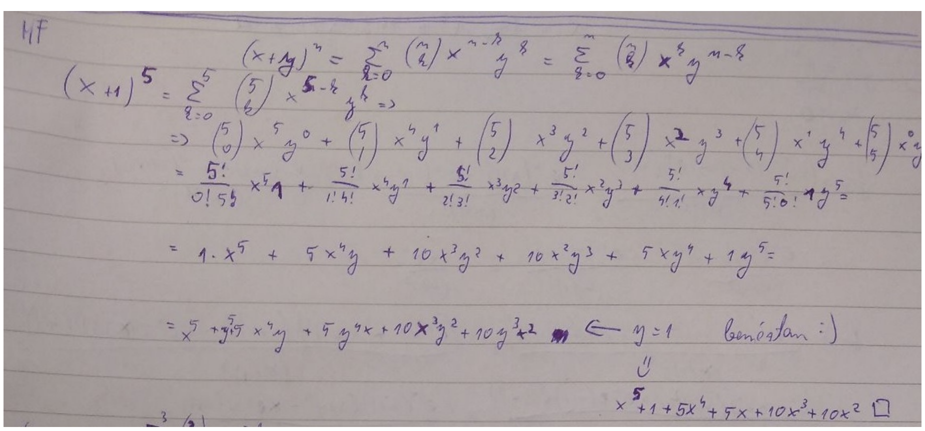
1 feladat

1.1 egyenlet megoldása Binomiális tétellel

$$(x+1)^5 \tag{1}$$

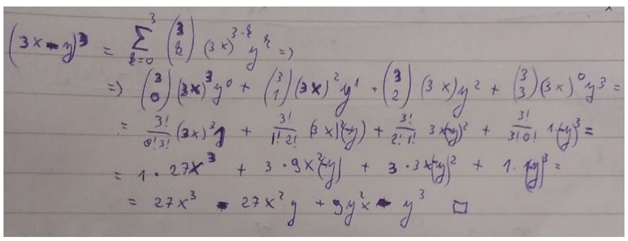
$$\begin{aligned}
&\sum_{k=0}^{5}\binom{5}{k}x^{5-k}1^k = \\
&= \binom{5}{0}x^5 1^0 + \binom{5}{1}x^4 1^1 + \binom{5}{2}x^3 1^2 + \binom{5}{3}x^2 1^3 + \binom{5}{4}x^1 1^4 + \binom{5}{5}x^0 1^5 = \\
&= 1x^5 + 5x^4 1 + 10x^3 1^2 + 10x^2 1^3 + 5x 1^4 + 1 1^5 = \\
&= x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1
\end{aligned} \tag{2}$$



1.2 egyenlet megoldása Binomiális tétellel

$$(3x-y)^3 \tag{3}$$

$$\sum_{k=0}^{3}\binom{3}{k}3x^{3-k}(-y)^k =$$
$$= \binom{3}{0}(3x)^3(-y)^0 + \binom{3}{1}(3x)^2(-y)^1 + \binom{3}{2}(3x)(-y)^2 + \binom{3}{3}(3x)^0(-y)^3 =$$
$$= 127x^3 + 39x^2(-y) + 33x(-y)^2 + 1(-y)^3 =$$
$$= 27x^3 - 27x^2y + 9y^2x - y^3 \quad (4)$$


$(3x - y)^3 = \sum_{k=0}^{3} \binom{3}{k} (3x)^{3-k} y^k \Rightarrow$

$\Rightarrow \binom{3}{0}(3x)^3 y^0 + \binom{3}{1}(3x)^2 y^1 + \binom{3}{2}(3x) y^2 + \binom{3}{3}(3x)^0 y^3 =$

$= \frac{3!}{0!3!}(3x)^3 + \frac{3!}{1!2!}(3x)^2(-y) + \frac{3!}{2!1!}3x(-y)^2 + \frac{3!}{3!0!}1(-y)^3 =$

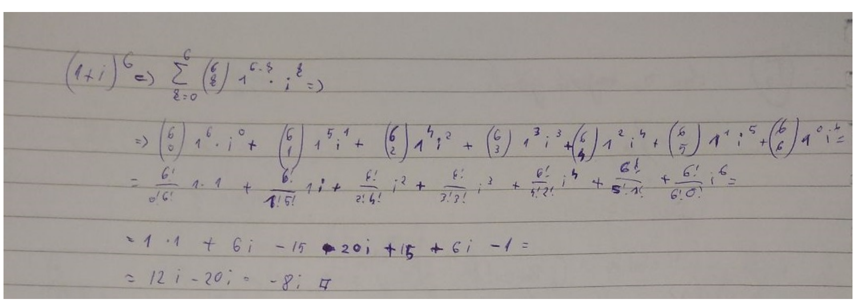
$= 1 \cdot 27x^3 + 3 \cdot 9x^2(-y) + 3 \cdot 3x(-y)^2 + 1 \cdot (-y)^3 =$

$= 27x^3 - 27x^2y + 9y^2x - y^3$ □

1.3 egyenlet megoldása Binomiális tétellel

$$(1+i)^6 \quad (5)$$

$$\sum_{k=0}^{6}\binom{6}{k}1^{6-k}i^k =$$
$$= \binom{6}{0}1^6i^0 + \binom{6}{1}1^5i^1 + \binom{6}{2}1^4i^2 + \binom{6}{3}1^3i^3 + \binom{6}{4}1^2i^4 + \binom{6}{5}1^1i^5 + \binom{6}{6}1^0i^6 =$$
$$= 1 + 6i - 15 - 20i + 15 + 6i - 1 =$$
$$= -8i \quad (6)$$


$(1+i)^6 \Rightarrow \sum_{k=0}^{6} \binom{6}{k} 1^{6-k} i^k \Rightarrow$

$\Rightarrow \binom{6}{0} 1^6 i^0 + \binom{6}{1} 1^5 i + \binom{6}{2} 1^4 i^2 + \binom{6}{3} 1^3 i^3 + \binom{6}{4} 1^2 i^4 + \binom{6}{5} 1^1 i^5 + \binom{6}{6} 1^0 i^6 =$

$= \frac{6!}{0!6!} 1 \cdot 1 + \frac{6!}{1!5!} 1 i + \frac{6!}{2!4!} i^2 + \frac{6!}{3!3!} i^3 + \frac{6!}{4!2!} i^4 + \frac{6!}{5!1!} + \frac{6!}{6!0!} i^6 =$

$= 1 \cdot 1 + 6i - 15 - 20i + 15 + 6i - 1 =$

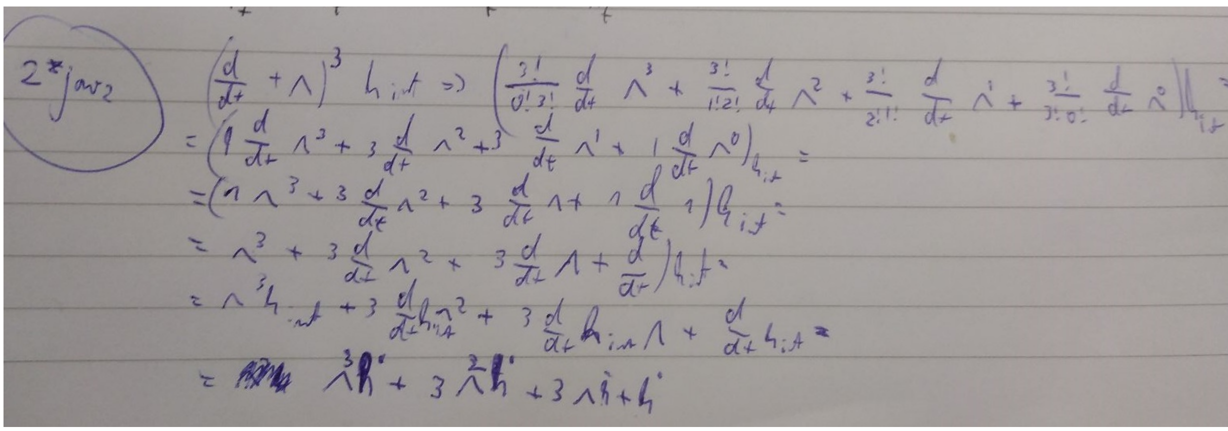
$= 12i - 20i = -8i$ □

2 feladat

$$\left(\frac{d}{dt} + \Lambda\right)^3 h_{int} \quad (7)$$

$\dot{h_{int}} + \Lambda)^3 h_{int} \Rightarrow$

$$
\begin{aligned}
&= (1\dot{h}\Lambda)^3 + 3\dot{h}\Lambda^2 + 3\ddot{h}\Lambda^1 + 1\dot{h}\Lambda^0)h_{int} = \\
&= (1\Lambda)^3 + 3\dot{h}\Lambda^2 + 3\ddot{h}\Lambda + 1\,\dddot{h}\,1)h_{int} = \\
&= \Lambda^3 h_{int} + 3\Lambda^2\dot{h} + 3\ddot{h}\Lambda + \ddot{h} = \\
&= \Lambda^3 h_{int} + 3\Lambda^2 + 3\dot{h}\Lambda + \ddot{h}
\end{aligned} \tag{8}
$$



2* jav2

$$\left(\frac{d}{dt} + \Lambda\right)^3 h_{int} \Rightarrow \left(\frac{3!}{0!3!}\frac{d}{dt}\Lambda^3 + \frac{3!}{1!2!}\frac{d}{dt}\Lambda^2 + \frac{3!}{2!1!}\frac{d}{dt}\Lambda^1 + \frac{3!}{3!0!}\frac{d}{dt}\Lambda^0\right)h_{int} =$$

$$= \left(1\frac{d}{dt}\Lambda^3 + 3\frac{d}{dt}\Lambda^2 + 3\frac{d}{dt}\Lambda^1 + 1\frac{d}{dt}\Lambda^0\right)h_{int} =$$

$$= \left(1\Lambda^3 + 3\frac{d}{dt}\Lambda^2 + 3\frac{d}{dt}\Lambda + 1\frac{d}{dt}1\right)h_{int} =$$

$$= \Lambda^3 + 3\frac{d}{dt}\Lambda^2 + 3\frac{d}{dt}\Lambda + \frac{d}{dt}\Big)h_{int} =$$

$$= \Lambda^3 h_{int} + 3\frac{d}{dt}h_{int}\Lambda^2 + 3\frac{d}{dt}h_{int}\Lambda + \frac{d}{dt}h_{int} =$$

$$= \Lambda^3\dot{h} + 3\Lambda^2\dot{h} + 3\Lambda\dot{h} + \dot{h}$$

3 feladat

(3) $f(x) = \frac{e^x}{x+1}$

$x_0 = 0$

$$f'(x) = \frac{(e^x)' \cdot (x+1) - e^x (x+1)'}{(x+1)^2} = \frac{e^x(x+1) - e^x(x' + 1')}{(x+1)^2} = \frac{e^x(x+1) - e^x(1+0)}{(x+1)^2} =$$

$$= \frac{e^x(x+1) - e^x}{(x+1)^2} = \frac{e^x((x+1) - 1)}{(x+1)^2} = \frac{xe^x}{(x+1)^2}$$

$$f''(x) = \frac{(xe^x)' \cdot (x+1)^2 - xe^x((x+1)^2)'}{(x+1)^{2^2}} = \frac{(x' e^x + x(e^x)')(x+1)^2 - xe^x\, 2(x+1)\,(x+1)'}{(x+1)^4} =$$

$$= \frac{(e^x + xe^x)(x+1)^2 - xe^x\, 2(x+1)(x' + 1')}{(x+1)^4} = \frac{e^x(x+1)(x+1)^2 - xe^x 2(x+1)}{(x+1)^4} =$$

$$= \frac{(x+1)^3 e^x - 2xe^x(x+1)}{(x+1)^4} = \frac{e^x((x+1)^3 - 2x(x+1))}{(x+1)^4}$$

Taylor: $\sum_{k=0}^{n} \frac{f^{k}(x_0)}{k!} (x - x_0)^k$

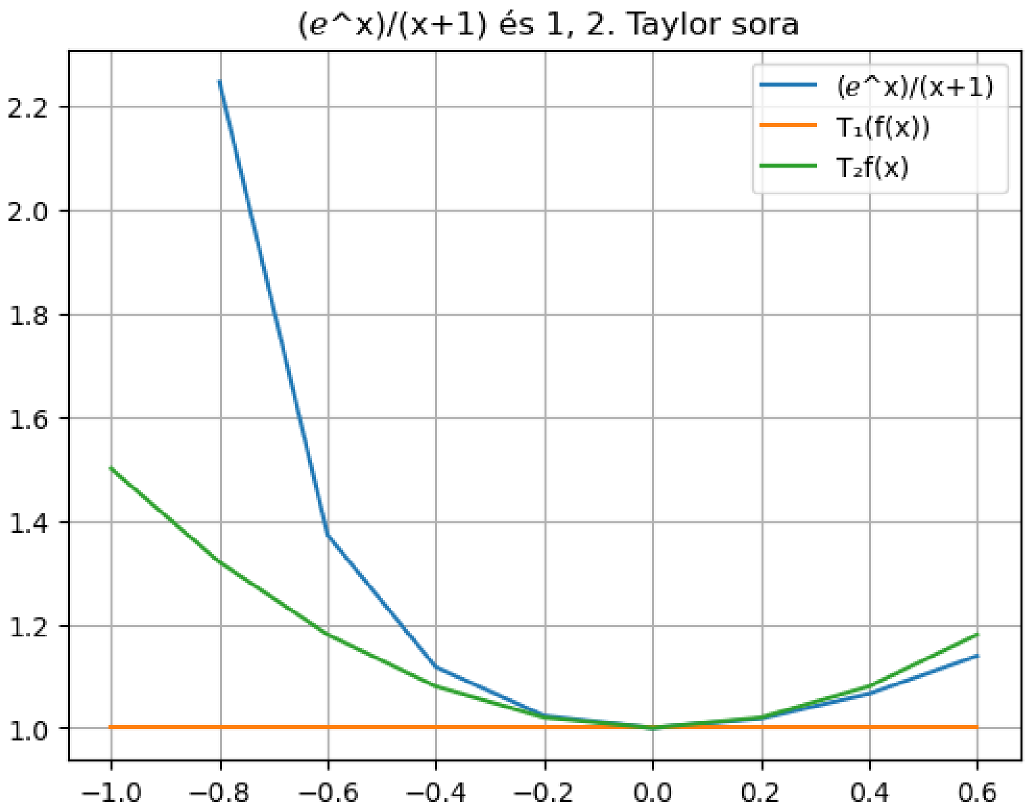
$f'(0) = \frac{0}{1^2} = 0$

$f'(0) = \frac{1-0}{1} = 1$

$T_1(f(x))$ $\frac{0}{0!}(x-0)^0 + \frac{1}{1!}(x-0)^1 = 1$

$T_2(f(x))$ $\frac{1}{0!}(x-0)^0 + \frac{0}{1!}(x-0)^1 + \frac{1}{1!}(x-1)^2 = 1 + \frac{x^2}{2}$

4 feladat



(e^x)/(x+1) és 1, 2. Taylor sora
(e^x)/(x+1)
T₁(f(x))
T₂f(x)
2.2
2.0
1.8
1.6
1.4
1.2
1.0
−1.0
−0.8
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6

5 feladat

(5) $g(x) = \sin(x)$

$x_0 = \frac{\pi}{4}$

$$\boxed{\sum_{k=0}^{n} \frac{f^{(k)}(x)}{k!}(x-x_0)^k}$$

$g'(x) = \cos(x)$

$g''(x) = -\sin(x)$

$g'''(x) = -\cos(x)$

$\Rightarrow$ $g(x_0) = 0.707 \approx \frac{\sqrt{2}}{2}$

$g''(x_0) = -0.707 \approx -\frac{\sqrt{2}}{2}$

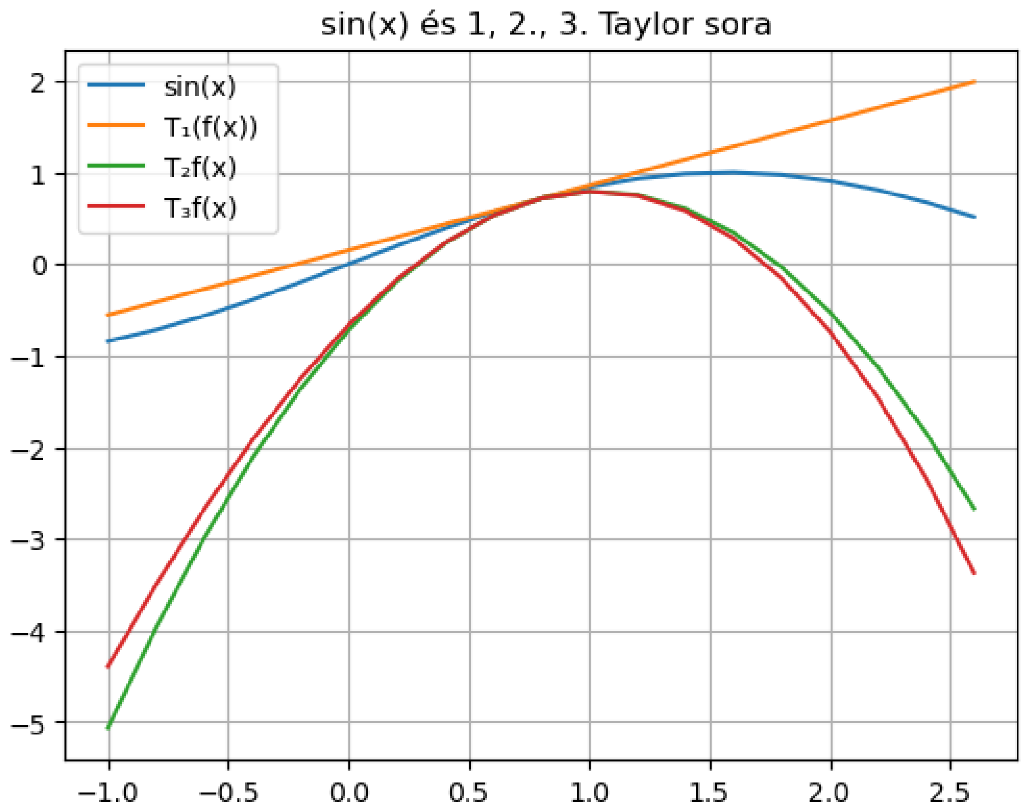
$g'''(x_0) = -0.707 \approx -\frac{\sqrt{2}}{2}$

$$T_{1(g(x_0))} = \frac{\frac{\sqrt{2}}{2}}{0!}\left(x-\frac{\pi}{4}\right)^0 + \frac{\frac{\sqrt{2}}{2}}{1!}\left(x-\frac{\pi}{4}\right)^1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right).$$

$$T_{2(g(x_0))} = \frac{\frac{\sqrt{2}}{2}}{0!}\left(x-\frac{\pi}{4}\right) + \frac{\frac{\sqrt{2}}{2}}{1!}\left(x-\frac{\pi}{4}\right)^1 + \frac{\frac{\sqrt{2}}{2}}{2!}\left(x-\frac{\pi}{4}\right)^2 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x-\frac{\pi}{4}\right)^2$$

$$T_3(g(x_0)) = \frac{\frac{\sqrt{2}}{2}}{0!}\left(x-\frac{\pi}{4}\right) + \frac{\frac{\sqrt{2}}{2}}{1!}\left(x-\frac{\pi}{4}\right)^1 + \frac{\frac{\sqrt{2}}{2}}{2!}\left(x-\frac{\pi}{4}\right)^2 + \frac{\frac{\sqrt{2}}{2}}{3!}\left(x-\frac{\pi}{4}\right)^3 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x-\frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{12}\left(x-\frac{\pi}{4}\right)^3$$

6 feladat


sin(x) és 1, 2., 3. Taylor sora
sin(x)
T₁(f(x))
T₂f(x)
T₃f(x)
2
1
0
−1
−2
−3
−4
−5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5