HL2027 - Mini Project 1 - Compressed Sensing

Gabriel Carrizo and Raissa Anderson

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1 Underlying Problems

Q: Summarize any information you find in them on the motivation for using CS in MRI.

*Note: not sure if we also need to discuss underlying problems with MRI or our code in general. Let me know, can add. I also wasn't sure if we should focus more on the problems with MRI or with CS. I have more on both, so we can adjust.

In Magnetic Resonance Imaging (MRI), it is desirable to have as low an acquisition time as possible in order to limit both the motion artefacts caused by patient movement, and the time the patient is required to be in the scanner (which contributes to the overall cost of the procedure) [1, 2]. (There is also the matter of limited volumetric coverage / spatiotemporal resolution because of the time constraint but not sure if this is worth mentioning or not.) The limiting factor in reducing the acquisition time in MRI is the k-space sampling requirement imposed by Nyquist's theorem. However, most imaging signals actually contain a large number of coefficients that are zero or near zero after transformation, and these elements are not required to faithfully reconstruct the image [3–5].

One of the techniques that has been proposed to decrease acquisition time is based on this concept; Compressed Sensing (CS) involves undersampling k-space while still attempting to maintain comparable image quality during reconstruction [3, 6]. The main issue with CS is that it often still results in artefacts in the image, and thus the quality of the image is not equal to what can be achieved with standard MRI [5, 6]. There have been a number of techniques proposed to address this problem, and they usually fall into one of three categories: some type of irregular sampling of k-space, the application of sparsity transforms, or optimization of the non-linear recovery of the signal [6].

There are several other problems with the standard CS technique that should be mentioned: the sparsification parameter is set globally for all of k-space rather than separately for high- and low-frequency information, and the algorithm itself is computationally demanding and requires large amounts of memory [5]. Several approaches have been proposed to address these issues over the last decade including iterative methods and combinations with other approaches like parallel imaging [2, 5–7]; however, for this project, we have applied a basic form of CS reconstruction from simulated, randomly undersampled k-space data.

2 Basic Theories

Q: Find out which are the particular steps composing the reconstruction algorithm in demo4.m.

In the report, present these steps using a suitable diagram.

-> I'm not sure what would be a suitable way to present the information below - any ideas?

Flowchart?

Compressed Sensing MRI Reconstruction (POCS: Projection Over Convex Sets):

1. Compute the Fourier Transform of the image and multiply it by the random 3-fold undersampling pattern generated from PDF_vardens.

$$DATA = fft2c(im).*mask\_vardens;$$

2. Divide by the PDF and compute the zero-filled Inverse Fourier Transform of the result to obtain the initial signal estimate.

$$im\_cs = ifft2c(DATA./pdf\_vardens);$$

3. Compute the Forward Wavelet Transform on the estimated signal, and then do soft thresholding to obtain the largest (λ·100)% of the coefficients and compute the Inverse Wavelet Transform. Then, take the Fourier Transform of the result, multiply by the inverse of the mask (where Data==0), and add to Data in order to alternate between enforcing data consistency and promoting sparsity. Finally, take the Inverse Fourier Transform of the result. Repeat this process 15 times.

for iter=1:15

im_cs = W'(SoftThresh(W*im_cs, λ));*

im_cs = ifft2c(fft2c(im_cs).(1-mask_vardens) + DATA);*

end

Q: Explain the concept of TPSF (see Eq. 2). (Transform Point Spread Function). What are the desired properties for the TPSF?

Transform Point Spread Function (TPSF)

The TPSF is a measure of how a single transform coefficient of the original object affects the other transform coefficients in the undersampled object; it is a tool to look at incoherence in the transform domain and is to the wavelet domain what the Point Spread Function (PSF) is to the image domain [8–10].

To begin, an individual point in the sparse (wavelet) domain is transformed to the image domain and then to Fourier space. Here it is undersampled, transformed back to the image domain and then transformed back to the wavelet domain where the side lobes can be examined as a measure of incoherence. The goal is for the TPSF to be small with random noise-like statistics [10].

Do we want to add the formula/mathematical background for the TPSF or no?

3 Solution Strategies

I wasn't really sure what we want to say here: that we started from the code from the wavelet lab? We can talk about this next week and I can fill it in after that.

4 Experimental Findings

Q: Provide Python code to reproduce the experiments in the spirit of Figures 4 and 5

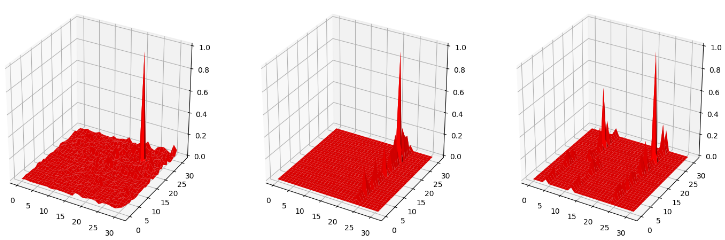

Figure 1: Plots in the spirit of Figures 4 and 5

Q: Provide Python code to perform the experiments in demo4.m.

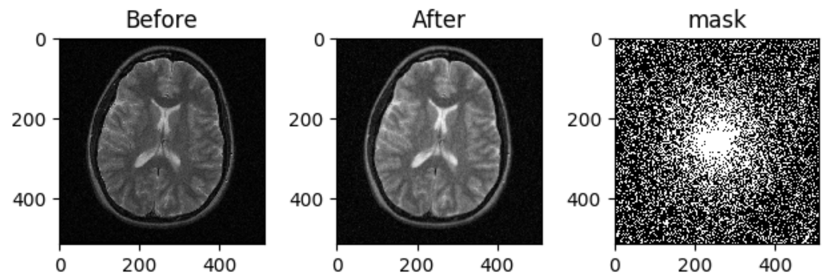

Figure 2: Output of demo4.m implementation in Python

Q: Investigate the effect of using different sampling patterns in k-space, i.e. vary used pdf, as well as modifying the thresholding approach.

I don't think we've done these two steps yet?

For varying PDF, we should see something like (directly from [2]):

1. Regular undersampling generates coherent replicas of the signal structure.

2. Random undersampling generates incoherent artefacts that appear like added noise.

3. Radial sampling permits undersampling along both spatial dimensions and thus enables a higher level of incoherence.

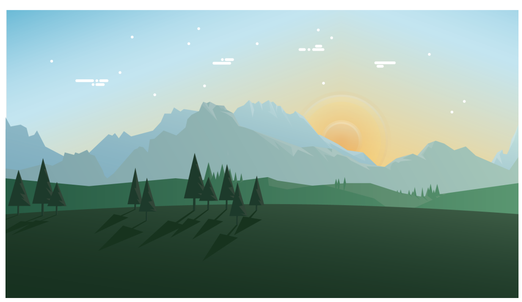

Figure 3: Effects of Varying PDF

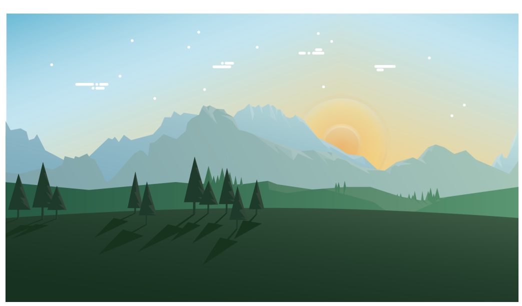

Figure 4: Effects of Modifying Thresholding Approach

5 Conclusions

Based on your experiences from Tasks 1-4, what are the key ingredients (or main principles) of the Compressed Sensing algorithm.

There are three main principles of the Compressed Sensing algorithm:

1. The images must be sparse in a known transform domain (wavelet in the case of MRI)
2. The aliasing artefacts that result from undersampling must be incoherent in the chosen domain
3. The image reconstruction must be non-linear in order to balance sparsity and consistency with acquired data

I'll add some other conclusion notes once we've finished everything.

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