HF 1

①

$$\begin{cases} y' - \frac{2xy}{x^2+1} = x^3 + x \\ y(0) = 1 \end{cases}$$

$$y'(x^2+1) - 2xy = (x^3+x)(x^2+1)$$

$$y'x^2 + y' - 2xy = x^5 + x^3 + x^3 + x$$

$$\begin{cases} y' \to pY - y(0) \\ y \to Y \\ x^n \to \frac{n!}{p^{n+1}} \end{cases} \xrightarrow{\text{alapján}} \begin{cases} x^1 \to \frac{1}{p^2} \\ x^2 \to \frac{2!}{p^3} = \frac{2}{p^3} \\ x^3 \to \frac{3!}{p^4} = \frac{6}{p^4} \\ x^4 \to \frac{4!}{p^5} = \frac{24}{p^5} \\ x^5 \to \frac{5!}{p^6} = \frac{120}{p^6} \\ x^6 \to \frac{6!}{p^7} = \frac{720}{p^7} \\ n=0 \Rightarrow 1 \to \frac{0!}{p} = \frac{1}{p} \end{cases}$$

$$(pY-1)\cdot\frac{2}{p^3} + pY - 1 - 2\left|\frac{1}{p^2}\right. Y = \frac{120}{p^6} + 2\frac{6}{p^4} + \frac{1}{p^2}$$

$$\frac{2}{p^2}Y - \frac{2}{p^3} + pY - 1 - \frac{2}{p^2}Y = \frac{120}{p^6} + \frac{12}{p^4} + \frac{1}{p^2}$$

$$pY = \frac{120}{p^6} + \frac{12}{p^4} + \frac{2}{p^3} + \frac{1}{p^2} + 1 \quad /\cdot\frac{1}{p}$$

$$Y = \frac{120}{p^7} + \frac{12}{p^5} + \frac{2}{p^4} + \frac{1}{p^3} + \frac{1}{p}$$

$$\frac{1}{6}\frac{720}{p^7} + \frac{1}{2}\cdot\frac{24}{p^5} + \frac{1}{3}\cdot\frac{6}{p^4} + \frac{1}{2}\cdot\frac{2}{p^3} + \frac{0!}{p}$$

$$\Rightarrow y = \frac{1}{6}x^6 + \frac{1}{2}x^5 + \frac{1}{3}x^3 + \frac{1}{2}x^2 + 1 =$$

$$= \frac{x^6}{6} + \frac{x^5}{2} + \frac{x^3}{3} + \frac{x^2}{2} + 1$$

②

$$\begin{cases} y'' - 4y' + 4y = 12x - 4 \\ y(0) = 3 \\ y'(0) = 8 \end{cases}$$

$$p^2Y - pf - 8 - 4pY + 12\frac{1}{p^2} + 4Y = 12\frac{1}{p^2} - 4\cdot\frac{1}{p}$$

$$\begin{cases} y'' \to p^2Y - py(0) - y'(0) \\ y' \to pY - y(0) \\ y \to Y \\ x \to \frac{1}{p^2} \\ 1 \to \frac{1}{p} \\ x^n \to \frac{n!}{p^{n+1}} \end{cases}$$

$$Y(p^2 - 4p + 4) = 12\cdot\frac{1}{p^2} - 4\frac{1}{p} - 12 + 8 + 4p$$

$$Y = \frac{12 - 4p - 4p^2 + 3p^3}{(p^2 - 4p + 4)p^2}$$

$$Y = \frac{3p^3 - 4p^2 - 4p + 12}{p^2(p-2)^2}$$

$$\frac{A}{p} + \frac{B}{p^2} + \frac{C}{p-2} + \frac{D}{(p-2)^2} = \frac{3p^3 - 4p^2 - 4p + 12}{p^2(p-2)^2}$$

$$\begin{cases} A = 3 \\ -4A + B + C + D = -4 \\ 4A - 4B - 2C = -4 \\ 4B = 12 \end{cases}$$

$$3p^3 - 4p^2 - p + 12 \Rightarrow Ap(p-2)^2 + B(p-2)^2 + Cp(p-2) + Dp^2 \to$$
$$\to Ap(p^2 - 4p + 4) + B(p^2 - 4p + 4) + Cp^2 - 2Cp + Dp^2 \to$$
$$\to Ap^3 - 4Ap^2 + 4Ap + Bp^2 - 4Bp + 4B + Cp^2 - 2Cp + Dp^2 \to$$
$$\to Ap^3 + p^2(-4A + B + C + D) + p(4A - 4B - 2C) + 4B \Rightarrow$$

$$\begin{cases} A = 3 \\ B = 3 \\ C + D = -4 + 12 - 3 = 5 \\ C = 2 \\ D = 5 \end{cases}$$

$e^{2x} \Rightarrow \frac{1}{p-2}$

$$Y = 3\cdot\frac{1}{p} + 3\cdot\frac{1}{p^2} + 2\cdot\frac{1}{p-2} + 5\cdot\frac{1}{(p-2)^2}$$

$$y = 3 + 3x + 2\cdot e^{2x} + 3x e^{2x}$$

$$\begin{cases} x \to \frac{1}{p^2} \\ x e^{-2x} \to \frac{1}{(p-2)^2} \end{cases}$$