Intelligens Fejlesztőeszkozok - 2. beadandó

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1 feladat

$$\begin{cases} y' - \frac{2xy}{x^2+1} = x^3 + x \\ y(0) = 1 \end{cases} \tag{1}$$

$$\begin{aligned} y'(x^2+1) - 2xy &= (x^3+x)(x^2+1) \\ y'x^2 + y' - 2xy &= x^5 + x^3 + x^3 + x \end{aligned} \tag{2}$$

$$\begin{cases} y' \Rightarrow pY - y(0) \\ y \Rightarrow Y \\ x^n \Rightarrow \frac{n!}{p^{n+1}} \end{cases} \Rightarrow \begin{cases} x^1 \Rightarrow \frac{1}{p^2} \\ x^2 \Rightarrow \frac{2!}{p^3} \Rightarrow \frac{2}{p^3} \\ x^3 \Rightarrow \frac{3!}{p^4} \Rightarrow \frac{6}{p^4} \\ x^4 \Rightarrow \frac{4!}{p^5} \Rightarrow \frac{24}{p^5} \\ x^5 \Rightarrow \frac{5!}{p^6} \Rightarrow \frac{120}{p^6} \\ x^6 \Rightarrow \frac{6!}{p^7} \Rightarrow \frac{720}{p^7} \\ n = 0 \Rightarrow 1 \rightarrow \frac{0!}{p} \Rightarrow \frac{1}{p} \end{cases} \tag{3}$$

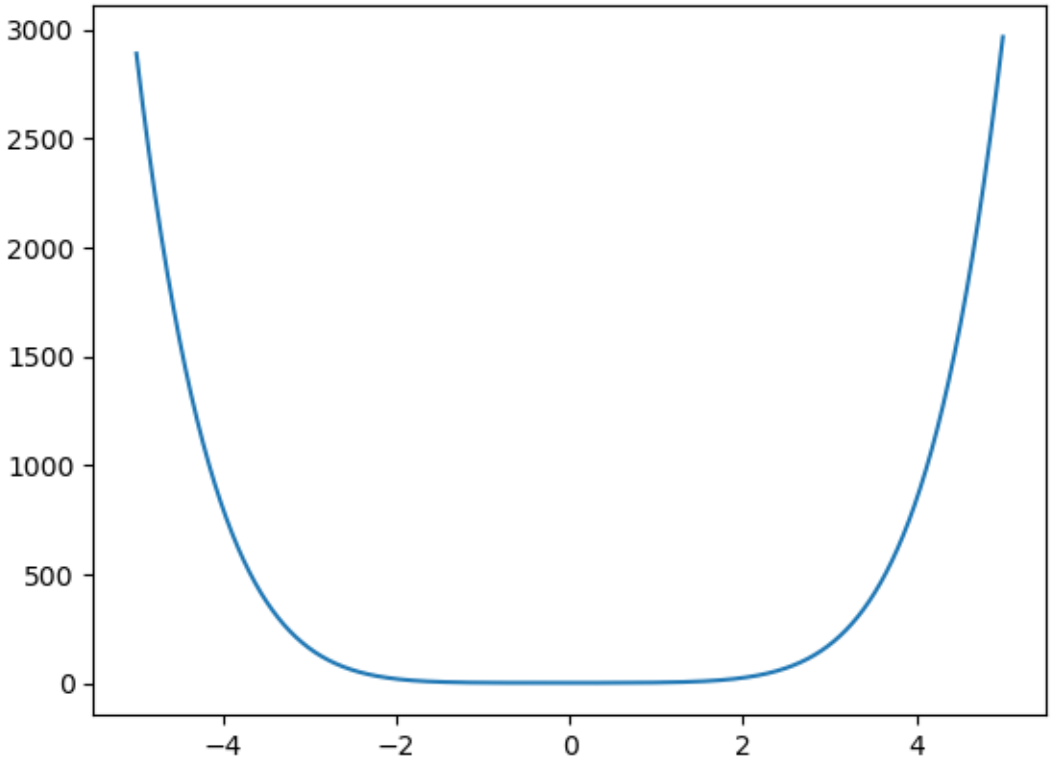
$$(pY-1)\frac{2}{p^3} + pY - 1 - 2\frac{1}{p^2}Y = \frac{120}{p^6} + 2\frac{6}{p4} + \frac{1}{p^2} \tag{4}$$

$$\frac{2}{p^2}Y - \frac{2}{p^3} + pY - 1 - \frac{2}{p^2}Y = \frac{120}{p^6} + \frac{12}{p^4} + \frac{1}{p^2} \tag{5}$$

$$pY = \frac{120}{p^6} + \frac{12}{p^4} + \frac{12}{p^3} + \frac{1}{p^2} + 1 \tag{6}$$

$$Y = \frac{120}{p^7} + \frac{12}{p^5} + \frac{12}{p^4} + \frac{1}{p^3} + \frac{1}{p} \tag{7}$$

$$\Rightarrow y = \frac{1}{6}x^6 + \frac{1}{2}x^4 + \frac{1}{3}x^3 + \frac{1}{2}x^2 + 1 = \frac{x^6}{6} + \frac{x^4}{2} + \frac{x^3}{3} + \frac{x^2}{2} + 1 \tag{8}$$



2 feladat

$$\begin{cases} y'' - 4y' + 4y = 12x - 4 \\ y(0) = 3 \\ y'(0) = 8 \end{cases} \tag{9}$$

$$p^2Y - p3 - 8 - 4pY + 12 + 4Y = 12\frac{1}{p^2} - 4\frac{1}{p} \tag{10}$$

$$\begin{cases} y'' \to p^2Y - py(0) - y'(0) \\ y' \to p4 - y(0) \\ y \to Y \\ x \to \frac{1}{p^2} \\ 1 \to \frac{1}{p} \\ x^n \to \frac{n!}{p^{n+1}} \end{cases} \tag{11}$$

$$Y(p^2 - 4p + 4) = 12\frac{1}{p^2} - 4\frac{1}{p} - 12 + 8 + 3p \tag{12}$$

$$Y = \frac{12 - 4p - 4p^2 + 3p^3}{(p^2 - 4p + 4)p^2} \tag{13}$$

$$Y = \frac{3p^2 - 4p^2 - 4p + 12}{(p^2(p-2)^2} \tag{14}$$

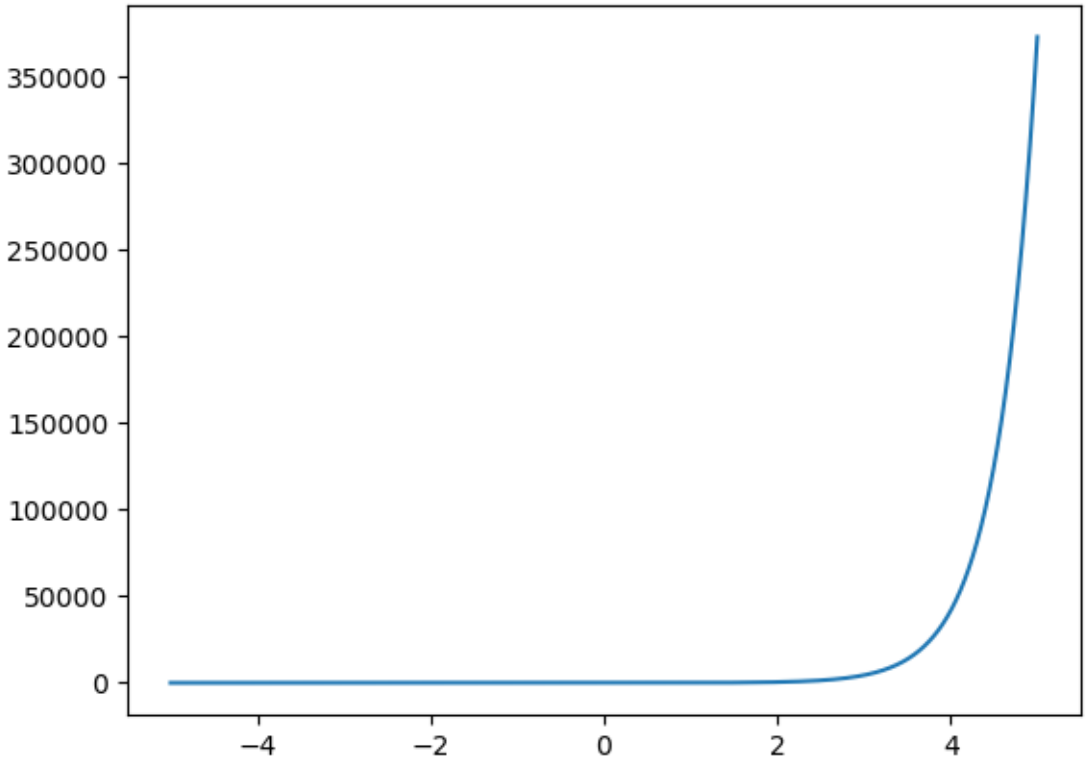
$$\frac{A}{p} + \frac{B}{p^2} + \frac{C}{p-2} + \frac{D}{(p-2)^2} = \frac{3p^3 - 4p^2 - 4p + 12}{(p^2(p-2)^2} \tag{15}$$

$$\begin{aligned} &3p^3 - 4p^2 - p + 12 \\ &\Rightarrow Ap(p-2)^2 + B(p-2)^2 + Cp(p-2) + Dp^2 \\ &\Rightarrow AP(p^3 - 4p + 4) + B(p^2 - 4p + 4) + Cp^2 - 2Cp + Dp^2 \\ &\Rightarrow Ap^3 - p^2(-4A + B + C + D) + p(4A + +4B - 2C) + 4B \end{aligned} \tag{16}$$

$$\begin{cases} A = 3 \\ -4A + B + C + D = -4 \\ 4A - 4B - 2C = -4 \\ 4B = 12 \end{cases} \Rightarrow \begin{cases} A = 3 \\ B = 3 \\ C + D = -4 + 12 - 3 = 5 \Rightarrow \begin{cases} C = 2 \\ D = 5 \end{cases} \end{cases} \tag{17}$$

$$\begin{cases} e^{2x} \frac{1}{p-2} \\ x \to \frac{1}{p^2} \\ xe^{-2x} \to \frac{1}{(p-2)^2} \end{cases} \tag{18}$$

$$\begin{aligned} Y = 3\frac{1}{p} + 3\frac{1}{p^2} + 2\frac{1}{p-2} + 5\frac{1}{(p-2)^2} \\ \Rightarrow y = 3 + 3x + 2e^{2x} + 3xe^{2x} \end{aligned} \tag{19}$$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