

Homework 9

OPTIONAL PROBLEMS

(No due date)

1 Written Problems

Adding an Edge

Given a graph G of V vertices and E edges AND a minimum spanning tree T of graph G , we would like to add a new edge e with weight w_e to G , forming a new graph G' . Describe an algorithm which constructs the minimum spanning tree of G' in $O(V)$ time.

Solution:

Given that we already have a minimum spanning tree, T , we can add e to T and know that all but one edge of this newly constructed tree, T' , will be included in our final minimum spanning tree. However, simply adding e to T necessarily creates a cycle by the definition of a tree, so in order to properly construct a final minimum spanning tree we need to remove an edge from T' . Because we know that there are exactly V edges in this new Graph ($V-1$ edges in a Tree plus the one added edge), then if we were to remove the highest weighted edge from the cycle we just created (which could only be composed of up to V edges) we could again have a tree, everything would be connected, and we would have removed the largest weighing edge from the only portion of the graph that we modified. What this means is that by iterating over all edges in the cycle, which we have shown to be $O(V)$, we can remove the highest weighing edges without changing the portion of the MST that would never have been affected by this added edge, resulting in a new MST with the added edge included (if it is not the heaviest edge in the cycle) or an otherwise unmodified MST (if it is).
