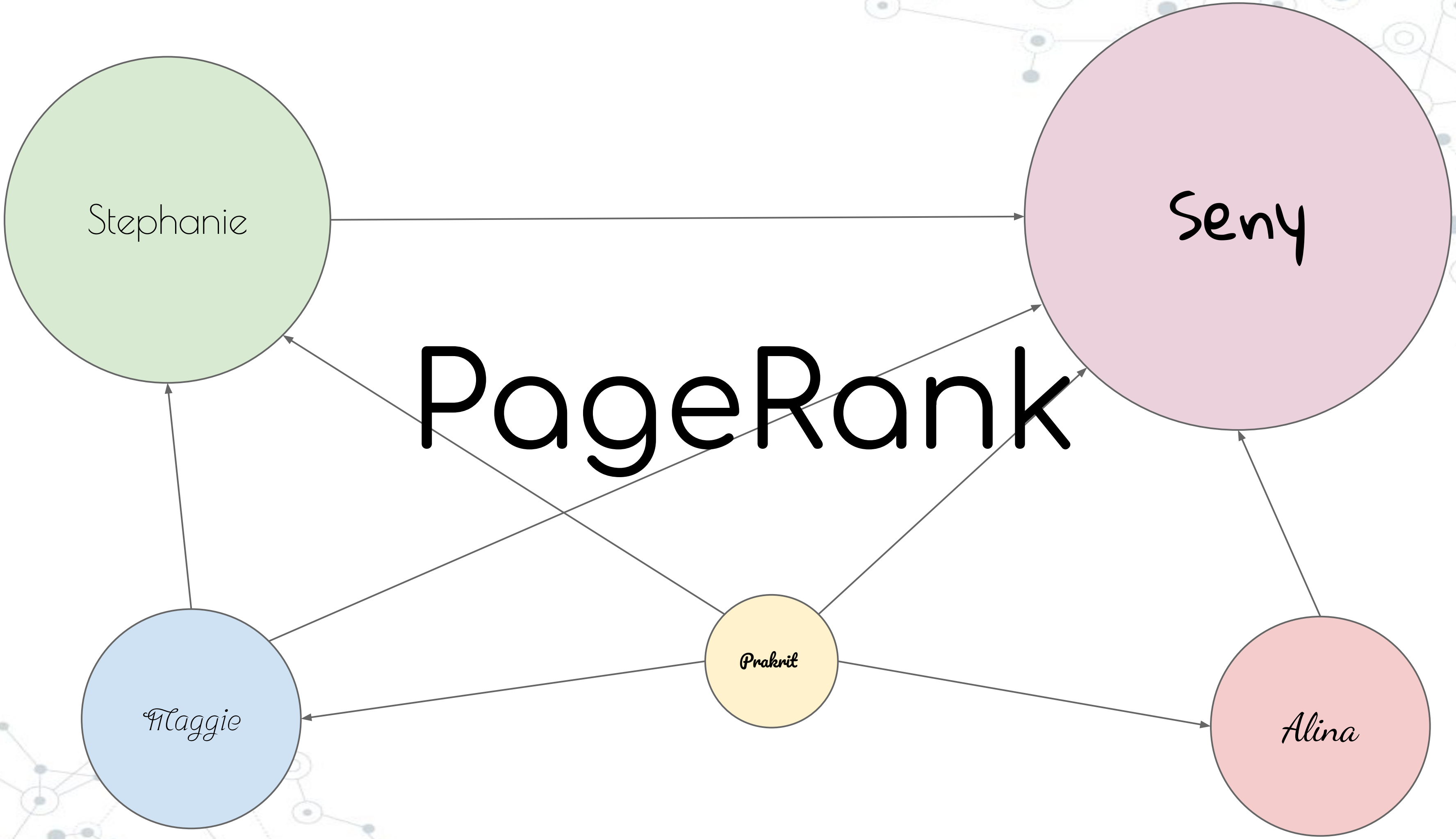
PageRank



Overview

- ➔ Description
- ➔ Simplified Algorithm
- ➔ Example
- ➔ Complete Algorithm - *What you'll be implementing!*
- ➔ Sinks
- ➔ Damping Factor
- ➔ Convergence
- ➔ Implementation
- ➔ Testing

Description

- ➔ Algorithm developed by L. Page and S. Brin (Google co-founders) used to determine the order of pages returned in response to a query
- ➔ According to Google:
  - ◆ "PageRank works by counting the number and quality of links to a page to determine a rough estimate of how important the website is. The underlying assumption is that more important websites are likely to receive more links from other websites."
- ➔ **Main idea:**
  - ◆ More important websites = more links from other websites
  - ◆ Ex: Wikipedia oftentimes is first result from Google search - this is because many pages reference Wikipedia

What is the “pagerank” of a page?

- The pagerank of a page represents its importance
- The pagerank of a page is a value between 0 and 1
- Each page starts with some amount of pagerank
- Think of pagerank as a “fluid” that is distributed among pages
  - the “pagerank” of a page is its total amount of “fluid”
  - the sum of all page’s pageranks is 1

Basic PageRank ***not implementing this***

$$PR(u) = \sum_{v \in in(u)} \frac{PR(v)}{|out(v)|}$$

➔ "the PageRank value for a page ***u*** is dependent on the PageRank values for each page ***v*** contained in the set ***in(u)*** (the set of all pages linking to page ***u***), divided by the number ***|out(v)|*** of outgoing links from page ***v***"

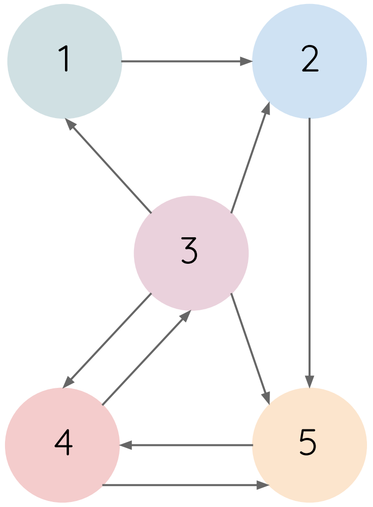
➔ ***We will now walk through an example using this simplified algorithm. It is not the algorithm we are using, but instead very similar. We will use this as it's easier to walk through the math with the simplified version.***

Steps for Simplified Algorithm

1. **Iteration 0:** Initialize all ranks to be 1/(number of total pages).
2. **Iteration 1:** For each page **u**, update **u**’s rank to be the sum of each adjacent page **v**’s rank from the previous iteration, divided by the number total number of links from page **v**

using basic algorithm (not implementing this)

Example 1



	Iteration 0	Iteration 1
P_1	1/5	1/20
P_2	1/5	5/20
P_3	1/5	1/10
P_4	1/5	5/20
P_5	1/5	7/20

1. Iteration 0: Initialize all pages to have rank ⅕.
2. Iteration 1:
3. **P_1:** has 1 link from P_3, and P_3 has 4 outbound links, so we take the rank of P_3 from iteration 0 and divide it by 4, which results in rank (⅕)/4 = 1/20 for P1

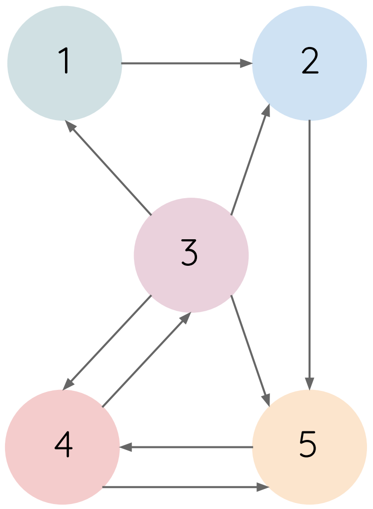
$$\mathbf{PR(P_1) = (1/5)/4 = 1/20}$$

4. **P_2:** has 2 links from P_1 and P_3, P_1 has 1 outbound link and P_3 has 4 outbound links, so we take (the rank of P_1 from iteration 0 and divide it by 1) and add that to (the rank of P_3 from iteration 0 and divided that by 4) to get ⅕ + 1/20 = 5/20 for P_2

$$\mathbf{PR(P_2) = 1/5 + (1/5)/4 = 5/20}$$

** using basic algorithm (not implementing this)**

Example 1: After 2 iterations



	Iteration 0	Iteration 1	Iteration 2	Final Rank
P_1	1/5	1/20	1/40	5
P_2	1/5	5/20	3/40	4
P_3	1/5	1/10	5/40	3
P_4	1/5	5/20	15/40	2
P_5	1/5	7/20	16/40	1

<u>Food for thought</u>

➔ Why do we calculate the pageranks multiple times?

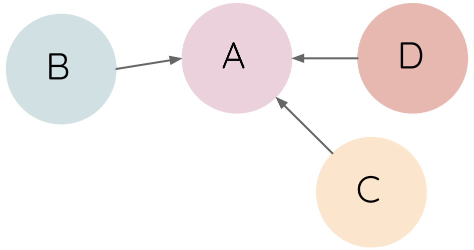
➔ When should we stop?

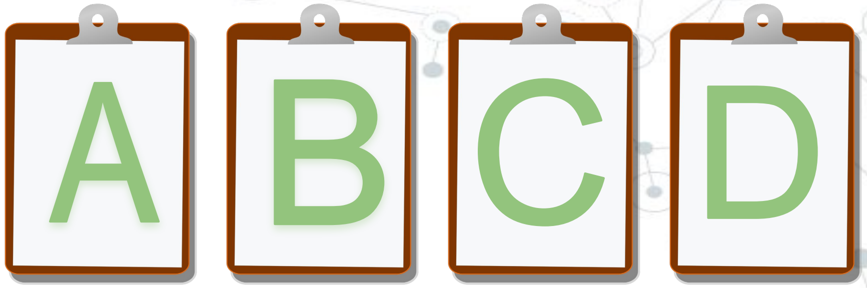
(to be explained later)

$PR(P_5) = \frac{1}{5} + \frac{1}{5} * \frac{1}{4} + \frac{1}{5} * \frac{1}{2} = 7/20$

****using basic algorithm (not implementing this)****

Example 2





- Say we have four pages: **A**, **B**, **C** and **D**.
- Links from a page to itself are ignored (there aren't any in this example).
- Multiple links from one page to another are treated as a single link.
- Each page's rank is initialized to be the same value: 1/(total number of pages)
  - In this example, every page would start out with a rank of 0.25
- Idea: Iterate and recalculate until the rank of each page converges.

** using basic algorithm (not implementing this)**

Example 2: Okay, so I've initialized the ranks. Now what?

$$PR(A) = \frac{PR_{prev}(B)}{|out(B)|} + \frac{PR_{prev}(C)}{|out(C)|} + \frac{PR_{prev}(D)}{|out(D)|}$$

- ➔ Apply the PageRank calculation, over and over until the ranks converge.
- ➔ At every iteration, for each page's PageRank, divide it by the number of outbound links from that page, and transfer that rank to each of the pages it points to.
- ➔ PR(A) = (PR(B) from previous iteration)/(number of outbound links from B) + (PR(C) from previous iteration)/(number of outbound links from C)

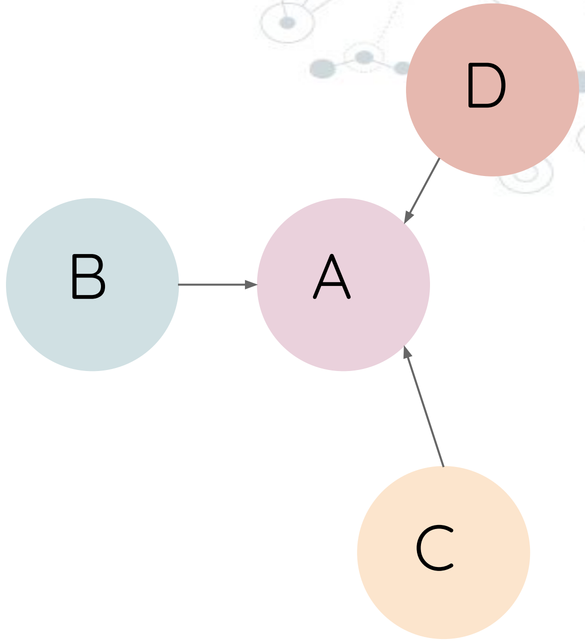
+ (PR(D) from previous iteration)/(number of outbound links from D)

*PR(x) = PageRank of x

** using basic algorithm (not implementing this)**

Example 2

- ➔ Since B, C, and D all have outbound links to A, the Pagerank of A will be **0.75** upon the first iteration
  - ◆ (B with rank of 0.25) + (C with rank of 0.25) + (D with rank of 0.25) would transfer all of those ranks to A
- ➔ But wait! What about ranks of pages B,C, and D? How does this add up to 1?
- ➔ ***This is why we are not using the simplified algorithm for our PageRank!***



Real PageRank Algorithm ***implementing this***

$$PR(p_i) = \frac{1-d}{N} + d \cdot \sum_{p_j \in in(p_i)} \frac{PR(p_j)}{|out(p_j)|}$$

- ➔ The PageRank of a page P_i is equal to (1 - damping factor)/(number of pages) + damping factor * (the sum of each page P_j's PageRank, where P_j links to P_i, divided over the number of outbound links of P_j)
- ➔ **P_1, P_2, … P_n** are the pages under consideration,
- ➔ ***in*(P$_i$)** is the set of pages that link to page P_i
- ➔ **|*out*(P$_j$)|** is the number of outbound links on page P_j
- ➔ **N** is total number of pages
- ➔ **d** is damping factor

Breaking Down the New Update Rule

➔ The PageRank of a page P_i is equal to **(1 - damping factor)/(number of pages)** + damping factor * (the sum of each page P_j's PageRank, where P_j links to P_i, divided over the number of outbound links of P_j)

$$\frac{1-d}{N}$$

➔ The PageRank of a page P_i is equal to (1 - damping factor)/(number of pages) + **damping factor * (the sum of each page P_j's PageRank, where P_j links to P_i, divided over the number of outbound links of P_j)**

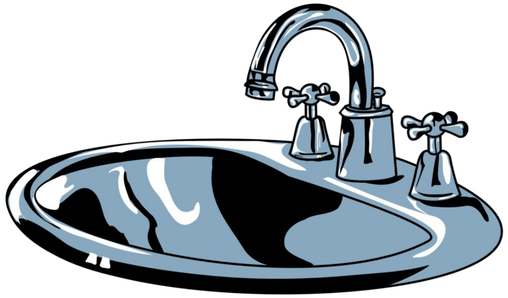
$$d \cdot \sum_{p_j \in in(p_i)} \frac{PR(p_j)}{|out(p_j)|}$$

Damping Factor

- For certain graphs, the simple update rule can cause pagerank to accumulate and get stuck in certain parts of the graphs
- We fix this by having each node (at every round)
  - Give a *d* fraction of its pagerank to its neighbors
  - Give a (*1-d*) fraction of its pagerank to everyone in the graph
- This also means that pages with no incoming links will get some pagerank
- *d* is called the damping factor
  - It is usually set to .85

Sinks

- ➔ If a page has no links to other pages, it is a **sink**
- ➔ Sinks can create problems for the PageRank algorithm
  - ◆ The pagerank (i.e., fluid) can accumulate and get stuck at sinks
- ➔ In the end, our goal is to distribute the rank of sink pages among all pages in the graph. There are two ways we can do this **(only choose one)**:
  - ◆ We can add outgoing edges from sinks to all pages
  - ◆ We can distribute a sink's pagerank evenly among all pages



Handle Sinks - Adding Edges Method

1. After you have determined which pages are sinks, add outgoing edges from each sink to every page in your graph, including the sinks. (This should be done before the main loop).
2. Remember to update the sinks' number of outgoing edges. (This should also be done before the main loop).
3. Within the main body of your loop, after you clone your **`_current`** array of pageranks into your **`_previous`** array, be sure to reset the entirety of **`_current`** to be 0.0.

Handle Sinks - Distribute Rank Method

Pseudocode for `handleSinks()` - you should call this at the start of every iteration of the algorithm and set that to be each current page's rank before updating it:

```
sinksDistribution = 0
for each sink page:
    updatedRank = sink's previous rank / number of pages
    add updatedRank to sinksDistribution
for each page:
    set the page's rank to be the sinksDistribution
```

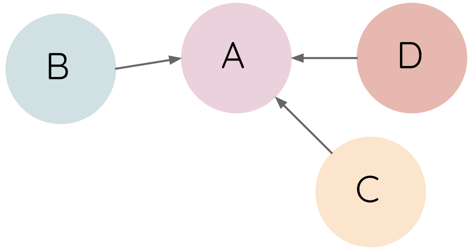
Convergence

- ➔ Is calculating the rank of each page one time enough?
- ➔ We iterate because pageranks can fluctuate. One round is not enough to tell how important a page is.
- ➔ We stop when the pageranks become steady or converge:
  - ◆ each page’s pagerank from the previous iteration differs from the current iteration by less than or equal to the margin of error (which we have set to 0.01)
  - ◆ OR once we have iterated 100 times, whichever comes first

Back to our original example

$$PR(A) = \frac{1-d}{N} + d\left(\frac{PR_{prev}(B)}{|out(B)|} + \frac{PR_{prev}(C)}{|out(C)|} + \frac{PR_{prev}(D)}{|out(D)|} + \ldots\right)$$

➔ PR(**A)** is equal to (1 - damping factor)/(number of pages) + damping factor * ((PR(**B**) from previous iteration)/(number of links from **B**) + (PR(**C**) from previous iteration)/(number of links from **C**) + (PR(**D**) from previous iteration)/(number of links from **D**))



Big Ideas

- ➔ Calculate pageranks of each page over and over again based on its incoming links!
- ➔ Don’t forget damping factor.
- ➔ Take sinks into account.
- ➔ Stop converging either at 100 iterations or when margin of error is at 0.01.

Implementation

- ➔ Use the graph's `vertices()` method to return an iterator of vertices!
- ➔ You should have some way to keep track of and store the vertices so that you can map them to their respective pageranks.
- ➔ Similarly, you should store the pageranks of each page! But wait…. you'll have to update them, but if you update the pagerank of page A, you might need the old pagerank for page B….. think about how to handle this!
- ➔ Take care of sinks! Make sure to distribute their ranks accordingly.
- ➔ Damping factor set to 0.85 and margin of error set to 0.01. *You should not adjust these parameters.*
- ➔ Make sure to stop calculating at 100 iterations or when margin of error hits 0.01 (whichever comes first).

RoadMap!

- ➔ Start out by figuring out what data structures and variables you need to store your ranks and vertices.
    - ◆ You will need to create two arrays to calculate ranks of pages: prev and curr.
    - ◆ Prev will contain the ranks of each page from the previous iteration and take into account pages that are sinks.
    - ◆ What other things will you need to keep track of?
        - Think about outgoing edges!
    - ◆ Make sure to initialize them in `calcPageRank(Graph<V> g)`!

RoadMap cont.

- Get the number of pages and store your vertices.
    - Remember that the `AdjacencyMatrixGraph`'s **`vertices()`** method returns an iterator of vertices - which is in no particular order.
    - You may want to make sure when storing the ranks of each page that they match up with the indexing in your vertices array.

RoadMap cont.

- Tackle the main method performing the PageRank calculation - `calcPageRank(Graph<V> g)`!
  - There is no constructor (reason has to do with visualizer) so you'll want to initialize all your variables here.
  - Initialize the rank of each page to be 1/(number of pages).
  - Enter the repeated rank calculation!
    - Food for thought: would a while loop, for loop, or do-while loop work best?
    - Considering an ending condition of the iterations is looking at the difference between a current and previous iteration, which would would fit most?

Main calculation

- ➔ At every iteration, you want to make sure your previous array is updated correctly.
    - ◆ You want your sinks to be updated for the current array.
- ➔ You want to then update the rank of each page using the previous array.
    - ◆ Maybe have a method to update the rank?
    - ◆ Don’t forget the damping factor!
- ➔ Write up a helper method to help check when you should stop iterating!

Testing

- All PageRanks should add up to 1!
- Pages with more links should have higher pageranks than pages with fewer links.
- Sinks should not have a pagerank of 0.

Citations

- ➔ https://en.wikipedia.org/wiki/PageRank
- ➔ http://www.cs.princeton.edu/~chazelle/courses/BIB/pagerank.htm
- ➔ https://www.quora.com/How-is-PageRank-calculated-What-was-the-initial-PageRank-How-does-the-algorithm-begin