Homework 1 - Coding Part

Lucas Sawade - The tables for the exponential series are in the Appendix at the very end.

Contents

Exercise 2

2. (a)/(b) - An accurate implementation of exp(x) Work out $e^{5.5}$ by calculating numerator and the denominator of the terms of the taylor expansion $e^x = \sum_{i=0}^{\inf} \frac{x^i}{i!}$

```
N = 31;
x = 5.5;
% Numerator and denominator
num  = ones(1,N);
den  = zeros(1,N);
% Terms
ter      = zeros(1,N);
iter     = zeros(1,N);
iter_sum = zeros(1,N);

for j=1:N

    iter(j) = j-1;
    temp_num = 1;
    temp_den = 1;

    for k=1:(j-1)
        temp_num = round(temp_num*x, 5, 'significant');
        temp_den = round(temp_den*k, 5, 'significant');
    end

    num(j)  = temp_num;
    den(j)  = temp_den;

    ter(j)   = round(num(j)/den(j), 5, 'significant');
    if j==1
        iter_sum(j) = ter(j);
    else
        iter_sum(j) = round(ter(j)+iter_sum(j-1),5, 'significant');
    end
end

abserr = abs(iter_sum-exp(x));
relerr = abs(iter_sum-exp(x))/exp(x);

fileID = fopen('exponential.txt','w');
```

```
fprintf(fileID,'%s %9s %12s %12s %12s %12s %12s \n','Term#','Numerator',...
             'Denominator','Term','Partial Sum','Abs. Error','Rel. Error');
fprintf(fileID,'-----------------------------------------------------------------------------
-----\n');
fprintf(fileID,'%2d%13.5g%13.5g%13.5g%13.5g%13.5g%13.5g\n',...
            [iter;num;den;ter;iter_sum;abserr;relerr]);
fclose(fileID);
```

The the error does not change after iteration term 17. and is accurate to to only 2 decimal places in the absolute error. The relative error turns out to be relatively small. $|\epsilon| < 10^{-4}$. See appendices table.

2. (c) repeat the exercise, but add the terms from right to left

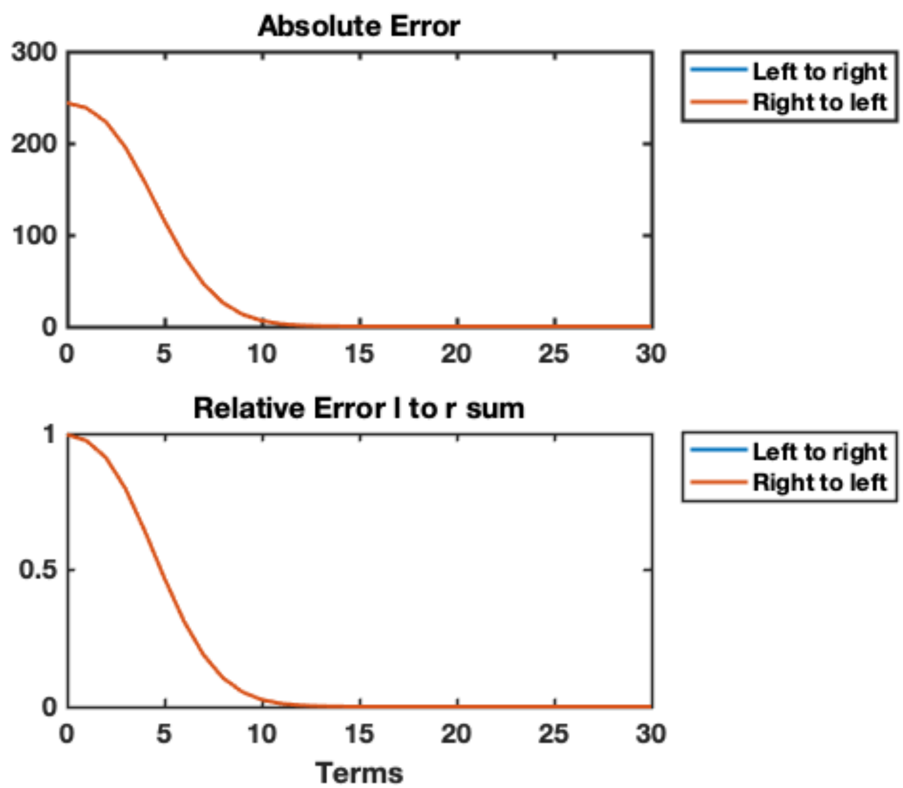
```
rl_iter_sum = zeros(1,N);

for j=1:N

    partial_sum = 0;

    for k = 1:j
            partial_sum = round(ter(j-k+1)+partial_sum,5, 'significant');
    end

    rl_iter_sum(j) = partial_sum;

end

rl_abserr = abs(rl_iter_sum-exp(x));
rl_relerr = abs(rl_iter_sum-exp(x))/exp(x);

fileID2 = fopen('exponential_right_left.txt','w');
fprintf(fileID2,'%s %9s %12s %12s %12s %12s %12s \n','Term#','Numerator',...
    'Denominator','Term','Partial Sum','Abs. Error','Rel. Error');
fprintf(fileID2,'----------------------------------------------------------------------------
------\n');
fprintf(fileID2,'%2d%13.5g%13.5g%13.5g%13.5g%13.5g%13.5g\n',...
                    [iter;num;den;ter;rl_iter_sum;rl_abserr;rl_relerr]);
fclose(fileID2);

max(abs(rl_iter_sum-iter_sum))
```

```
ans =

    0.0100
```

It takes more computations to make get closer to the actual number; specifically 4 more terms. In the plot, the errors are so close together that one is plotted on top of the other and therefore invisible.

```
lw = 2;
fig = figure(1);
subplot(2,1,1)
plot(iter,abserr,'Linewidth',lw)
hold on
plot(iter,rl_abserr,'Linewidth',lw)
```

```
hold off
title('Absolute Error')
legend('Left to right','Right to left','Location','NorthEastOutside')
set(gca,"Fontsize",14,'Fontweight','bold','Linewidth',lw)

subplot(2,1,2)
plot(iter,relerr,'Linewidth',lw)
hold on
plot(iter,rl_relerr,'Linewidth',lw)
hold off
xlabel('Terms')
legend('Left to right','Right to left','Location','NorthEastOutside')
title('Relative Error l to r sum')
set(gca,"Fontsize",14,'Fontweight','bold','Linewidth',1.5)
```



Clear Cariables before next computations

```
clearvars
```

2 (d) -- Convergence of the different methods -- (i) Left to right summation

```
N = 31;
x = 5.5;
% Numerator and denominator
num  = ones(1,N);
den  = zeros(1,N);
% Terms
ter      = zeros(1,N);
iter     = zeros(1,N);
iter_sum = zeros(1,N);
```

```
for j=1:N

    iter(j) = j-1;

    % odd parts negative
    if mod(j-1,2)==1
        temp_num = -1;
        temp_den = 1;
    else
        temp_num = 1;
        temp_den = 1;
    end

    for k=1:(j-1)
        temp_num = round(temp_num*x, 5, 'significant');
        temp_den = round(temp_den*k, 5, 'significant');
    end

    num(j)  = temp_num;
    den(j)  = temp_den;

    ter(j)   = round(num(j)/den(j), 5, 'significant');
    if j==1
        iter_sum(j) = ter(j);
    else
        iter_sum(j) = round(ter(j)+iter_sum(j-1),5, 'significant');
    end
end

abserr = abs(iter_sum-exp(-x));
relerr = abs(iter_sum-exp(-x))/exp(-x);

fileID = fopen('exponential-.txt','w');
fprintf(fileID,'%s %9s %12s %12s %12s %12s %12s \n','Term#','Numerator',...
    'Denominator','Term','Partial Sum','Abs. Error','Rel. Error');
fprintf(fileID,'---------------------------------------------------------------------------
-----\n');
fprintf(fileID,'%2d%13.5g%13.5g%13.5g%13.5g%13.5g%13.5g\n',...
                [iter;num;den;ter;iter_sum;abserr;relerr]);
fclose(fileID);
```

2. (d) (ii) right to left summation

```
rl_iter_sum = zeros(1,N);

for j=1:N

    partial_sum = 0;

    for k = 1:j
        partial_sum = round(ter(j-k+1)+partial_sum,5, 'significant');
    end

    rl_iter_sum(j) = partial_sum;

end

rl_abserr = abs(rl_iter_sum-exp(-x));
```

```
rl_relerr = abs(rl_iter_sum-exp(-x))/exp(-x);

fileID2 = fopen('exponential-_right_left.txt','w');
fprintf(fileID2,'%s %9s %12s %12s %12s %12s %12s \n','Term#','Numerator',...
            'Denominator','Term','Partial Sum','Abs. Error','Rel. Error');
fprintf(fileID2,'--------------------------------------------------------------------------
------\n');
fprintf(fileID2,'%2d%13.5g%13.5g%13.5g%13.5g%13.5g%13.5g\n',...
                    [iter;num;den;ter;rl_iter_sum;rl_abserr;rl_relerr]);
fclose(fileID2);
```

(iii) left to right split positive and negative sums

```
tot_sum = nan(1,N);

for j=1:N

    partial_sum = 0;

    pos_sum = 0;
    neg_sum = 0;

    for k = 1:j

        %even
        if mod(k-1,2)==0
            pos_sum = round(pos_sum + ter(k),5,'significant');
        %odd
        else
            neg_sum = round(neg_sum + ter(k),5,'significant');
        end

        partial_sum = round(pos_sum + neg_sum,5, 'significant');

    end

    tot_sum(j) = partial_sum;

end

posneg_abserr = abs(tot_sum-exp(-x));
posneg_relerr = abs(tot_sum-exp(-x))/exp(-x);

fileID2 = fopen('exp-posneg_left_to_right.txt','w');
fprintf(fileID2,'%s %9s %12s %12s %12s %12s %12s \n','Term#','Numerator',...
            'Denominator','Term','Partial Sum','Abs. Error','Rel. Error');
fprintf(fileID2,'--------------------------------------------------------------------------
------\n');
fprintf(fileID2,'%2d%13.5g%13.5g%13.5g%13.5g%13.5g%13.5g\n',...
                [iter;num;den;ter;tot_sum;posneg_abserr;posneg_relerr]);
fclose(fileID2);
```

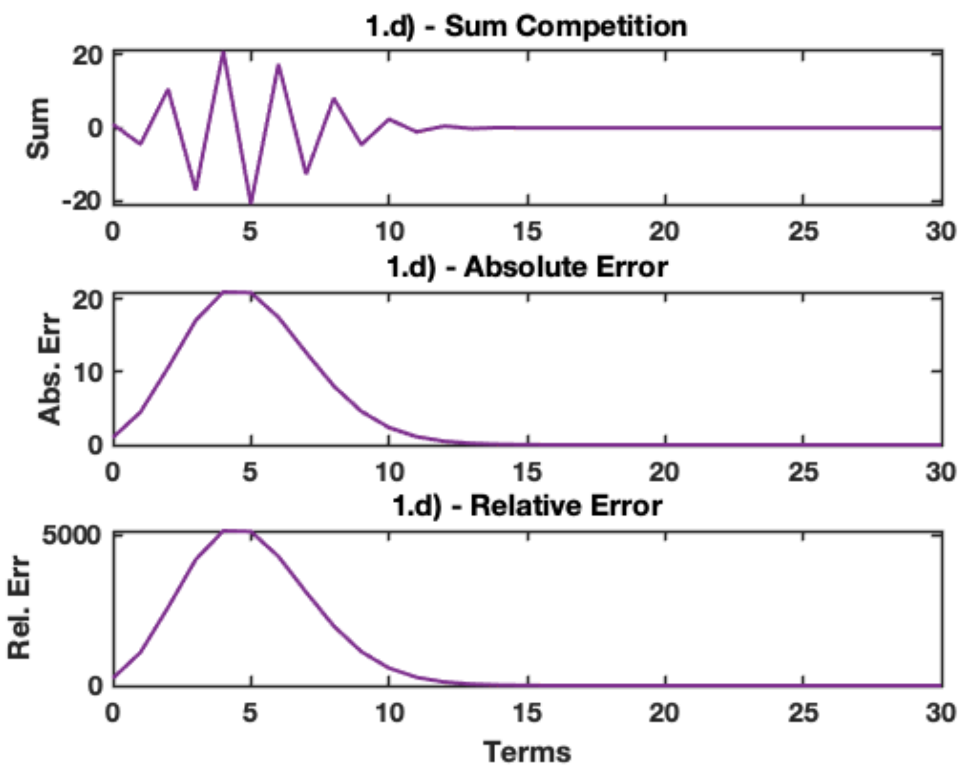
(iv) right to left sum, split positive and negative sums

```
rl_tot_sum = nan(1,N);

for j=1:N
```

```
    partial_sum = 0;

    pos_sum = 0;
    neg_sum = 0;

    for k = 1:j

        %even
        if mod(k-1,2)==0
            pos_sum = round(pos_sum + ter(j-k+1),5,'significant');
        %odd
        else
            neg_sum = round(neg_sum + ter(j-k+1),5,'significant');
        end

        partial_sum = round(pos_sum + neg_sum,5, 'significant');

    end

    rl_tot_sum(j) = partial_sum;

end

rl_posneg_abserr = abs(rl_tot_sum-exp(-x));
rl_posneg_relerr = abs(rl_tot_sum-exp(-x))/exp(-x);

fileID2 = fopen('exp-posneg_right_to_left.txt','w');
fprintf(fileID2,'%s %9s %12s %12s %12s %12s %12s \n','Term#','Numerator',...
            'Denominator','Term','Partial Sum','Abs. Error','Rel. Error');
fprintf(fileID2,'---------------------------------------------------------------------------
------\n');
fprintf(fileID2,'%2d%13.5g%13.5g%13.5g%13.5g%13.5g%13.5g\n',...
          [iter;num;den;ter;rl_tot_sum;rl_posneg_abserr;rl_posneg_relerr]);
fclose(fileID2);
```

```
lw = 2; % lineweight
% Plotting the competition in one plot
figure(3)
subplot(3,1,1)
plot(iter,iter_sum,'Linewidth',lw)
hold on
plot(iter,rl_iter_sum,'Linewidth',lw)
plot(iter,tot_sum,'Linewidth',lw)
plot(iter,rl_tot_sum,'Linewidth',lw)
hold off
ylabel('Sum')
title('1.d) - Sum Competition')
set(gca,"Fontsize",14,'Fontweight','bold','Linewidth',1.5)

subplot(3,1,2)
plot(iter,abs(iter_sum-exp(-x)),'Linewidth',lw)
hold on
plot(iter,abs(rl_iter_sum-exp(-x)),'Linewidth',lw)
plot(iter,abs(tot_sum-exp(-x)))
plot(iter,abs(rl_tot_sum-exp(-x)),'Linewidth',lw)
hold off
ylabel('Abs. Err')
title('1.d) - Absolute Error')
```

```
set(gca,"Fontsize",14,'Fontweight','bold','Linewidth',1.5)

subplot(3,1,3)
plot(iter,abs(iter_sum-exp(-x))/exp(-x),'Linewidth',lw)
hold on
plot(iter,abs(rl_iter_sum-exp(-x))/exp(-x),'Linewidth',lw)
plot(iter,abs(tot_sum-exp(-x))/exp(-x),'Linewidth',lw)
plot(iter,abs(rl_tot_sum-exp(-x))/exp(-x),'Linewidth',lw)
hold off
xlabel('Terms')
ylabel('Rel. Err')
title('1.d) - Relative Error')
set(gca,"Fontsize",14,'Fontweight','bold','Linewidth',1.5)
```



See appendices for attached table of the computations. Comparing the first two approaches: For exp(-x) it takes much less terms for the right to left summation. In fact, until the summations become insignificant it takes 19 terms where as the left to right summation takes 25. Additionally, the final error of the right to left summation is ~1/3 of the size of the left to right summation's error which is interesting

The standard left to right sum never gets closer to 0 than $|\epsilon| \leq 3.8e-4$. And only reaches the insignificance of terms after 22 terms have been added. Likewise, the right to left sum reaches the limit of significant addition by term 20, which is 2 less terms. However, the error is slightly larger $|\epsilon| \leq 4e-4$. The Split sum are not only more efficient, they are also more accurate. Both reach 0 after 18 terms have been added and, hence, show 0 error. However, The right to left sum is returning an error of $|\epsilon| \leq 1e-2 after summation of higher order terms, which is weird. What's interesting that the relative error when computing the split sums approaches 1 since the partial sums seem to lose significant figures and are simpy set to 0 after a certain amount of steps.

2 (e) One way of calculating $e^{-5.5}$ is by computing $e^{5.5}$ and take the reciprocal. Let's see how that goes:

```
N = 31;
x = 5.5;
```

```
% Numerator and denominator
num  = ones(1,N);
den  = zeros(1,N);
% Terms
ter      = zeros(1,N);
iter     = zeros(1,N);
iter_sum = zeros(1,N);

for j=1:N

    iter(j) = j-1;
    temp_num = 1;
    temp_den = 1;

    for k=1:(j-1)
        temp_num = round(temp_num*x, 5, 'significant');
        temp_den = round(temp_den*k, 5, 'significant');
    end

    num(j)  = temp_num;
    den(j)  = temp_den;

    ter(j)   = round(num(j)/den(j), 5, 'significant');
    if j==1
        iter_sum(j) = ter(j);
    else
        iter_sum(j) = round(ter(j)+iter_sum(j-1),5, 'significant');
    end
end

% Division operation 1
iter_sum = round(1./iter_sum,5, 'significant');

abserr = abs(iter_sum-exp(-x));
relerr = abs(iter_sum-exp(-x))/exp(-x);

fileID = fopen('neg_exponential.txt','w');
fprintf(fileID,'%s %9s %12s %12s %12s %12s %12s \n','Term#','Numerator',...
            'Denominator','Term','Partial Sum','Abs. Error','Rel. Error');
fprintf(fileID,'---------------------------------------------------------------------------
-----\n');
fprintf(fileID,'%2d%13.5g%13.5g%13.5g%13.5g%13.5g%13.5g\n',...
                        [iter;num;den;ter;iter_sum;abserr;relerr]);
fclose(fileID);
```

The table is found in the appendix. The error stays the same after approximately the same amount of steps, but what's amazing is the accuracy compared to computing the negative exponential directly through its series. Here, the relative error is of the order of e(-5) and the absolute error of the order of e(-7). Both significantly less than when computing the series directly.

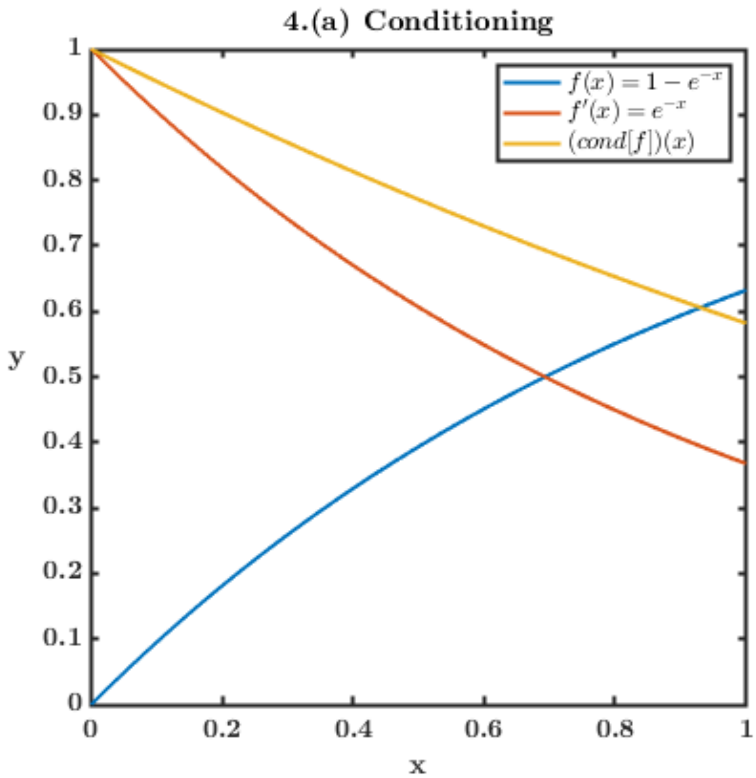
Clear variables

```
clearvars
```

Exercise 4

(a) Condition

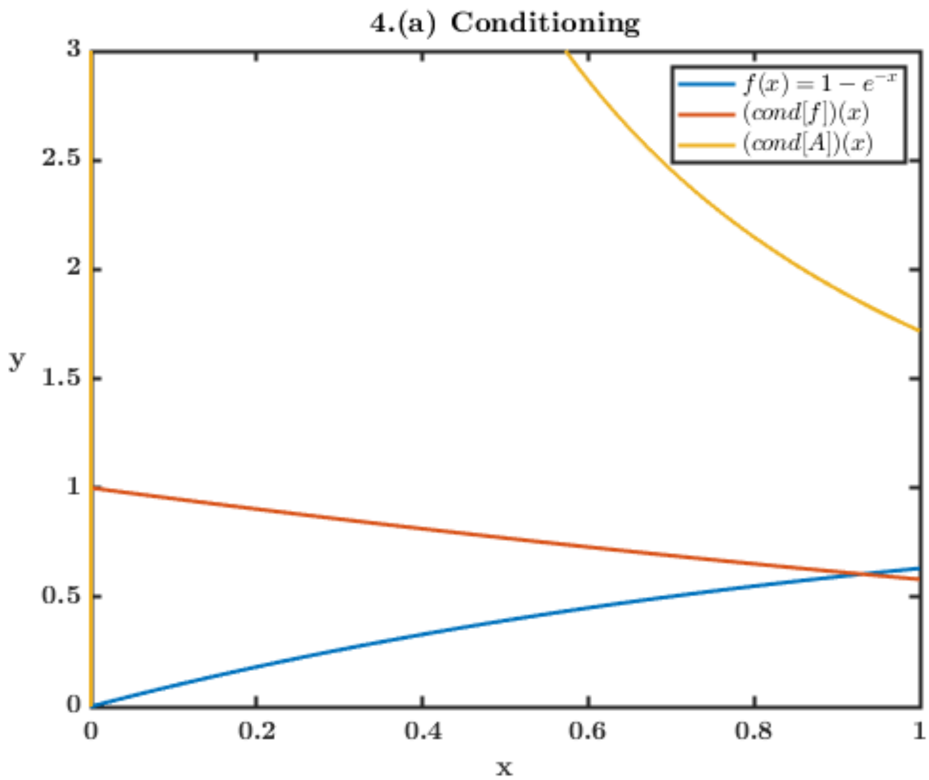
```
x = linspace(-2,2,1002);

% Function
y = 1-exp(-x);

% Condition if eps=1
condy = abs(exp(-x).*x)./abs(exp(-x)-1);

% Derivative
dydx = exp(-x);

% Plots
lw = 2; % Linewidth
figure(4)
plot(x,y,x,dydx,'Linewidth',lw)
hold on
plot(x,condy,'Linewidth',lw)
axis equal
axis([0 1 0 1])
hold off

legend({"$f(x) = 1-e^{-x}$","$f'(x) = e^{-x}$","$(cond[f])(x)$"},...
       'Interpreter','latex')
xlabel('\bf{x}','Interpreter','latex')
ylabel('\bf{y}  ','Interpreter','latex','Rotation',0,...
       'HorizontalAlignment','right')
title('\bf{4.(a) Conditioning}','Interpreter','latex')
set(gca,"Fontsize",14,'Fontweight','bold','Linewidth',lw,...
                      'TickLabelInterpreter','latex')
axesH = gca;
axesH.XAxis.TickLabelFormat      = '\\textbf{%g}';
axesH.YAxis.TickLabelFormat      = '\\textbf{%g}';
```



(c)

```
x = linspace(-2,2,1002);

% Function
y = 1-exp(-x);

% Condition if eps=1
condy = abs(exp(-x).*x)./abs(exp(-x)-1);

% Condition if eps=1
condA = (exp(1)-1)./x;

% Plots
lw = 2; % Linewidth
figure(4)
plot(x,y,'Linewidth',lw)
hold on
plot(x,condy,'Linewidth',lw)
plot(x,condA,'Linewidth',lw)
% axis equal
axis([0 1 0 3])
hold off

legend({"$f(x) = 1-e^{-x}$","$(cond[f])(x)$","$(cond[A])(x)$"},...
        'Interpreter','latex')
xlabel('\bf{x}','Interpreter','latex')
ylabel('\bf{y}  ','Interpreter','latex','Rotation',0,...
        'HorizontalAlignment','right')
title('\bf{4.(a) Conditioning}','Interpreter','latex')
set(gca,"Fontsize",14,'Fontweight','bold','Linewidth',lw,...
        'TickLabelInterpreter','latex')
```

```
axesH = gca;
axesH.XAxis.TickLabelFormat       = '\\textbf{%g}';
axesH.YAxis.TickLabelFormat       = '\\textbf{%g}';
```



(f) The function is ill-conditioned due the subtraction of a number very close to one but not 1 from 1 as $f(x)$ approaches 0. In the condition itself, it's the division by something close to zero due to the denominator approaching 0.

clear variables

```
clearvars
```

Exercise 5

The simplest thing is to do a while loop that checks whether the real value $\epsilon \leq |\lim_{n+1\to\infty} \left(1+\frac{1}{n+1}\right)^{n+1} - \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{n}|$, where $\epsilon = e - 12$.

```
% real value
real_value = exp(1);

% starting n
n = 10^0;

% Tolerance
eps = 1e-12;

% Approximation function, Matlab equivalent to lambda, in this case ok I
% think
f = @(n) (1 + 1/n)^(n);
```

```
% Some counter
counter = 1;

% First approximation value
approx = f(n(counter));

% No error can be computed
err = NaN;

% Some counter
counter = 2;

% Compute the specified n
n(counter) = 10^(counter-1);

% Compute approximation
approx(counter) = f(n(counter));

% Calculate error between second and first approximation
err(counter) = abs(approx(counter)-approx(counter-1));


% while err(counter)>eps
for k=1:30
    % increase step
    counter  = counter + 1;

    if counter == 30
        break
    end

    % Compute the specified n
    n(counter) = 10^(counter-1);

    % Compute approximation
    approx(counter) = f(n(counter));

    % Compute Error
    err(counter) = abs(approx(counter)-approx(counter-1));

end
```

```
fileID2 = fopen('exponential_series_approx.txt','w');

fprintf(fileID2,'%16s%16s%16s\n','n','e(x)_approx','Absolute Error');
fprintf(fileID2,'------------------------------------------------\n');
fprintf(fileID2,'%16.5g%16.5g%16.5g\n',[n',approx',err']');
fclose(fileID2);
```

Results:

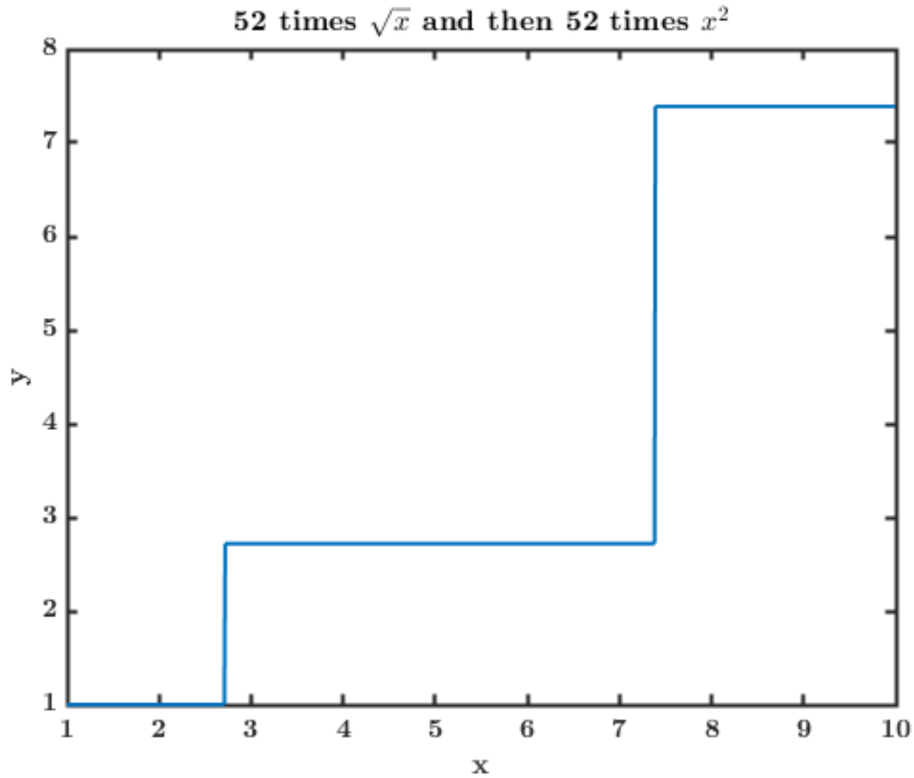
```
fprintf('nstop: %g\n',n(end))
fprintf('Final Approximation of e: %f\n',approx(end))

clearvars
```

```
nstop: 1e+28
Final Approximation of e: 1.000000
```

Exercise 6

Squareroot thing

```
x = linspace(1,10,1001);

y = x;

N = 52;
for i=1:N
   y = sqrt(y);
end

for i=1:N
   y = y.^2;
end

lw = 2;
figure(4)
plot(x,y,'Linewidth',lw)
xlabel('\textbf{x}','Interpreter','latex')
ylabel('\textbf{y}','Interpreter','latex')
title('\bf{52 times $\sqrt{x}$ and then 52 times ${x}^2$}',...
    'Interpreter','latex')
set(gca,"Fontsize",14,'Fontweight','bold','Linewidth',lw,...
        'TickLabelInterpreter','latex')
axesH = gca;
axesH.XAxis.TickLabelFormat      = '\\textbf{%g}';
axesH.YAxis.TickLabelFormat      = '\\textbf{%g}';
```



Exercise 7

(a) Wilkinson polynomial roots

```
wilk_root = 1:20;

% Get coefficients from a polynomial
c  = poly(wilk_root);

% Derivative coefficients
dc = polyder(c);
```

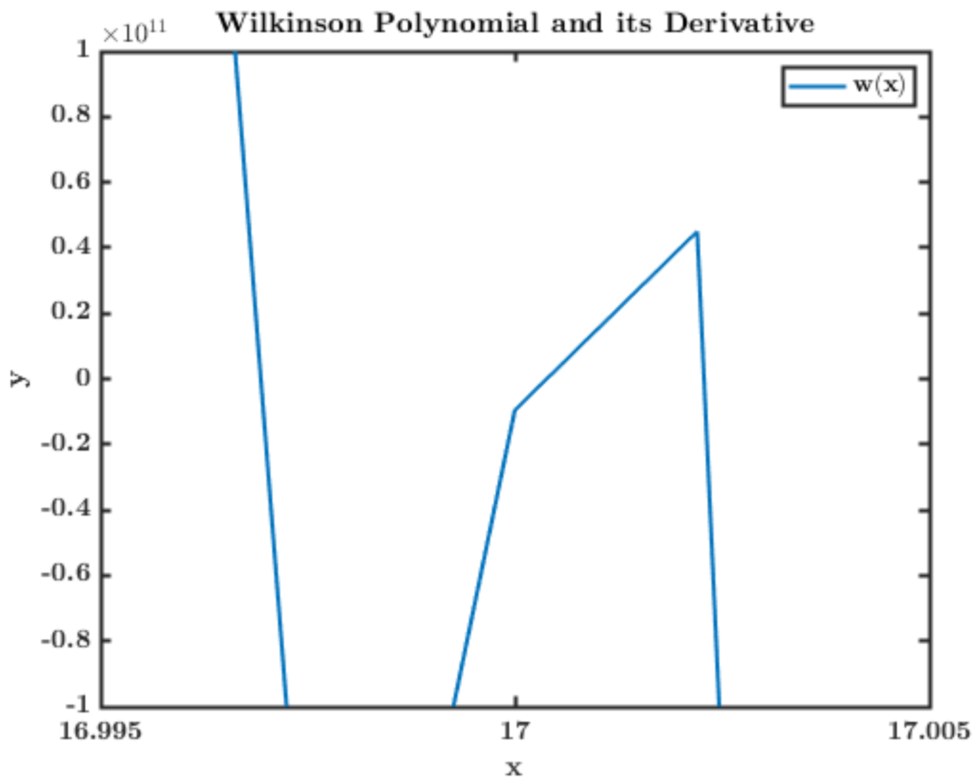
(b)

```
w  = @(x) polyval(c, x); % Wilkinson polynomial
dw = @(x) polyval(dc,x); % derivative of wilkinson polynomial
```

Plot fucntion and derivative on $x\epsilon[0,20]$

```
x   = linspace(-1,21,10000);
fw  = nan(1,numel(x));
dfw = nan(1,numel(x));


for k = 1:numel(x)
    fw(k)  =  w(x(k));
    dfw(k) = dw(x(k));
end
```

```
lw = 2;
figure(5)
plot(x,fw,'LineWidth',lw)
% xlim([3.95,4.05])
axis([16.995,17.005,-10^11,10^11])
legend({'$\mathbf{w(x)}$'},'Interpreter','latex')
xlabel('\textbf{x}','Interpreter','latex')
ylabel('\textbf{y}','Interpreter','latex')
title('\bf{Wilkinson Polynomial and its Derivative}','Interpreter','latex')
set(gca,"Fontsize",14,'Fontweight','bold','Linewidth',lw,...
            'TickLabelInterpreter','latex')
axesH = gca;
axesH.XAxis.TickLabelFormat      = '\\textbf{%g}';
axesH.YAxis.TickLabelFormat      = '\\textbf{%g}';
```




```
% Mini Newton
N = 1;
x0 = 21;
for k=1:N
    x1 = x0-w(x0)/dw(x0);
    x0 = x1;
end
```

Relative error between Newton root and real root at x = 20 and after 20 iterations

```
err_N = abs((x1-20)/20)
```

```
err_N =
```

```
    0.0361
```

Matlabs built in root finder

```
wilk_roots = roots(c)
```

```
wilk_roots =

   19.9999
   19.0013
   17.9937
   17.0185
   15.9597
   15.0593
   13.9302
   13.0627
   11.9589
   11.0225
    9.9912
    9.0027
    7.9994
    7.0001
    6.0000
    5.0000
    4.0000
    3.0000
    2.0000
    1.0000
```

Error between Matlab Root and real root at x = 20:

```
err_m = abs((wilk_roots(1)-20)/20)
```

```
err_m =

   6.2972e-06
```

After the statement 'Does it?' in the exercise, I was genuinely surprised that the simplest form of the root finder, coded by hand, found it immediately (to a certain accuracy). I also tried a bunch of other numbers within and close to the interval that all worked. I mean, it was just trial and error. I could have easily missed one where it loops forever.

That was until I realized that the function oscillates quite a lot and seems to have multiple roots (more than 20). This gets worse the closer one gets to the highest root of $x = 20$. The zoomed-in figure shows that the Wilkinson Polynomial fluctates alot. Meaning, that a root finder has a really hard time since the graph crosses the x axis a lot.

7 (c)

```
delta = 10^(-2);
```

```
% Wilkinson polynomial roots
wilk_rootc = 1:20;

% Get coefficients from a polynomial
cc   = poly(wilk_rootc);
cc(1) = cc(1) + delta;   % cc(1) is the a = 1;

% Derivative coefficients
dcc = polyder(cc);

% Create Polynomials
wc   = @(x) polyval(cc, x); % Wilkinson polynomial
dwc = @(x) polyval(dcc,x); % derivative of wilkinson polynomial

% Mini Newton
Nc = 10000;
x0c = 21;
for k=1:Nc
    x1c = x0c-wc(x0c)/dwc(x0c);
    x0c = x1c;
end
```

For $\delta = 10^{-8}$, the algorithm does not converge to 20; $x_{10^4} \approx 9.5855$ after 10000, which is not a root andnot unexpected when thinking about the results that follow in exercise part (e). The possible error is supposed to be large.

Also for $\delta = 10^{-6}$, the algorithm does not converges to 20 her either, $x_{10^4} \approx 7.7527$ after 10000

Also for $\delta = 10^{-4}$, the algorithm does not converge to 20, but it does converge to another root, i.e., 6. Seems alot closer to a root than the previous 2 solutions with $x_{10^4} \approx 5.9693$ after 10000.

Also for $\delta = 10^{-2}$, the algorithm does not converge either with a value that is quite a lot off; $x_{10^4} \approx 5.4696$ after 10000 iterations, which is a larger margin to the actual value.

My explanation for these fluctuations is that the the values actually not represent roots but close neighbors. it seems like the polynomial has more than 20 roots due to fluctuations

(d)

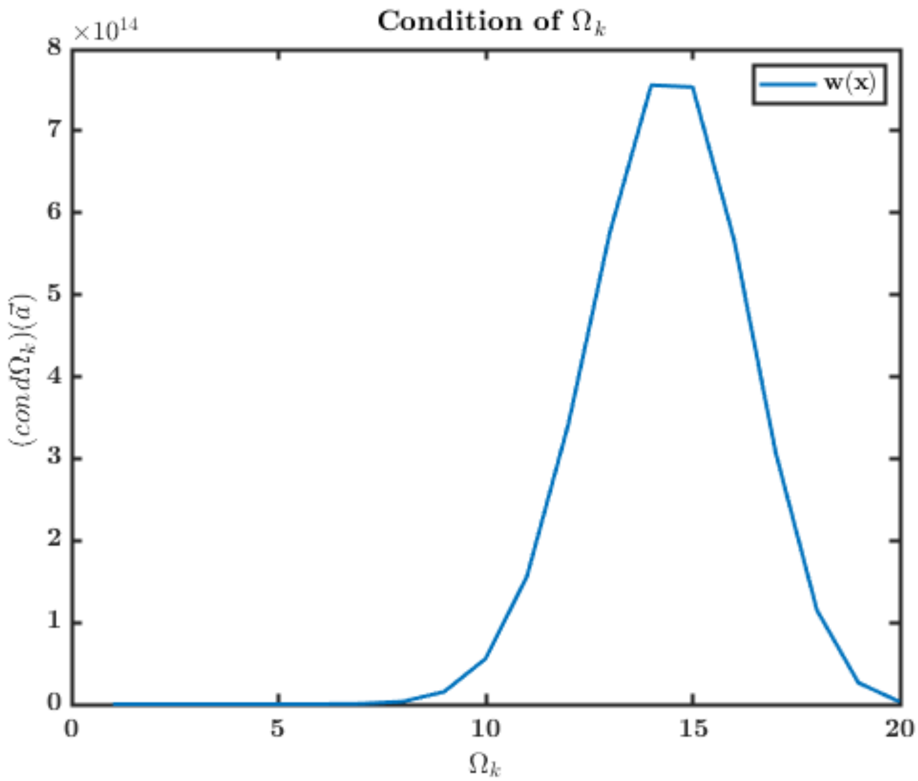
```
delta = -2^(-53);

% Wilkinson polynomial roots
wilk_rootd = 1:20;

% Get coefficients from a polynomial
cd   = poly(wilk_rootd);
cd(2) = cd(2) + delta;   % cd(2) is the a_19 = -210;

% Derivative coefficients
dcd = polyder(cd);

% Create Polynomials
wd   = @(x) polyval(cd, x); % Wilkinson polynomial
dwd = @(x) polyval(dcd,x); % derivative of wilkinson polynomial

% Mini Newton
Nd = 10000;
x0d = 21;
```

```
for k=1:Nd
    x1d = x0d-wd(x0d)/dwd(x0d);
    x0d = x1d;
end
```

For $\delta = -2^{-53}$ on a_{19}, the algorithm converges safely to 20; $x_{10^4} \approx 20.5171$, which is unexpected when thinking about the results that follow in exercise part (e).

(e) - (ii) Compute the condition for

```
cond_omega = zeros(1,20);

% polynomial coefficients
a      = fliplr(c);

% predefined root
omega = 1:20;
for k=1:20
    for l=1:20
        cond_omega(k) = cond_omega(k) + abs(a(l)*omega(k)^(l-1)/dw(omega(k)));
    end
end

lw = 2;
figure(6)
plot(omega,cond_omega,'LineWidth',lw)
legend({'$\mathbf{w(x)}$'},'Interpreter','latex')
xlabel('\textbf{$\Omega_k$}','Interpreter','latex')
ylabel('\textbf{ $(cond \Omega_k) (\vec{a}) $}','Interpreter','latex')
title('\textbf{ Condition of $\Omega_k$}','Interpreter','latex')
set(gca,"Fontsize",14,'Fontweight','bold','Linewidth',lw,...
                'TickLabelInterpreter','latex')
axesH = gca;
axesH.XAxis.TickLabelFormat      = '\\textbf{%g}';
axesH.YAxis.TickLabelFormat      = '\\textbf{%g}';
```



The condition number for the roots $[14, 16, 17, 20]$ are simply bad. It apparently is worst at 14 and best at 0. Even though it looks like the condition goes to 0 at 20 it merely goes to 10^{12}, whereas at Ω_k it is actually significantly smaller $O(10^2)$.

(iii) As seen in the above results, the problem is ill-conditioned, on the domain from $[1, 20]$. Since the problem itself is ill-posed, there is no way of getting around the issue. This is large du to the large factors of the polynomial.

Exercise 8

(c) Brute Force computing Stirling's approximation to get a guess'timate of $\frac{k!}{\epsilon}$

```
for N=29:34
    N;
    N*log(N)-N;
end

% Set N to a smaller range of numbers
N = factorial([30 31 32 33]);

k = 20;
kOverEps = factorial(k)*2^52;

% Calculate least close
(N-kOverEps)
```

```
ans =

   1.0e+36 *

  -0.0107   -0.0027    0.2522    8.6724
```

Smallest N can be to get a good approximation of y_k is $N = 32$.

(d)

```
N  = 32;
yN = 0;

k = 20;

for l = fliplr(k:1:N-1)
    yk = (exp(1)-yN)/(1+l);
    yN = yk;
end

yk
```

```
yk =

    0.1238
```

From Wolfram alpha I got ca. 0.123804 which is pretty exactly what I am getting here. That's crazy. And testing different starting values shows that N = 32 is the least upper bound of N. After that the 10^{13} significant digit does not change anymore.

Appendix

Exercise 2. All tables on exercise 2 and its error propagation

exp(x) left to right summation

Term#	Numerator	Denominator	Term	Partial Sum	Abs. Error	Rel. Error
0	1	1	1	1	243.69	0.99591
1	5.5	1	5.5	6.5	238.19	0.97344
2	30.25	2	15.125	21.625	223.07	0.91162
3	166.38	6	27.73	49.355	195.34	0.7983
4	915.09	24	38.129	87.484	157.21	0.64247
5	5033	120	41.942	129.43	115.26	0.47105
6	27682	720	38.447	167.88	76.812	0.31391
7	1.5225e+05	5040	30.208	198.09	46.602	0.19045
8	8.3738e+05	40320	20.768	218.86	25.832	0.10557
9	4.6056e+06	3.6288e+05	12.692	231.55	13.142	0.053708
10	2.5331e+07	3.6288e+06	6.9805	238.53	6.1619	0.025182
11	1.3932e+08	3.9917e+07	3.4902	242.02	2.6719	0.01092
12	7.6626e+08	4.79e+08	1.5997	243.62	1.0719	0.0043807
13	4.2144e+09	6.227e+09	0.67679	244.3	0.39193	0.0016017
14	2.3179e+10	8.7178e+10	0.26588	244.57	0.12193	0.00049831
15	1.2748e+11	1.3077e+12	0.097484	244.67	0.021932	8.9632e-05
16	7.0114e+11	2.0923e+13	0.03351	244.7	0.0080677	3.2971e-05
17	3.8563e+12	3.5569e+14	0.010842	244.71	0.018068	7.3839e-05
18	2.121e+13	6.4024e+15	0.0033128	244.71	0.018068	7.3839e-05
19	1.1666e+14	1.2165e+17	0.00095898	244.71	0.018068	7.3839e-05
20	6.4163e+14	2.433e+18	0.00026372	244.71	0.018068	7.3839e-05
21	3.529e+15	5.1093e+19	6.907e-05	244.71	0.018068	7.3839e-05
22	1.941e+16	1.124e+21	1.7269e-05	244.71	0.018068	7.3839e-05
23	1.0676e+17	2.5852e+22	4.1297e-06	244.71	0.018068	7.3839e-05
24	5.8718e+17	6.2045e+23	9.4638e-07	244.71	0.018068	7.3839e-05

25	3.2295e+18	1.5511e+25	2.0821e-07	244.71	0.018068	7.3839e-05
26	1.7762e+19	4.0329e+26	4.4043e-08	244.71	0.018068	7.3839e-05
27	9.7691e+19	1.0889e+28	8.9715e-09	244.71	0.018068	7.3839e-05
28	5.373e+20	3.0489e+29	1.7623e-09	244.71	0.018068	7.3839e-05
29	2.9552e+21	8.8418e+30	3.3423e-10	244.71	0.018068	7.3839e-05
30	1.6254e+22	2.6525e+32	6.1278e-11	244.71	0.018068	7.3839e-05

exp(x) right to left summation

Term#	Numerator	Denominator	Term	Partial Sum	Abs. Error	Rel. Error
0	1	1	1	1	243.69	0.99591
1	5.5	1	5.5	6.5	238.19	0.97344
2	30.25	2	15.125	21.625	223.07	0.91162
3	166.38	6	27.73	49.355	195.34	0.7983
4	915.09	24	38.129	87.484	157.21	0.64247
5	5033	120	41.942	129.43	115.26	0.47105
6	27682	720	38.447	167.88	76.812	0.31391
7	1.5225e+05	5040	30.208	198.09	46.602	0.19045
8	8.3738e+05	40320	20.768	218.86	25.832	0.10557
9	4.6056e+06	3.6288e+05	12.692	231.55	13.142	0.053708
10	2.5331e+07	3.6288e+06	6.9805	238.53	6.1619	0.025182
11	1.3932e+08	3.9917e+07	3.4902	242.02	2.6719	0.01092
12	7.6626e+08	4.79e+08	1.5997	243.62	1.0719	0.0043807
13	4.2144e+09	6.227e+09	0.67679	244.29	0.40193	0.0016426
14	2.3179e+10	8.7178e+10	0.26588	244.56	0.13193	0.00053918
15	1.2748e+11	1.3077e+12	0.097484	244.66	0.031932	0.0001305
16	7.0114e+11	2.0923e+13	0.03351	244.69	0.0019323	7.8967e-06
17	3.8563e+12	3.5569e+14	0.010842	244.7	0.0080677	3.2971e-05
18	2.121e+13	6.4024e+15	0.0033128	244.7	0.0080677	3.2971e-05
19	1.1666e+14	1.2165e+17	0.00095898	244.7	0.0080677	3.2971e-05
20	6.4163e+14	2.433e+18	0.00026372	244.7	0.0080677	3.2971e-05
21	3.529e+15	5.1093e+19	6.907e-05	244.71	0.018068	7.3839e-05
22	1.941e+16	1.124e+21	1.7269e-05	244.71	0.018068	7.3839e-05
23	1.0676e+17	2.5852e+22	4.1297e-06	244.71	0.018068	7.3839e-05
24	5.8718e+17	6.2045e+23	9.4638e-07	244.71	0.018068	7.3839e-05
25	3.2295e+18	1.5511e+25	2.0821e-07	244.71	0.018068	7.3839e-05
26	1.7762e+19	4.0329e+26	4.4043e-08	244.71	0.018068	7.3839e-05
27	9.7691e+19	1.0889e+28	8.9715e-09	244.71	0.018068	7.3839e-05
28	5.373e+20	3.0489e+29	1.7623e-09	244.71	0.018068	7.3839e-05
29	2.9552e+21	8.8418e+30	3.3423e-10	244.71	0.018068	7.3839e-05
30	1.6254e+22	2.6525e+32	6.1278e-11	244.71	0.018068	7.3839e-05

exp(-x) left to right summation

Term#	Numerator	Denominator	Term	Partial Sum	Abs. Error	Rel. Error
0	1	1	1	1	0.99591	243.69
1	-5.5	1	-5.5	-4.5	4.5041	1102.1
2	30.25	2	15.125	10.625	10.621	2598.9
3	-166.38	6	-27.73	-17.105	17.109	4186.5
4	915.09	24	38.129	21.024	21.02	5143.4
5	-5033	120	-41.942	-20.918	20.922	5119.5
6	27682	720	38.447	17.529	17.525	4288.2
7	-1.5225e+05	5040	-30.208	-12.679	12.683	3103.4
8	8.3738e+05	40320	20.768	8.089	8.0849	1978.3
9	-4.6056e+06	3.6288e+05	-12.692	-4.603	4.6071	1127.3
10	2.5331e+07	3.6288e+06	6.9805	2.3775	2.3734	580.76
11	-1.3932e+08	3.9917e+07	-3.4902	-1.1127	1.1168	273.27
12	7.6626e+08	4.79e+08	1.5997	0.487	0.48291	118.16

13	-4.2144e+09	6.227e+09	-0.67679	-0.18979	0.19388	47.44
14	2.3179e+10	8.7178e+10	0.26588	0.07609	0.072003	17.619
15	-1.2748e+11	1.3077e+12	-0.097484	-0.021394	0.025481	6.2349
16	7.0114e+11	2.0923e+13	0.03351	0.012116	0.0080292	1.9647
17	-3.8563e+12	3.5569e+14	-0.010842	0.001274	0.0028128	0.68826
18	2.121e+13	6.4024e+15	0.0033128	0.0045868	0.00050003	0.12235
19	-1.1666e+14	1.2165e+17	-0.00095898	0.0036278	0.00045897	0.11231
20	6.4163e+14	2.433e+18	0.00026372	0.0038915	0.00019527	0.047781
21	-3.529e+15	5.1093e+19	-6.907e-05	0.0038224	0.00026437	0.06469
22	1.941e+16	1.124e+21	1.7269e-05	0.0038397	0.00024707	0.060456
23	-1.0676e+17	2.5852e+22	-4.1297e-06	0.0038356	0.00025117	0.06146
24	5.8718e+17	6.2045e+23	9.4638e-07	0.0038365	0.00025027	0.061239
25	-3.2295e+18	1.5511e+25	-2.0821e-07	0.0038363	0.00025047	0.061288
26	1.7762e+19	4.0329e+26	4.4043e-08	0.0038363	0.00025047	0.061288
27	-9.7691e+19	1.0889e+28	-8.9715e-09	0.0038363	0.00025047	0.061288
28	5.373e+20	3.0489e+29	1.7623e-09	0.0038363	0.00025047	0.061288
29	-2.9552e+21	8.8418e+30	-3.3423e-10	0.0038363	0.00025047	0.061288
30	1.6254e+22	2.6525e+32	6.1278e-11	0.0038363	0.00025047	0.061288

exp(-x) right to left summation

Term#	Numerator	Denominator	Term	Partial Sum	Abs. Error	Rel. Error
0	1	1	1	1	0.99591	243.69
1	-5.5	1	-5.5	-4.5	4.5041	1102.1
2	30.25	2	15.125	10.625	10.621	2598.9
3	-166.38	6	-27.73	-17.105	17.109	4186.5
4	915.09	24	38.129	21.024	21.02	5143.4
5	-5033	120	-41.942	-20.918	20.922	5119.5
6	27682	720	38.447	17.529	17.525	4288.2
7	-1.5225e+05	5040	-30.208	-12.679	12.683	3103.4
8	8.3738e+05	40320	20.768	8.089	8.0849	1978.3
9	-4.6056e+06	3.6288e+05	-12.692	-4.603	4.6071	1127.3
10	2.5331e+07	3.6288e+06	6.9805	2.378	2.3739	580.88
11	-1.3932e+08	3.9917e+07	-3.4902	-1.113	1.1171	273.34
12	7.6626e+08	4.79e+08	1.5997	0.487	0.48291	118.16
13	-4.2144e+09	6.227e+09	-0.67679	-0.19	0.19409	47.491
14	2.3179e+10	8.7178e+10	0.26588	0.076	0.071913	17.597
15	-1.2748e+11	1.3077e+12	-0.097484	-0.021	0.025087	6.1385
16	7.0114e+11	2.0923e+13	0.03351	0.012	0.0079132	1.9363
17	-3.8563e+12	3.5569e+14	-0.010842	0.001	0.0030868	0.75531
18	2.121e+13	6.4024e+15	0.0033128	0.005	0.00091323	0.22346
19	-1.1666e+14	1.2165e+17	-0.00095898	0.004	8.6771e-05	0.021232
20	6.4163e+14	2.433e+18	0.00026372	0.004	8.6771e-05	0.021232
21	-3.529e+15	5.1093e+19	-6.907e-05	0.004	8.6771e-05	0.021232
22	1.941e+16	1.124e+21	1.7269e-05	0.004	8.6771e-05	0.021232
23	-1.0676e+17	2.5852e+22	-4.1297e-06	0.004	8.6771e-05	0.021232
24	5.8718e+17	6.2045e+23	9.4638e-07	0.004	8.6771e-05	0.021232
25	-3.2295e+18	1.5511e+25	-2.0821e-07	0.004	8.6771e-05	0.021232
26	1.7762e+19	4.0329e+26	4.4043e-08	0.004	8.6771e-05	0.021232
27	-9.7691e+19	1.0889e+28	-8.9715e-09	0.004	8.6771e-05	0.021232
28	5.373e+20	3.0489e+29	1.7623e-09	0.004	8.6771e-05	0.021232
29	-2.9552e+21	8.8418e+30	-3.3423e-10	0.004	8.6771e-05	0.021232
30	1.6254e+22	2.6525e+32	6.1278e-11	0.004	8.6771e-05	0.021232

exp(-5.5) computed via 1/x

Term#	Numerator	Denominator	Term	Partial Sum	Abs. Error	Rel. Error
0	1	1	1	1	0.99591	243.69

1	5.5	1	5.5	0.15385	0.14976	36.646
2	30.25	2	15.125	0.046243	0.042156	10.315
3	166.38	6	27.73	0.020261	0.016174	3.9577
4	915.09	24	38.129	0.011431	0.0073442	1.7971
5	5033	120	41.942	0.0077262	0.0036394	0.89054
6	27682	720	38.447	0.0059566	0.0018698	0.45753
7	1.5225e+05	5040	30.208	0.0050482	0.00096143	0.23525
8	8.3738e+05	40320	20.768	0.0045691	0.00048233	0.11802
9	4.6056e+06	3.6288e+05	12.692	0.0043187	0.00023193	0.056751
10	2.5331e+07	3.6288e+06	6.9805	0.0041923	0.00010553	0.025822
11	1.3932e+08	3.9917e+07	3.4902	0.0041319	4.5129e-05	0.011043
12	7.6626e+08	4.79e+08	1.5997	0.0041048	1.8029e-05	0.0044114
13	4.2144e+09	6.227e+09	0.67679	0.0040933	6.5286e-06	0.0015975
14	2.3179e+10	8.7178e+10	0.26588	0.0040888	2.0286e-06	0.00049637
15	1.2748e+11	1.3077e+12	0.097484	0.0040871	3.2856e-07	8.0396e-05
16	7.0114e+11	2.0923e+13	0.03351	0.0040866	1.7144e-07	4.195e-05
17	3.8563e+12	3.5569e+14	0.010842	0.0040865	2.7144e-07	6.6419e-05
18	2.121e+13	6.4024e+15	0.0033128	0.0040865	2.7144e-07	6.6419e-05
19	1.1666e+14	1.2165e+17	0.00095898	0.0040865	2.7144e-07	6.6419e-05
20	6.4163e+14	2.433e+18	0.00026372	0.0040865	2.7144e-07	6.6419e-05
21	3.529e+15	5.1093e+19	6.907e-05	0.0040865	2.7144e-07	6.6419e-05
22	1.941e+16	1.124e+21	1.7269e-05	0.0040865	2.7144e-07	6.6419e-05
23	1.0676e+17	2.5852e+22	4.1297e-06	0.0040865	2.7144e-07	6.6419e-05
24	5.8718e+17	6.2045e+23	9.4638e-07	0.0040865	2.7144e-07	6.6419e-05
25	3.2295e+18	1.5511e+25	2.0821e-07	0.0040865	2.7144e-07	6.6419e-05
26	1.7762e+19	4.0329e+26	4.4043e-08	0.0040865	2.7144e-07	6.6419e-05
27	9.7691e+19	1.0889e+28	8.9715e-09	0.0040865	2.7144e-07	6.6419e-05
28	5.373e+20	3.0489e+29	1.7623e-09	0.0040865	2.7144e-07	6.6419e-05
29	2.9552e+21	8.8418e+30	3.3423e-10	0.0040865	2.7144e-07	6.6419e-05
30	1.6254e+22	2.6525e+32	6.1278e-11	0.0040865	2.7144e-07	6.6419e-05

Exercise 5. Significance when computing e.

n	e(x)_approx	Absolute Error
1	2	NaN
10	2.5937	0.59374
100	2.7048	0.11107
1000	2.7169	0.01211
10000	2.7181	0.001222
1e+05	2.7183	0.00012231
1e+06	2.7183	1.2232e-05
1e+07	2.7183	1.225e-06
1e+08	2.7183	1.0422e-07
1e+09	2.7183	2.5366e-07
1e+10	2.7183	1.2232e-09
1e+11	2.7183	1.2232e-10
1e+12	2.7185	0.00024144
1e+13	2.7161	0.0024135
1e+14	2.7161	1.2168e-13
1e+15	3.035	0.31893
1e+16	1	2.035
1e+17	1	0
1e+18	1	0
1e+19	1	0
1e+20	1	0
1e+21	1	0
1e+22	1	0
1e+23	1	0
1e+24	1	0

```
1e+25               1               0
1e+26               1               0
1e+27               1               0
1e+28               1               0
```

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