

Homework 3

OPTIONAL PROBLEMS

(No due date)

1 Written Problems

1.1 Longest Increasing Subset

NOTE: this is a challenge problem

Given an array of integers, find the length of the longest increasing subsequence of these integers. (This can be done with dynamic programming in $O(n \log n)$ time!!) For example,

1. $[0, 3, 6, 2, 10, 1, 5, 33]$ would return 5, because the longest increasing subsequence, $[0, 3, 6, 10, 33]$ is of length 5
2. $[14, 2, 15, 11]$ would return 2, because the longest increasing subsequences, $[14, 15]$, $[2, 11]$ and $[2, 15]$ are all of length 2

1.2 Sharpay's Fan Club Meeting

Silly premise: Once every year, Sharpay's fan club gets together in East High Schools Cafeteria to discuss Sharpay's beauty and flawlessness. After years of being in love with Sharpay, Zeke James Baylor, a basketball player, has finally convinced himself of declaring his love to her. However, he faces the challenge of finding her within thousands of Sharpay look alikes in the cafeteria. Zeke can't get the wrong girl! He only brought one portion of his famous creme brulee made specifically for his loved Sharpay. Therefore, in order to help Zeke, you need to find a way to discover which girl in the cafeteria is actually Sharpay.

You know that at the club meeting, there is only one real Sharpay. Within a group of n girls, *Sharpay* is defined as a girl who is known by everyone, but knows no one. At the club meeting, you are only allowed to ask questions of the form "Excuse me, but do you know *that* girl over there?" Your job is to determine whether Sharpay is actually in the group. Clearly you can do this by asking each of the n girls about each of the other $n - 1$ girls, a total of $n(n - 1)$ questions. But you'd like to do much better!! Note: If a group has size $n = 1$, you should assume the sole member of the group is Sharpay (after all, she knows no one, and everyone knows her). Note 2: As creepy as it may sound in real-life, for this question you should not assume that girl A knowing girl B implies girl B knows girl A, even if both are not Sharpay!

(a) Explain why there can be at most one *Sharpay* in this club meeting, and describe a group of size n (for every $n > 1$) in which there's no such person.

(b) Suppose you ask A whether they know B , and A says "Yes." What (if anything) can you conclude about the identities of A and B ? What if A says "No"? What can you conclude then (if anything)?

(c) Describe an algorithm that, given a group of n girls, either finds Sharpay or determines that Sharpay isn't in the group of fans using $O(n)$ questions. A short paragraph explanation will suffice.

(d) Prove by induction that your algorithm uses $O(n)$ questions. By this we mean use the same logical structure as an inductive proof in your written explanation (i.e. with a base case, inductive step, etc.). In order to use $O(n)$ questions, you may ask only a constant number of questions for each girl in the fan club meeting. You will lose points if you don't use induction.

2 Python Problems

2.1 Climbing Stairs

Given a set of stairs comprised of n steps, and that you can climb 1, 2, or 3 steps at a time, write a function that returns the number of ways to climb that set of stairs. For example,

1. `climber(2)` would return 2, because the sequence of steps could be `[1, 1]` or `[2]`
2. `climber(3)` would return 4, because the sequence of steps could be `[1, 2]`, `[2, 1]`, `[3]`, or `[1, 1, 1]`

You do not need to return the possible sequences of steps, just the number.

2.2 Climbing Stairs++

Modify your `climber(n)` function to print out the possible sequences of steps, such that `climber(3)` would print out `"[1, 2], [2, 1], [3], [1, 1, 1]"`