(1)

$$y' - \frac{2xy}{x^2+1} = x^3 + x$$

$$y' - \frac{2xy}{x^2+1} = 0$$

$$y' = \frac{2xy}{x^2+1}$$

$$\frac{dy}{dx} = \frac{2xy}{x^2+1}$$

$$\frac{dy}{y} = \frac{2x\,dx}{x^2+1}$$

$$\int \frac{1}{y}\,dy = \int \frac{2x}{x^2+1}\,dx$$

$$\ln y = \ln x^2+1 + C$$

$$y = e^C (x^2+1)$$

$$y_n = C(x^2+1)$$

$$y_n = g(x)(x^2+1)$$

$$y'_p = g'(x)(x^2+1) + 2g(x)$$

$$g'(x)(x^2+1) = x^3 + x$$

$$g'(x) = x$$

$$g'(x) = u'(x)$$

$$u'(x) = x$$

$$\frac{du}{dx} = x$$

$$du = x\,dx$$

$$\int 1\,du = \int x\,dx$$

$$u = \frac{x^2}{2} + C$$

$$y = \frac{x^2(x^2+2C)}{2} + \frac{x^2+2C}{2}$$

$$y = \frac{x^4}{2} + \frac{x^2}{2} + C\left(\frac{x^2}{2} + \frac{1}{2}\right)$$

$$y(0) = 1$$

$$1 = \frac{C}{2}$$

$$C = 2$$

$$y = \frac{x^4}{2} + \frac{x^2}{2} + 2\left(\frac{x^2}{2} + \frac{1}{2}\right)$$

② $y'' - 4y' + 4y = 12x - 4$

$\lambda^2 - 4\lambda + 4 = 0$

$\frac{4 \pm \sqrt{4^2 - 4 \cdot 4}}{2}$

$y_h = (C_1 x + C) e^{2x}$

$y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

$4Ax + 4B - 4A = 12x - 4$

$4A = 12$

$A = 3$

$4B - 4A = -4$

$B = 2$

$y = (C_1 x + C) e^{2x} + 3x + 2$

$y = C_1 x e^{2x} + C e^{2x} + 3x + 2$