APC523 – Homework #1

2.

a.

The first 31 terms in e^(5.5)

1

5.5

15.125

27.73

38.129

41.942

38.447

30.208

20.768

12.692

6.9805

3.4902

1.5997

0.67679

0.26588

0.097484

0.03351

0.010842

0.0033128

0.00095898

0.00026372

6.907e-05

1.7269e-05

4.1297e-06

9.4638e-07

2.0821e-07

4.4043e-08

8.9715e-09

1.7623e-09

3.3423e-10

6.1278e-11

b.

The partial sums converge to an answer of 244.71 at n=18. The “true solution” computed by doing exp(5.5) in the computer is 244.6919…, and the relative error in the estimate by using the partial sums is 7.3839\*10^-5. In this case the relative error is small.

The partial sum values are shown here:

1

6.5

21.625

49.355

87.484

129.43

167.88

198.09

218.86

231.55

238.53

242.02

243.62

244.3

244.57

244.67

244.7

244.71

244.71

244.71

244.71

244.71

244.71

244.71

244.71

244.71

244.71

244.71

244.71

244.71

244.71

c.

If you add the terms in the partial sum from right to left you converge at the same answer of 244.71. However, it takes longer to converge and k=22 before we get convergence. The relative error is still the same as part b since the final answer is the same.

The partial sums for part C are shown below:

1

6.5

21.625

49.355

87.484

129.43

167.88

198.09

218.86

231.55

238.53

242.02

243.62

244.29

244.56

244.66

244.69

244.7

244.7

244.7

244.7

244.71

244.71

244.71

244.71

244.71

244.71

244.71

244.71

244.71

244.71

d.

i) adding each term left to right the answer converges to 0.0038363, compared to 0.00408677… for the true solution. The relative error in this estimate is much higher than in the case of exp(5.5) since we have to subtract so many terms. For adding left to right, the relative error is 0.0613. In this case we converge to the solution much more slowly at k=26.

The partial sums of part i are shown below:

1

-4.5

10.625

-17.105

21.024

-20.918

17.529

-12.679

8.089

-4.603

2.3775

-1.1127

0.487

-0.18979

0.07609

-0.021394

0.012116

0.001274

0.0045868

0.0036278

0.0038915

0.0038224

0.0038397

0.0038356

0.0038365

0.0038363

0.0038363

0.0038363

0.0038363

0.0038363

0.0038363

ii) Adding right to left this time the answer converges to 0.004 compared to the true solution of 0.00408677… This method results in a lower relative error since you add terms of similar size first so that they do not get wiped out when larger terms are ultimately added. The relative error in this case is 0.0212. In this case we converge to the solution at k=20.

The partial sums in this case are shown:

1

-4.5

10.625

-17.105

21.024

-20.918

17.529

-12.679

8.089

-4.603

2.378

-1.113

0.487

-0.19

0.076

-0.021

0.012

0.001

0.005

0.004

0.004

0.004

0.004

0.004

0.004

0.004

0.004

0.004

0.004

0.004

iii) In this method the solution converges to zero so the relative error is 100%. This may be because in adding all the negative terms left to right and adding all the positive terms left to right you end up with two things that have very similar magnitude so that when you combine them the result goes to zero (with 5 significant figures). The result converges to zero around k=18.

The partial sums are shown:

1

-4.5

10.625

-17.105

21.024

-20.918

17.529

-12.679

8.09

-4.6

2.38

-1.11

0.49

-0.19

0.08

-0.02

0.01

0

0

0

0

0

0

0

0

0

0

0

0

0

iv) In this case, adding all positive the terms right to left, adding all the negative terms right to left, and then combining them gives a solution that converges to 0.01. This is very different from the true solution and has a high relative error of 1.4469. The solution converges around k=19. In this case the solution is very off because I think combining the positive and negative terms at the end wipes out a lot of degrees of accuracy since they are of similar size.

The partial sums are shown:

1

-4.5

10.625

-17.105

21.024

-20.918

17.529

-12.679

8.09

-4.6

2.38

-1.11

0.49

-0.19

0.08

-0.02

0.01

0

0.01

0.01

0.01

0.01

0.01

0.01

0.01

0.01

0.01

0.01

0.01

0.01

In general, the method with the lowest error is to add all the terms right to left within each partial sum. This yields a relative error of roughly 0.02. However, this is still much larger than the relative error resulting from doing exp(5.5), which had a relative error on the order of 10^-5. This is because doing all the subtraction reduces the accuracy of the estimate.

e.

We could instead compute exp(-5.5) = 1/exp(5.5), using the same expansion method as in part a for the denominator and then simply doing one over the final result.

By doing this method we get an answer of exp(-5.5) = 0.0040856 and a relative error of 6.64\*10^-5. This is much lower than the error for any of the other methods we used to calculate exp(-5.5).