Structure of an Induction Proof – CS16 Spring 2019

The left side of the handout is a step-by-step stencil for an inductive proof. The right side walks through these steps given an example.

To prove: *write out statement here...*	To prove: if $a > -1$, then for every integer $n \geq 1$, $(1+a)^n \geq 1 + an$
We'll prove this by induction. Let $P(n)$ be the statement: *fill in predicate here...*	We'll prove this by induction. Let $P(n)$ be the statement: $$(1+a)^n \geq 1 + an$$
Base case: We'll first prove that $P(1)$ is true. *write out P(1) here, and give an explanation of why it's true.*	Base case: We'll first prove that $P(1)$ is true. $$(1+a)^1 \geq 1 + a \cdot 1$$ $P(1)$ is true because it simplifies to $1 + a \geq 1 + a$
Inductive step: Assume $P(k)$ is true for some positive integer k: *write out* $P(k)$ *here*	Inductive step: Assume $P(k)$ is true for some positive integer k: $$P(k) : (1+a)^k \geq 1 + ak$$
We'll show that $P(k)$ implies $P(k+1)$: *write out* $P(k+1)$ *here.*	We'll show that $P(k)$ implies $P(k+1)$: $$P(k+1) : (1+a)^{k+1} \geq 1 + a(k+1)$$
Start from $P(k)$ *and argue the truth of* $P(k+1)$.	We've assumed: $$(1+a)^k \geq 1 + ak$$ Multiplying both sides by $1+a$, we get: $$(1+a)^k(1+a) \geq (1+ak)(1+a)$$ $$(1+a)^{k+1} \geq 1 + a + ak + a^2k$$ Refactoring the right hand side, we get: $$(1+a)^{k+1} \geq 1 + a(k+1) + a^2k$$ If this is true, then the following is also true, because a^2k is positive $$(1+a)^{k+1} \geq 1 + a(k+1)$$ which is exactly the statement $P(k+1)$, which we promised to prove.
Since $P(1)$ is true, and $P(k) \rightarrow P(k+1)$, by induction $P(n)$ is true for all integers $n \geq 1$	Since $P(1)$ is true, and $P(k) \rightarrow P(k+1)$, by induction $P(n)$ is true for all integers $n \geq 1$